Particle-core coupling in nuclei:
new insights, fundamental misconceptions

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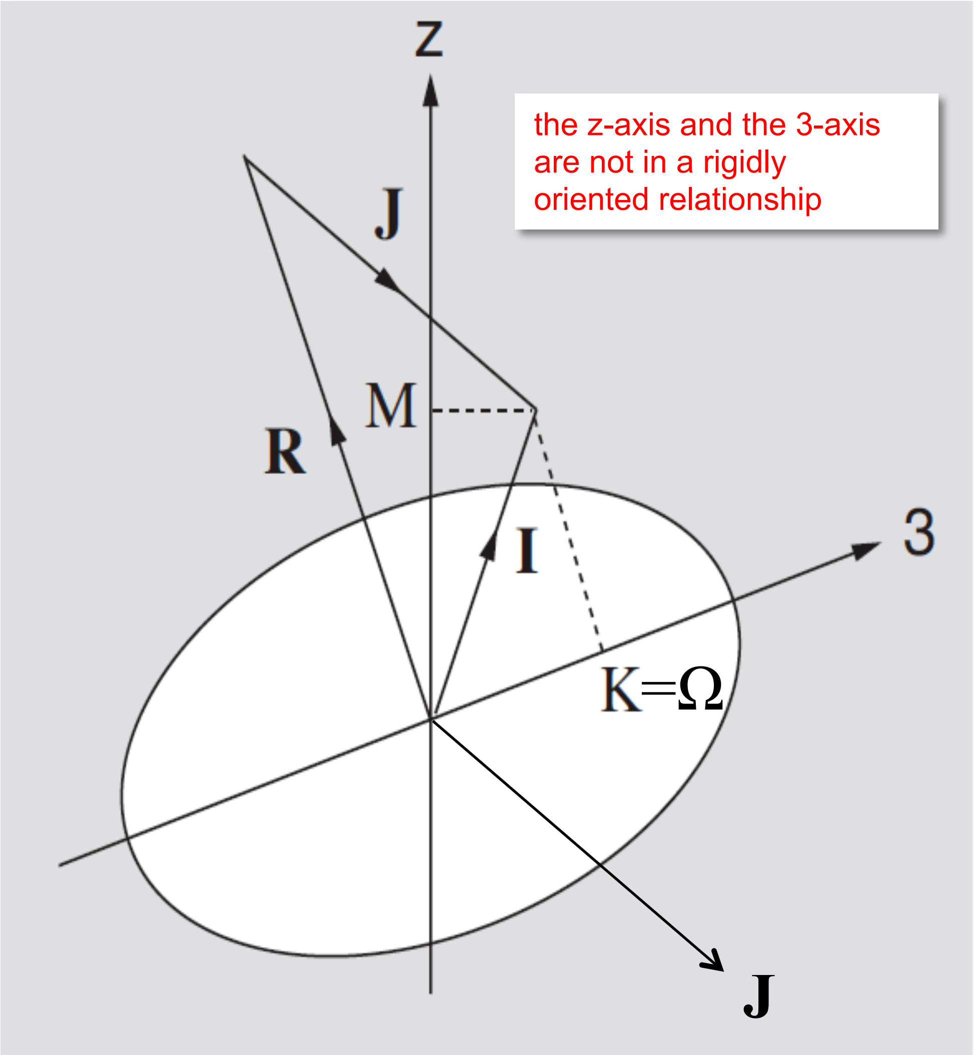
Symmetric-top model:
quantum numbers



R: collective angular momentum

J: intrinsic spin

I: total spin / angular momentum

M: laboratory-frame,
z-component of I

K: body-frame (symmetry axis),
3-component of I; K = Ω

"Coriolis" interaction:

$$\mathbf{R}\cdot\mathbf{R} = (\mathbf{I} - \mathbf{J})\cdot(\mathbf{I} - \mathbf{J})$$
$$= \mathbf{I}\cdot\mathbf{I} - 2\,\mathbf{I}\cdot\mathbf{J} + \mathbf{J}\cdot\mathbf{J}$$

Nilsson model plus rotations

$$\hat{H} = \frac{\hbar^2 \hat{\mathbf{I}}^2}{2\Im} + \hat{h} + \frac{\hbar^2 \hat{\mathbf{j}}^2}{2\Im} - \frac{\hbar^2}{\Im} \hat{\mathbf{I}} \cdot \hat{\mathbf{j}}.$$

$$\langle \vec{I} \cdot \vec{j} \rangle = \langle \frac{1}{2}(\hat{I}_+ \hat{j}_- + \hat{I}_- \hat{j}_+) + \hat{I}_z \hat{j}_z \rangle$$

$I_+ := I_x + iI_y$, $I_- = I_x - iI_y$
$j_+ := j_x + ij_y$, $j_- = j_x - ij_y$

$$|KIM\rangle + \varepsilon(-1)^{I+K} | -K, IM\rangle$$

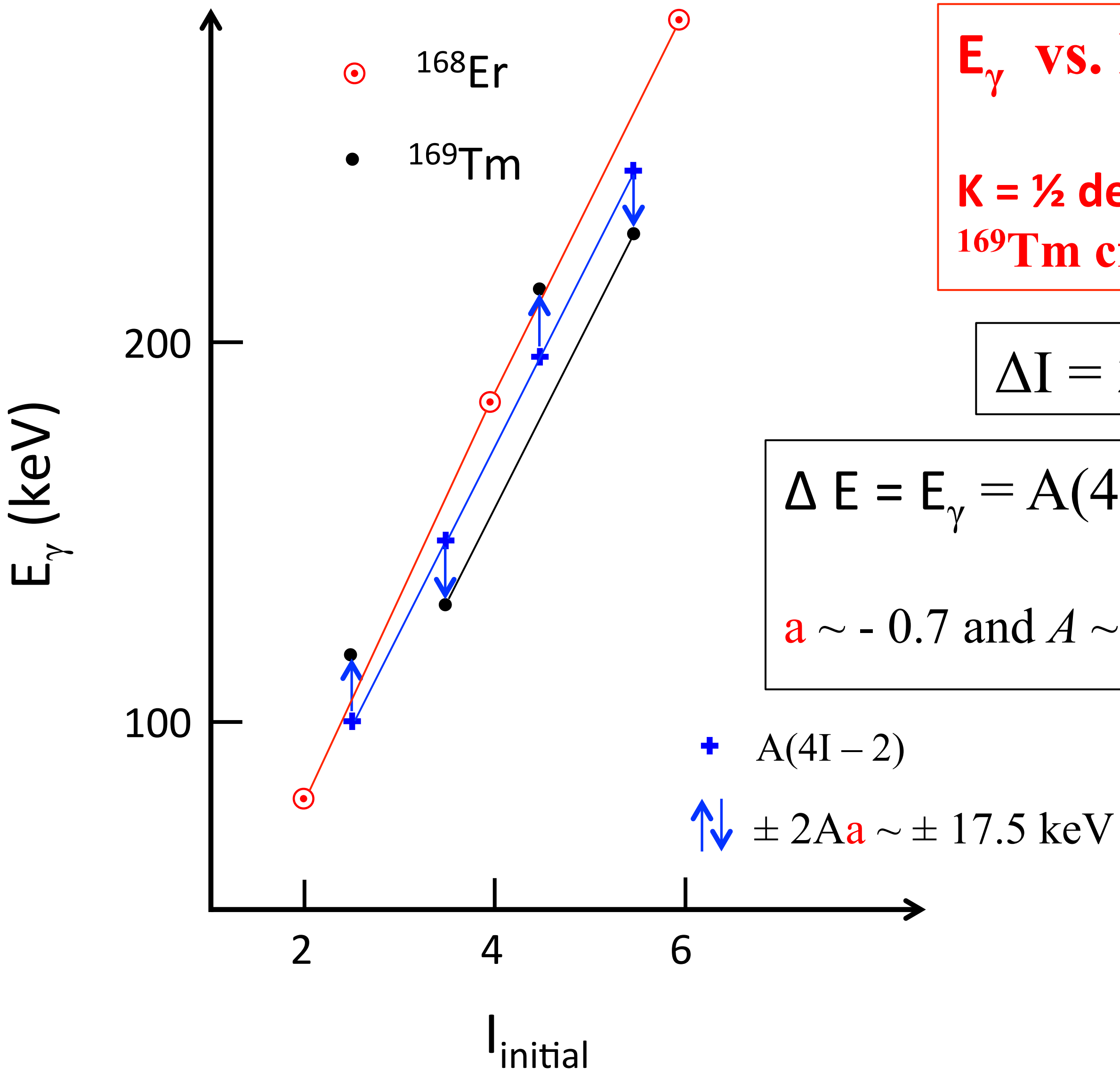
ε = +1 reflection symmetric
ε = -1 reflection asymmetric

$$E_I = E_0 + A[I(I+1) + (-1)^{I+1/2}(I + 1/2)\, a\, \delta_{K,1/2}]$$

$\delta_{K,1/2} = 1,\ K = ½$
$\delta_{K,1/2} = 0$ otherwise

^{169}Tm

½$^{+}$ [411]



E_γ vs. $I_{initial}$ plot

K = ½ decoupled band

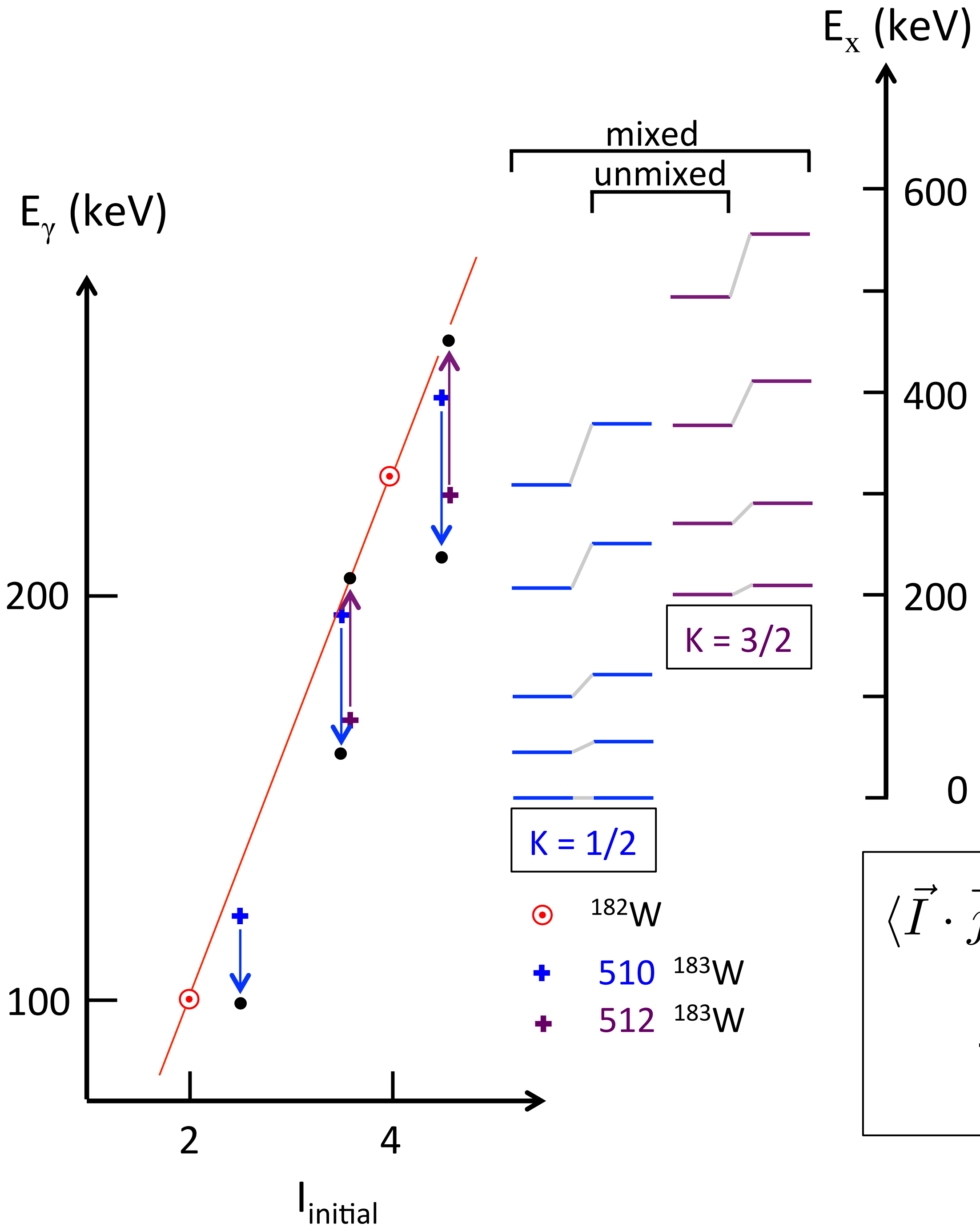
^{169}Tm cf. ^{168}Er

$\Delta I = 2$ transitions

$\Delta E = E_\gamma = A(4I - 2) \pm 2 A a$

$a \sim -0.7$ and $A \sim 12.5$ keV

$$E_I = E_0 + A[I(I+1) + (-1)^{I+1/2}(I + 1/2)\, a\, \delta_{K,1/2}]$$



E_γ vs. $I_{initial}$ plot

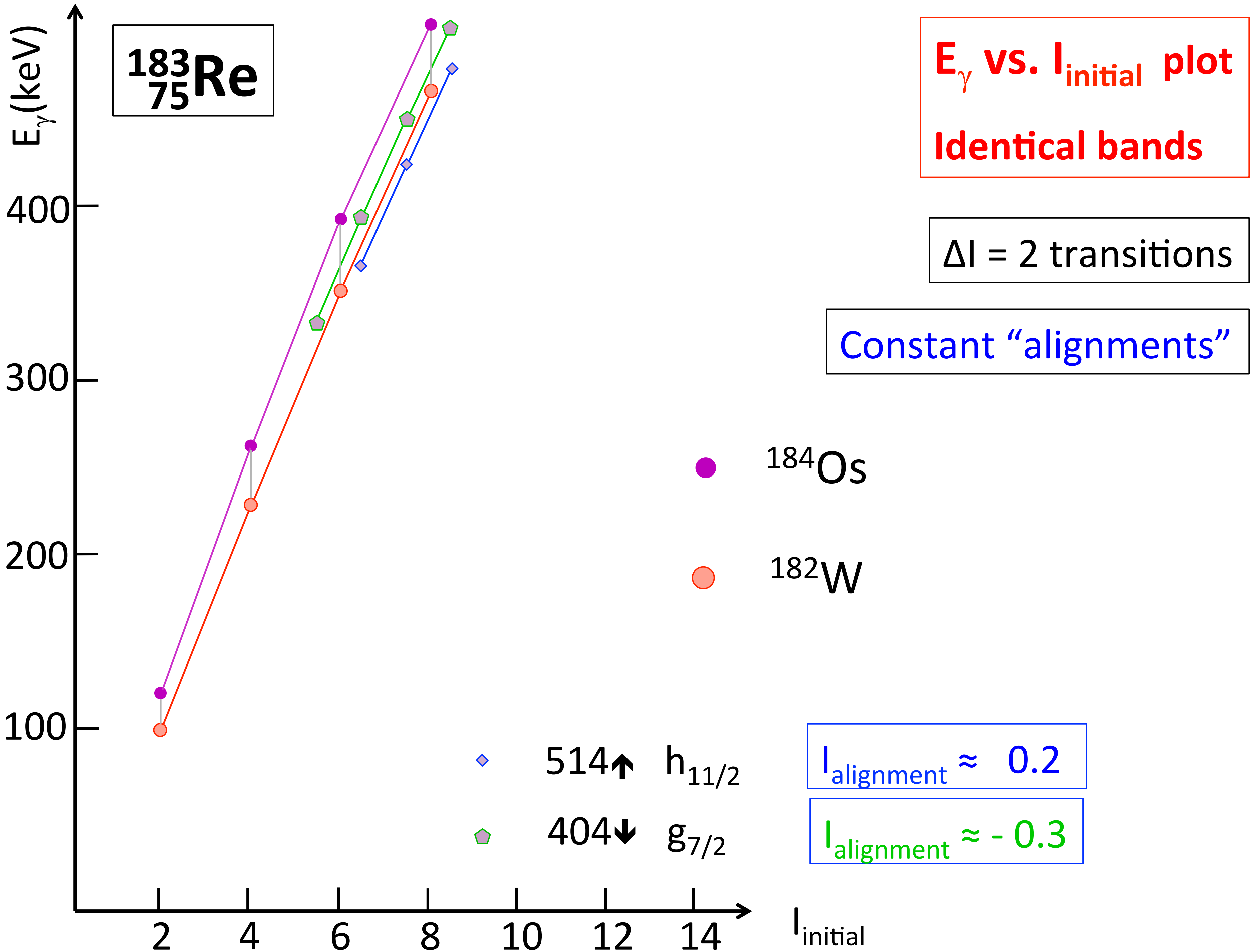
$\Delta K = 1$ band mixing
$1/2^-$ 510 + $3/2^-$ 512
^{183}W cf. ^{182}W

$\Delta I = 2$ transitions

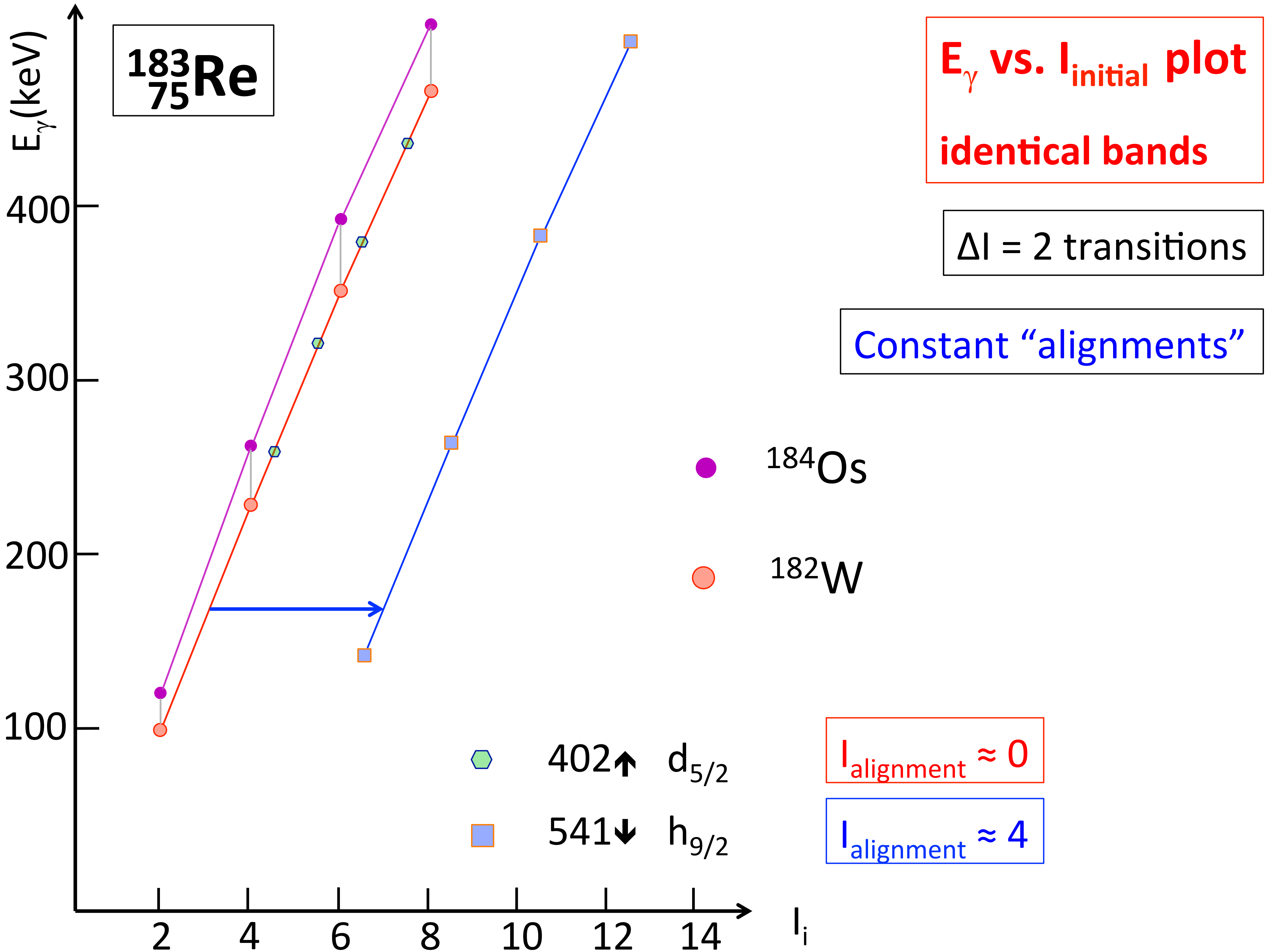
E_γ “shifts” ~ 35 keV

$$\langle \vec{I} \cdot \vec{j} \rangle = \langle \frac{1}{2}(\hat{I}_+ \hat{j}_- + \hat{I}_- \hat{j}_+) + \hat{I}_z \hat{j}_z \rangle$$

