THEORY OF ALPHA-PARTICLE CONDENSATION IN NUCLEAR SYSTEMS
A nuclear Quantum Phase Transition

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P. Schuck (IPN Orsay and LPMMC Grenoble)

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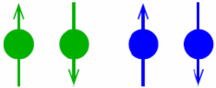
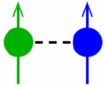
Clusters important aspect and richness of nuclear systems due to 4 Fermions :

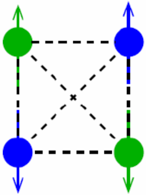


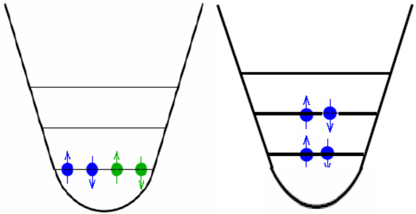
Dimer :



$\frac{E}{A} = 1$ MeV

Quartet :





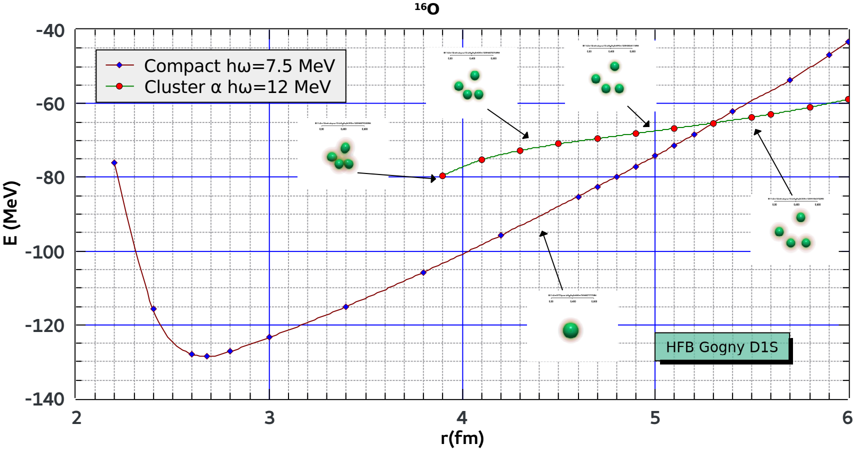
$\frac{E}{A} = 7$ MeV, $E^{*} = 20$ MeV

Proposal :

Trapping of 4 different species of Fermionic atoms.

Bi-Excitons

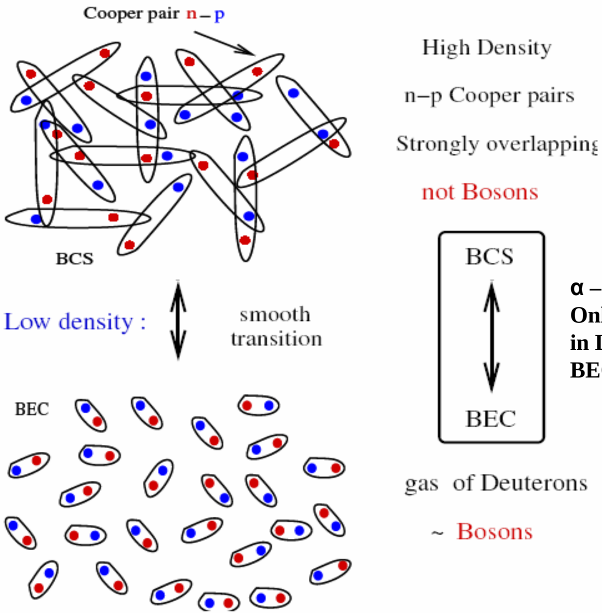
EOS with Spherical Expansion of 16O nucleus vs alpha-clustering



Tetrahedron = cristal

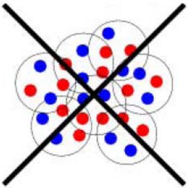
Mean field: quite wrong

In reality rather: "alpha-condensation"!

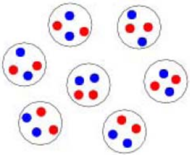


α – Particles
Only Exist
in Low Density
BEC Phase

Quartetting



No BCS phase (dense phase) of α-particles possible!

