Experimental information on axial symmetry breaking in heavy nuclei

and microscopic calculations

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Level densities - n-tof data

Spectroscopy of odd nuclei – energies and rates

Quadrupole observables in even nuclei

Splitting of giant dipole resonances

Photon strength and n-capture

Rotor models

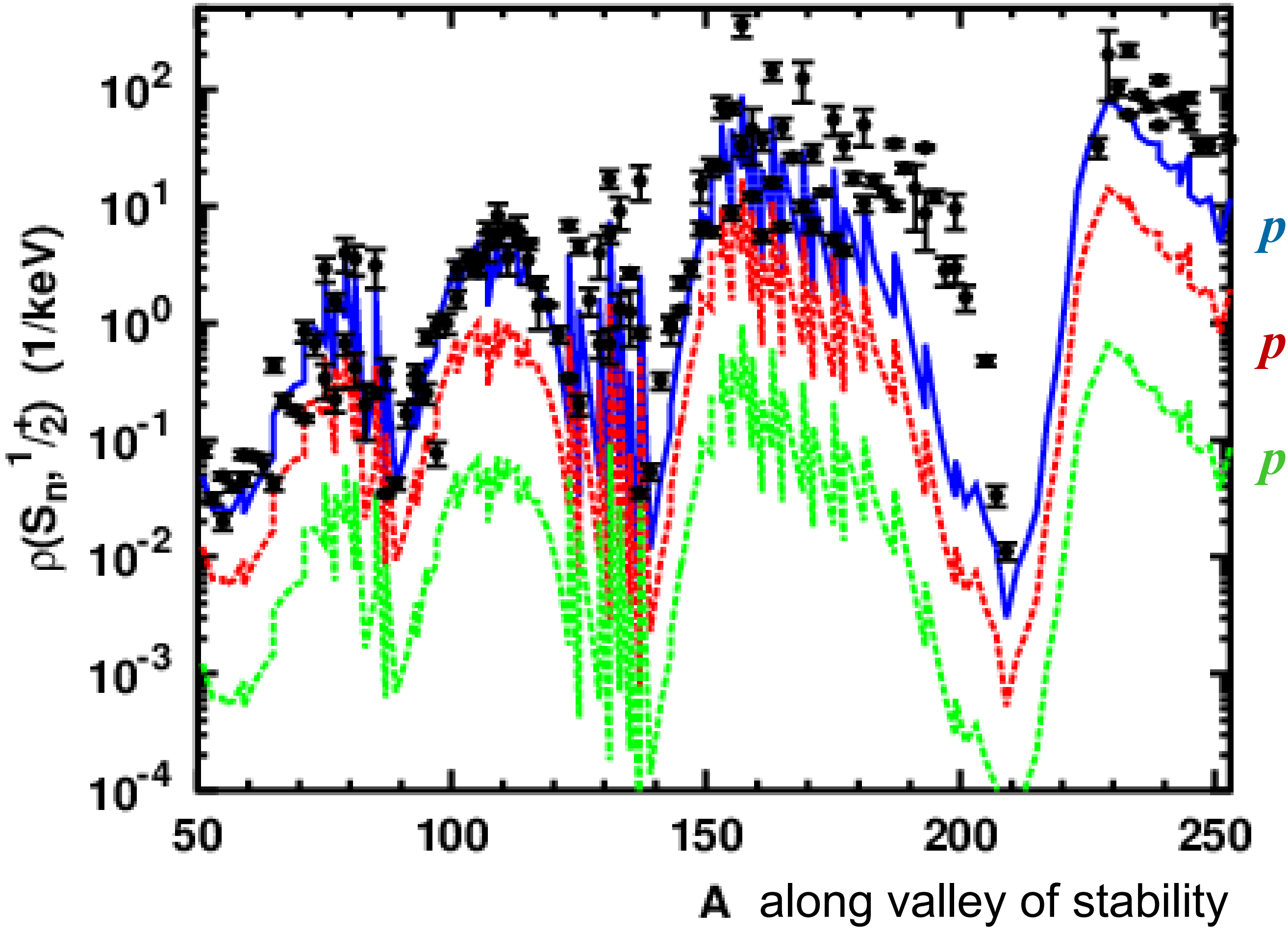
QHFB and RPA

*together with : **A.R. Junghans**, Institute of Radiation Physics, Helmholtz-Zentrum Dresden-Rossendorf*

Level densites ρ(E,I) *in heavy nuclei result from collective enhancement (group theory)*

of intrinsic state density ω(E); account for spin dispersion and cut off compensates E_{rot} .

Accurate data stem from n-capture resonances just above $\boldsymbol{S_n}$*:*



prediction for broken axiality

prediction assuming axiality

prediction for spherical case,

absolute scale,

no parameters adjusted.

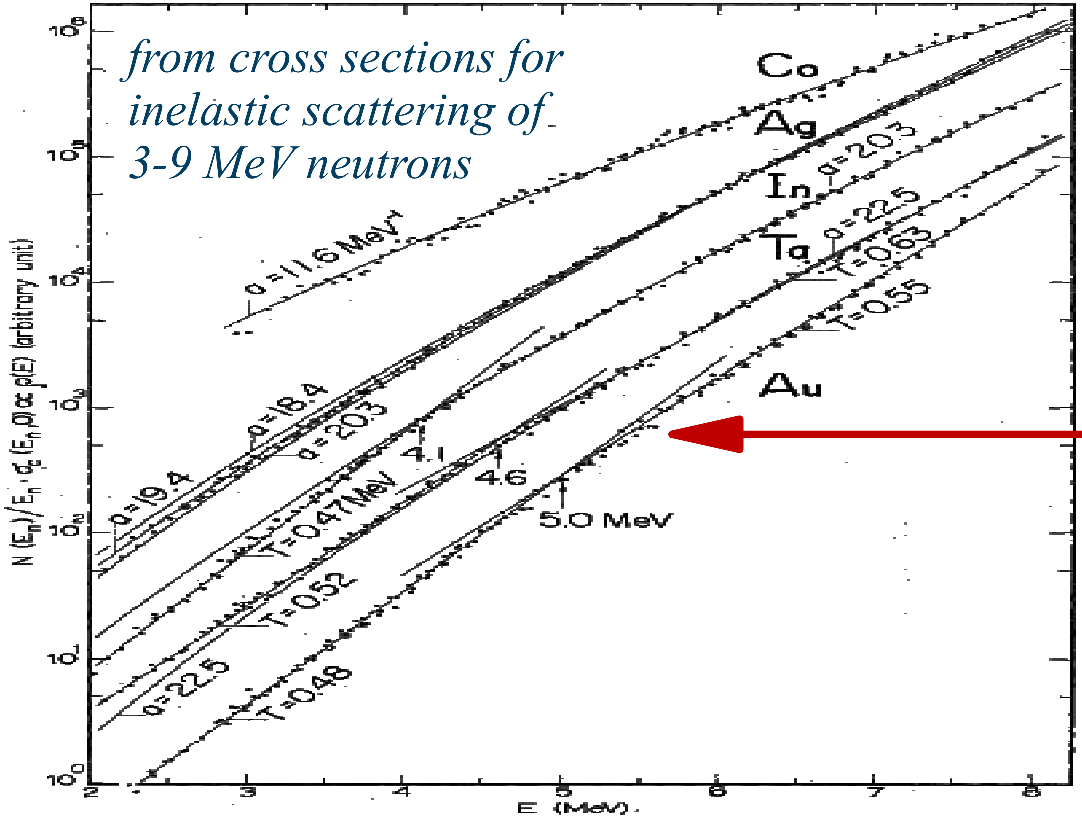
Data from RIPL-3 H.Bethe, Rev. Mod. Phys. 9 (1937) 69; S.Bjørnholm, A.Bohr, B.Mottelson; Rochester conf. 1974, IAEA-SM-l74/205

Level densites* $\rho(E,I)$** *in heavy nuclei indicate a kind of phase transition between a* ***Fermi gas *above* $t_c = \Delta_0 \cdot e^C/\pi = 0.567 \cdot \Delta_0$ *with* $\omega_{qp}(E_x) = \frac{\sqrt{\pi} \cdot exp\left(2\sqrt{\tilde{a}(E_x - E_{bs})}\right)}{12\, \tilde{a}^{1/4} (E_x - E_{bs})^{5/4}}$ *(FGM)*

and below a regime influenced by ***pairing and shell effects****,*

approximated by an exponential rise: $\omega_{qp}(E_x) = \omega_{qp}(0)\, exp\left(\frac{E_x}{T_{ct}}\right)$ *(CTM).*

Values for Δ *and* $\tilde{a}$ *taken from nuclear matter;* E_{bs} *from LD-mass fit, no fit to level density!*



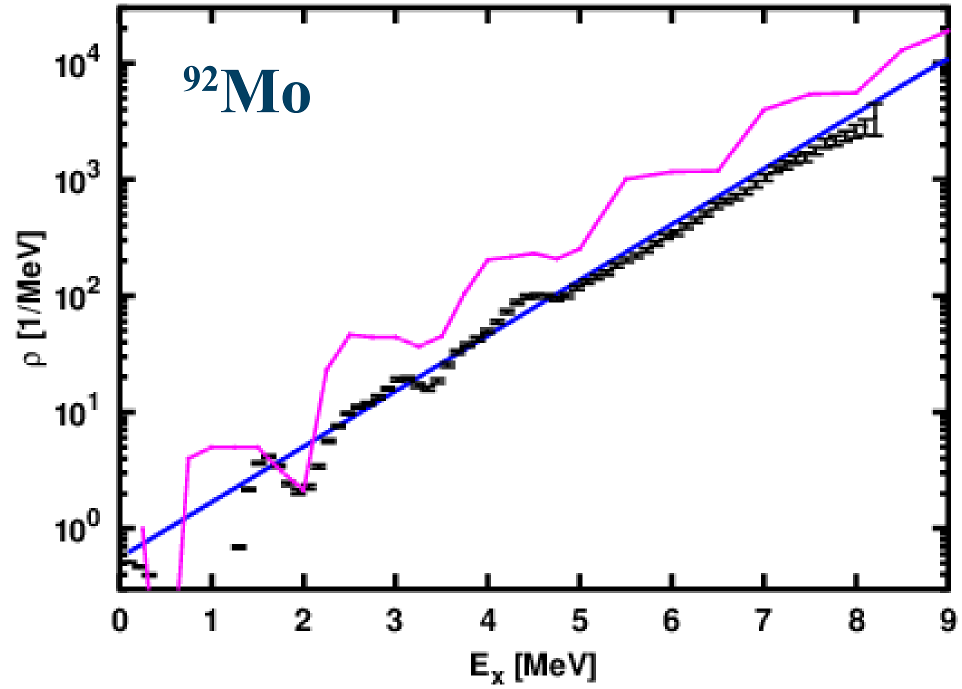
from cross sections for inelastic scattering of 3-9 MeV neutrons