--produces $\Delta K = \Delta \Omega = 1$ mixing



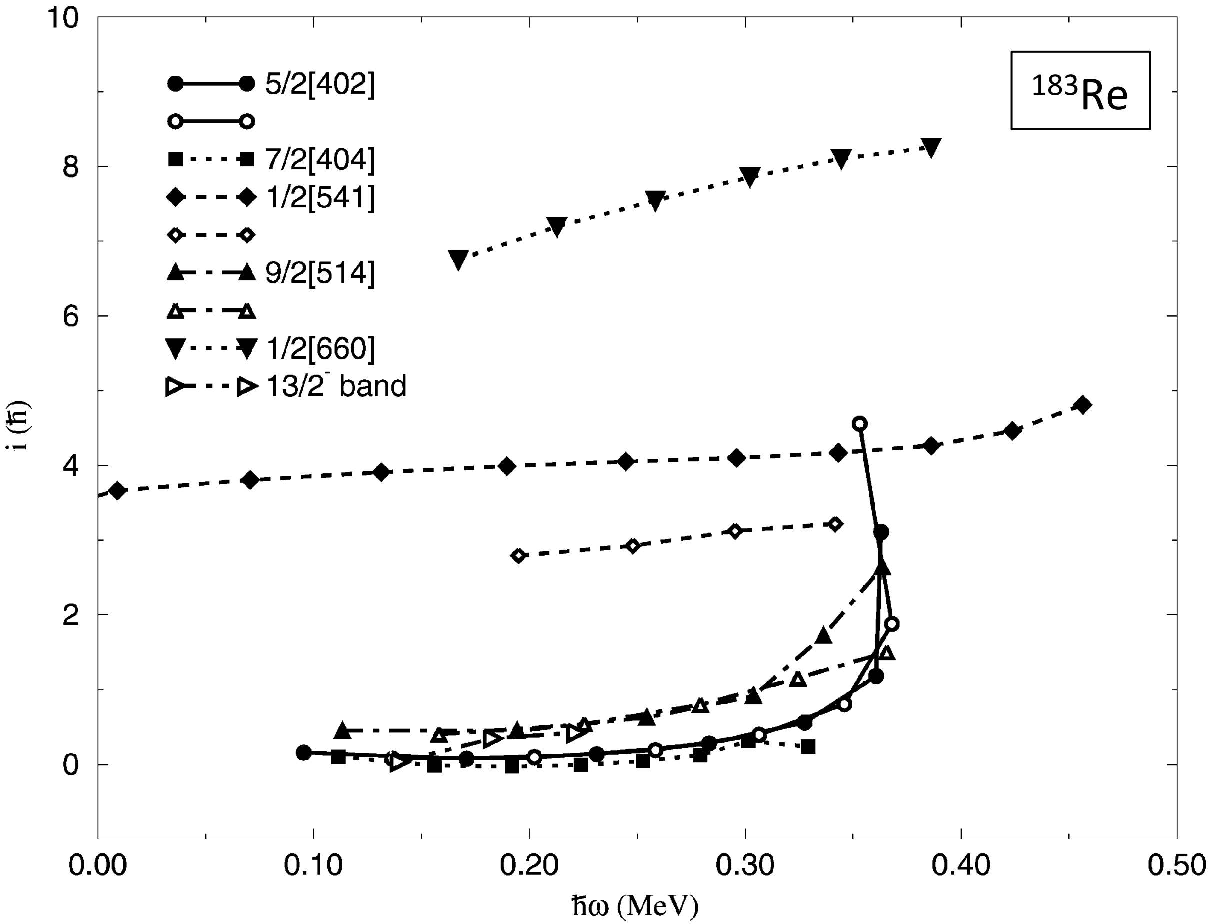
E_γ(keV)
$^{183}_{75}$Re
400
300
200
100
2
4
6
8
10
12
14
$I_{initial}$
E_γ vs. $I_{initial}$ plot
Identical bands
ΔI = 2 transitions
Constant “alignments”
^{184}Os
^{182}W
514↑ $h_{11/2}$
404↓ $g_{7/2}$
$I_{alignment}$ ≈ 0.2
$I_{alignment}$ ≈ - 0.3



$^{183}_{75}Re$
E_γ(keV)
400
300
200
100
2
4
6
8
10
12
14
I_i
E_γ vs. $I_{initial}$ plot
identical bands
ΔI = 2 transitions
Constant "alignments"
^{184}Os
^{182}W
402↑ $d_{5/2}$
541↓ $h_{9/2}$
$I_{alignment} \approx 0$
$I_{alignment} \approx 4$

"Standard" alignment plots for ^{183}Re

$$i = (\mathcal{J}_0 + \omega^2 \mathcal{J}_1)\omega$$ Harris formula



Purry
NP A672 54

Alignment of 402 and 404 recognized under name "identical bands"—25 years ago

VOLUME 69, NUMBER 10 PHYSICAL REVIEW LETTERS 7 SEPTEMBER 1992

Low-Spin Identical Bands in Neighboring Odd-*A* and Even-Even Nuclei: A Possible Challenge to Mean-Field Theories

C. Baktash, J. D. Garrett, D. F. Winchell, and A. Smith
Physics Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831-6371
(Received 3 February 1992)

A comprehensive study of odd-*A* rotational bands in normally deformed rare-earth nuclei indicates that a large number of seniority-one configurations (30% for odd-*Z* nuclei) at low spin have moments of inertia nearly identical to that of the seniority-zero configuration of the neighboring even-even nucleus with one less nucleon. It is difficult to reconcile these results with conventional models of nuclear pair correlation, which predict variations of about 15% in the moments of inertia of configurations differing by one unit in seniority.

Annu. Rev. Nucl. Part. Sci. 1995. 45:485–541

IDENTICAL BANDS IN DEFORMED AND SUPERDEFORMED NUCLEI

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Moments of inertia, MoI and pairing --sample statements

- "The identical bands are found to be associated with up-sloping particle states, suggesting that the cause may be a cancellation between pairing and deformation decreases."

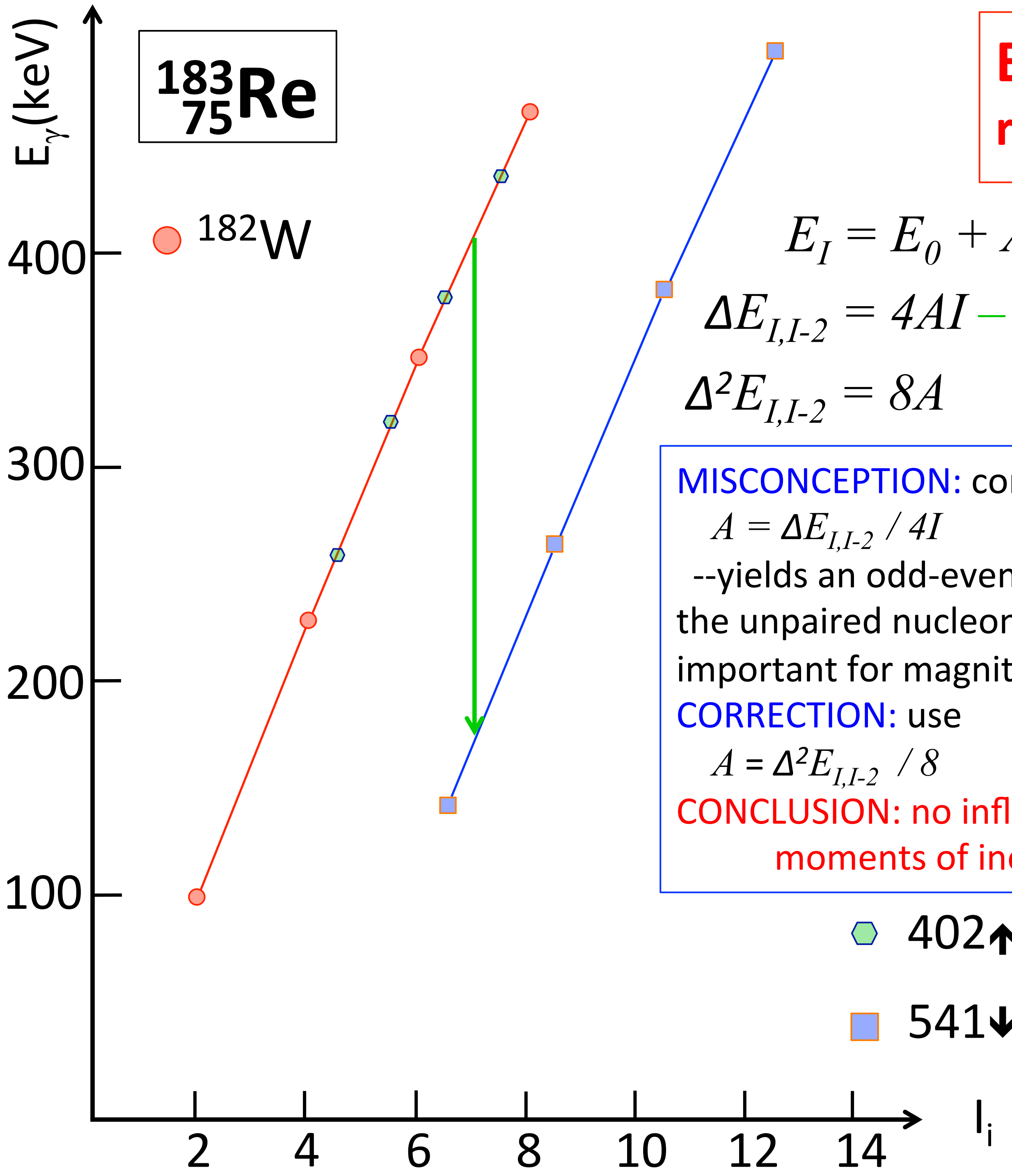
  --PRL 69 3448 (1992).

- "...BCS [pairing] theory can qualitatively reproduce the experimental large fluctuations in the MoI which is helpful to understand the appearance of the normally deformed identical bands in an odd-A nucleus and its even-even neighbors..."

  —PR C63 047306 (2001).

- "In particular, the reduction of the BCS pairing correlations due to the blocking of one and two orbitals implies large changes (up to 30%) in the moments of inertia and cannot be reconciled with these [identical band] systematics."

  --Annu. Rev. Nucl. Part. Sci. 45 485 (1995).



E_γ vs. $I_{initial}$ as a rotation alignment plot

$$E_I = E_0 + AI(I+1) - 2A\langle I \bullet J \rangle$$

$$\Delta E_{I,I-2} = 4AI - const.$$

$$\Delta^2 E_{I,I-2} = 8A$$

MISCONCEPTION: commonly, use

$A = \Delta E_{I,I-2} / 4I$

--yields an odd-even staggering and conclusion that the unpaired nucleon blocks pairing which is important for magnitudes of moments of inertia.

CORRECTION: use

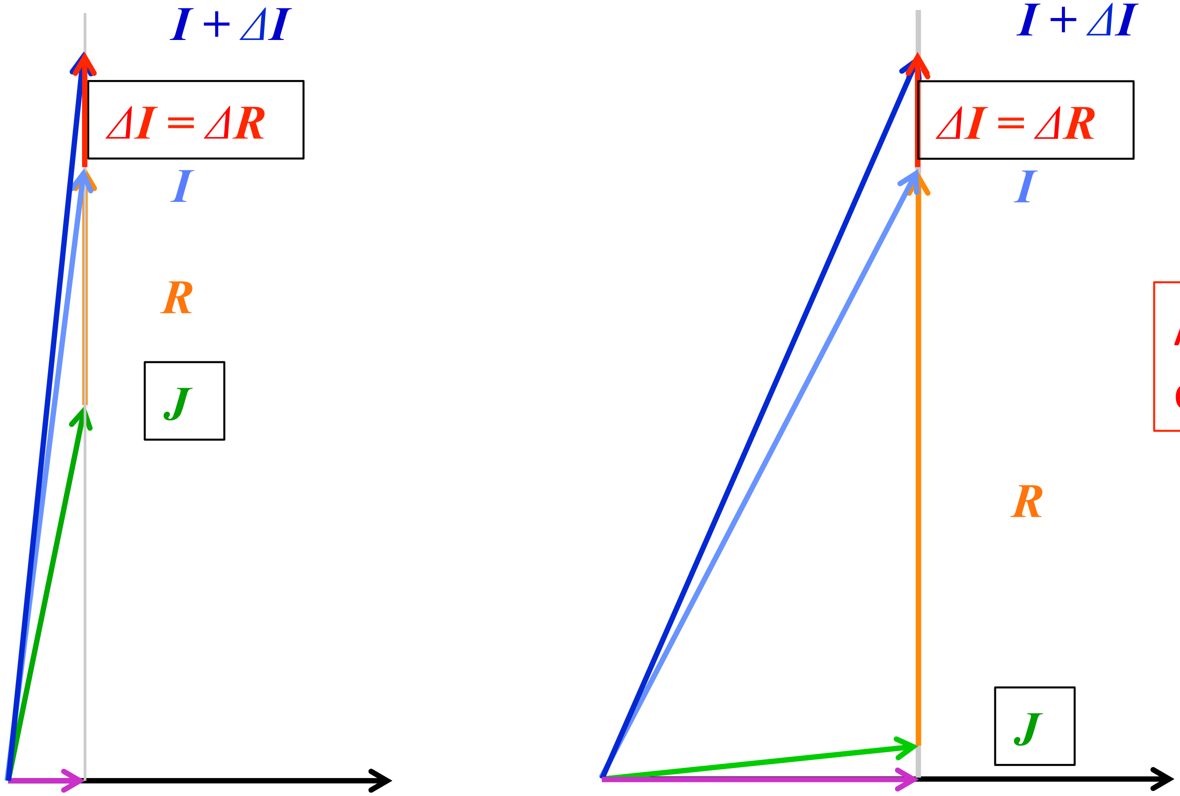
$A = \Delta^2 E_{I,I-2} / 8$

CONCLUSION: no influence of pairing on moments of inertia

402↑ $d_{5/2}$ $const. = 0$ keV

541↓ $h_{9/2}$ $const. = -230$ keV

“Coriolis” contribution to energy *differences*



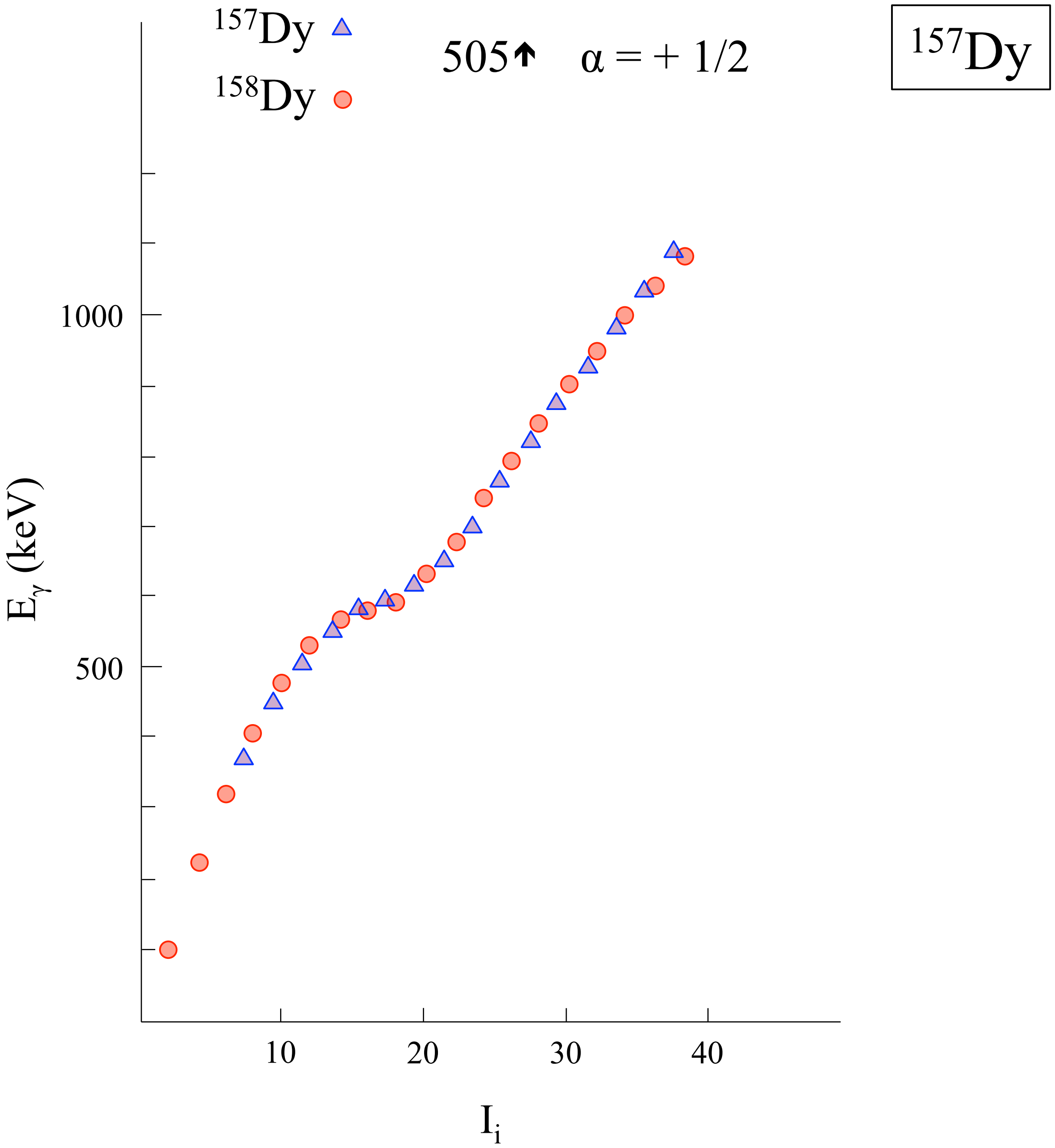
A vectorial difference effect

K = Ω = 1/2

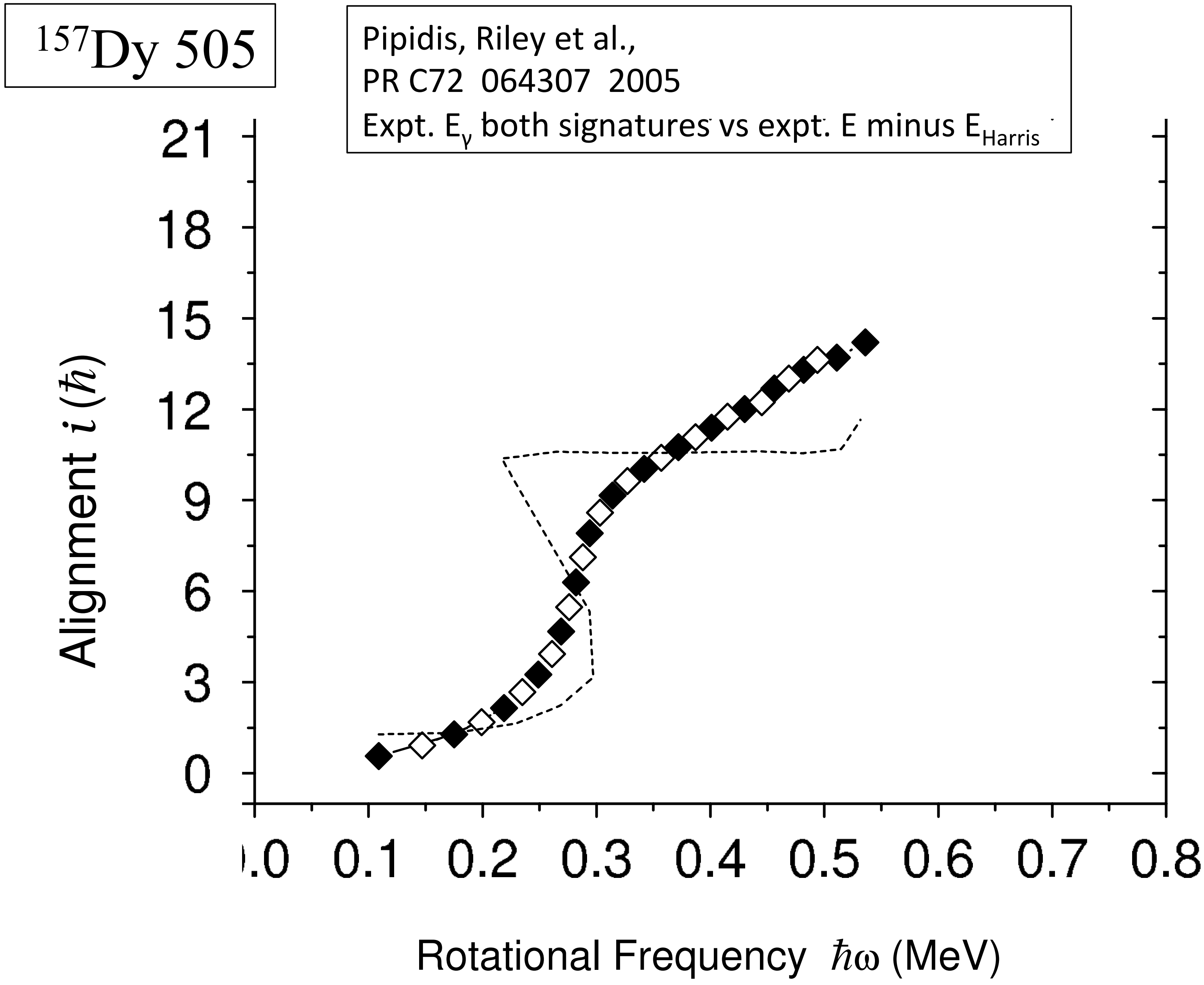
$\langle \Delta I \bullet J \rangle$ *large*