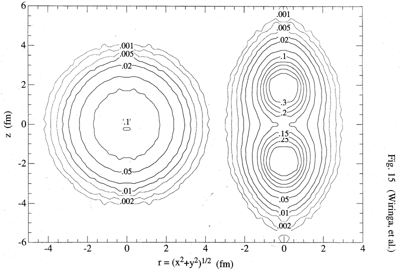


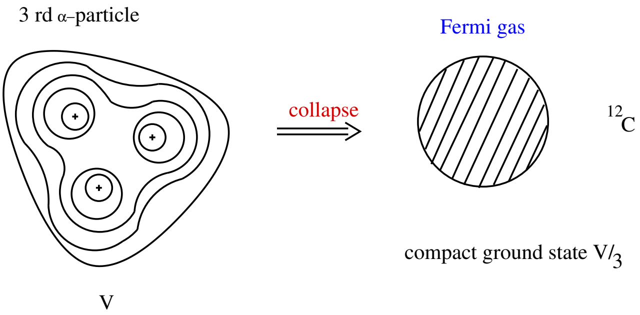
Bose-Einstein-Condensation of α-particles (dilute)

Finite nuclei ?

Exact ^{8}Be :

Density : $\frac{\rho_0}{3}$





Does a dilute 3α $^{12}C^*$ state exist ?
Similar to $^8Be + \alpha$?

The
Fred Hoyle
story

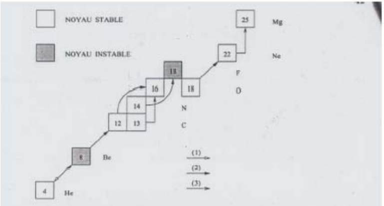
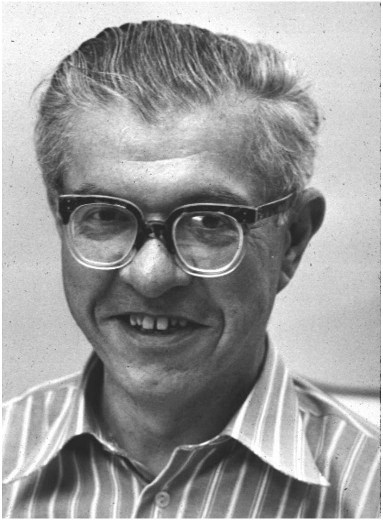


Figure 26. Réactions principales impliquées dans la combustion non explosive de He. Les flèches de type (1), (2) et (3) se rapportent respectivement à des réactions (γ, α), (α, γ), et (α, n)

At $T = 10^8 K$ helium burning
thermal equilibrium

$$\alpha + \alpha + \alpha \rightarrow {}^8Be \rightarrow \alpha \rightarrow {}^{12}C^* \quad O_2^+$$

O_2^+ : dilute 3α state hypothesis !



Fred Hoyle

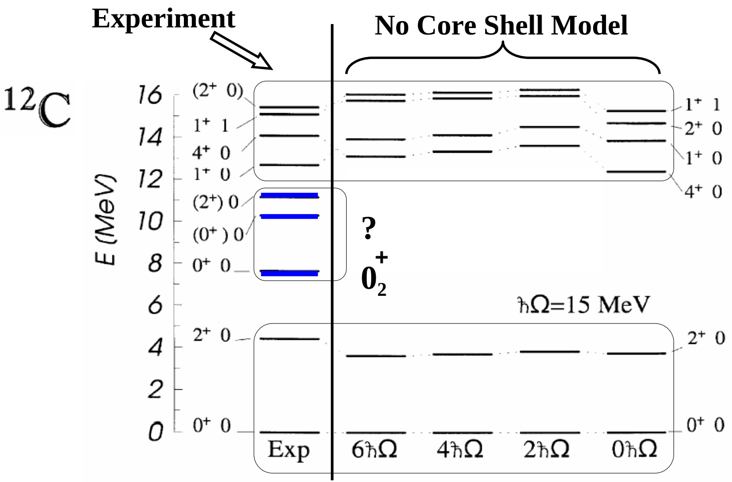
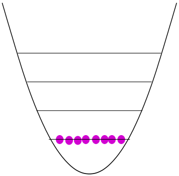


Figure 4. *Experimental and NCSM excitation spectra for ^{12}C for different model space sizes.*

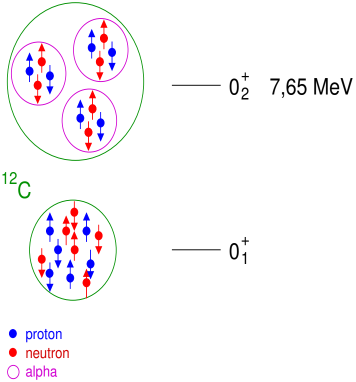
it seems impossible to get Hoyle state from shell model calculation !