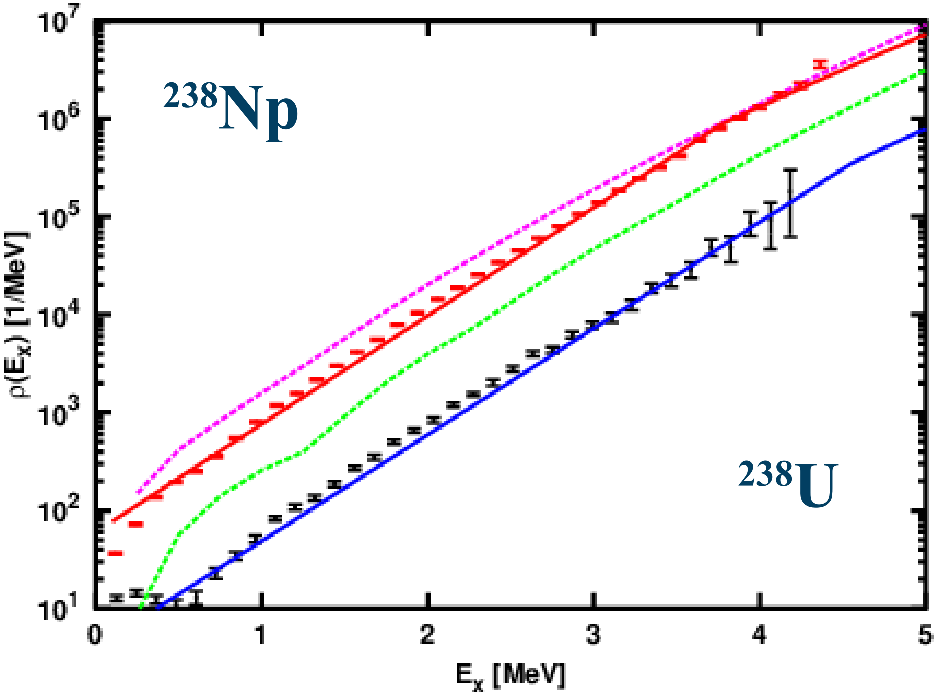
phase transition @ t_c

Tsukada et al., NP78 (1966) 369

Gilbert and Cameron, Can. Journ. of Phys., 43 (1965) 1446

Level densites ρ(E,I) *as predicted by TU-Dresden - HZDR collaboration agree well on absolute scale to measurements performed at Oslo cyclotron, using compound reaction cross sections* (http://www.mn.uio.no/fysikk)*:*





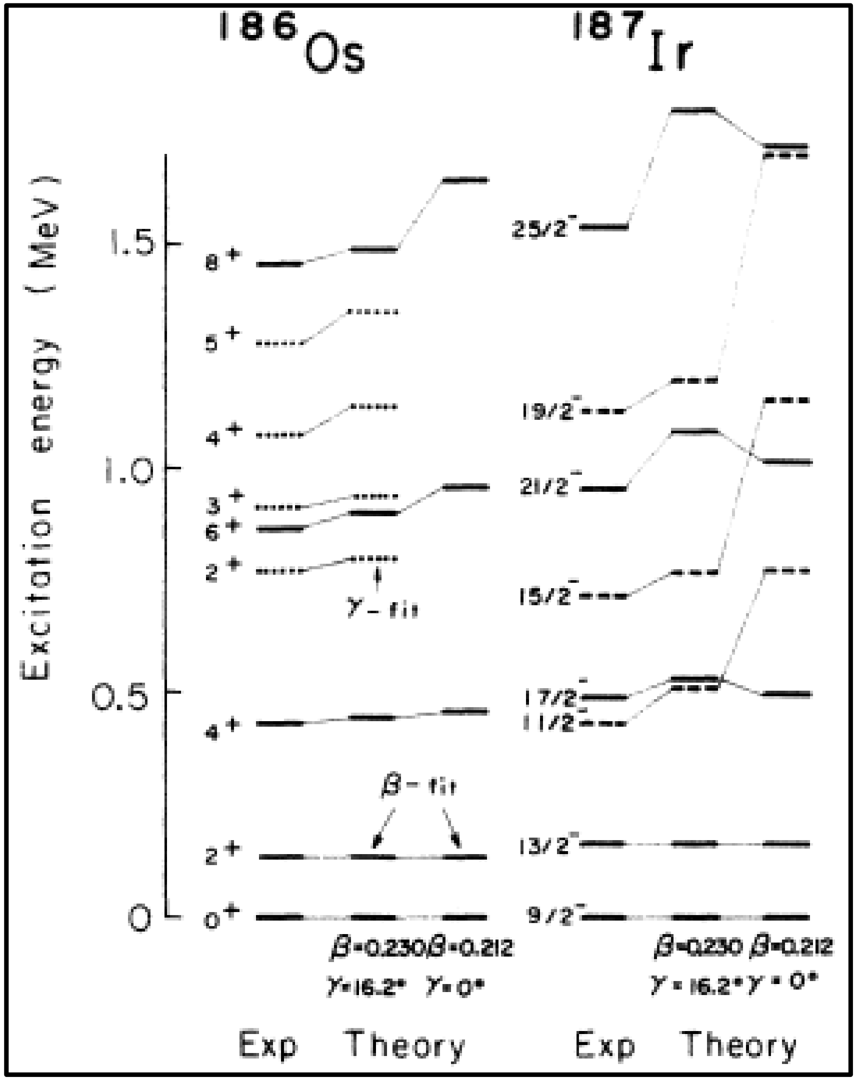
collective enhancement 3.6 ***~16***

phase transition at E ~ 8 MeV ***phase transition at E ~ 4 MeV***

Microscopic calculations: Goriely, Hilaire and Koning, Phys. Rev. C78 (2008) 064307

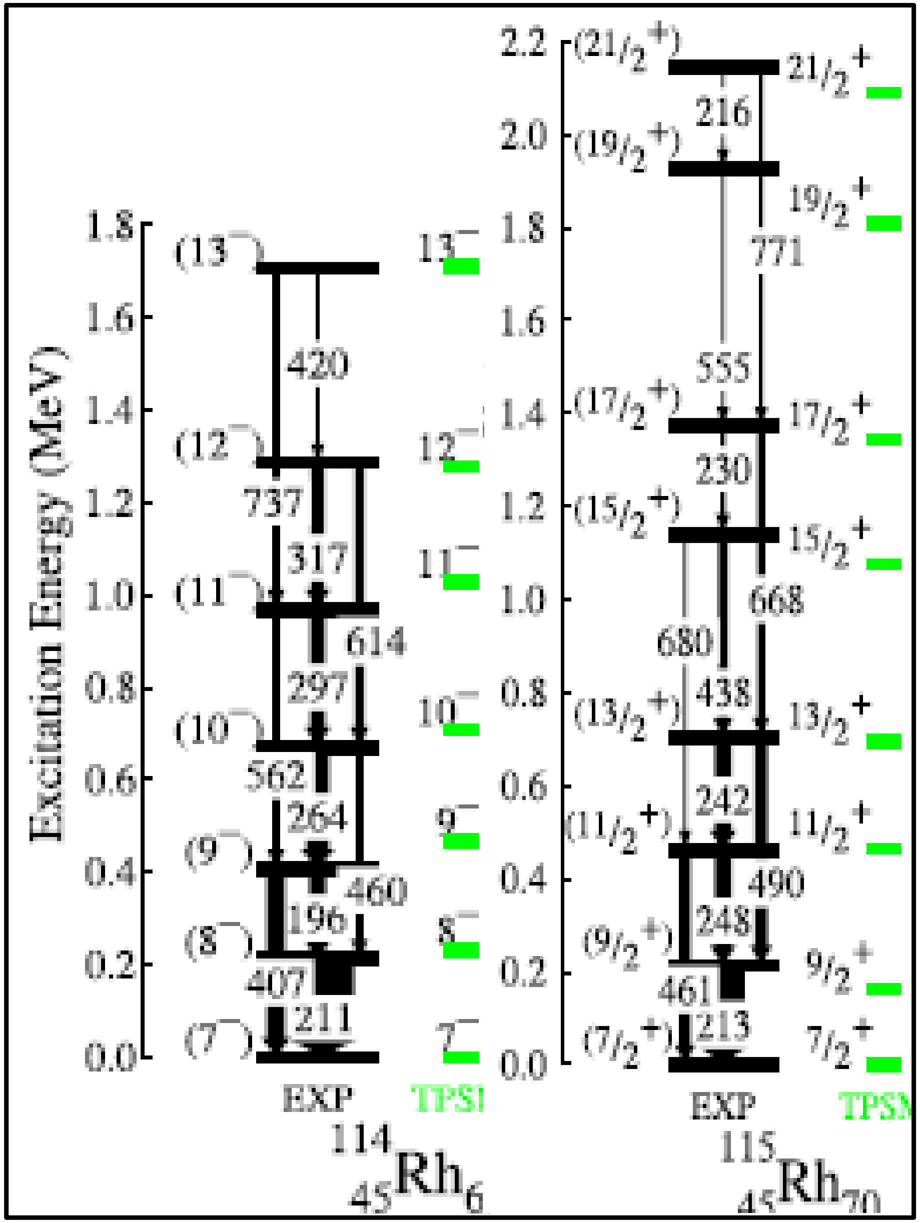
Level sequence in odd nuclei indicates triaxiality of core

chirality, wobbling, parallel bands, . .



J. Meyer ter Vehn, F. S. Stephens, and R. M. Diamond

Phys. Rev. Lett. 32, 383 (1974)



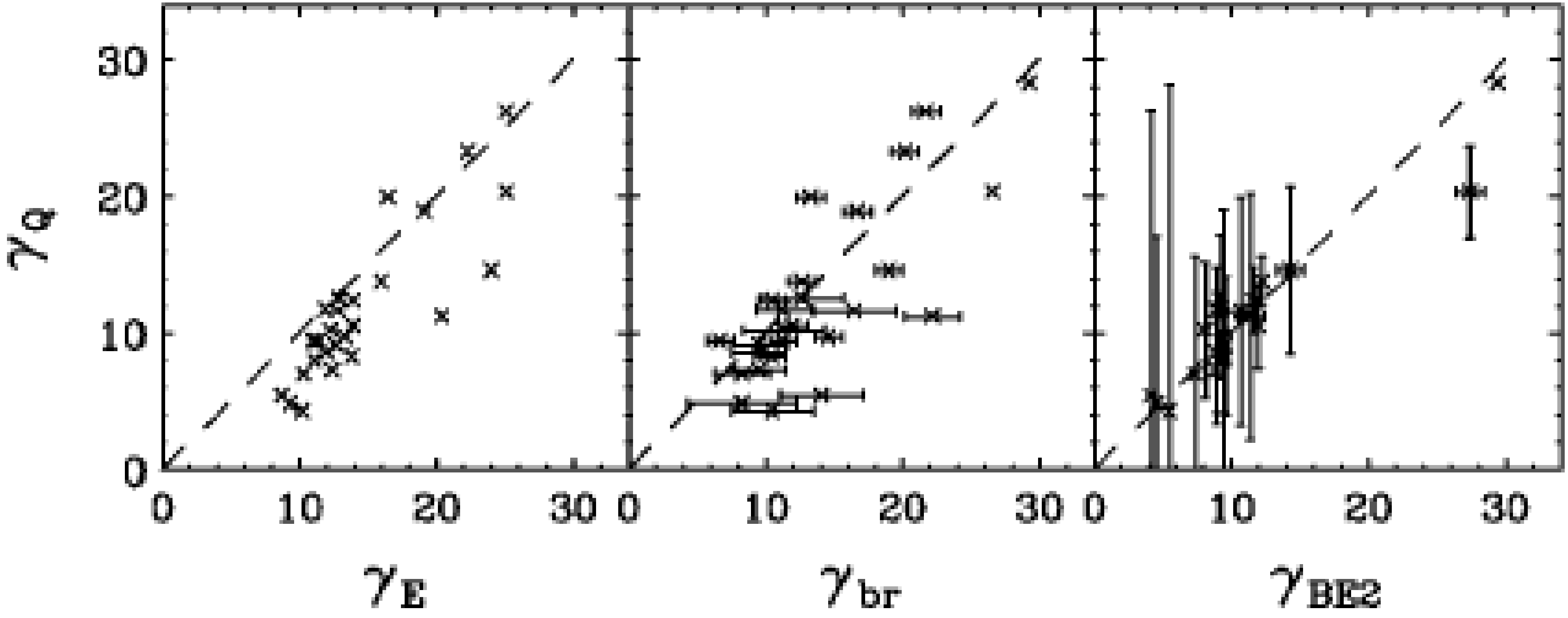
A. Navin et al. / Physics Letters B 767 (2017) 480

GANIL: fission fragment spectroscopy

Comparison of empirical γ-values for nuclei with $50 < Z < 82$

The three panels compare γ_Q obtained from IBA-1 fits to the data with γ_E, γ_{br}, and γ_{BE2} values obtained from the Davydov model relating γ to the empirical energy ratio, branching ratio, and B(E2) ratios, respectively.