K = Ω = max

$\langle \Delta I \bullet J \rangle \approx 0$



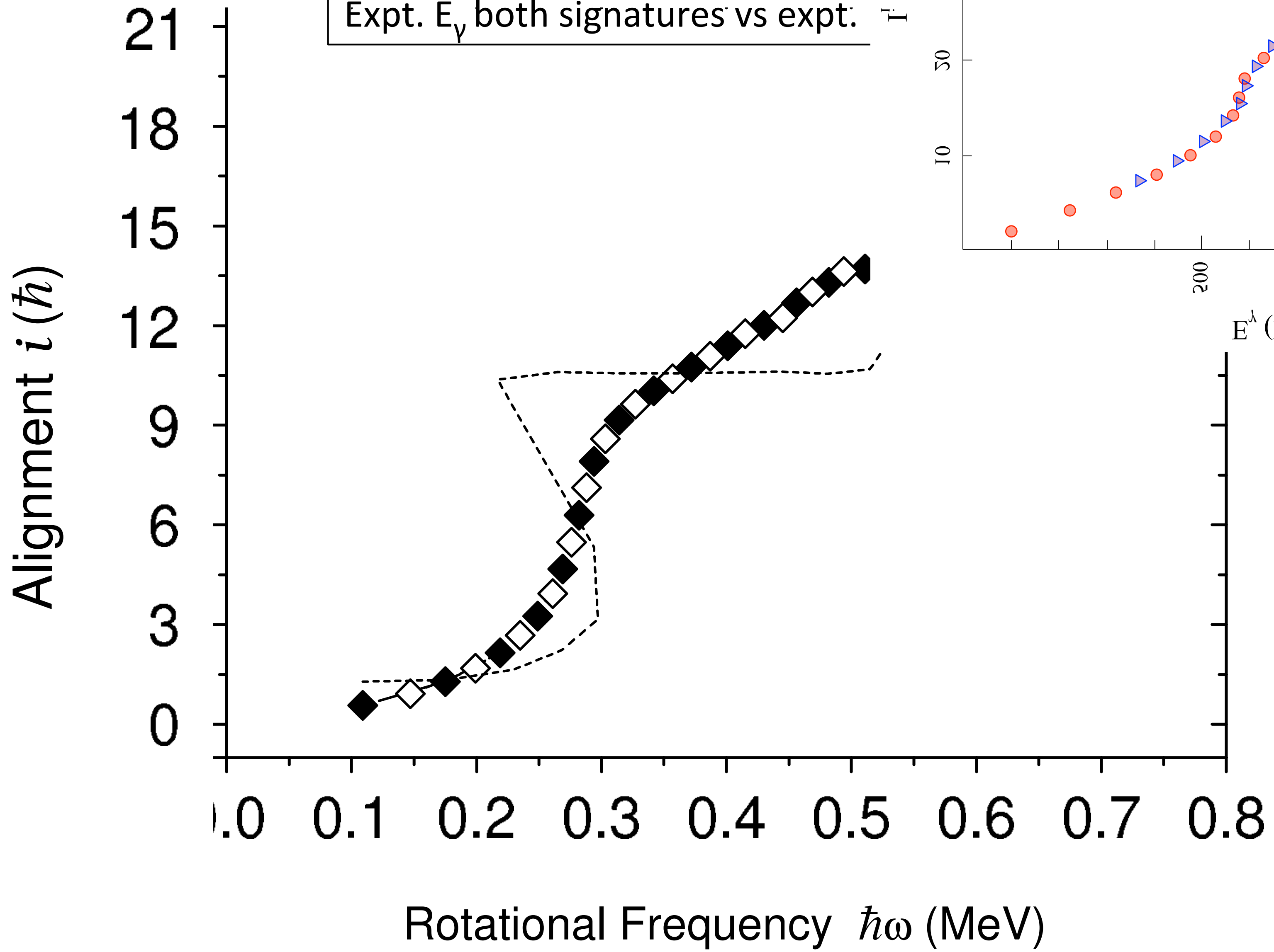
^{157}Dy
^{158}Dy
505↑ α = + 1/2
^{157}Dy
1000
500
10
20
30
40
E_γ (keV)
I_i

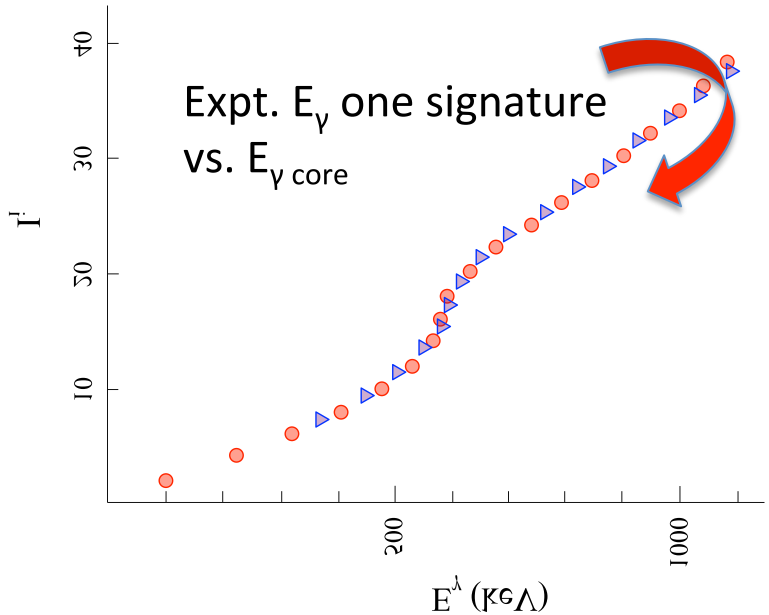


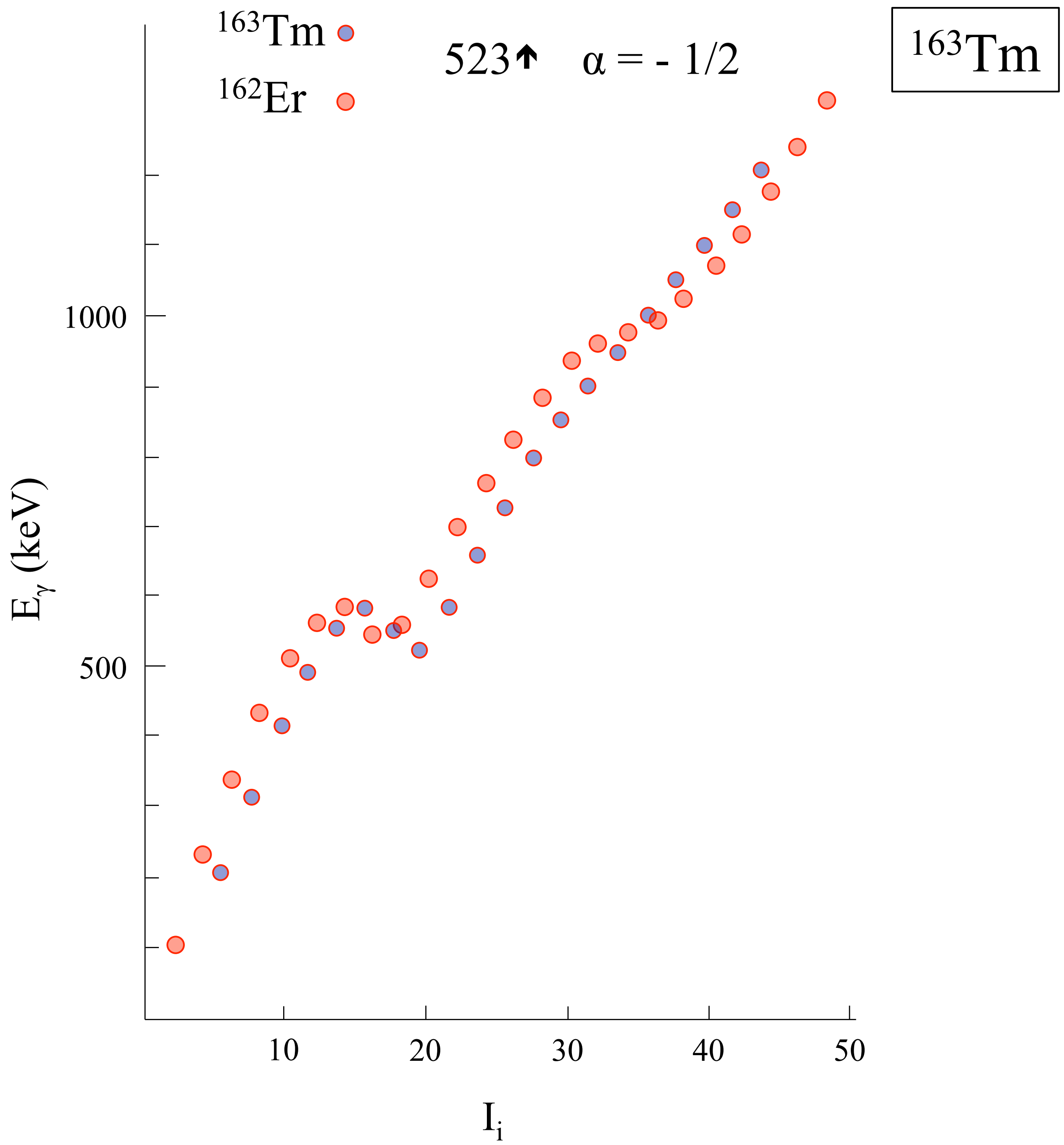
^{157}Dy 505
Pipidis, Riley et al.,
PR C72 064307 2005
Expt. E_γ both signatures vs expt. E minus E_{Harris}
Alignment i ($\hbar$)
21
18
15
12
9
6
3
0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Rotational Frequency $\hbar\omega$ (MeV)

^{157}Dy 505

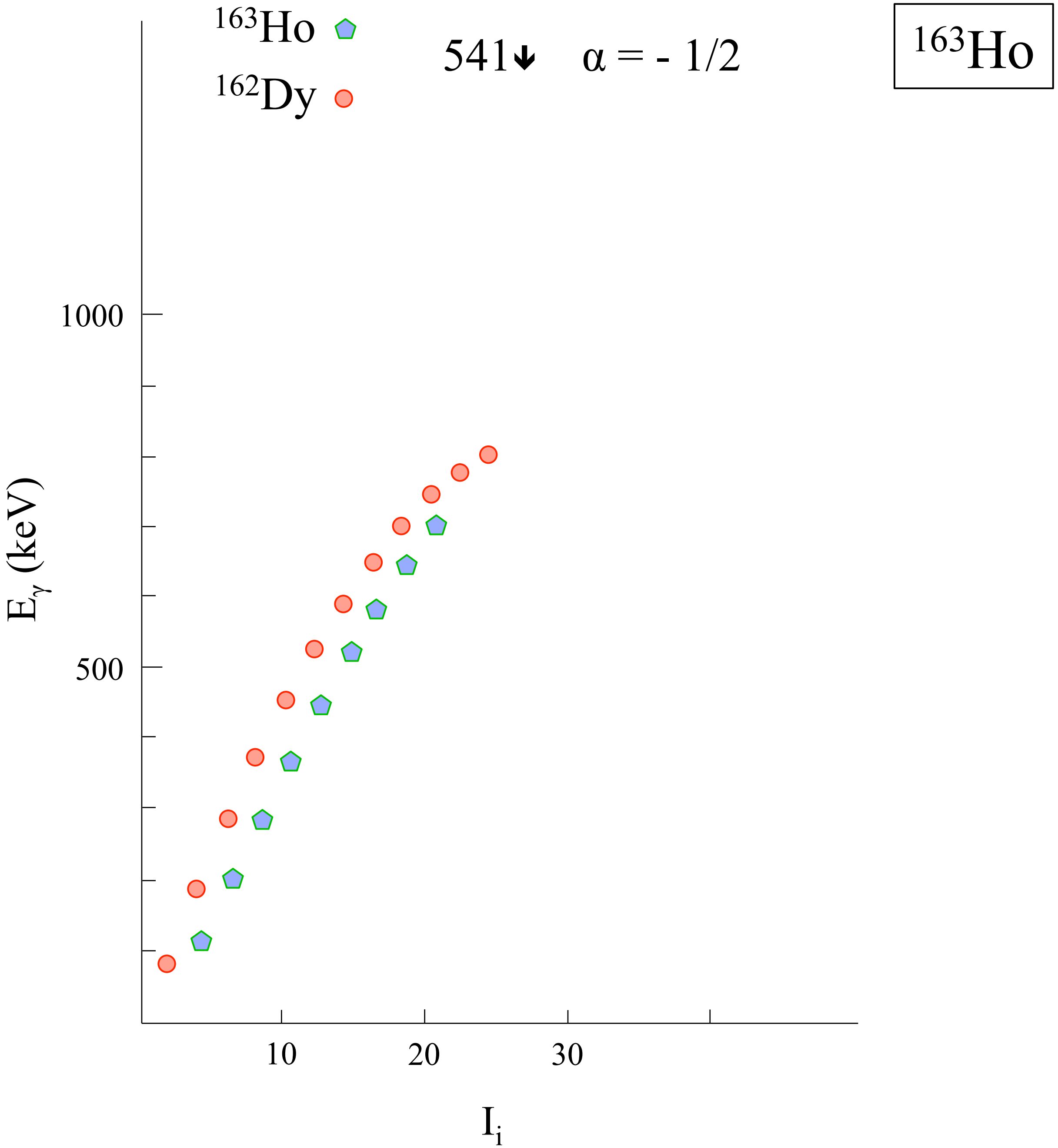
Pipidis, Riley et al.,
PR C72 064307 2005
Expt. E_γ both signatures vs expt.







163Tm
162Er
523↑ α = - 1/2
163Tm
1000
500
Eγ (keV)
10
20
30
40
50
Ii



163Ho
162Dy
541↓ α = - 1/2
163Ho
1000
500
Eγ (keV)
10
20
30
Ii

CONCLUSIONS

- The rotational-particle coupling term produces effects that have been overlooked.
- The moments of inertia in an odd nucleus are essentially the same as those in the neighboring even nucleus with the greatest deformation.
- There is no odd-particle "blocking" effect of moments of inertia.
- There is no "deformation-driving effect" of unpaired nucleons.
- A reassessment of our understanding of the magnitudes of moments of inertia is needed: there are models that attribute "rotational" energies to potential-energy sources, e.g., Q•Q interactions in SU(3) coupling schemes.
- If "rotational" energy is partly potential, Coriolis effects are attenuated—this solves a long-standing problem.

ELSEVIER

Nuclear Physics A 662 (2000) 125–147

PHYSICS A

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SU(3) quasi-dynamical symmetry as an organizational mechanism for generating nuclear rotational motions

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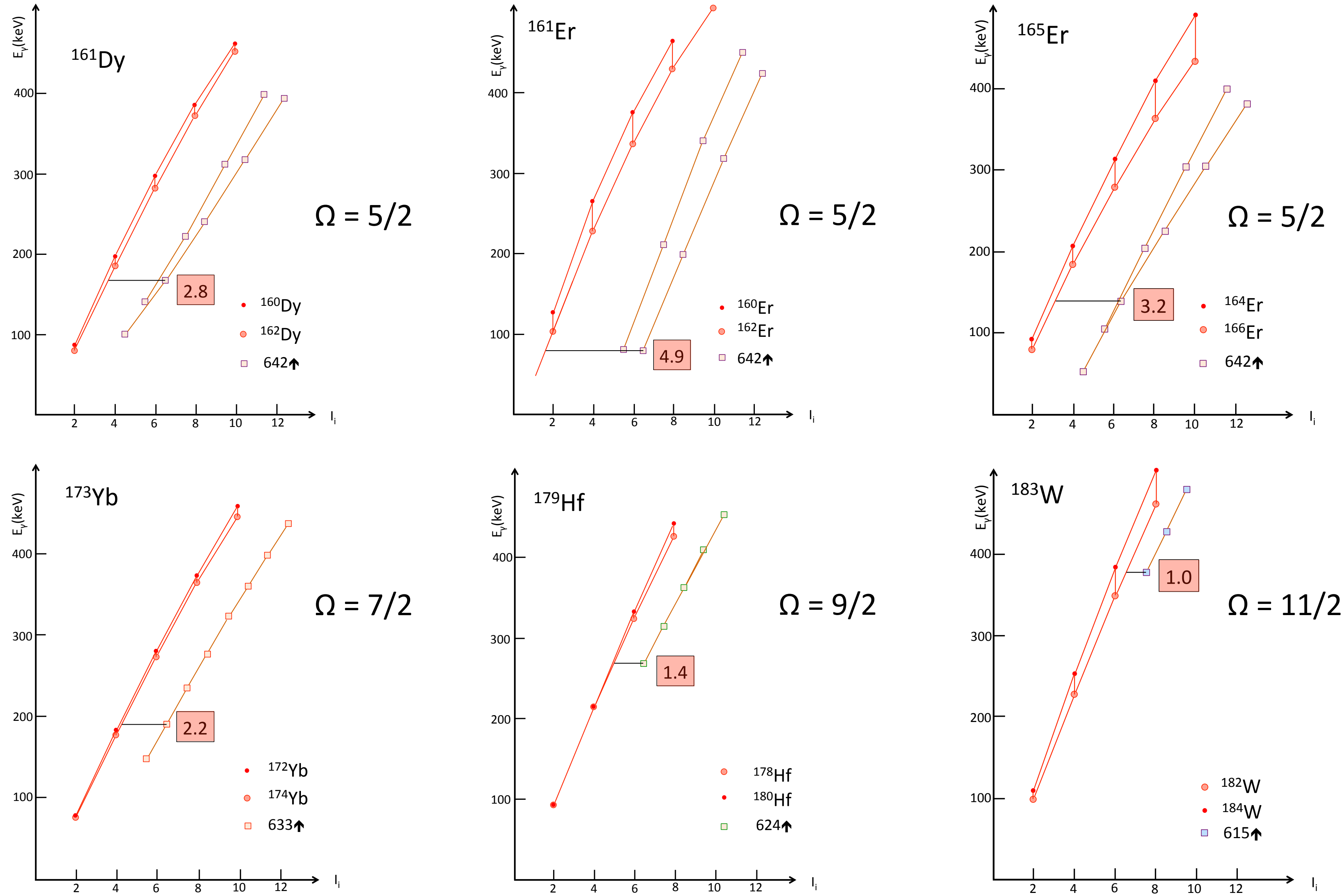
Department of Physics, University of Toronto, Toronto, Ontario M5S 1A7, Canada

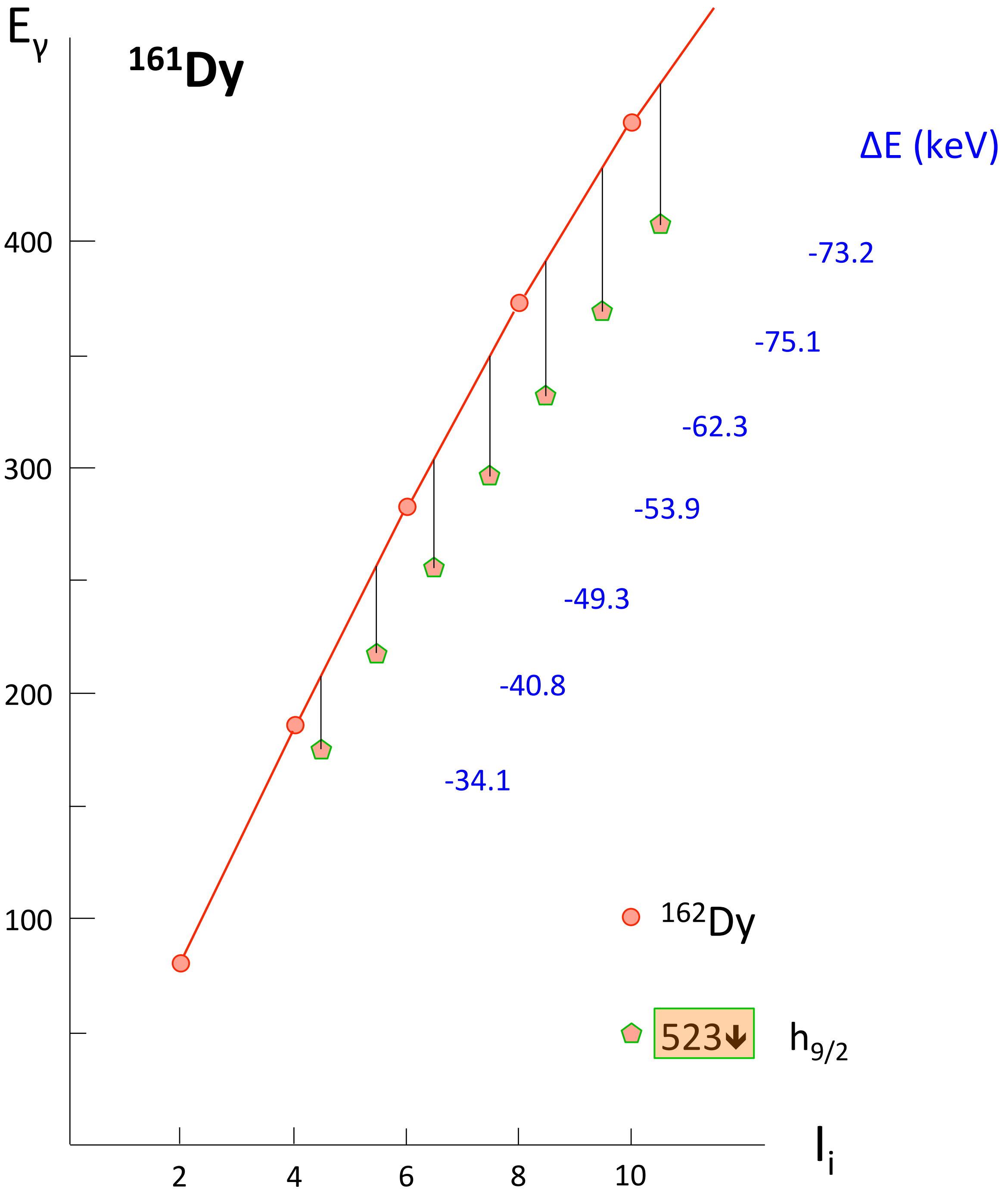
"A particularly interesting challenge was to learn how a model, without pair correlations, could give correct moments of inertia when it is known that the cranking model is only successful when pairing correlations are included. The early calculations of Park et al. indicated that the dominant contribution to rotational energies came from the potential energy part of the Hamiltonian, thus calling into question the very concept of the moment of inertia as an inverse coefficient of the L2 term in the kinetic energy. The results of the present calculation indicate that the inclusion of only stretched states, as in the calculation of Park et al.,# tends to exaggerate this effect. Nevertheless, it confirms that the dominant component of the rotational energies comes from the potential energy; for the self-consistent value of x only about 20% of the rotational energy comes from the kinetic energy in the present calculation."