45 MeV B. Barret

Bosons

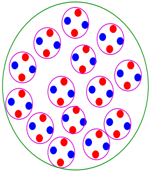


Back to nuclei



many α's

→ condensate



strong cluster phenomena in lighter nuclei

Theoretical Description

Ideal Bose condensate : $\left|0\right\rangle = b_0^\dagger b_0^\dagger \cdots b_0^\dagger \left|vac\right\rangle$

α-particle condensate : $\left|\Phi_{\alpha C}\right\rangle = C_\alpha^\dagger C_\alpha^\dagger \cdots C_\alpha^\dagger \left|vac\right\rangle$

In r-space :

$$\left\langle \vec{r}_1, \vec{r}_2, \cdots, \vec{r}_{4n} \middle| \Phi_{\alpha C} \right\rangle = \mathcal{A}\left\{ \Phi\left(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4\right) \Phi\left(\vec{r}_5, \vec{r}_6, \vec{r}_7, \vec{r}_8\right) \cdots \Phi\left(\vec{r}_{4n-3}, \vec{r}_{4n-2}, \vec{r}_{4n-1}, \vec{r}_{4n}\right) \right\}$$

In comparison with pairing :

$$\left\langle \vec{r}_1, \vec{r}_2, \cdots \middle| \mathrm{BCS} \right\rangle = \mathcal{A}\left\{ \Phi\left(\vec{r}_1, \vec{r}_2\right) \Phi\left(\vec{r}_3, \vec{r}_4\right) \cdots \right\}$$

Variational ansatz for $\Phi(\vec{r}_1,\vec{r}_2,\vec{r}_3,\vec{r}_4)$: $\Phi(\vec{r}_1,\vec{r}_2,\vec{r}_3,\vec{r}_4) = e^{-\frac{2}{B^2}\vec{R}^2}\phi_\alpha(\vec{r}_i - \vec{r}_j)$

Center of mass : $\vec{R} = \frac{1}{4}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4)$

Intrinsic α-wave function :

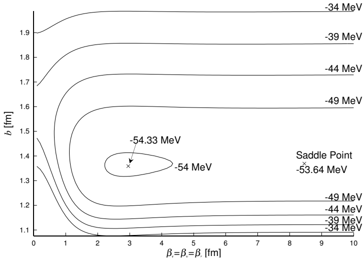
$$\phi_\alpha(\vec{r}_i - \vec{r}_j) = e^{-\frac{1}{8b^2}\left\{(\vec{r}_4-\vec{r}_1)^2+(\vec{r}_4-\vec{r}_2)^2+(\vec{r}_4-\vec{r}_3)^2+\cdots\right\}}$$

Two variational parameters : $B,\ b$

Two limits : $B = b$ $\quad |\Phi_{\alpha C}\rangle =$ Slater determinant

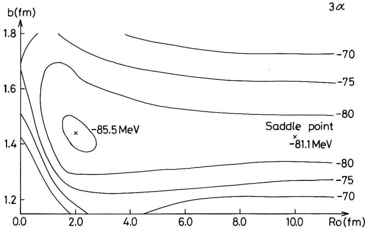
$B \gg b$ $\quad |\Phi_{\alpha C}\rangle =$ gas of independent α-particles

Two dimensional surface : $E(B,b) = \frac{\langle\Phi_{\alpha C}|H|\Phi_{\alpha C}\rangle}{\langle\Phi_{\alpha C}|\Phi_{\alpha C}\rangle}$



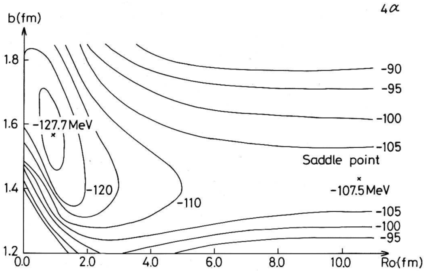
-34 MeV
-39 MeV
-44 MeV
-49 MeV
-54.33 MeV
-54 MeV
Saddle Point
-53.64 MeV
-49 MeV
-44 MeV
-39 MeV
-34 MeV
b [fm]
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
0 1 2 3 4 5 6 7 8 9 10