The uncertainties in γ_Q are the same in each of these panels and shown in only one of them



The IBA-1 suggests that axial asymmetry arises from γ-softness

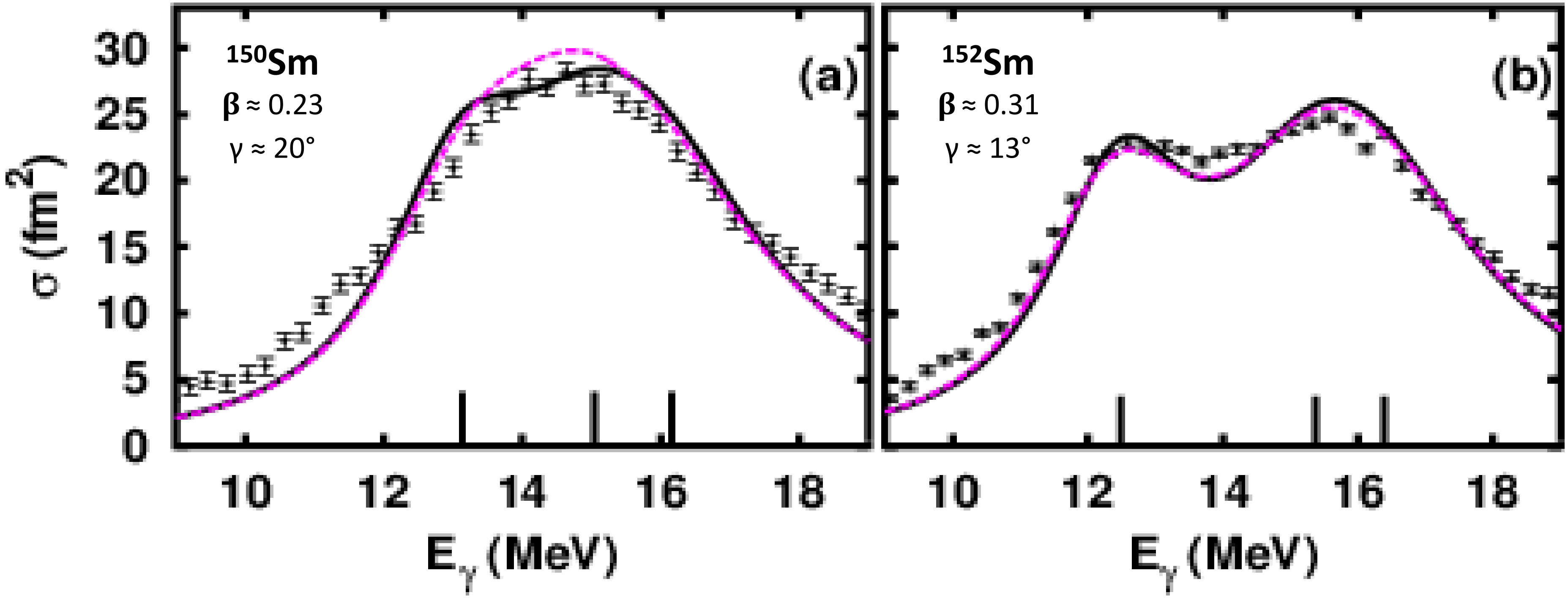
Zhang, Casten, and Zamfir, PRC 60 (99) 021304

IVGDR's in neighbor nuclei indicate axial symmetry breaking

energies from LDM and widths from surface dissipation model

deformation-*parameters from HFB/GCM*

incl. shape sampling



with deformation ***β,γ*** *from* ***QHFB/GCM*** *and* ***global*** *fixing of the width* $\Gamma = c_w \cdot E_r^{1.6}$

2-pole fit seems impossible for **^{150}Sm** *but* *may be possible for* **^{152}Sm**

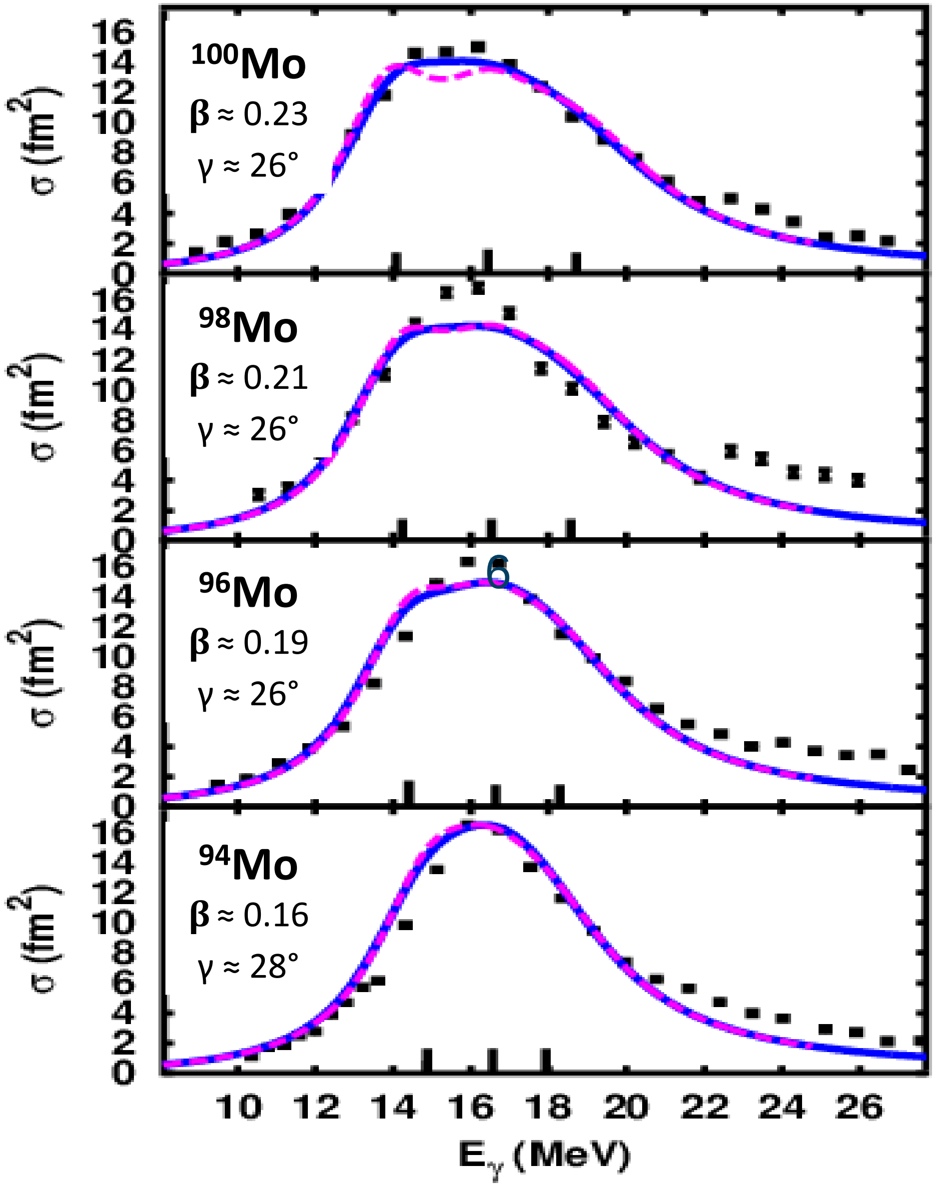
Carlos et al., Nucl. Phys. A 225, 171 (1974)

Bush and Alhassid, Nucl. Phys. A 531, 27 (1991)