#--1984

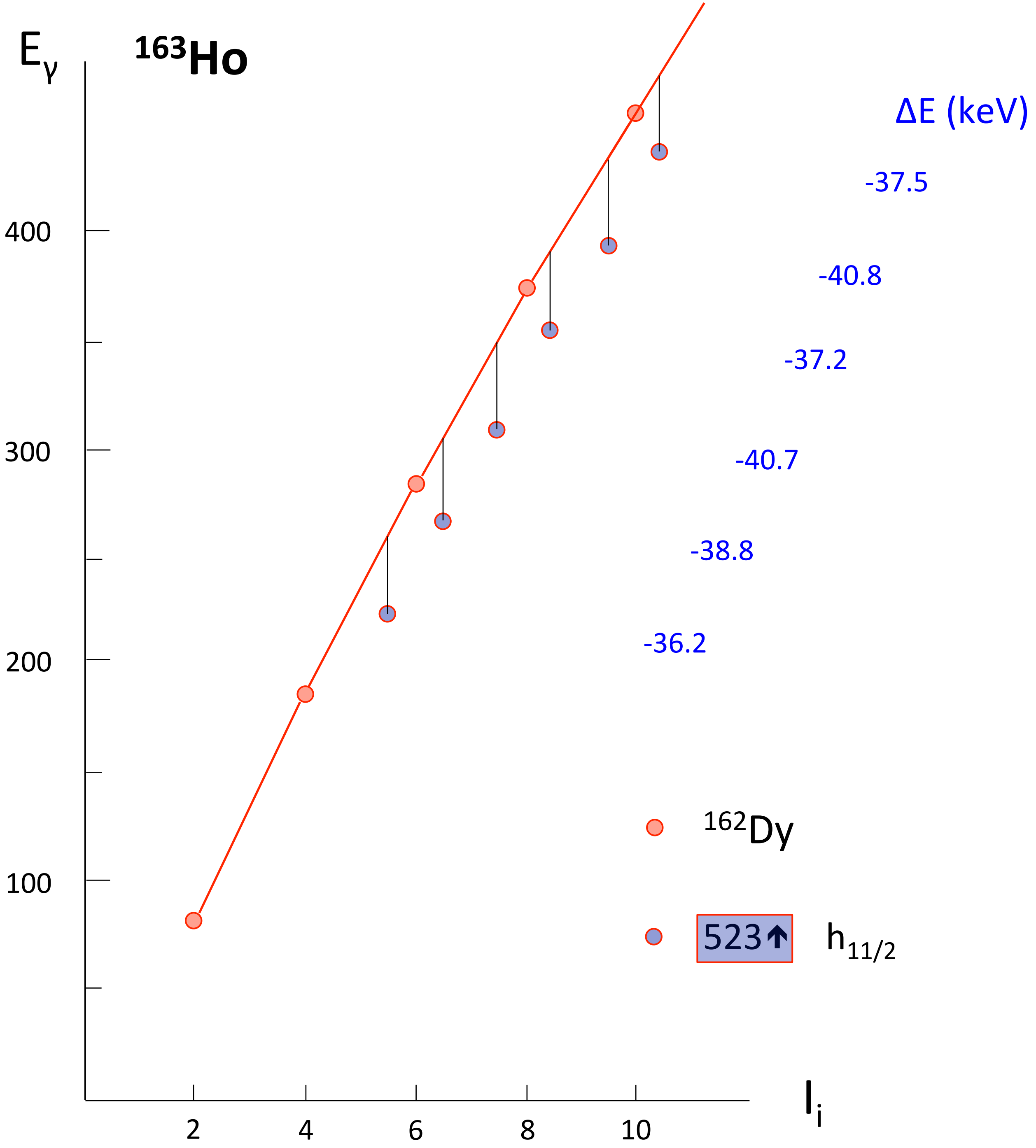


$i_{13/2}$

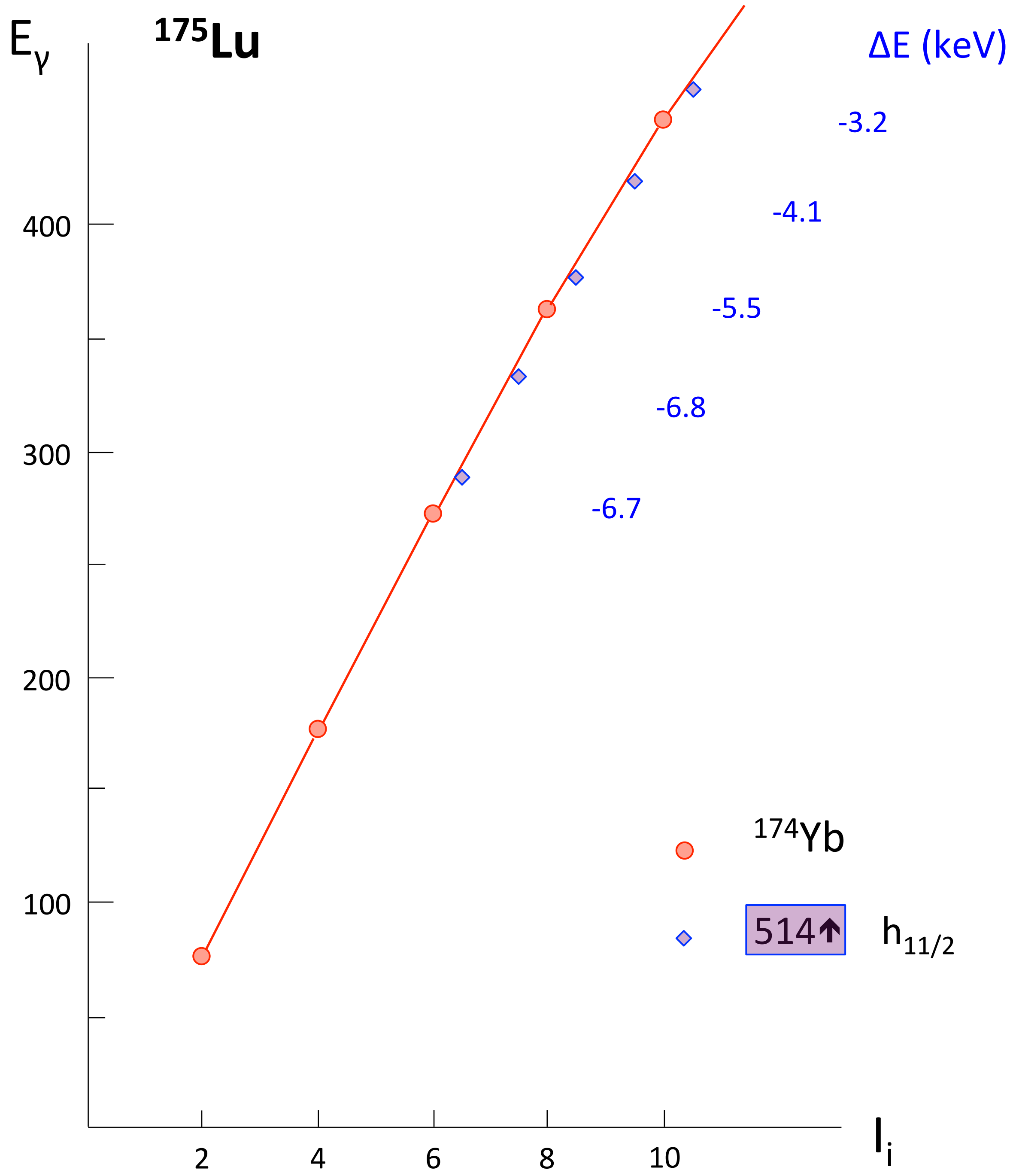
alignment decreases with:
- increasing deformation
- increasing Ω



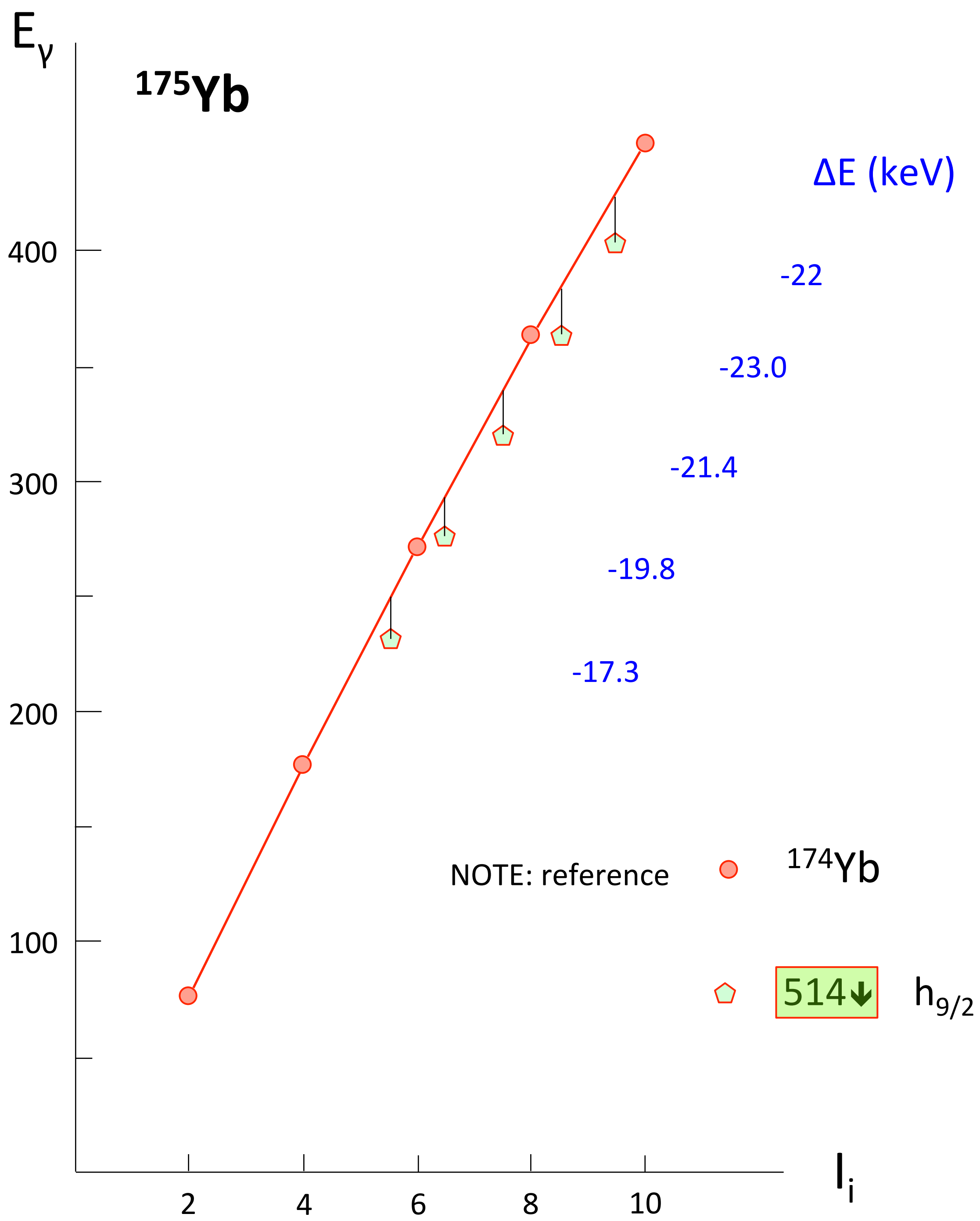
***ΔI •J* increasing**



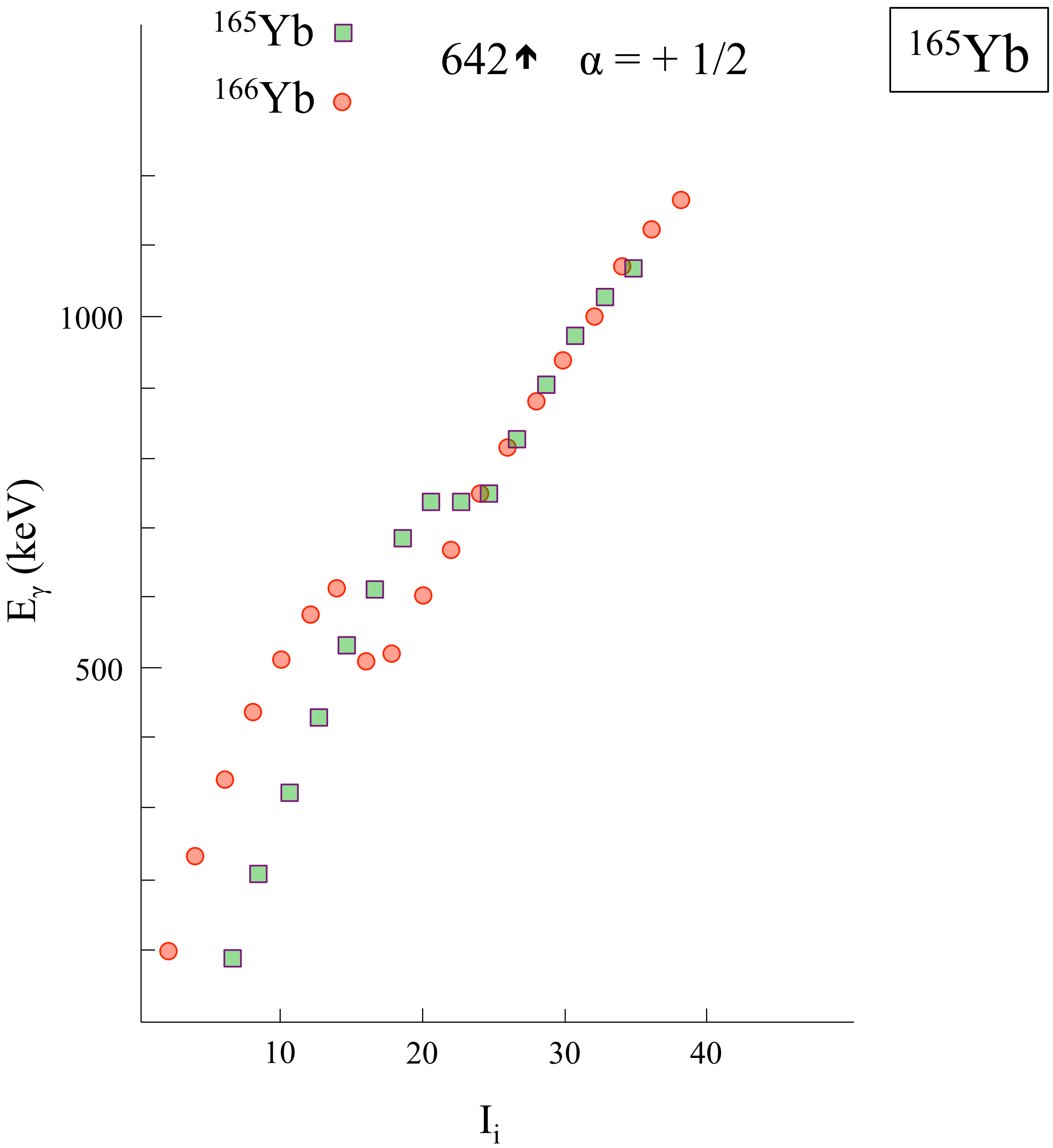
E_γ
^{163}Ho
ΔE (keV)
-37.5
-40.8
-37.2
-40.7
-38.8
-36.2
400
300
200
100
2
4
6
8
10
I_i
^{162}Dy
523↑
$h_{11/2}$

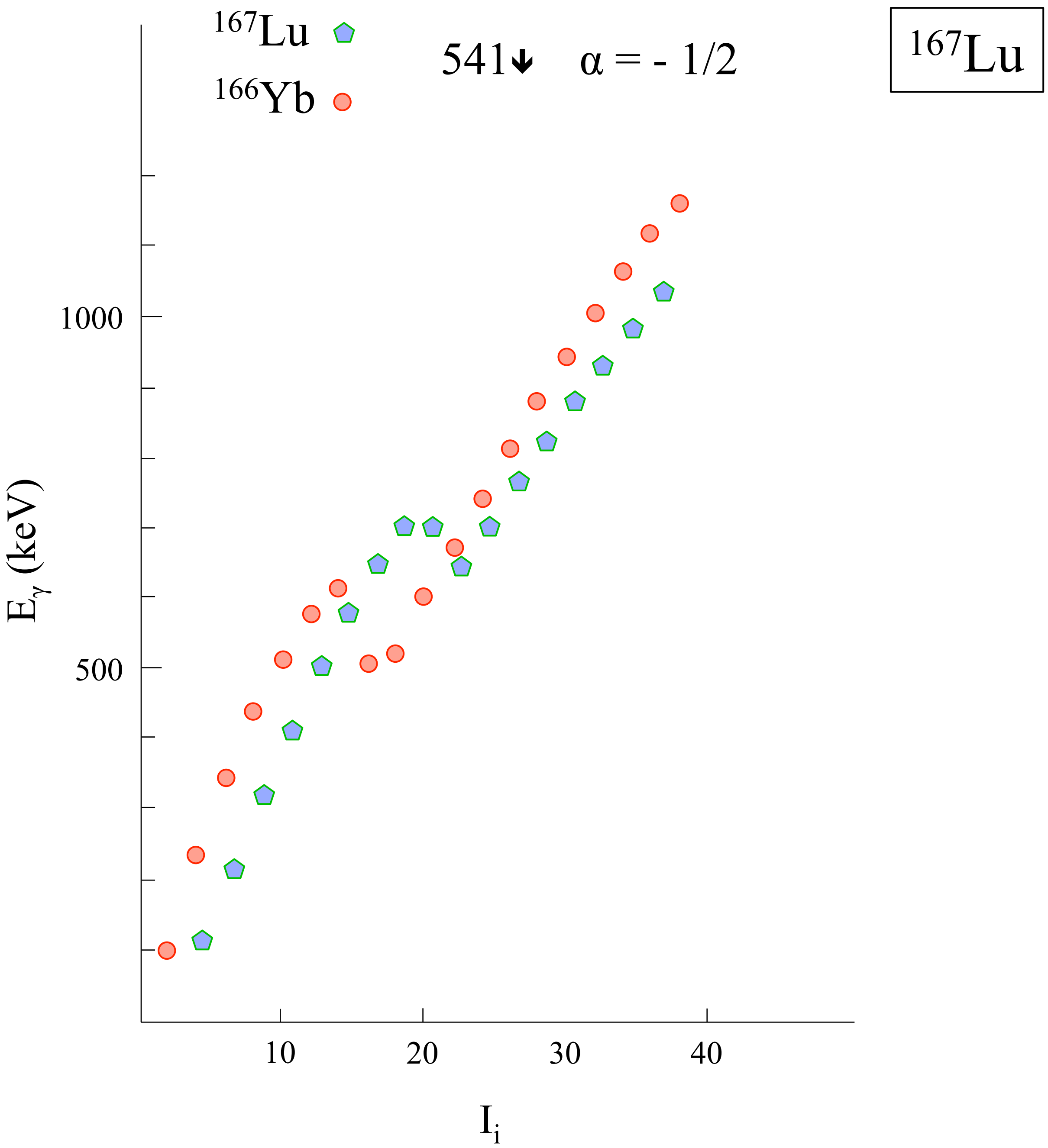


E_γ
^{175}Lu
ΔE (keV)
-3.2
-4.1
-5.5
-6.8
-6.7
400
300
200
100
2
4
6
8
10
I_i
^{174}Yb
514↑
$h_{11/2}$