^{8}B



b(fm)
3α
1.8
1.6
1.4
1.2
-70
-75
-80
-85.5 MeV
Saddle point
-81.1 MeV
-80
-75
-70
0.0 2.0 4.0 6.0 8.0 10.0
Ro(fm)

^{12}C



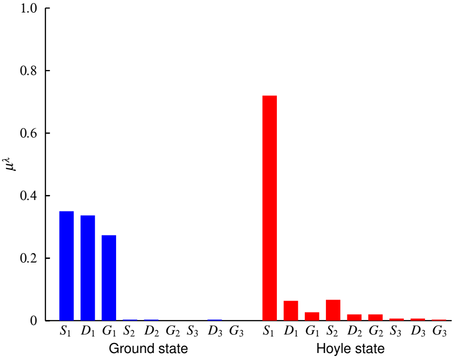
b(fm)
4α
1.8
1.6
1.4
1.2
-90
-95
-100
-105
-127.7 MeV
-120
-110
Saddle point
-107.5 MeV
-105
-100
-95
0.0 2.0 4.0 6.0 8.0 10.0
Ro(fm)

^{16}O

Boson occupancies:

α-particle density matrix: $\rho_\alpha(\mathbf{R}, \mathbf{R}')$, $\mathbf{R}$: c.o.m. of α

diagonalisation



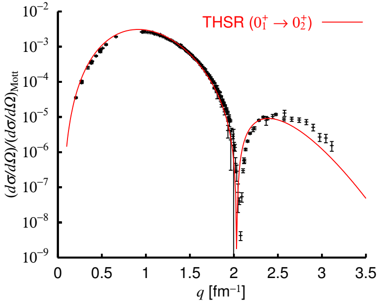
Comparison with experiment: inelastic form factor to Hoyle

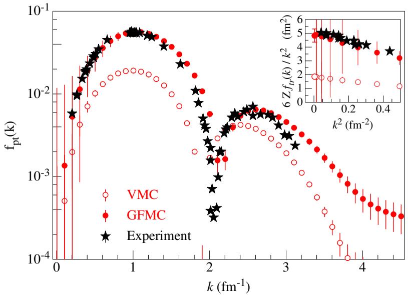
THSR and **GFMC**(Pieper et al.)

THSR

(Tohsaki, Horiuchi, Schuck, Roepke)

Vol_Hoyle
--------------   ~ 4 !!
Vol_12C

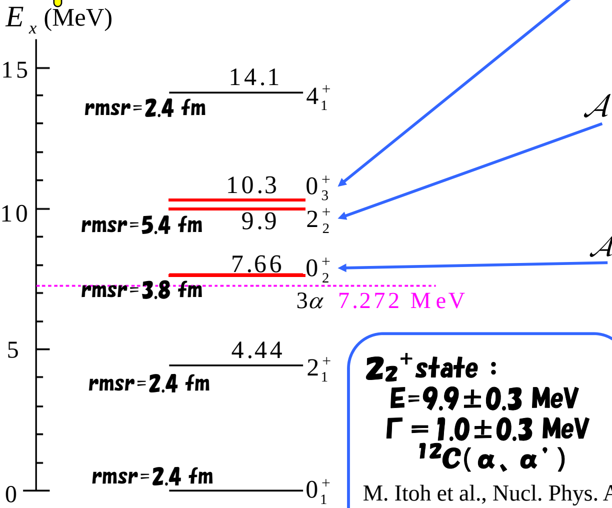


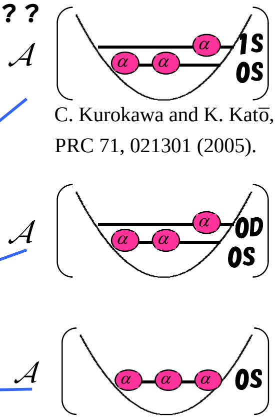


GFMC (Wiringa, Pieper et al., RMP 2017)