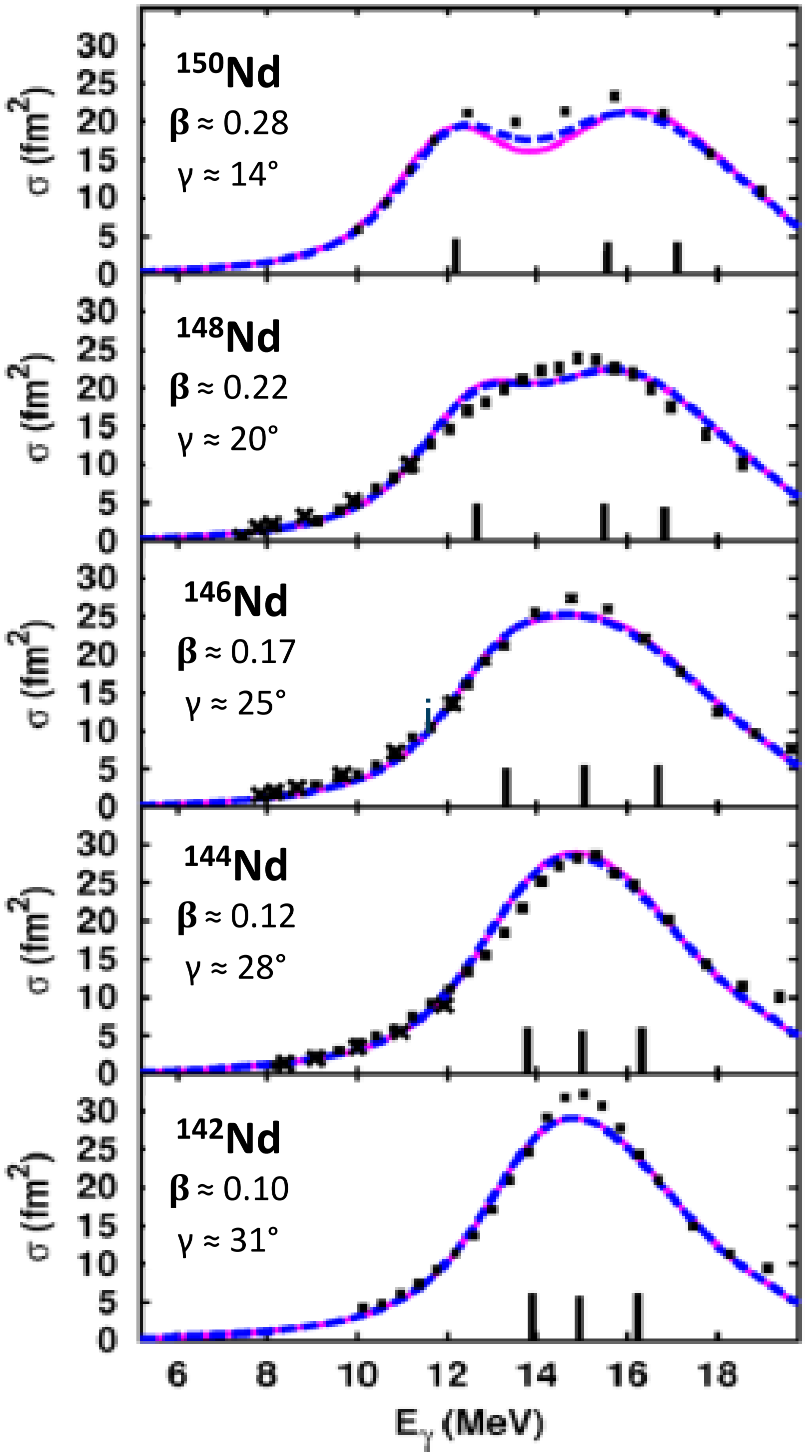
IVGDR's *and* ***deformation***

parameters from HFB/GCM

incl. shape sampling



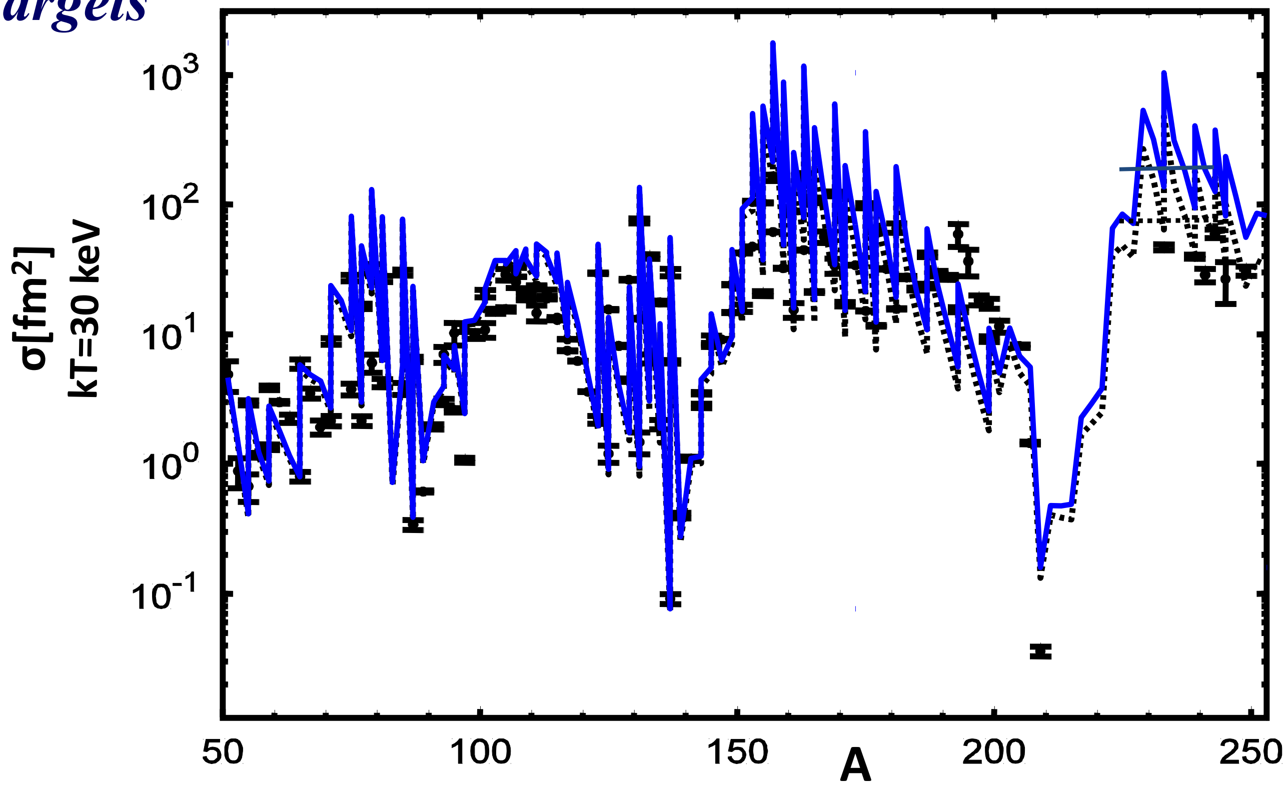
Beil *et al.*, Nucl. Phys. A 227, 427 (1974)



Carlos et al., Nucl. Phys. A172, 437 (1971)

Maxwellian average capture cross-sections, *at stellar temperatures of* ***$3\cdot10^8$ K,***

for J=0 targets



TLO + ***minor strength***

only **E1** *from* ***TLO***

Good agreement for >130 nuclei on absolute scale calculated from global predictions for

average ***level densities $\rho(E_r)$, obtained by admitting broken axiality*** *and*

photon widths *for* ***radiative neutron capture from an extrapolation of TLO-fits to IVGDR's***

=> simultaneous ***test of broken axiality for photon strength*** *and the* ***level density prediction***

Data from: Dillmann et al., PRC 81 (10) 015801; Pritychenko et al., At.D. and Nucl.D. Tabl. 96 (10) 645

Sensitivity of n-capture is dominated

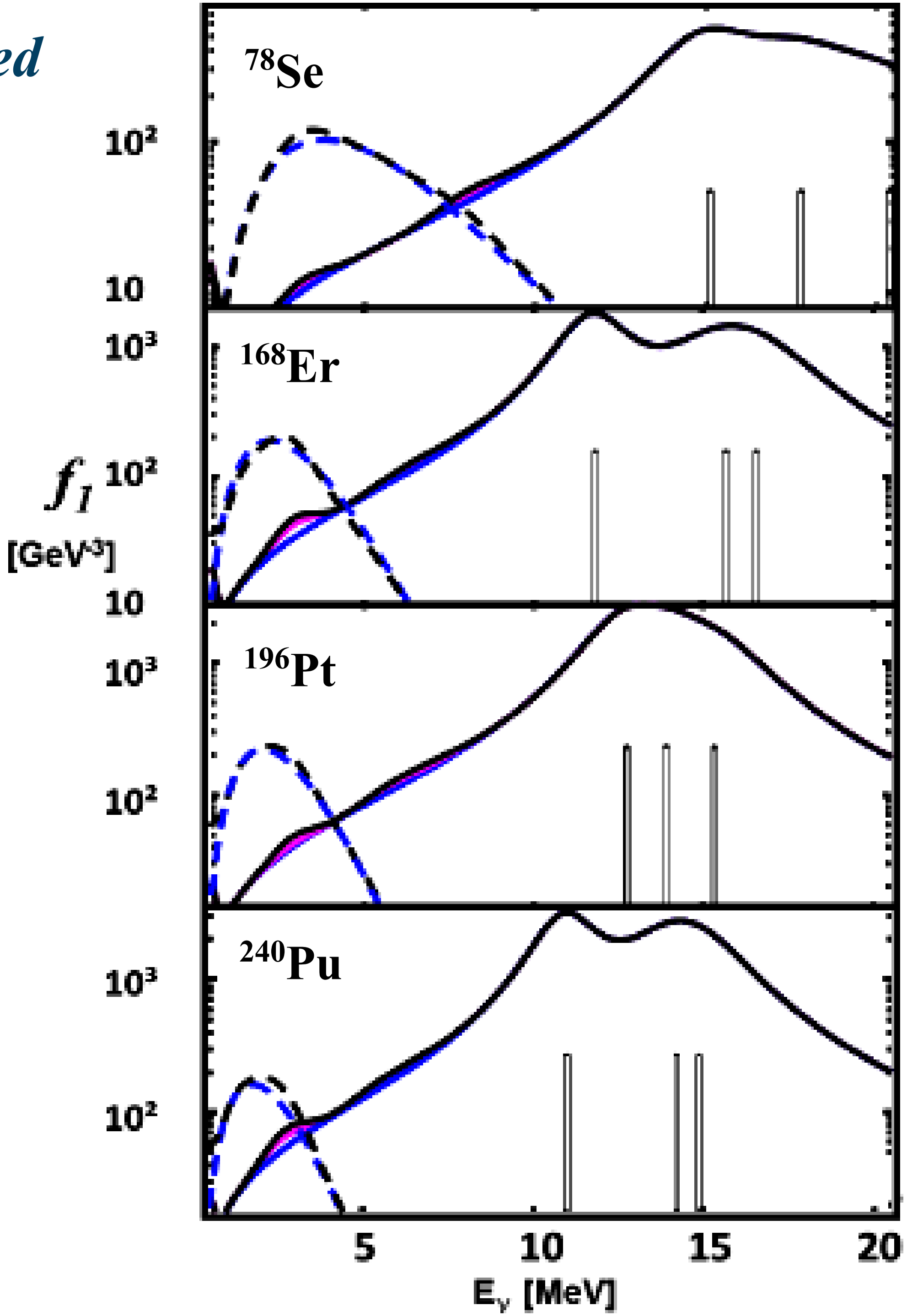
by emission of 1st photon:

convolution of statistical level density

and photon strength

results in peak at very low energy,

a fact often neglected.



Broken axial symmetry indicated by experimental data on:

(a) level densities, *esp. for low spins near* S_n

(b) level energies and transitions, *esp in odd nuclei*

(c) splitting of giant dipole resonances, *if global width is assumed to scale triaxially*

(d) n-capture cross sections, *if* **TLO** *is extrapolated to low energies*

Theoretical models assume axiality very often, but:

(a) rigid 3-ax rotor *does not*

(b) cranking of 3-ax body *is possible*

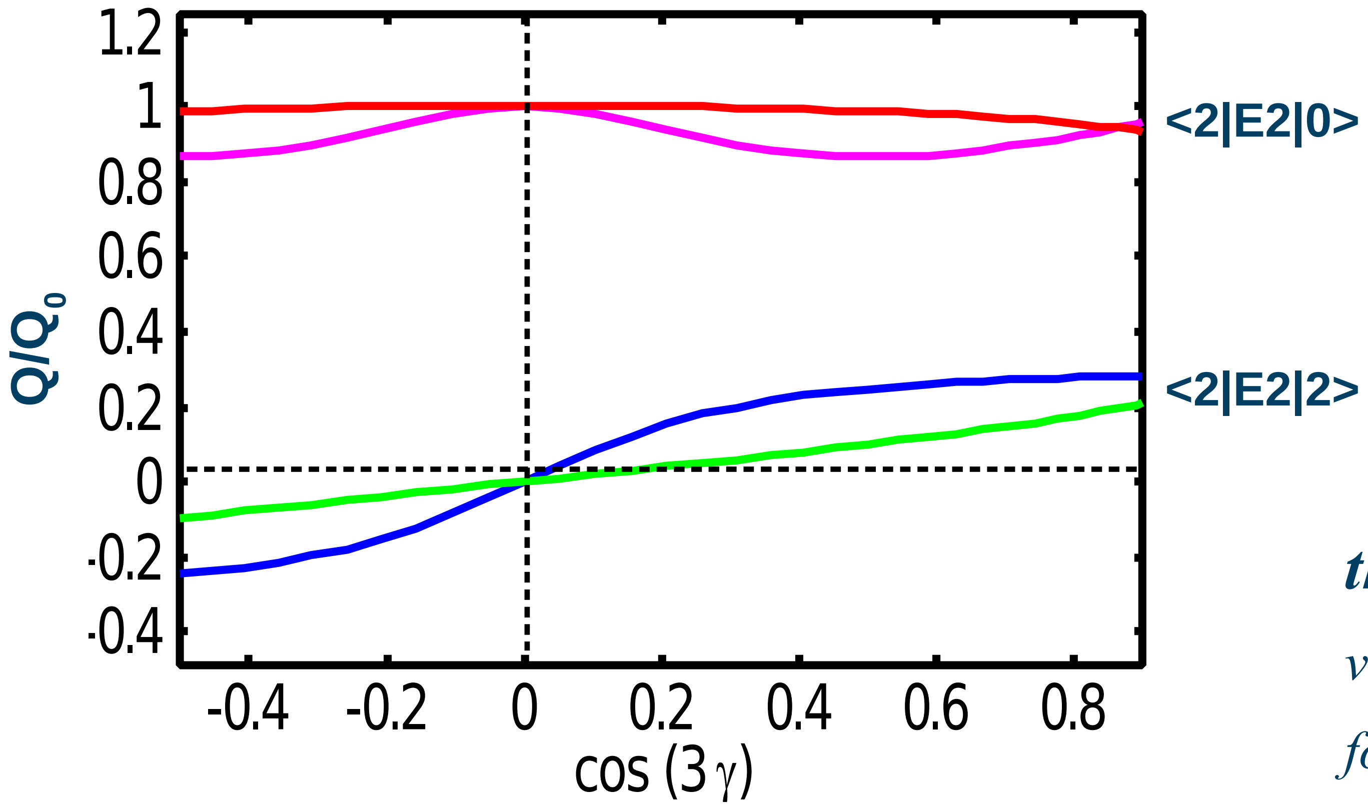
(c) HF-variation after projection enhances broken axiality

(d) QHFB+GCM *(GognyD1S)* **creates triaxiality; combined to LDM for TLO**

(e) RPA+OM predicts GDR in ^{208}Pb with 3 MeV width; scaled for TLO

(f) RPA+QHFB produce GDR with 1 or 2 poles plus fragments

3-axial rotor, rigid & with cranking



***cos(3γ) ⇒** indicator for axiality*

***Q** is especially sensitive to it*

***the two models** make*
very similar predictions
for the two observables
Q(2$^+$)** and **B(E2, 0$^+$→2$^+$);
this does not help to find
best approach to treat
axial symmetry breaking

Davydov and Fillipov, Nucl. Phys. A 8, 237 (1958)

Ring, Hayashi, Hara, Emling and Grosse, Ph.Lett.B 110, 423 (1982)

Probabely, the rareness in predictions of ***triaxiality***

(e.g. in ***QHFB*** *calculations)*

may result from ***not*** *performing the*

variation ***<u>after</u>*** *the* ***projection***

on angular momentum.