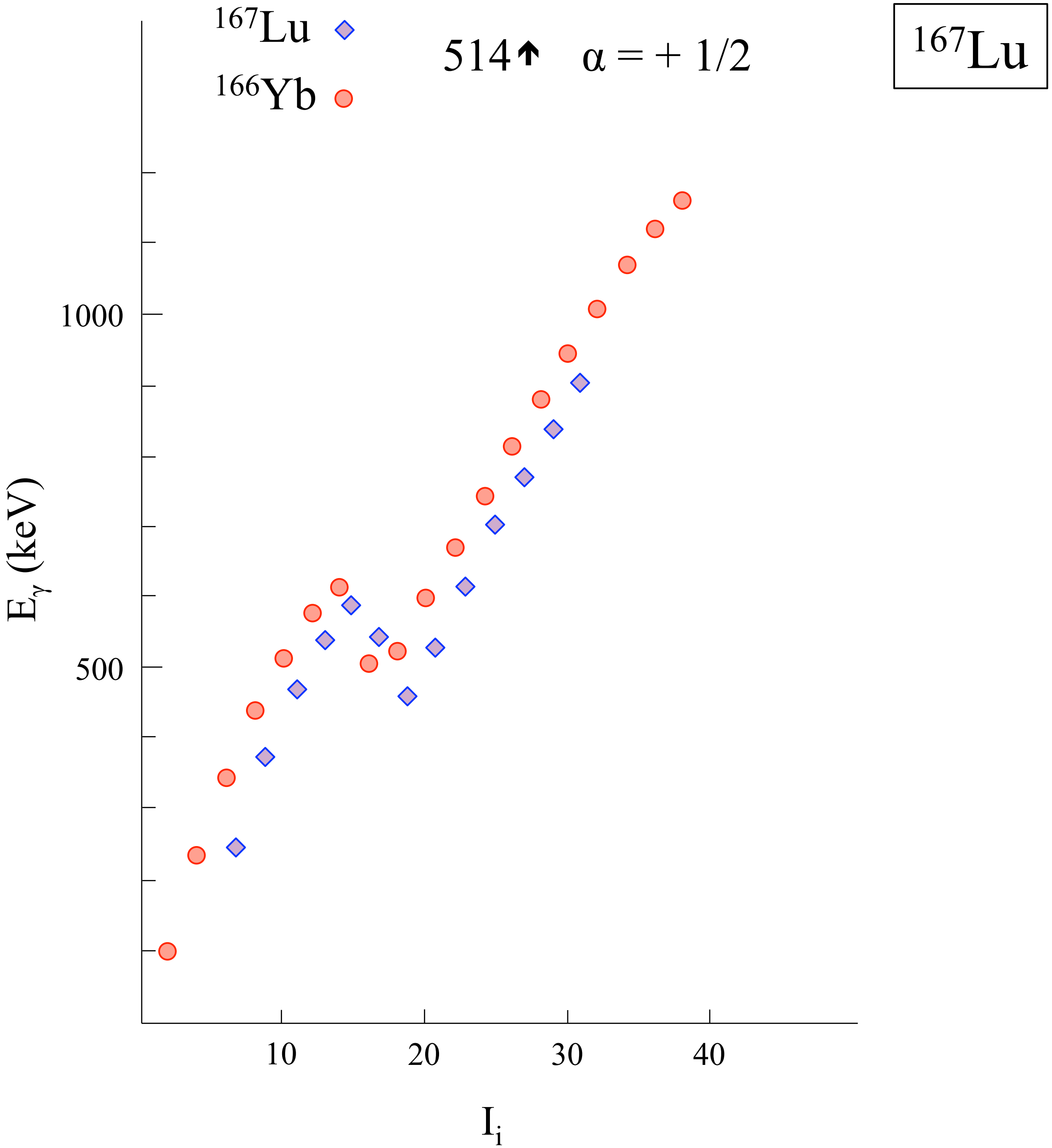


E_γ
^{175}Yb
ΔE (keV)
400
300
200
100
-22
-23.0
-21.4
-19.8
-17.3
NOTE: reference
^{174}Yb
514↓
$h_{9/2}$
2
4
6
8
10
I_i

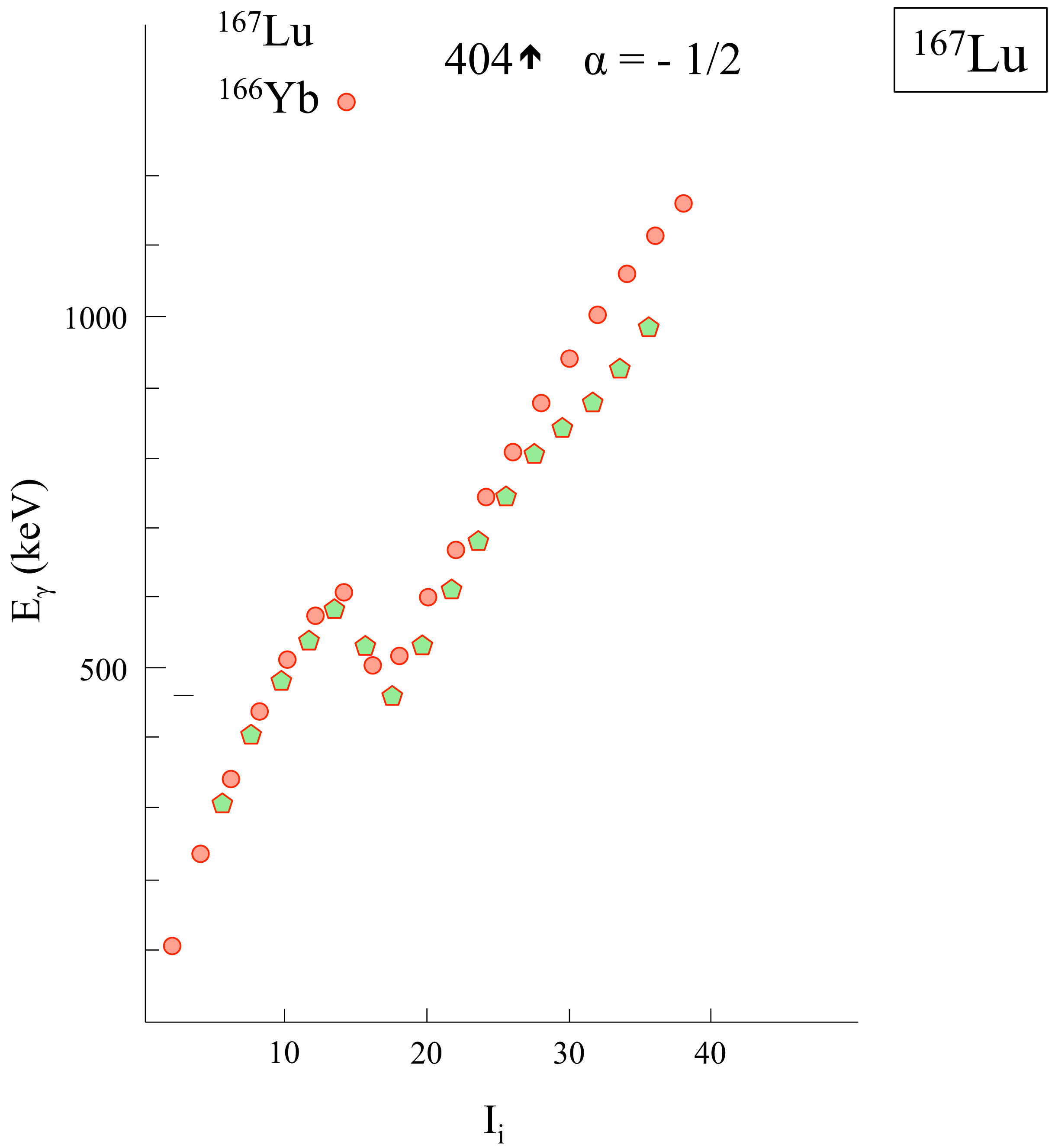

165Yb
166Yb
642↑ α = + 1/2
165Yb
1000
500
Eγ (keV)
10
20
30
40
Ii



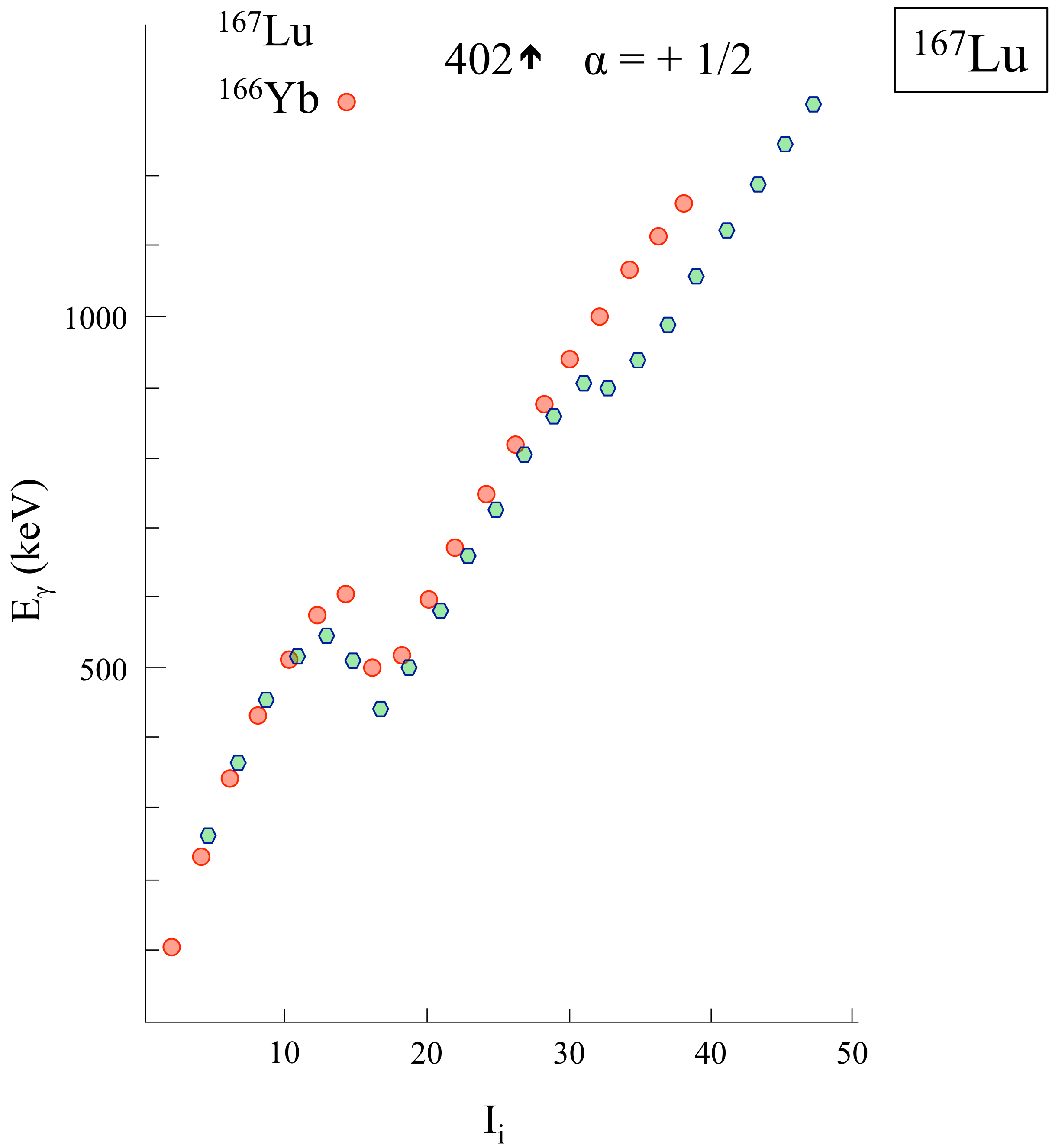
^{167}Lu
^{166}Yb
541↓ α = - 1/2
^{167}Lu
1000
500
E_γ (keV)
10
20
30
40
I_i



^{167}Lu
^{166}Yb
514↑ α = + 1/2
^{167}Lu
1000
500
E_γ (keV)
10
20
30
40
I_i

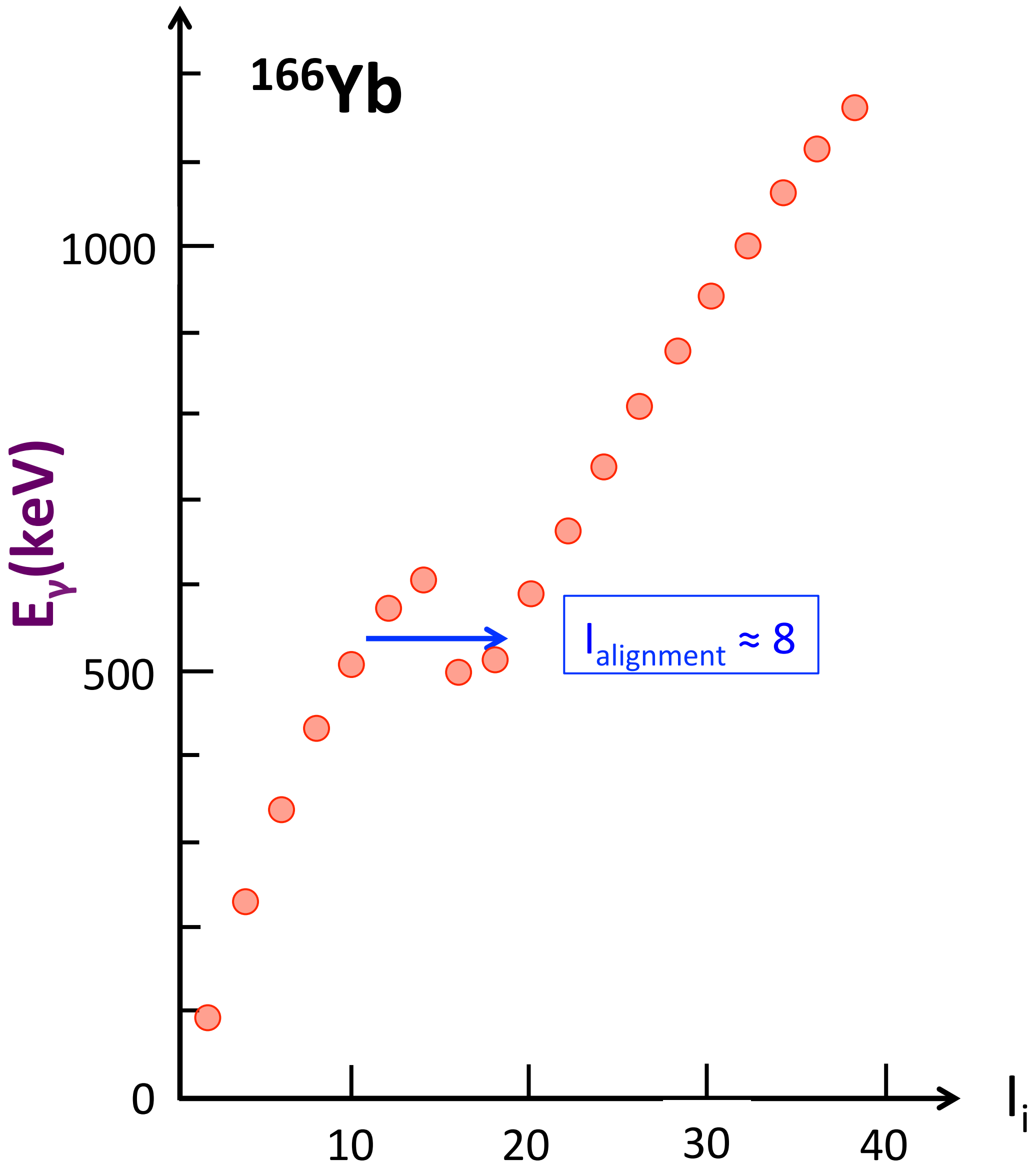


^{167}Lu
^{166}Yb
404↑ α = - 1/2
^{167}Lu
1000
500
E_γ (keV)
10
20
30
40
I_i



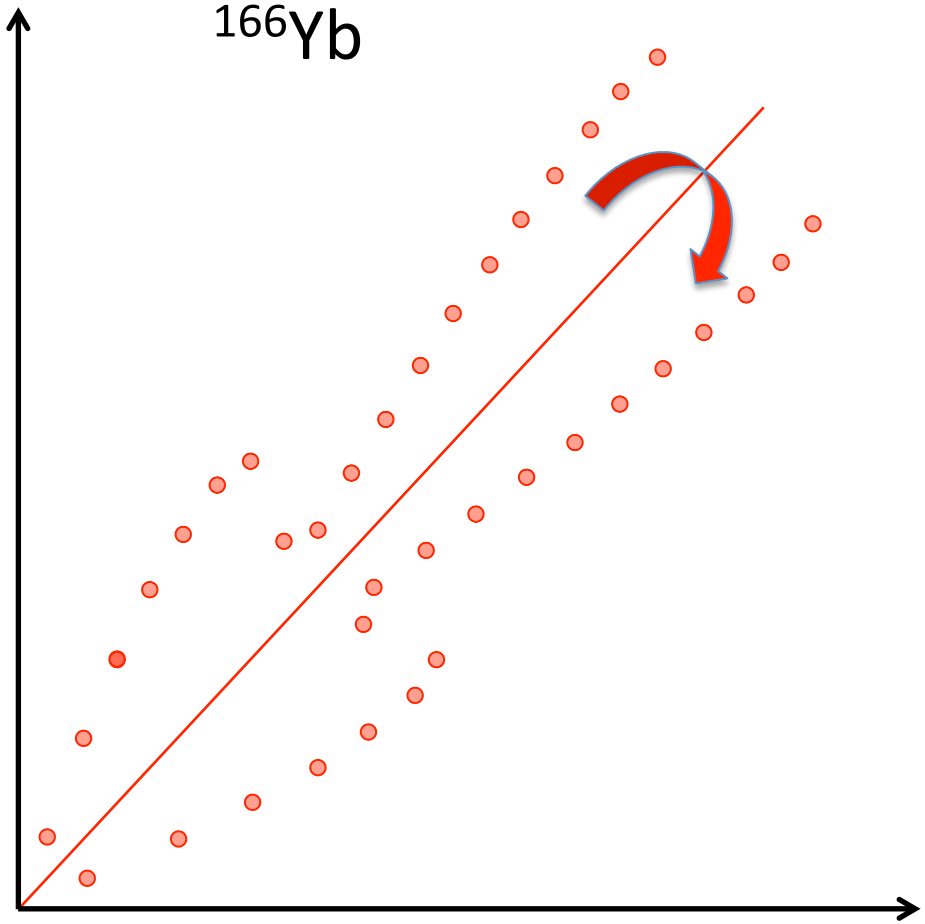
^{167}Lu
^{166}Yb
402↑ α = + 1/2
^{167}Lu
1000
500
E_γ (keV)
10
20
30
40
50
I_i

E_γ(keV) vs. I_i plots:
"backbending"

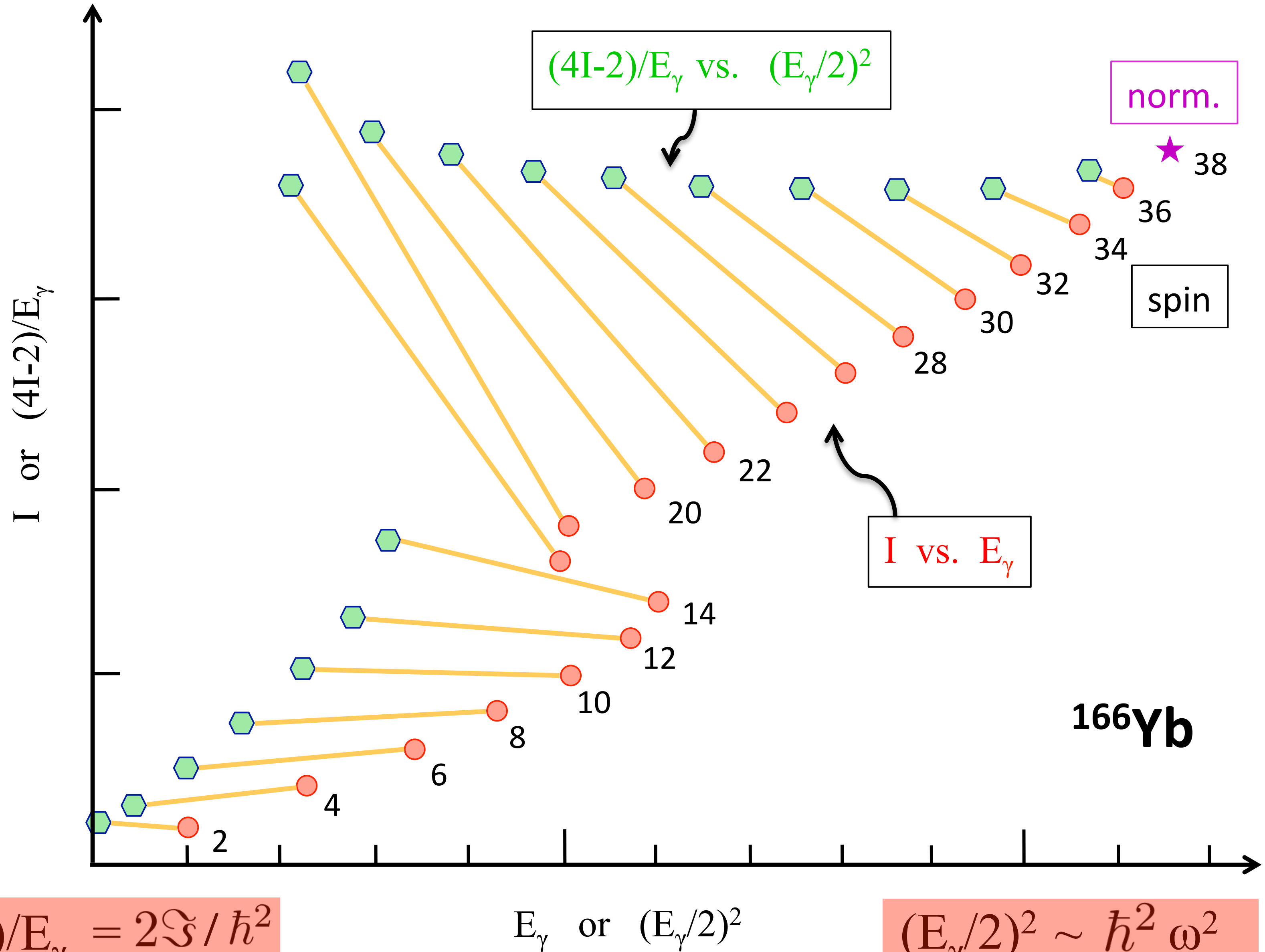


Relationships for plotting spin vs. γ-ray energy:

transformation from E_γ vs. I_i plot to I_i vs. E_γ plot



Relationships for plotting spin vs. γ-ray energy

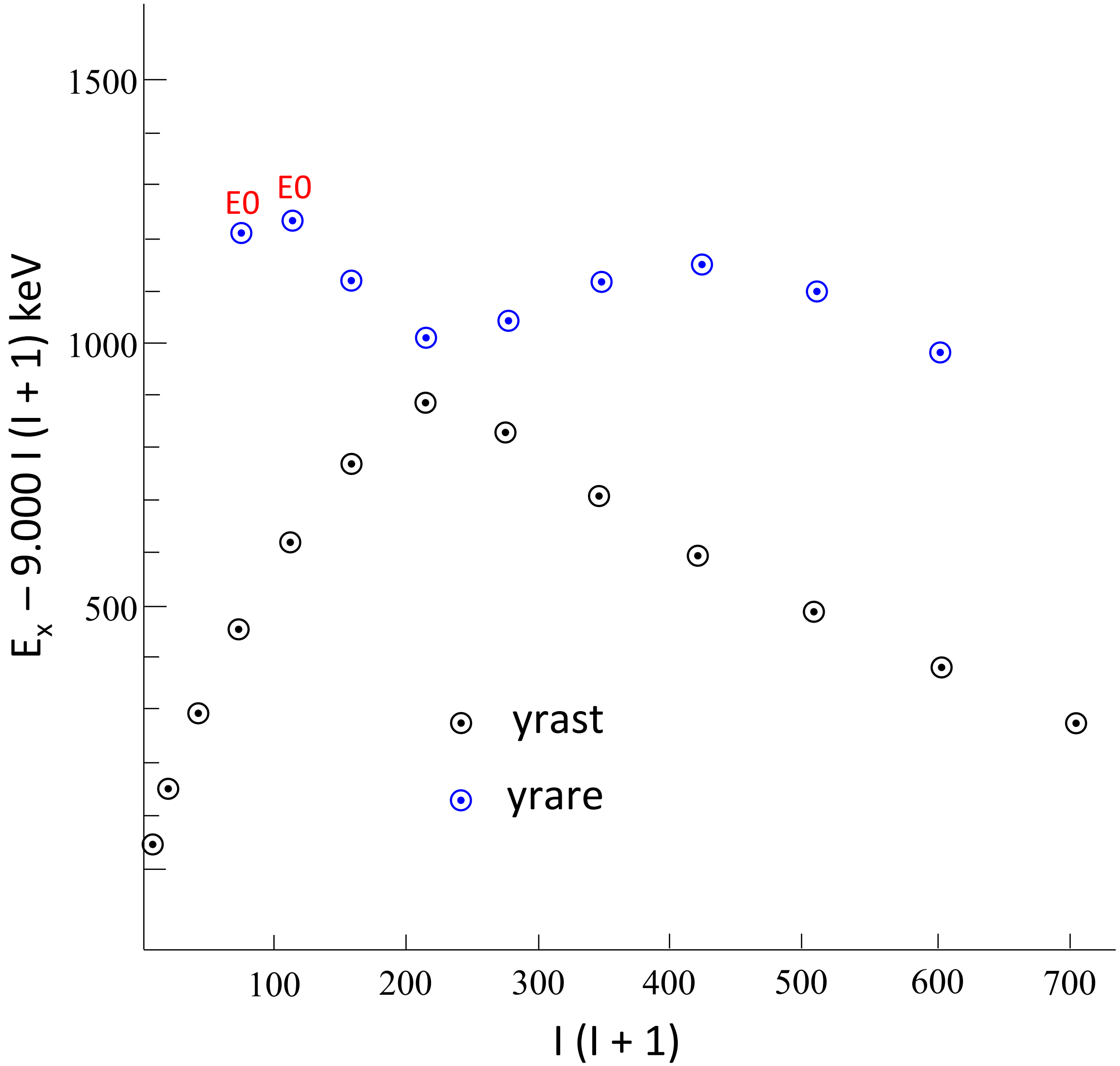


$(4I\text{-}2)/E_\gamma = 2\Im/\hbar^2$

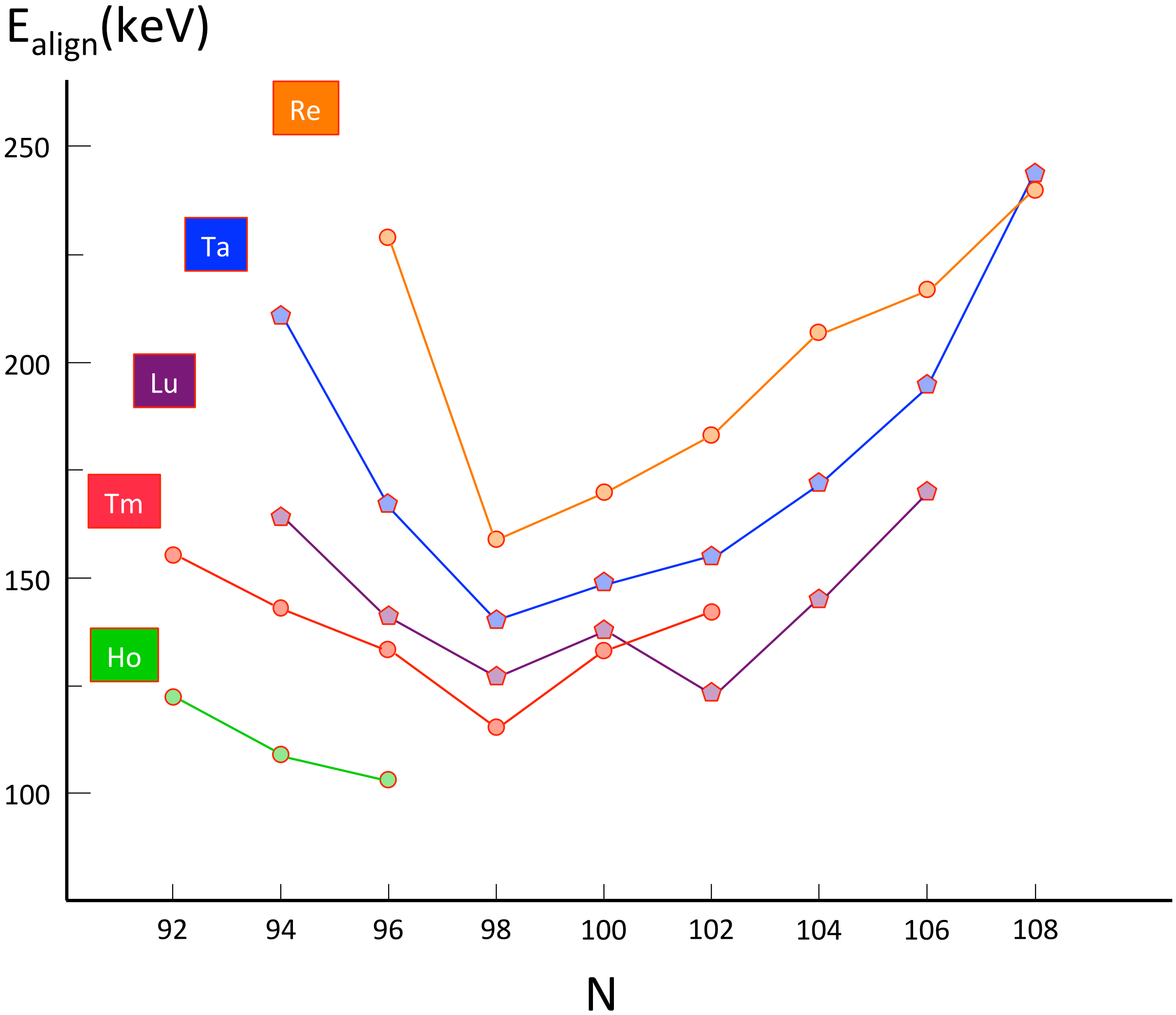
$(E_\gamma/2)^2 \sim \hbar^2\omega^2$

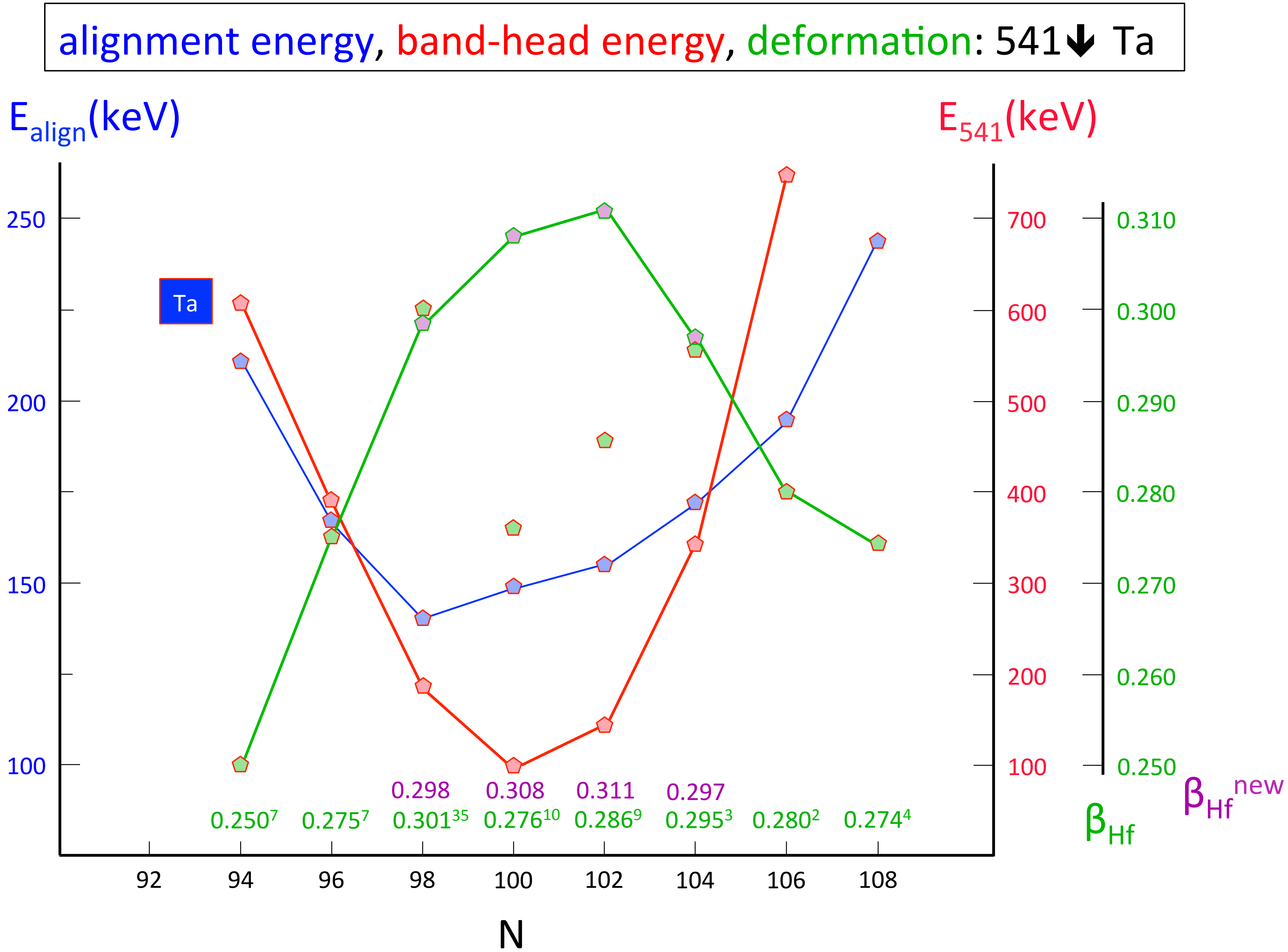
166Yb

E0's: C. Fields et al., PL **B130** 157 ; NP **A431** 473



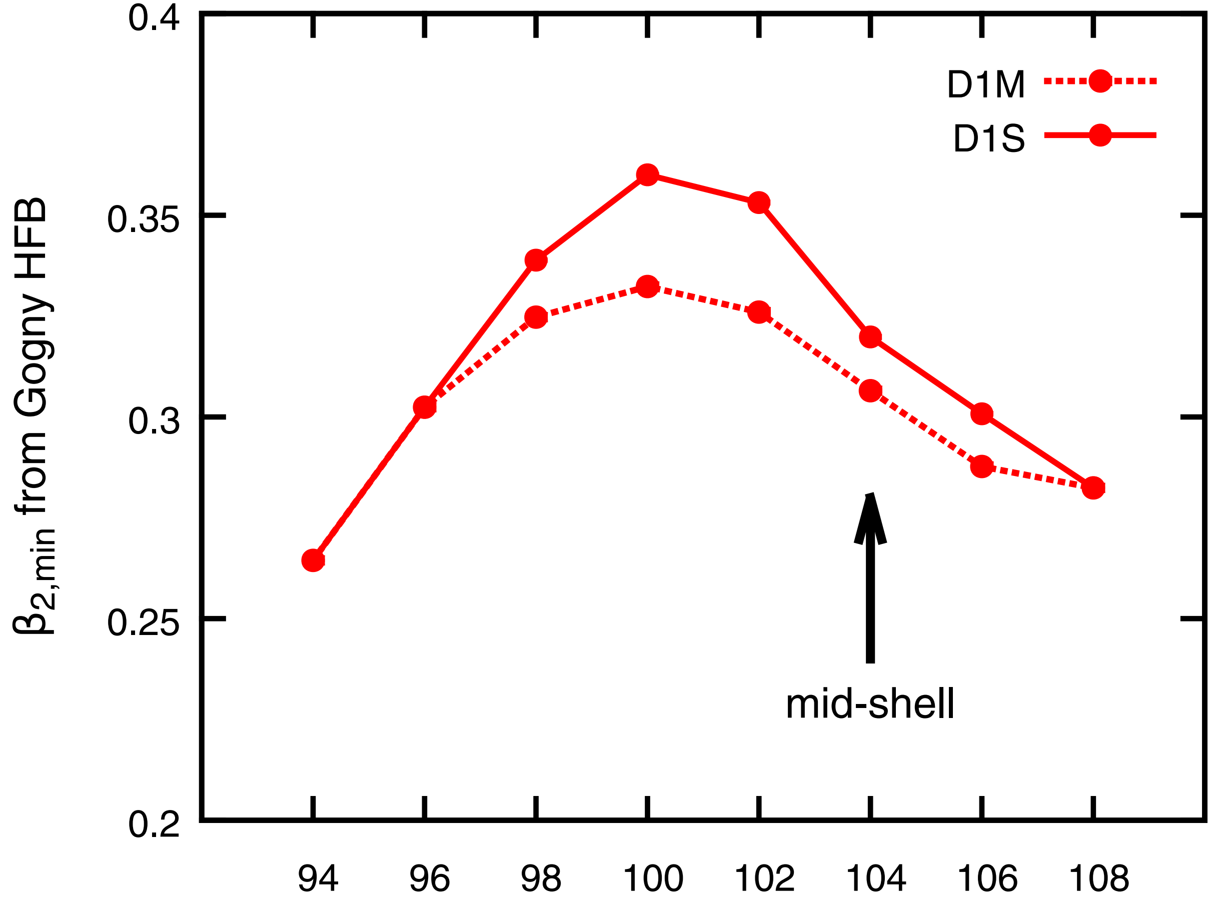
Alignment energy (lowering) for 541↓ band in rare-earth nuclei



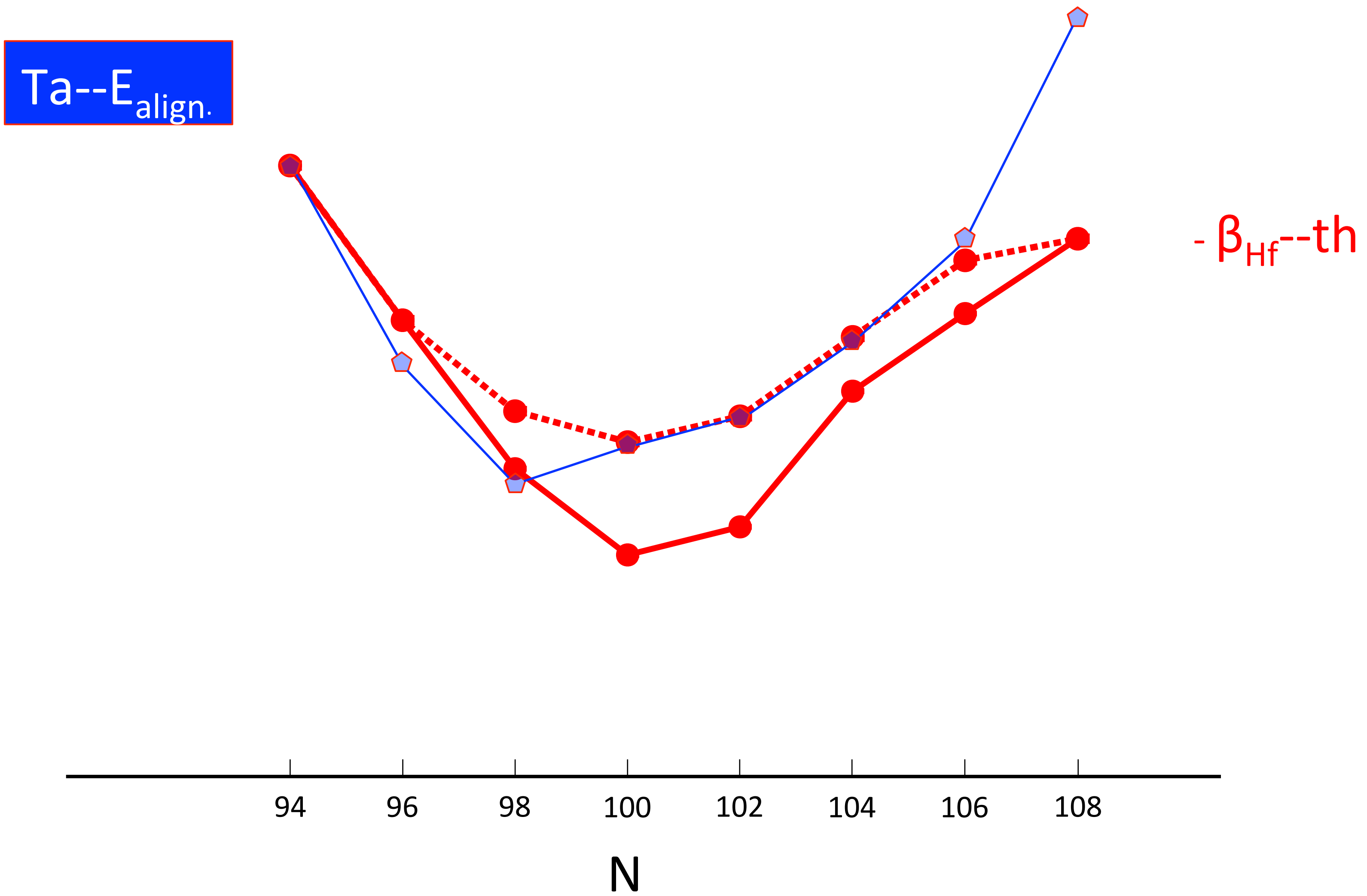

alignment energy, band-head energy, deformation: 541↓ Ta
E_align (keV)
E_541 (keV)
250
200
150
100
700
600
500
400
300
200
100
0.310
0.300
0.290
0.280
0.270
0.260
0.250
Ta
0.298
0.308
0.311
0.297
0.250⁷
0.275⁷
0.301³⁵
0.276¹⁰
0.286⁹
0.295³
0.280²
0.274⁴
β_Hf
β_Hf^new
92
94
96
98
100
102
104
106
108
N