"BEC" in ^{12}C

Observed levels of ^{12}C





2_2^+ state :
E=9.9±0.3 MeV
Γ =1.0±0.3 MeV
$^{12}C(\alpha, \alpha')$

M. Itoh et al., Nucl. Phys. A 738 (2004) 268-272

α cond. +ACCC
E=9.38 MeV
Γ =0.64 MeV
Volkov No. 1 force is adopted

Y. F. et al., EPJA 24, 321 (2005).

Infinite nuclear matter:

critical temperature

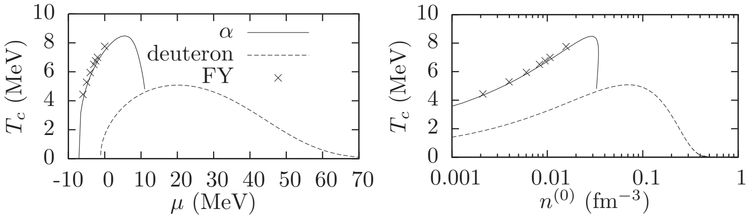
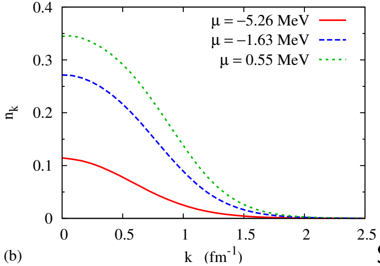


Figure: Critical temperature for α condensation as a function of chemical potential (left panel) and as a function of uncorrelated density (right panel) compared to the one of neutron-proton (deuteron) pairing (broken line). The crosses correspond to a full solution of the in medium Faddeev-Yakubovsky equations with the Malfliet-Tjohn potential [?]

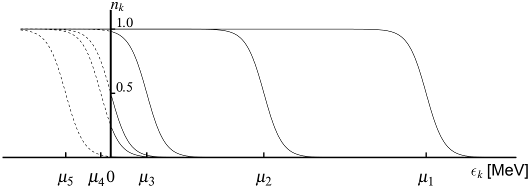
Fully self-consistent quartet alpha-order parameter

$\langle c_1 c_2 c_3 c_4 \rangle \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_4)$

at $T = 0$



Schematic view in case of BCS:



Alpha condensation stops at $\mu \simeq 0.55$ MeV

This corresponds to $\rho \sim \rho_0/5 \rightarrow$ Quantum Phase Transition!

Why does it stop?

Three-hole level densities:

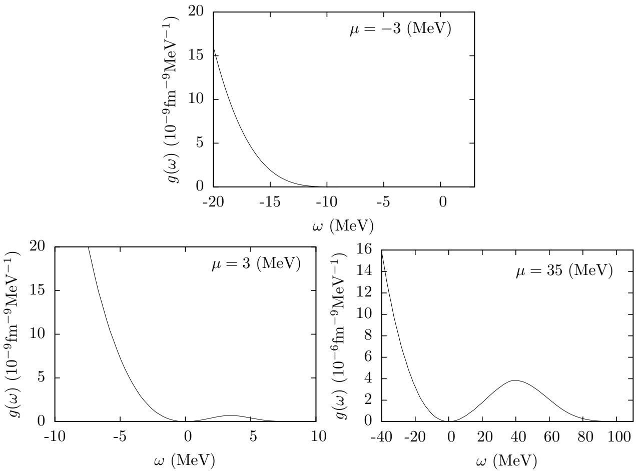


Figure: $3h$-level density for negative (top) and two positive (bottom) chemical potentials [?].

CONCLUSIONS

Hoyle state precursor of α particle condensation in nuclear matter

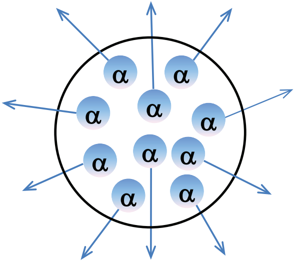
Alpha (quartet) condensation only exists at low density ($\rho < \rho_0/5$)

Alpha-Condensation is a Quantum Phase Transition with density as control parameter.

Stems from vanishing four particle level density at Fermi-energy

Alpha condensation predicted in heavier nuclei. E.g. ^{16}O: 6-th 0^+ state at 15.1 MeV

Dream: Coulomb explosion of ^{40}Ca



THANK YOU !