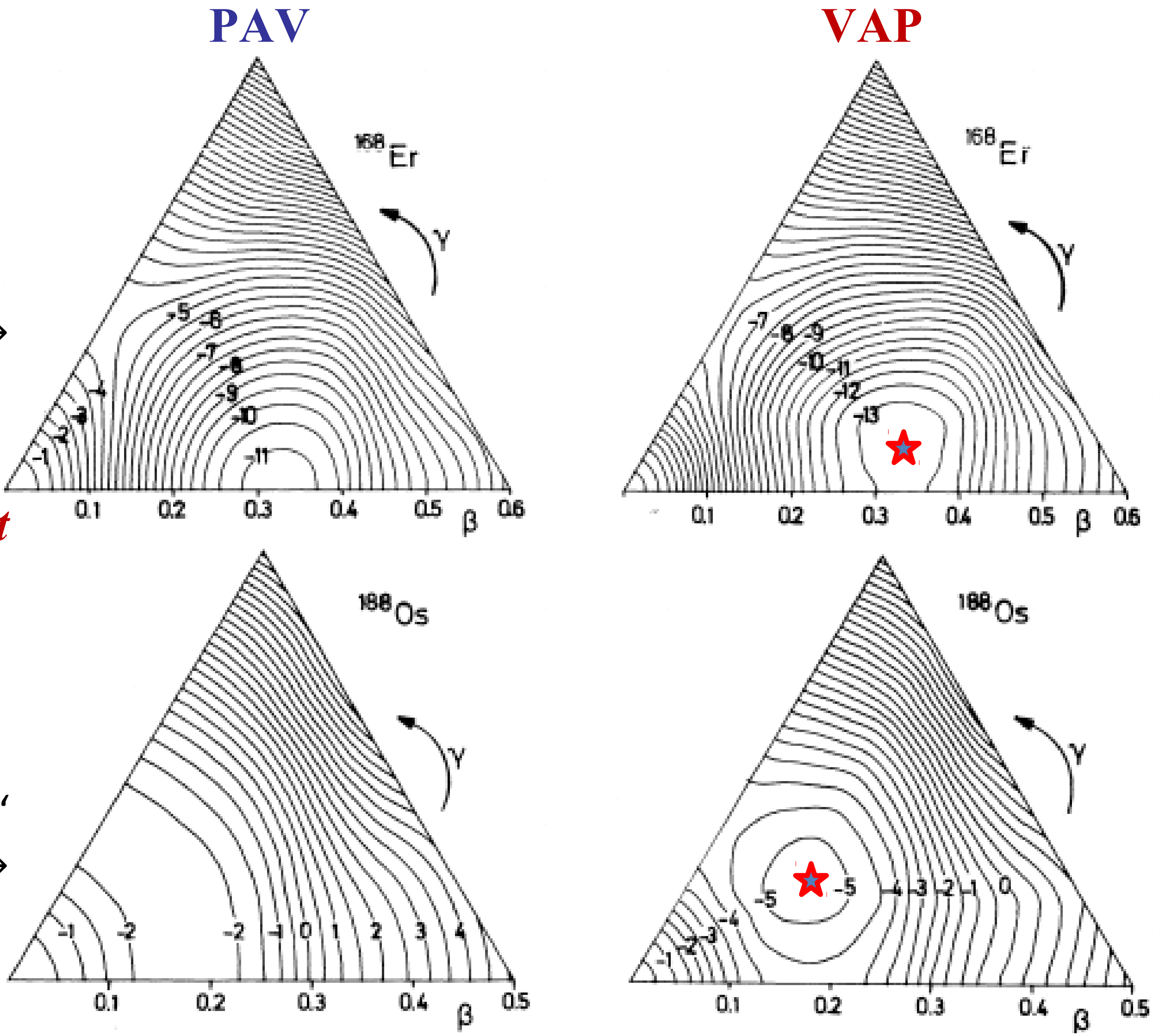
Often a projection is carried out

only after the HFB-variation

(neglecting quantum mechanics ?).