Rudigier et al., PR C91 044301 2015

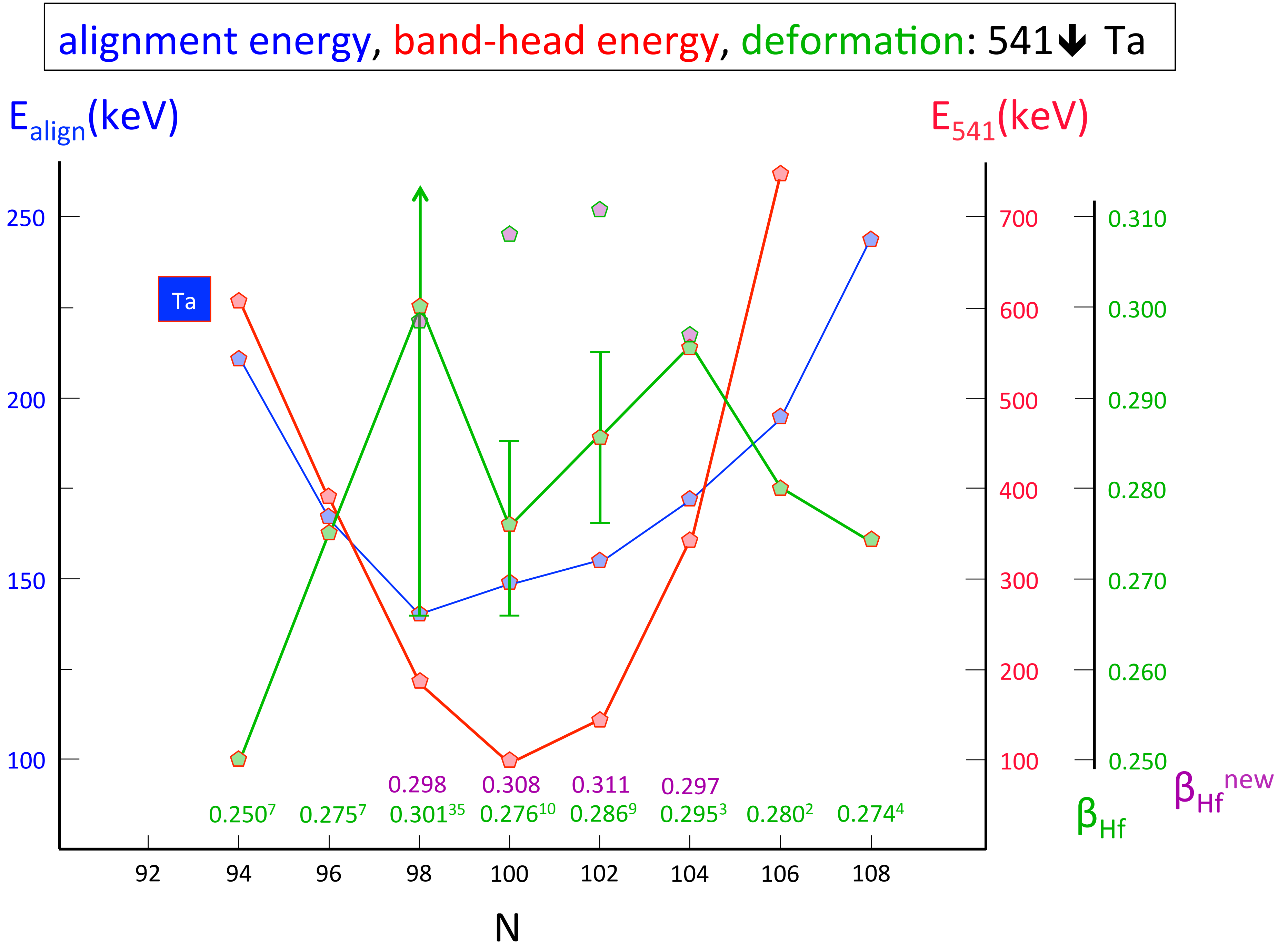
And see Smith et al., PR C87 044317 2013

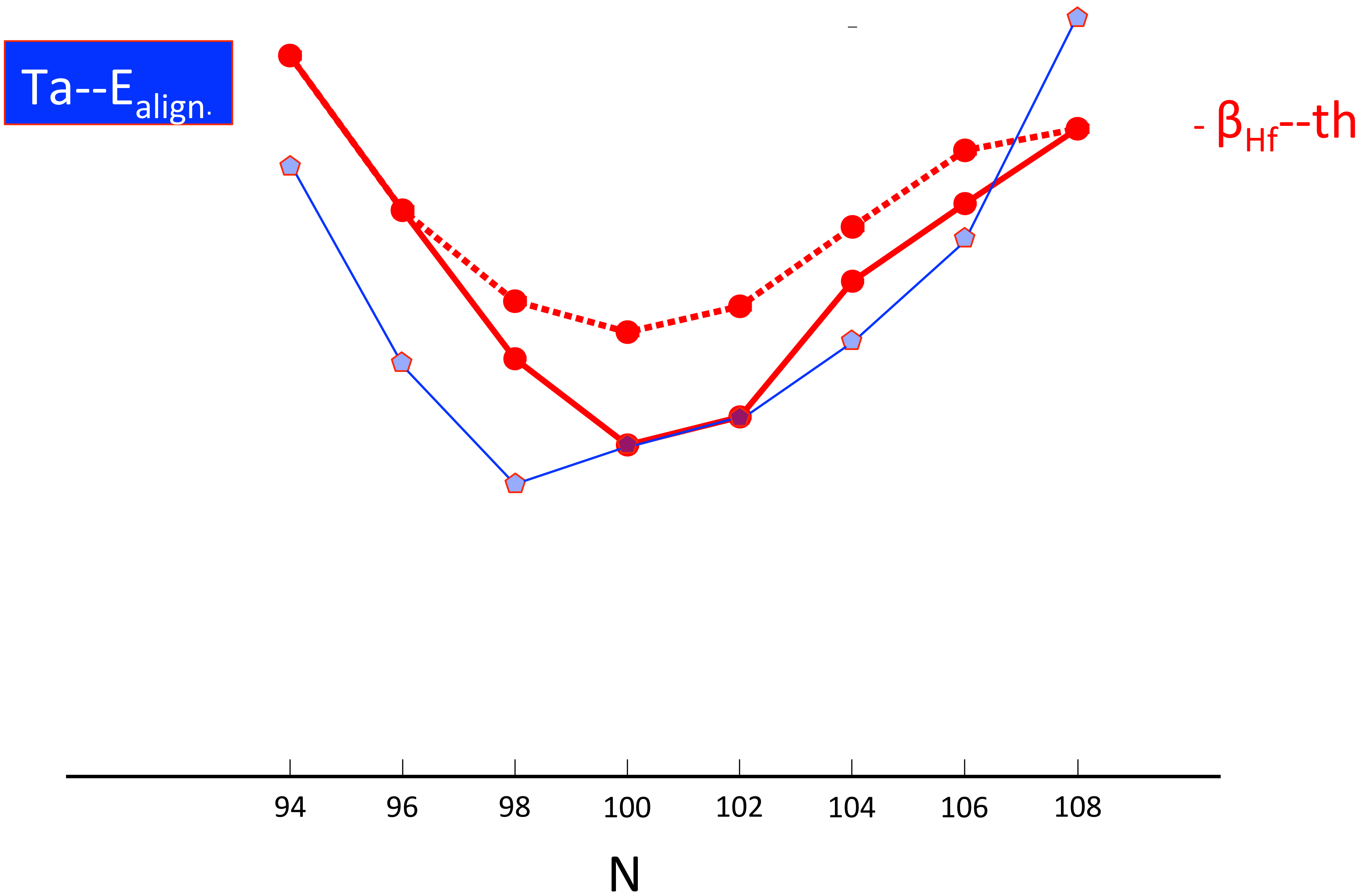


Ram.		
170	185	181[6]
172	156	194[6]
174	168	199[6]
176	179	182[7]

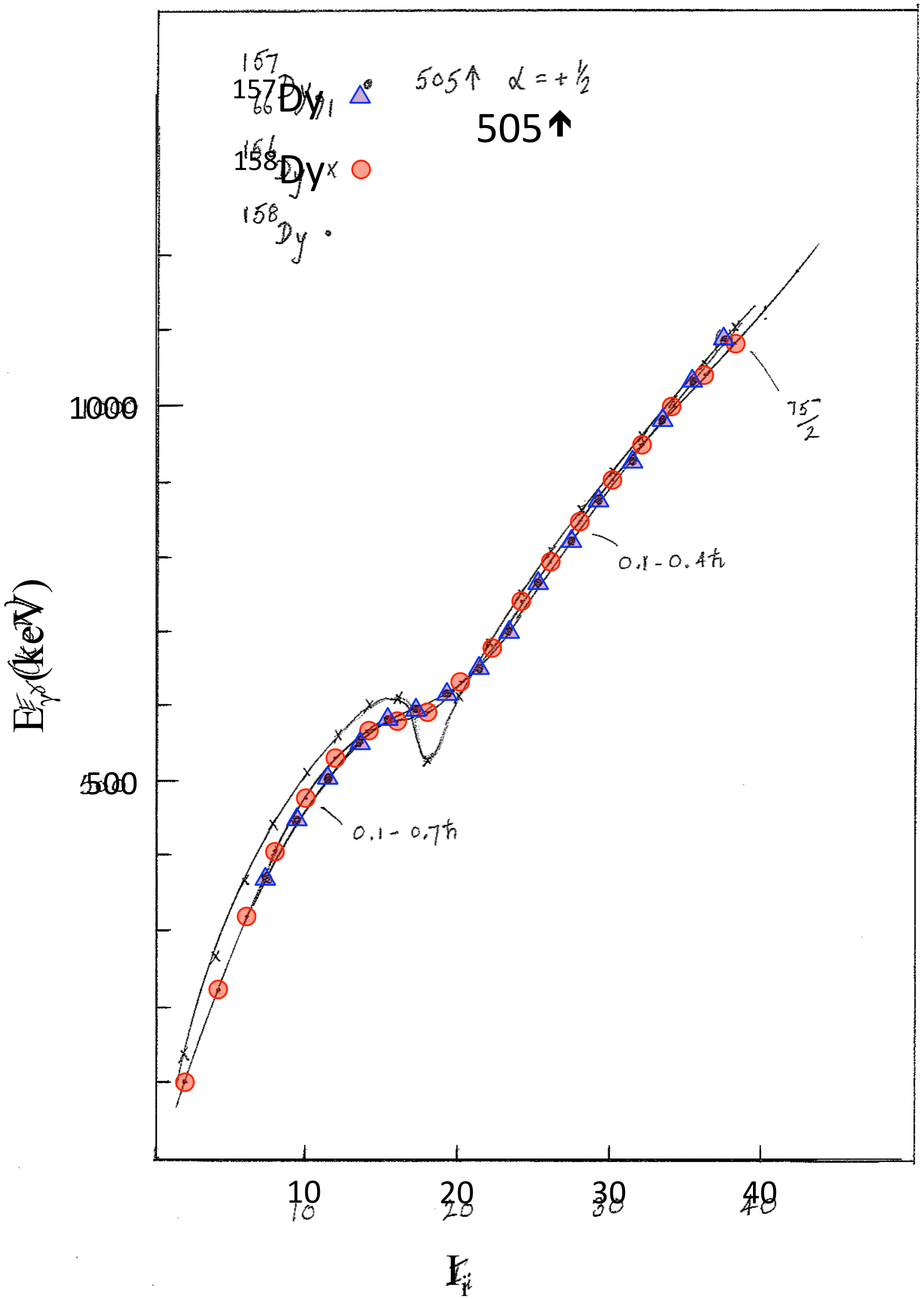


Ta--E$_{align.}$
- β$_{Hf}$--th
94
96
98
100
102
104
106
108
N


alignment energy, band-head energy, deformation: 541↓ Ta
E_{align}(keV)
E_{541}(keV)
Ta
250
200
150
100
700
600
500
400
300
200
100
0.310
0.300
0.290
0.280
0.270
0.260
0.250
0.298
0.308
0.311
0.297
0.250^{7}
0.275^{7}
0.301^{35}
0.276^{10}
0.286^{9}
0.295^{3}
0.280^{2}
0.274^{4}
β_{Hf}
β_{Hf}^{new}
92
94
96
98
100
102
104
106
108
N

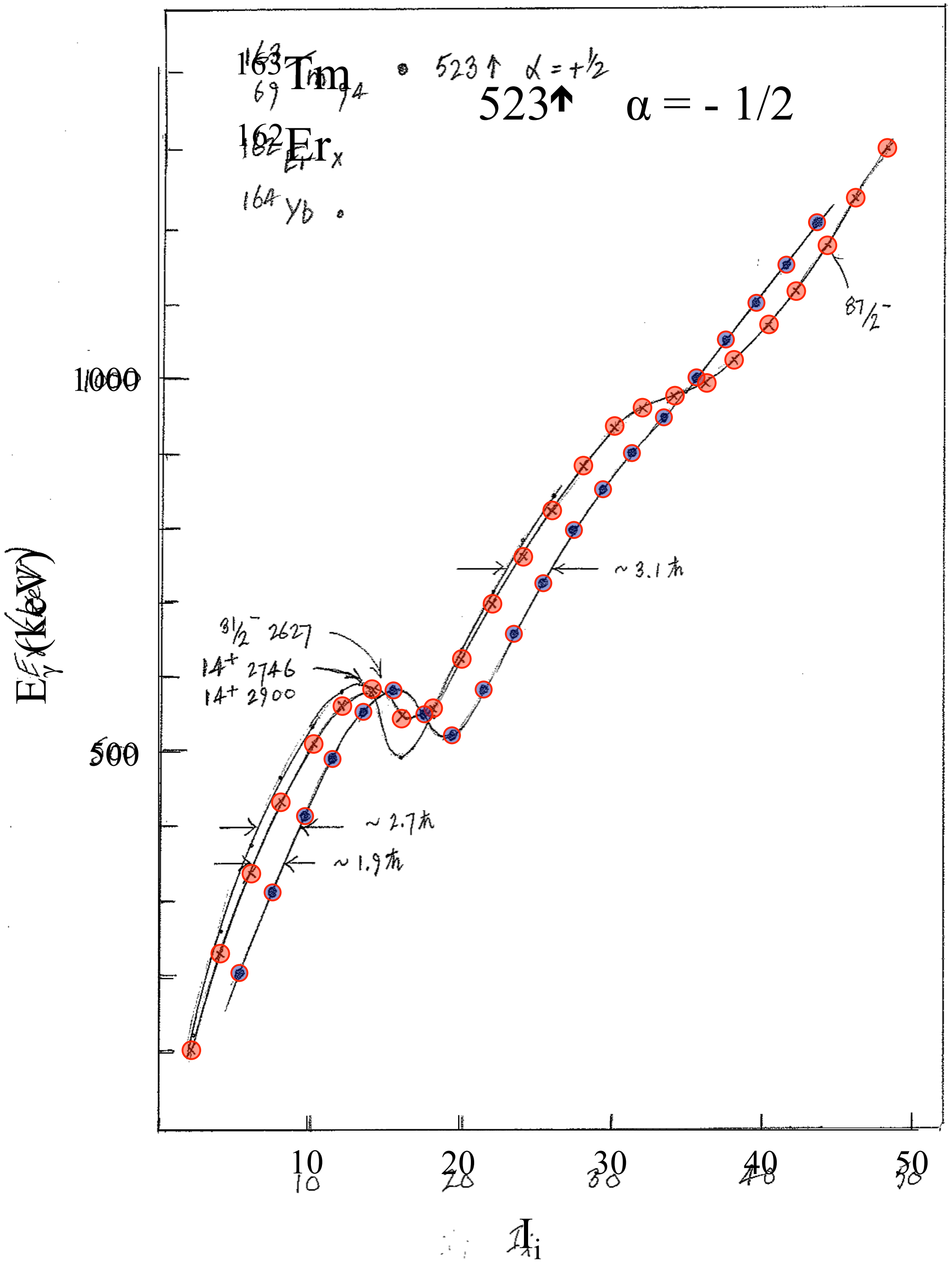


Ta--E align.
- β Hf--th
94
96
98
100
102
104
106
108
N



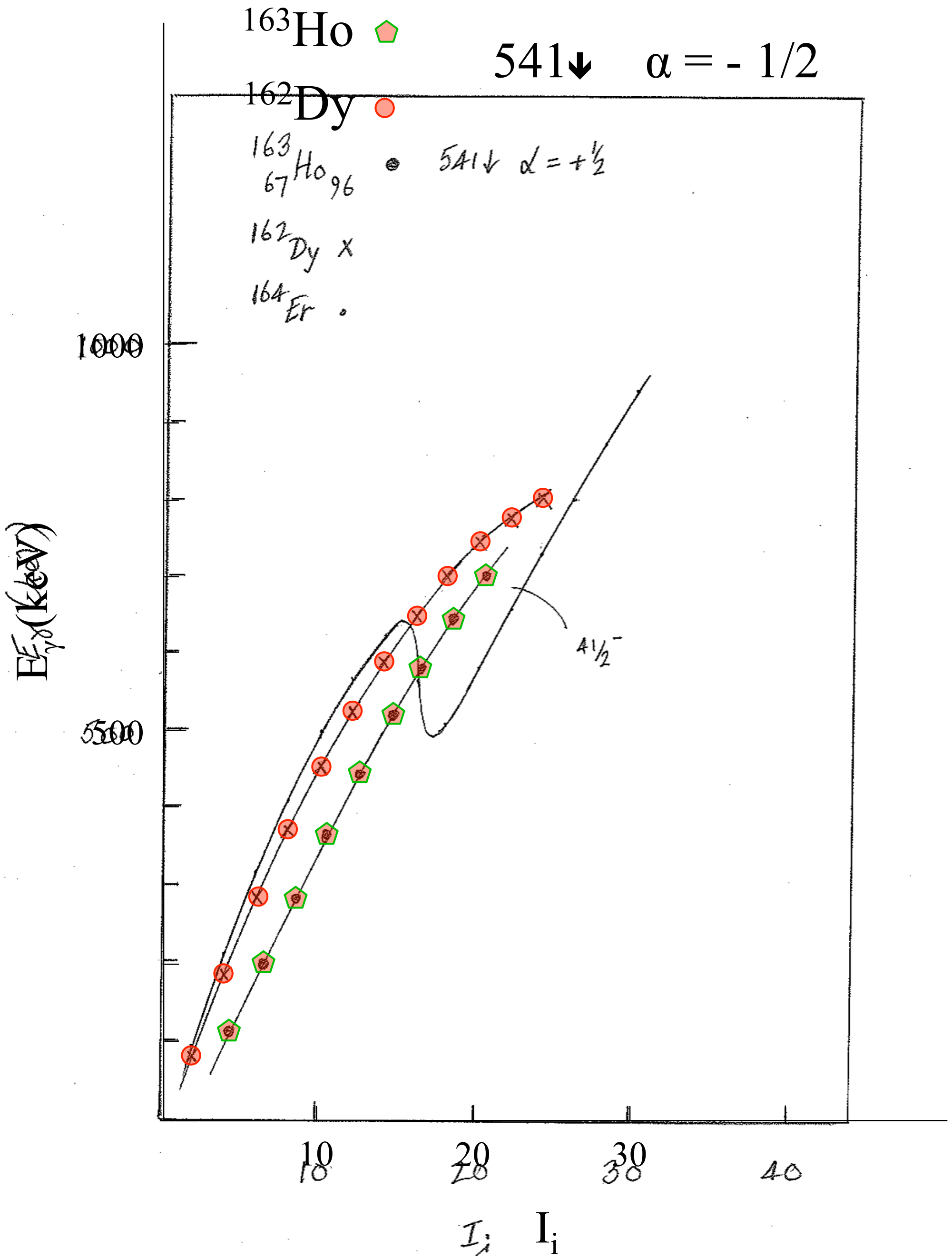
157Dy
158Dy
505↑
$\alpha = +1/2$
0.1 - 0.7ħ
0.1 - 0.4ħ
75/2
1000
500
10
20
30
40
E_γ (keV)
I_i