‘axially deformed‘ nucleus →

‘γ-soft‘ nucleus →



Hayashi, Hara, Ring , Phys. Rev. Lett. 53 (1984) 337

Shell model + RPA

schematic calculation,

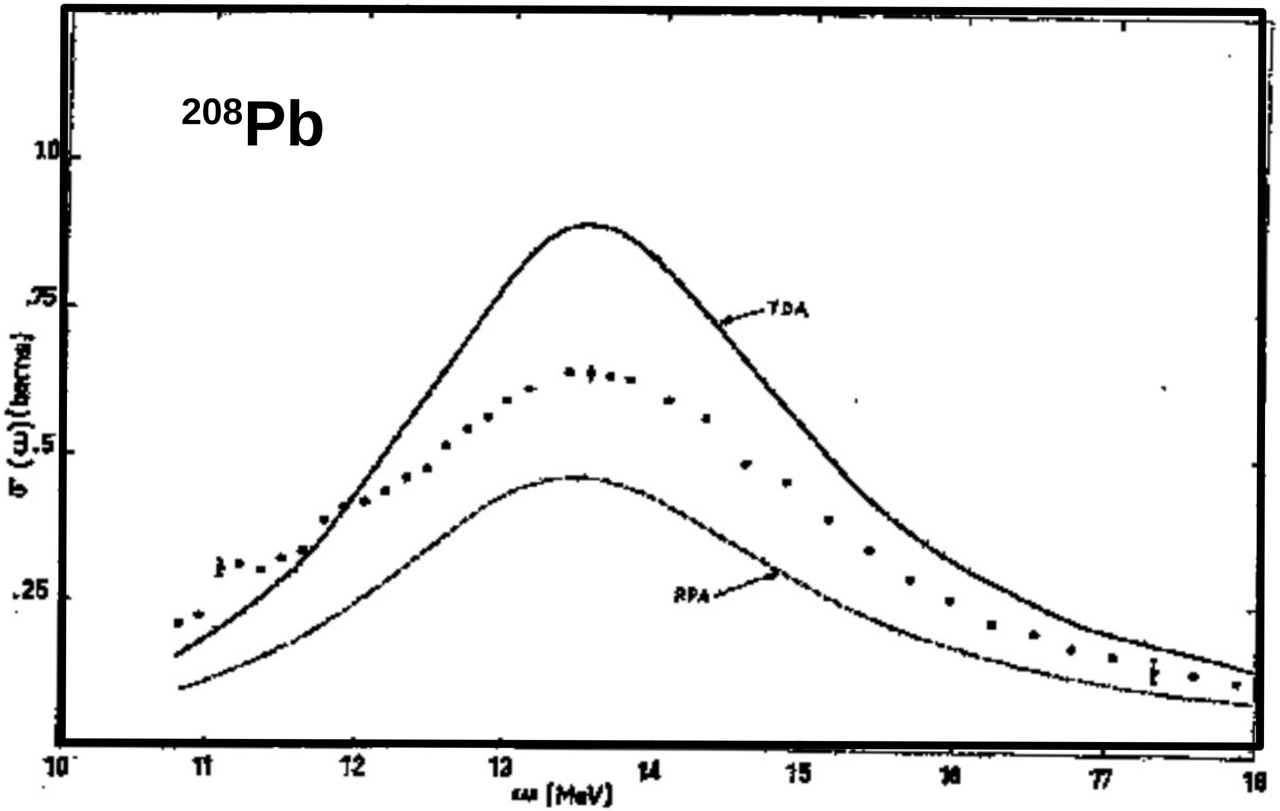
for ^{208}Pb, E_r adjusted,

strength integral depends

on gs-corr. (RPA),

width is used by TLO

after scaling by $(E/E_{208})^{1.6}$

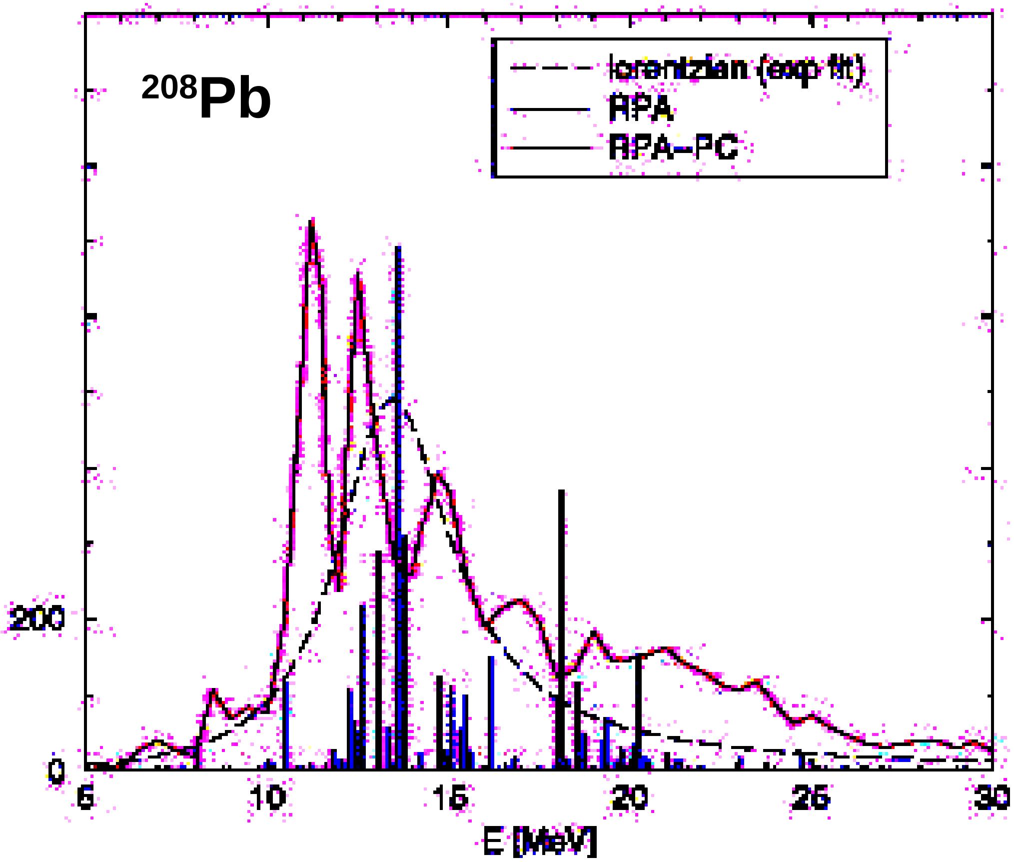


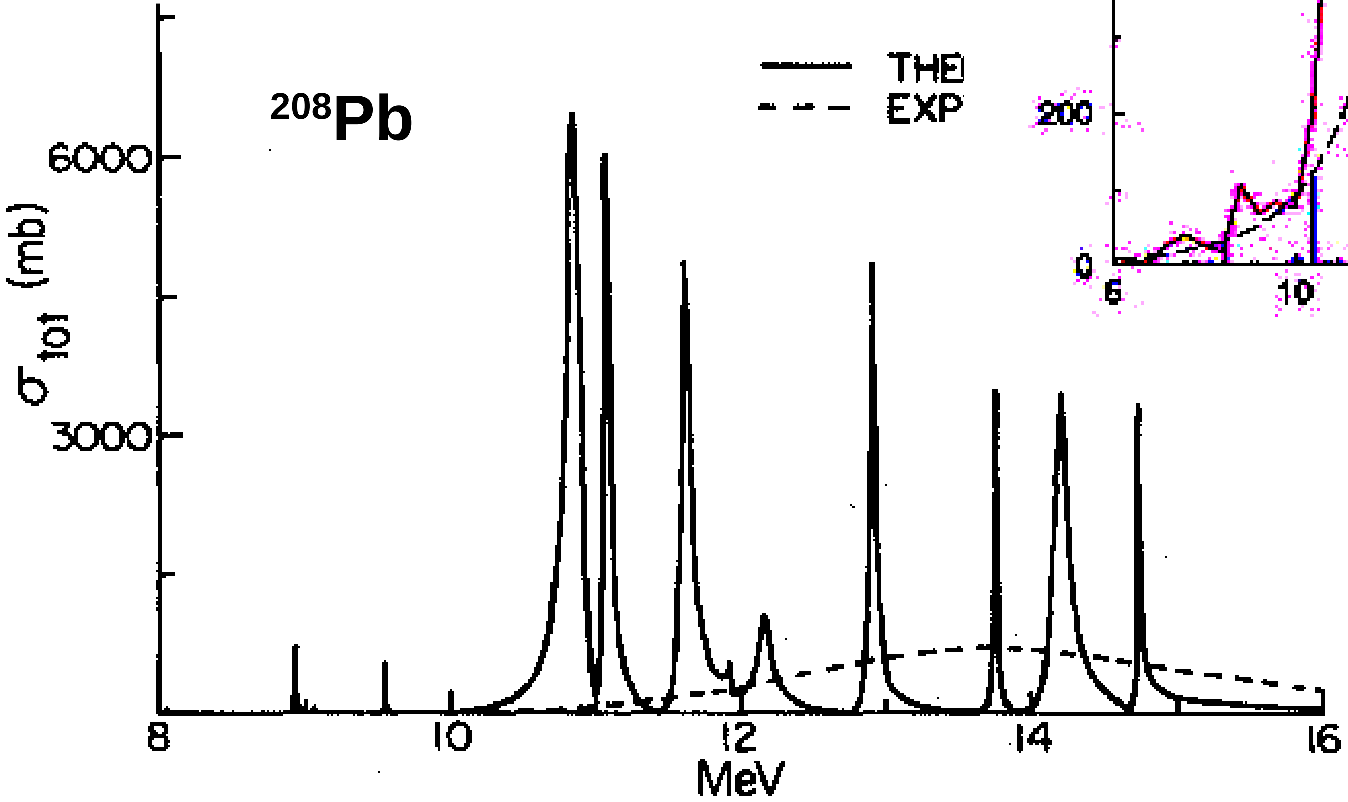
Bush and Alhassid, Nucl. Phys. A 531, 27 (1991)

Dover et al., Ann of Ph (NY) 70 (1972) 458

QRPA-HFB *(Hartree-Fock-Bogolyubov)*

***calcul's** show distinct fragmentation with spreading clearly exceeding escape widths; often reduced by phonon coupling or smeared by additional broadening*



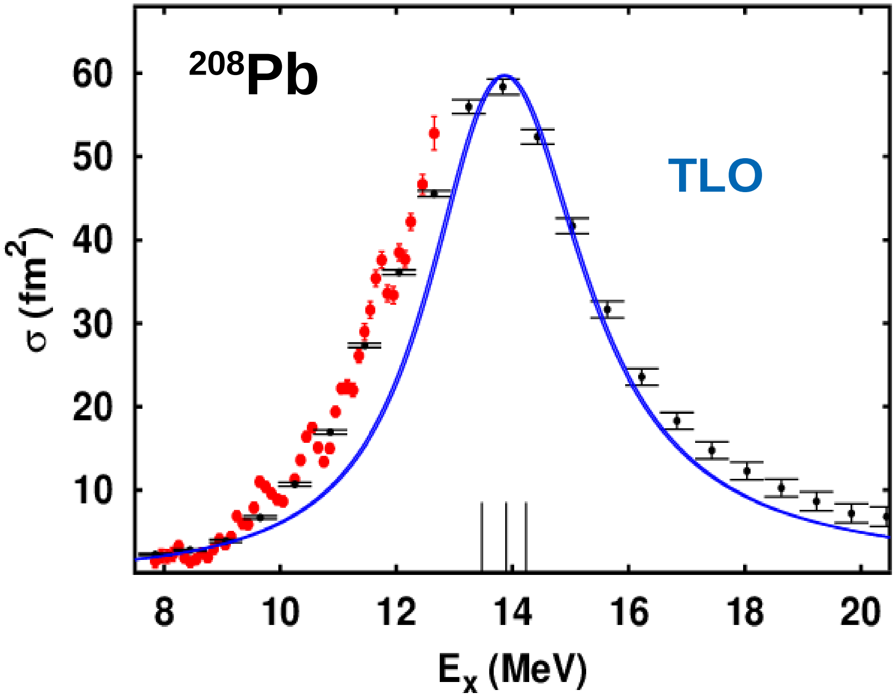


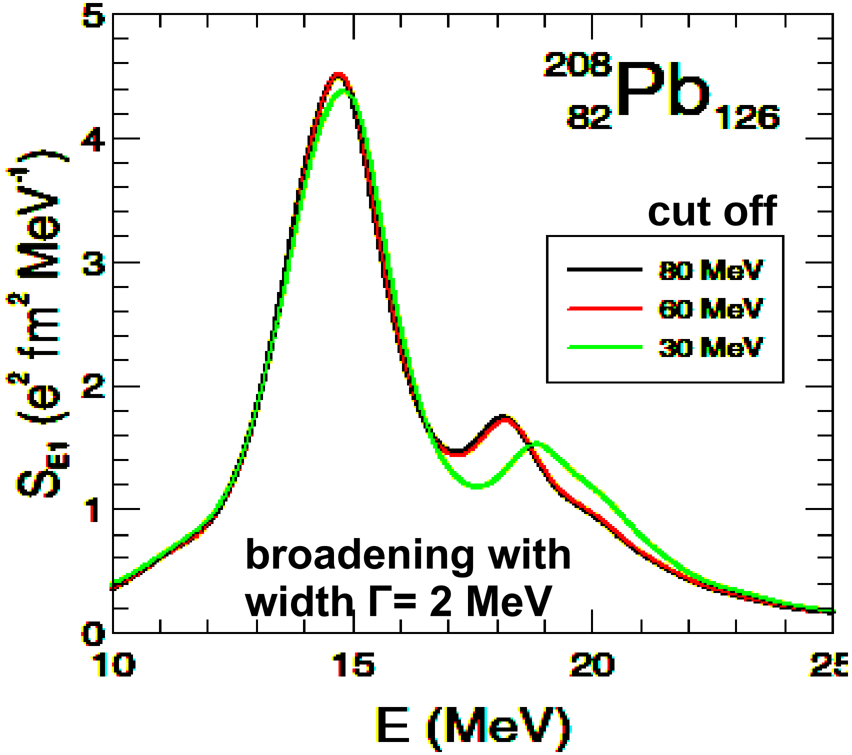
Bertsch et al., Rev. Mod. Phys. 55, 287 (1983)
Shlomo and Bertsch, NPA 243,507(1975)

Sarchi, Bortignon, and Colo, PLB **601**, 27 (2004)

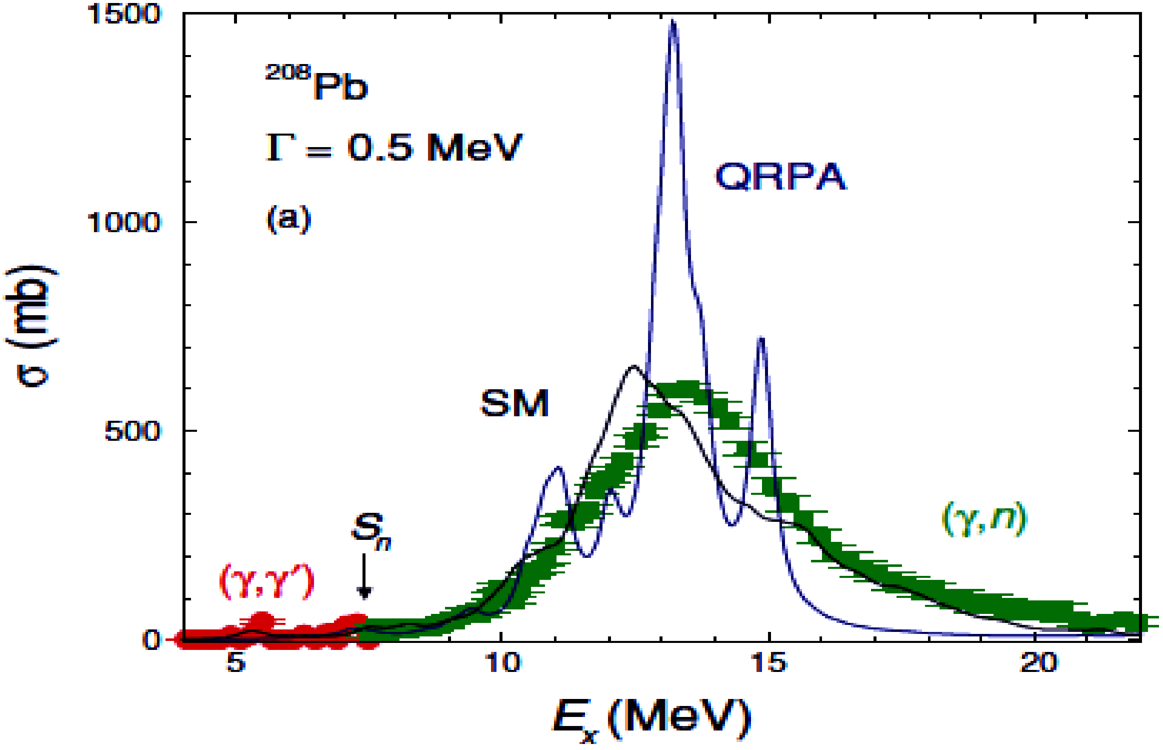
HFB-QRPA-calc's show distinct fragmentation (p-h, Landau damping), many apply additional broadening (2p-2h); experimental data show much less:

(Van de Vyver et al., Z.Phys. A 284, 91 (1978))





Martini et al. , Phys. Rev. C 94, 014304 (2016)