157Dy



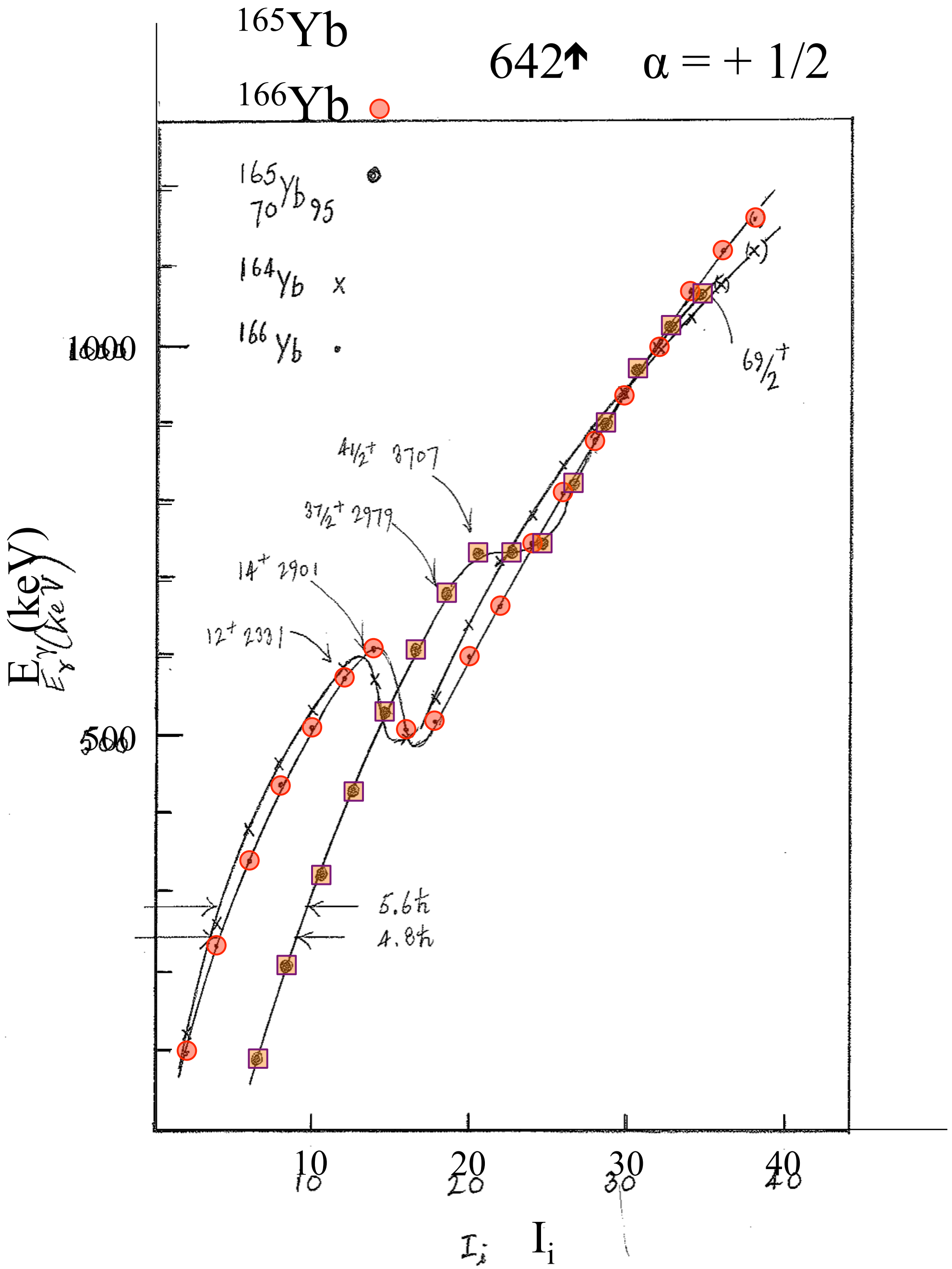
^{163}Tm

Note: reassignment of signature

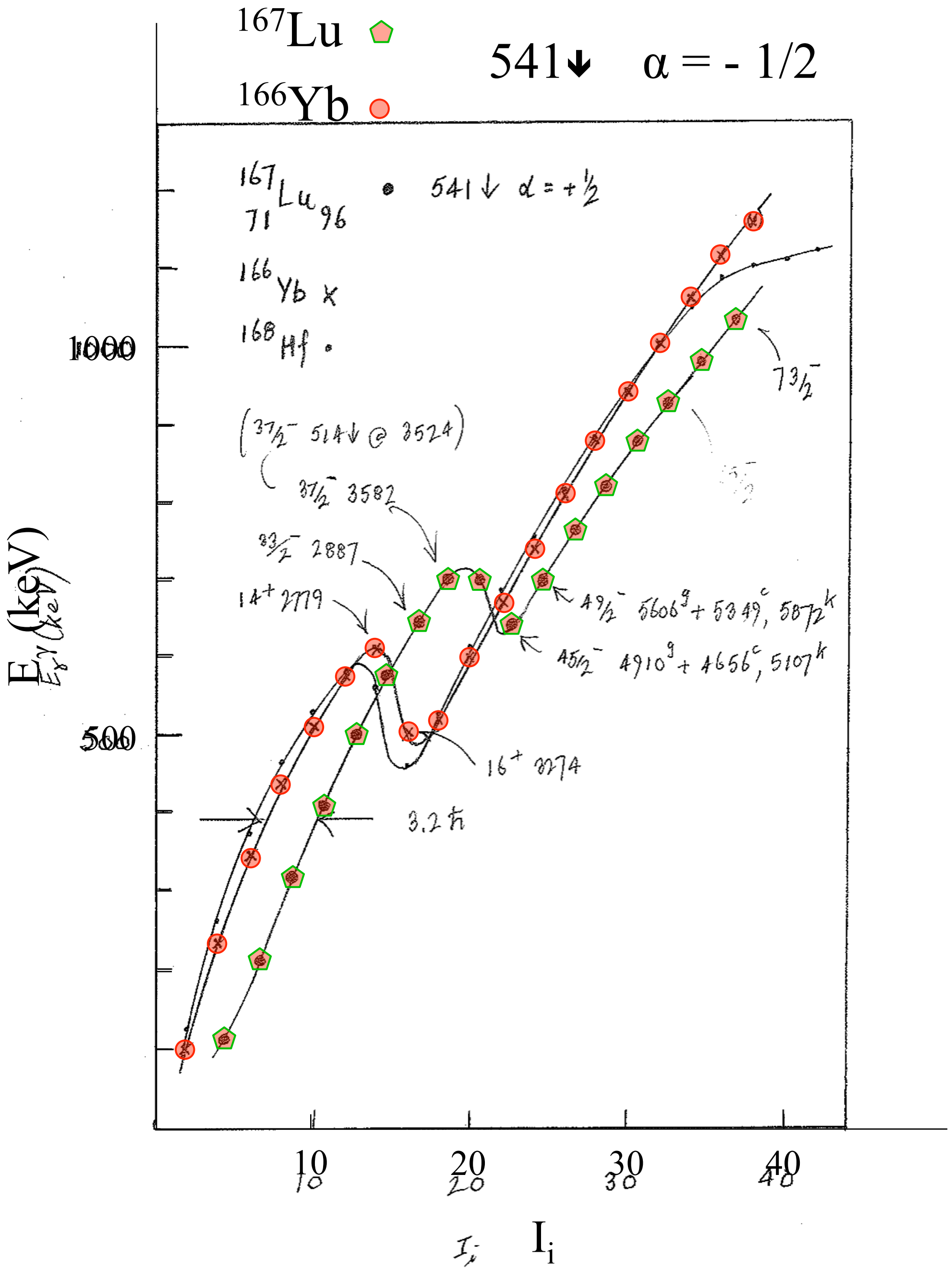


^{163}Ho
^{162}Dy
541↓ α = - 1/2
$^{163}_{67}$Ho$_{96}$ 541↓ α = +½
^{162}Dy x
^{164}Er
1000
500
E_γ (keV)
41/2$^-$
10
20
30
40
I_i

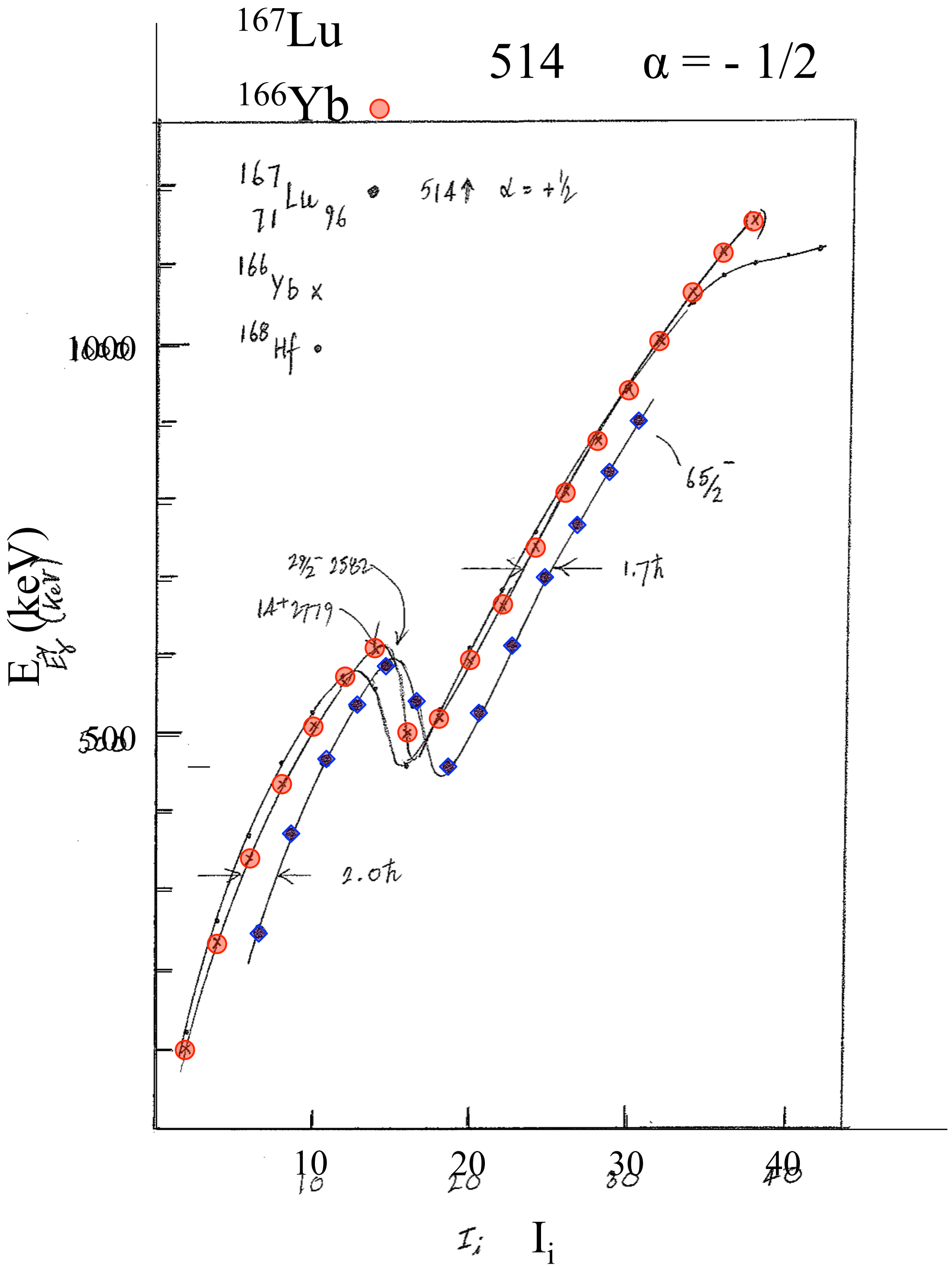
^{165}Yb



165Yb
642↑ α = + 1/2
166Yb
165 70 Yb 95
164Yb x
166Yb ·
1000
500
E γ (keV)
41/2+ 3707
37/2+ 2979
14+ 2901
12+ 2331
69/2+
(x)
5.6ħ
4.8ħ
10
20
30
40
I i



167Lu
166Yb
541↓ α = - 1/2
1000
500
Eγ (keV)
10
20
30
40
Ii

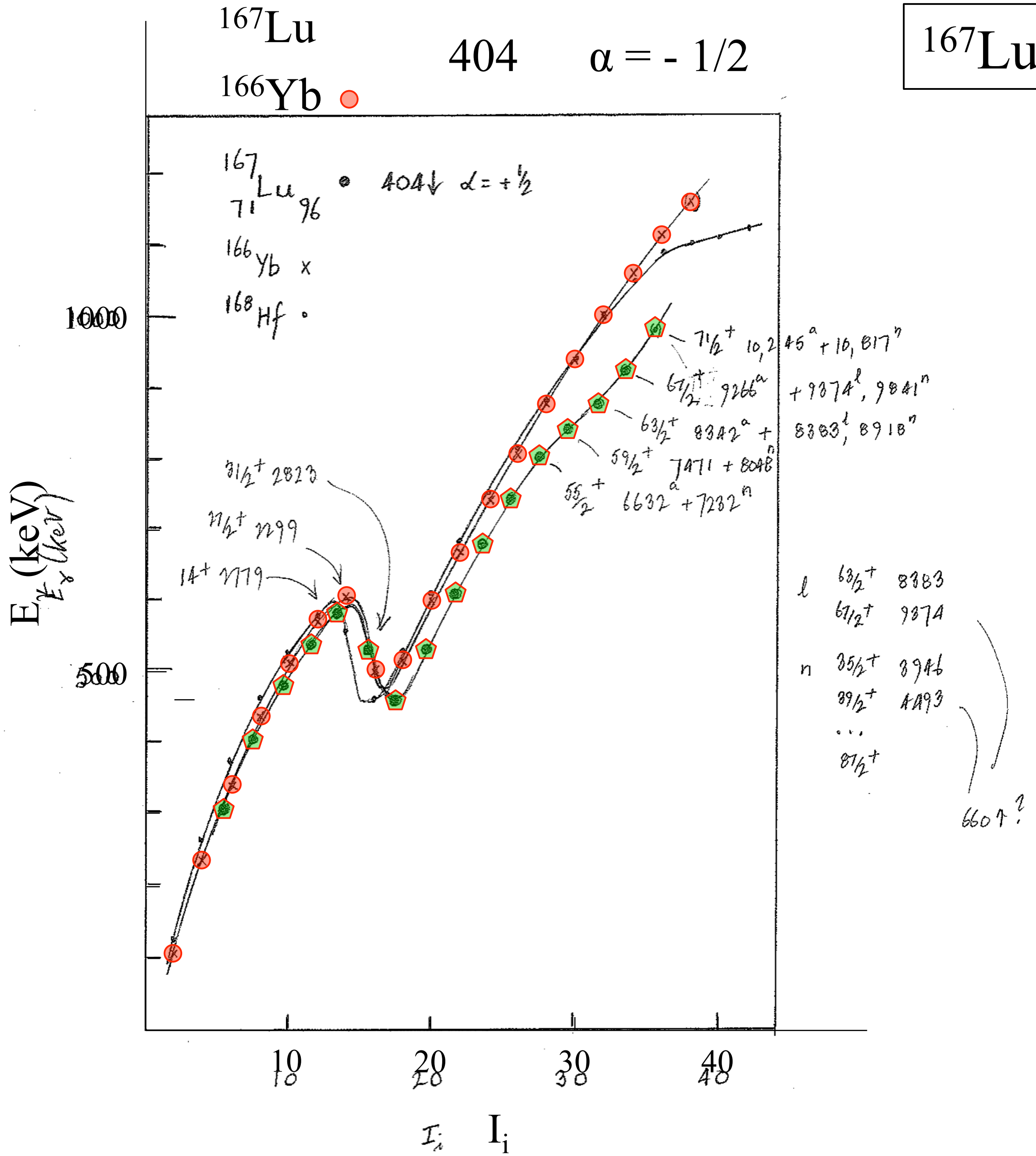


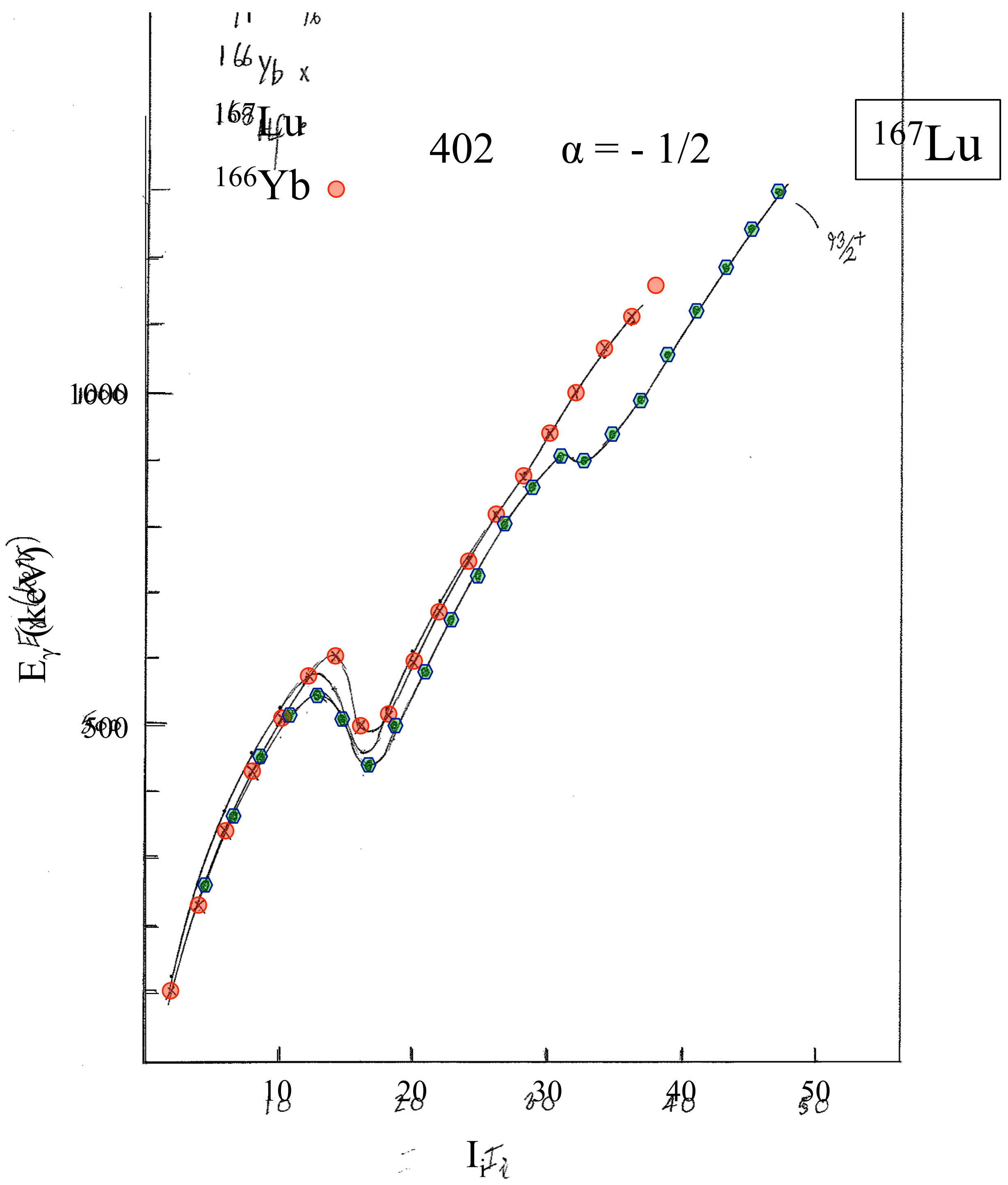
^{167}Lu
^{166}Yb
514
α = - 1/2
1.7ħ
2.0ħ
65/2⁻
1000
500
10
20
30
40
E_γ (keV)
I_i

^{167}Lu

^{167}Lu

404 α = - 1/2



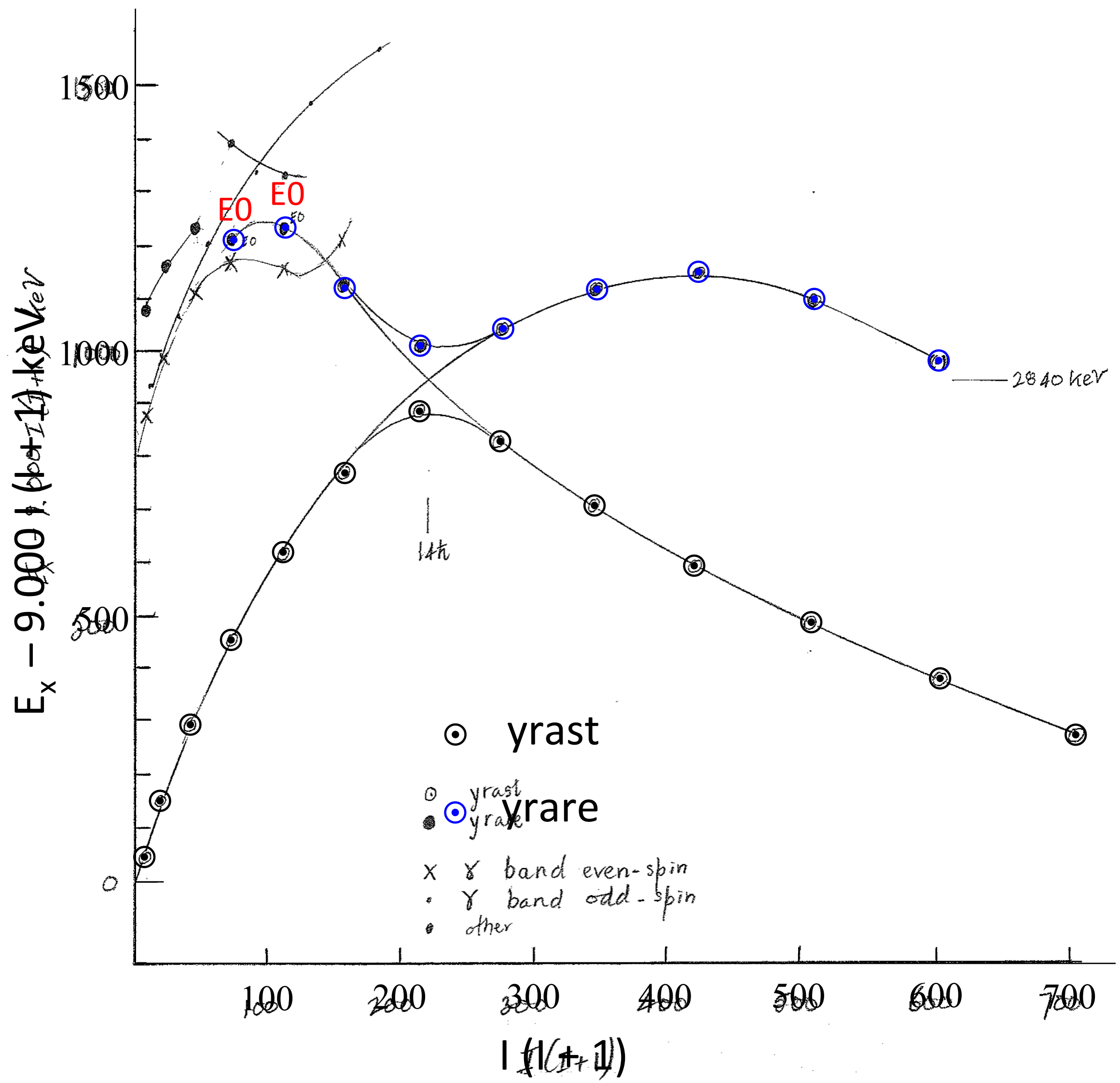


166Yb x
167Lu
166Yb
402 α = - 1/2
167Lu
93/2+
1000
500
Eγ (keV)
10
20
30
40
50
Ii

^{166}Yb E0's: C. Fields et al., PL **B130** 157 ; NP **A431** 473

3/19/13

S band near identical to ^{164}Er
E0's. Fields, PL B130, 157; NP A431, 473



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IDENTICAL BANDS IN DEFORMED AND SUPERDEFORMED NUCLEI

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"In particular, the reduction of the BCS pairing correlations due to the blocking of one and two orbitals implies large changes (up to 30%) in the moments of inertia and cannot be reconciled with these [identical band] systematics."