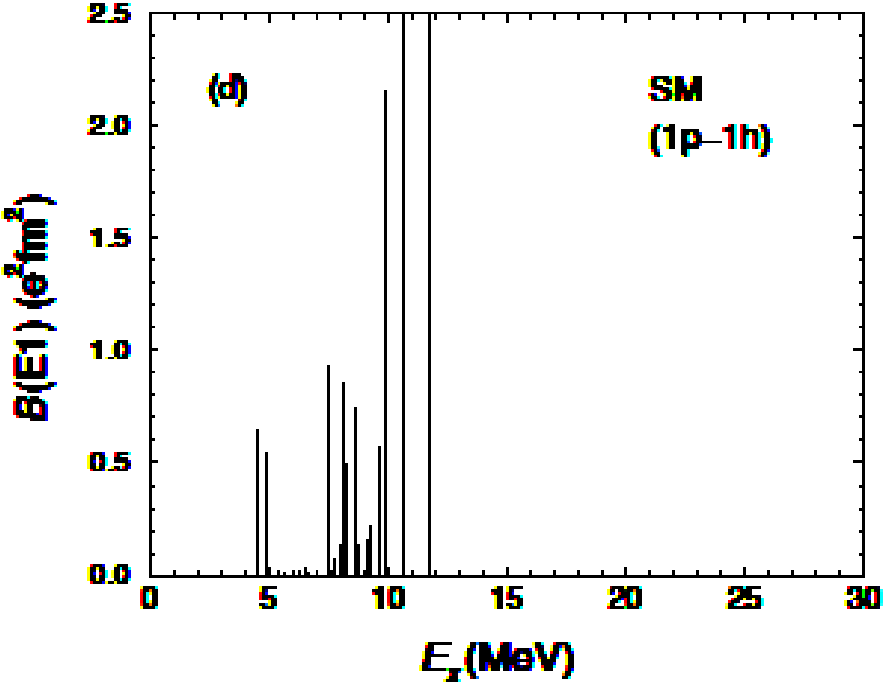


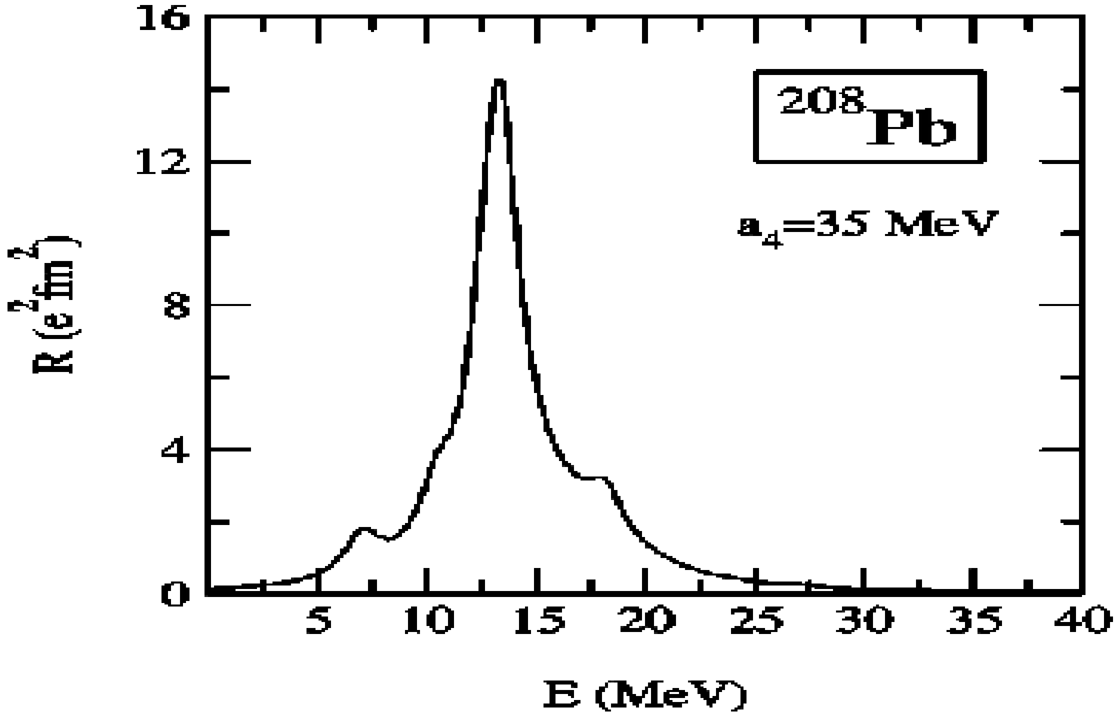
Schwengner et al., PRC **81**, 054315 (2010)

HFB-QRPA-calcul‘s show distinct fragmentation, indicating strong spreading

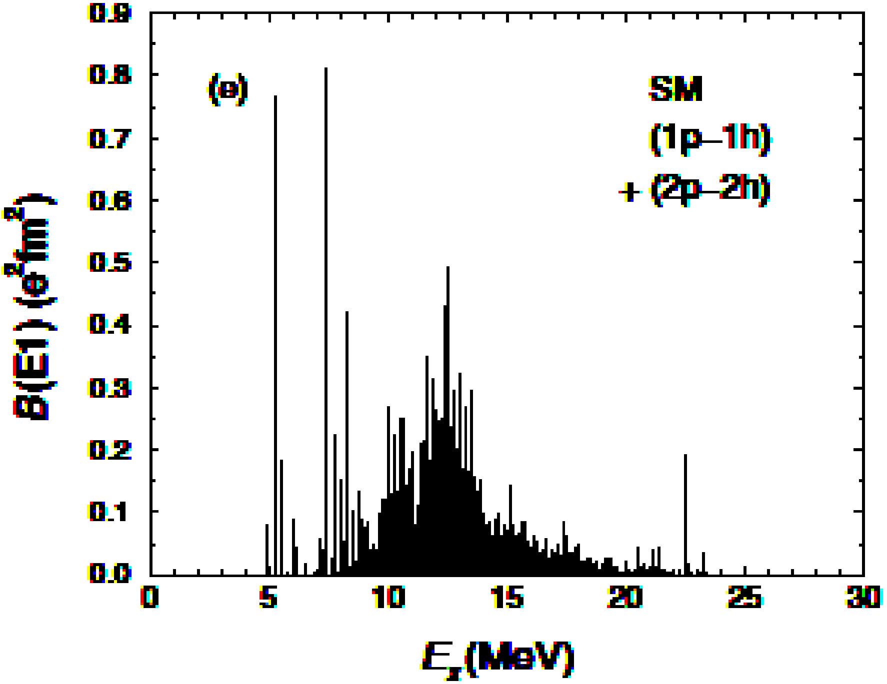
covariant (relativistic with meson coupling) or shell model calculations

show less of it





Niksic, Vretenar, and Ring, Phys. Rev. C **66**, 064302 (2002).



Brown, Phys. Rev. Lett. **85**, 5300 (2000).

Schwengner et al., PRC **81**, 054315 (2010)

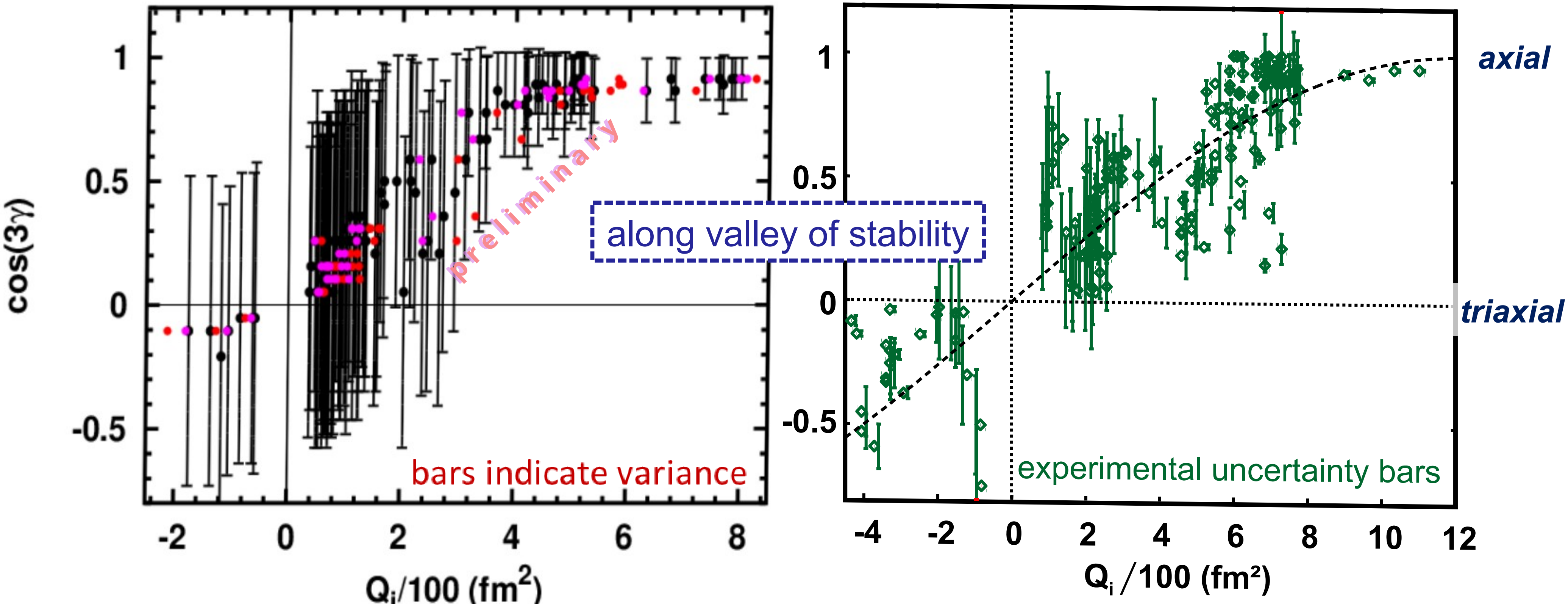
***Prediction** for even heavy nuclei*
– avoiding axial symmetry postulate – by :

*Constrained **HFB + GCM***

***cos(3γ)** ⇒ indicator for axiality*

⇒

*multiple Coulex by **Q - invariants** or*
*B(E2)-ratios by **shape invariants***



Bertsch et al., PRL 99 (2007) 032502; Delaroche et al., Phys. Rev. C 81 (2010) 014303; Grosse et al., Phys. Scr. 2018

Conclusions:

Many experimental facts indicate broken axial symmetry for heavy nuclei :

1. Level densities *predicted on* ***absolute scale***

2. Level sequences and transition rates

3. *Coulomb reorientation and multiple excitation*

4. Split *of the* ***giant dipole resonance***

5. Neutron capture *cross sections (via 1 & 3)*

Theoretical calculations *may impose* ***triaxiality as property of a rotor****, but many assume axiality and predict level densities, photon strength or GDR shapes.*

Axial symmetry breaking is found by

1. Angular momentum projection ***before*** *the Hartree-Fock-Bogolyubov-variation*

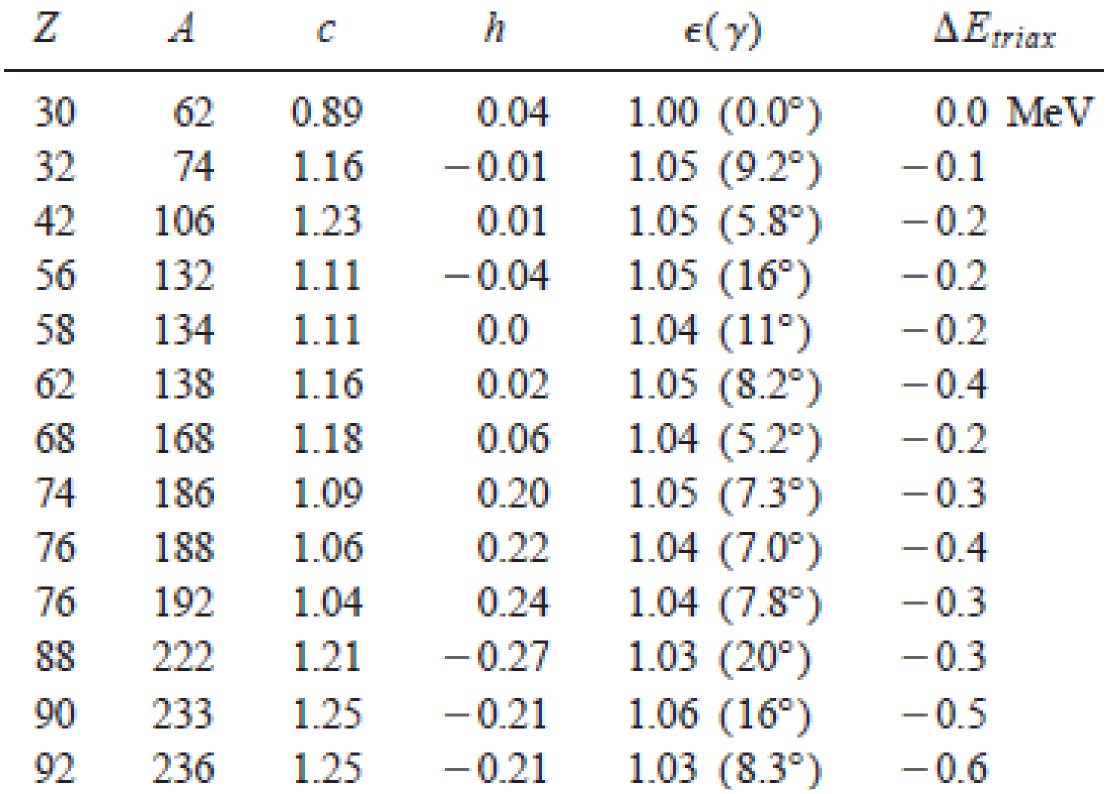
2. HFB *calculations with mapping onto a 5D collective quadrupole Hamiltonian (****GCM****)*

3. *Jahn-Teller effect:****symmetric configurations do not always have the lowest energy***

- ***All heavy nuclei are triaxial, some are more deformed and less triaxial than others***

Thomas-Fermi (ETFSI) method used to calculate nuclear masses

(randomly selected in valley of stability).



Z	A	c	h	$\epsilon(\gamma)$	ΔE_{triax}
30	62	0.89	0.04	1.00 (0.0°)	0.0 MeV
32	74	1.16	−0.01	1.05 (9.2°)	−0.1
42	106	1.23	0.01	1.05 (5.8°)	−0.2
56	132	1.11	−0.04	1.05 (16°)	−0.2
58	134	1.11	0.0	1.04 (11°)	−0.2
62	138	1.16	0.02	1.05 (8.2°)	−0.4
68	168	1.18	0.06	1.04 (5.2°)	−0.2
74	186	1.09	0.20	1.05 (7.3°)	−0.3
76	188	1.06	0.22	1.04 (7.0°)	−0.4
76	192	1.04	0.24	1.04 (7.8°)	−0.3
88	222	1.21	−0.27	1.03 (20°)	−0.3
90	233	1.25	−0.21	1.06 (16°)	−0.5
92	236	1.25	−0.21	1.03 (8.3°)	−0.6

When triaxiality is admitted in the calculations, ground state energy is lowered by less than 0.5 MeV.

But axial symmetry is broken anyhow. And it is also broken if triaxiality is only dynamic.

Dutta, Pearson and Tondeur, PRC 61 054303 (2000)

Rotational enhancement of nuclear level density vs. symmetry class

The intrinsic ***quasi-particle state density*** *in a finite nucleus* $\boldsymbol{\omega_{qp}(E_x)}$ *is* ***not*** *yet the observable density of nuclear levels with* ***well defined spin*** $\rho(E_x, J = I_{rot}+j\,, \pi)$.

To fix J the underlying collective symmetry has to be introduced:

small J limit

1. *spherical* ⇒ *only q-p states* $$\rho(E_x, J, \pi) \longrightarrow \frac{2J+1}{2\cdot\sqrt{8\pi}\,\sigma^3}\,\omega_{qp}(E_x)$$
2. *axial* ⇒ *q-p states & rotation ⊥ axis* $$\rho \longrightarrow \frac{2J+1}{2\cdot\sqrt{8\pi}\,\sigma}\,\omega_{qp}(E_x)$$
3. ***non-axial*** *(triax)* ⇒ ***q-p states & rotation about any axis*** $$\rho \longrightarrow \frac{2J+1}{2\cdot 4}\,\omega_{qp}(E_x)$$
4. *no reflection symmetry* ⇒ *q-p states & octupole deform.* $$\rho \longrightarrow \frac{2J+1}{2}\,\omega_{qp}(E_x)$$

1 parity

Thomas-Fermi Model ⇒ $$\sigma^2 = \frac{\tilde{a}\cdot t}{11}A^{2/3} \approx \frac{A^{5/3}}{143}\cdot t$$

H.Bethe, Rev. Mod. Phys. 9 (1937) 69; S.Bjørnholm, A.Bohr, B.Mottelson; Rochester conf. 1974, IAEA-SM-174/205

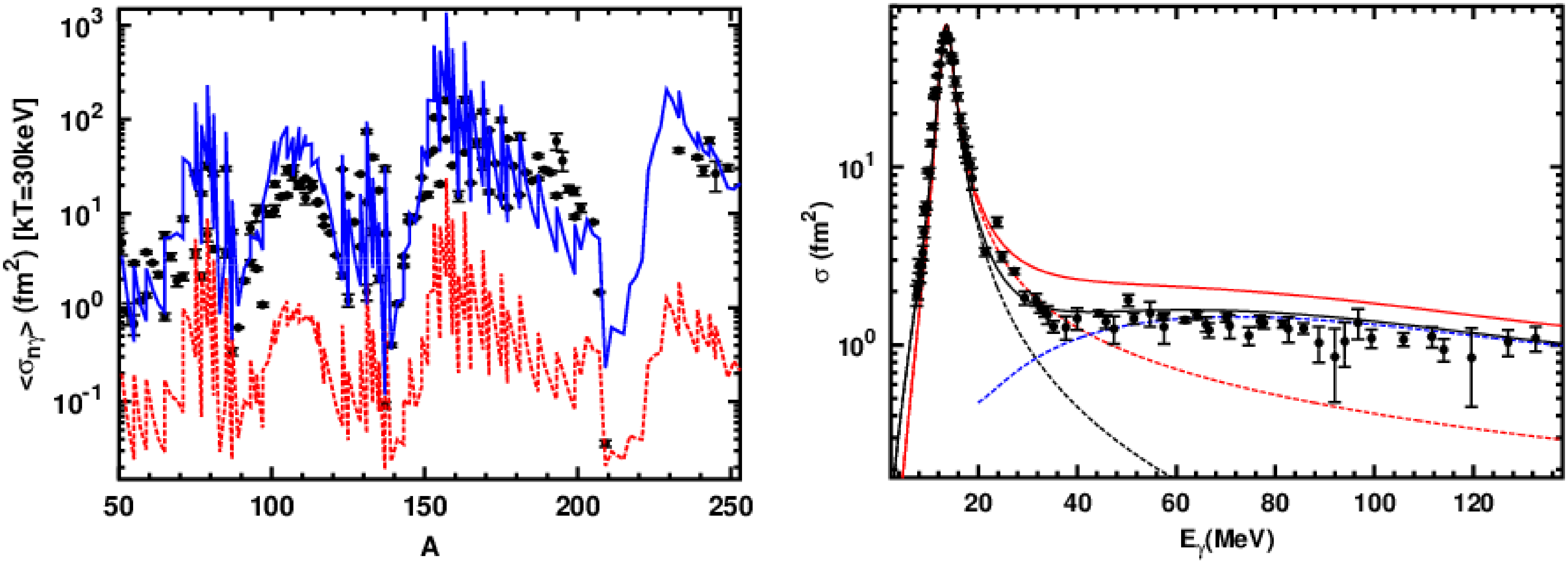
GDR's, ***their widths Γ_i and low energy tail***

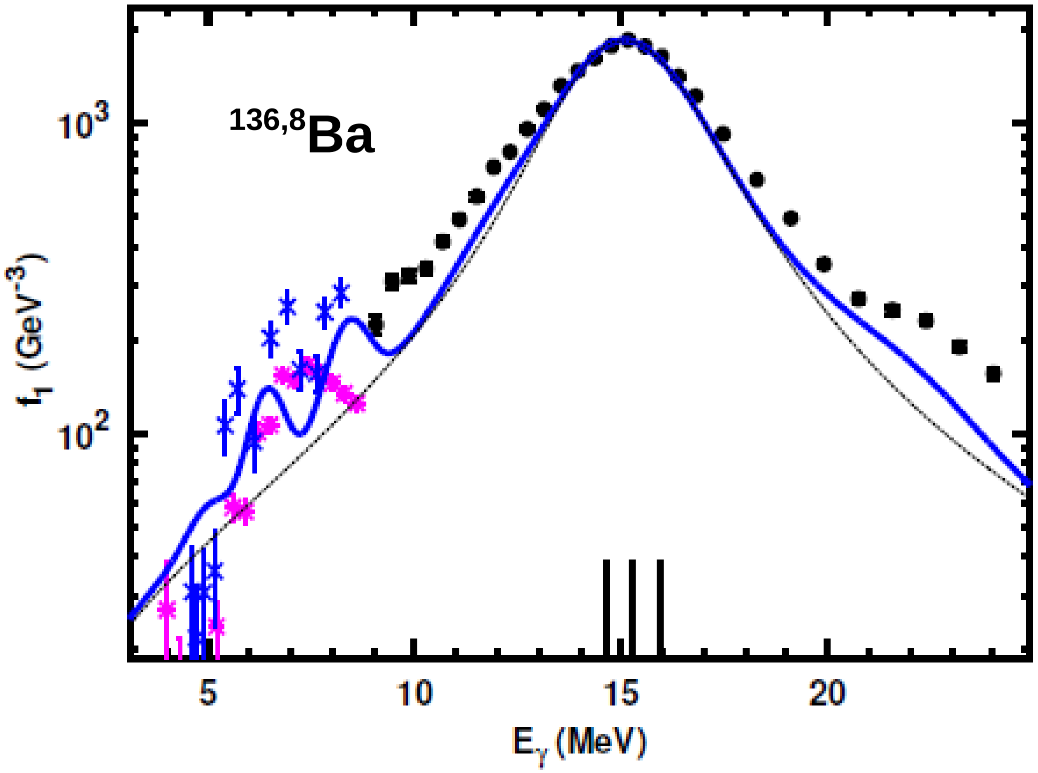
As proposed 1983 by Kadmenskii, Markushev and Furmann for n-capture resonances Γ_i *vary with* E_i *.*

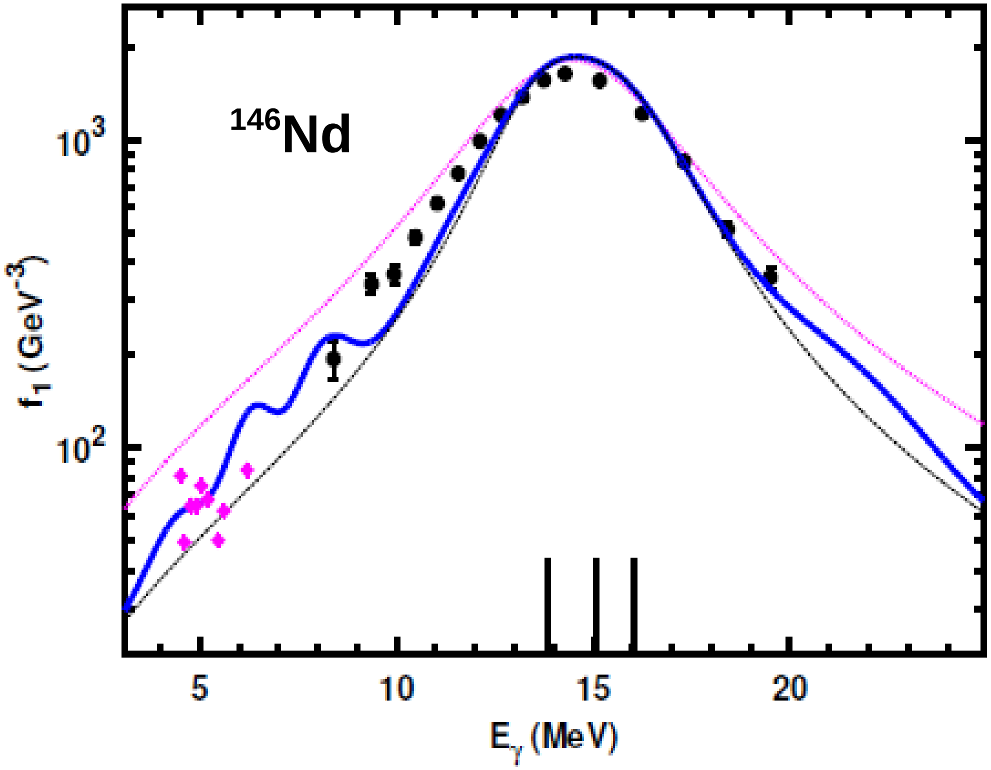
A false application often labelled ***KMF*** *proposes to apply this to GDR's with a dependence of* Γ_i *on* E_γ*;*

this results in a ***low*** *prediction for σ(n,γ), if the TLO fit is used [left panel] - and a* ***surplus*** *above the GDR,*

where one sees effect of ***quasi-deuteron break up,*** *calculated 1991 by Chadwick et al [right panel].*



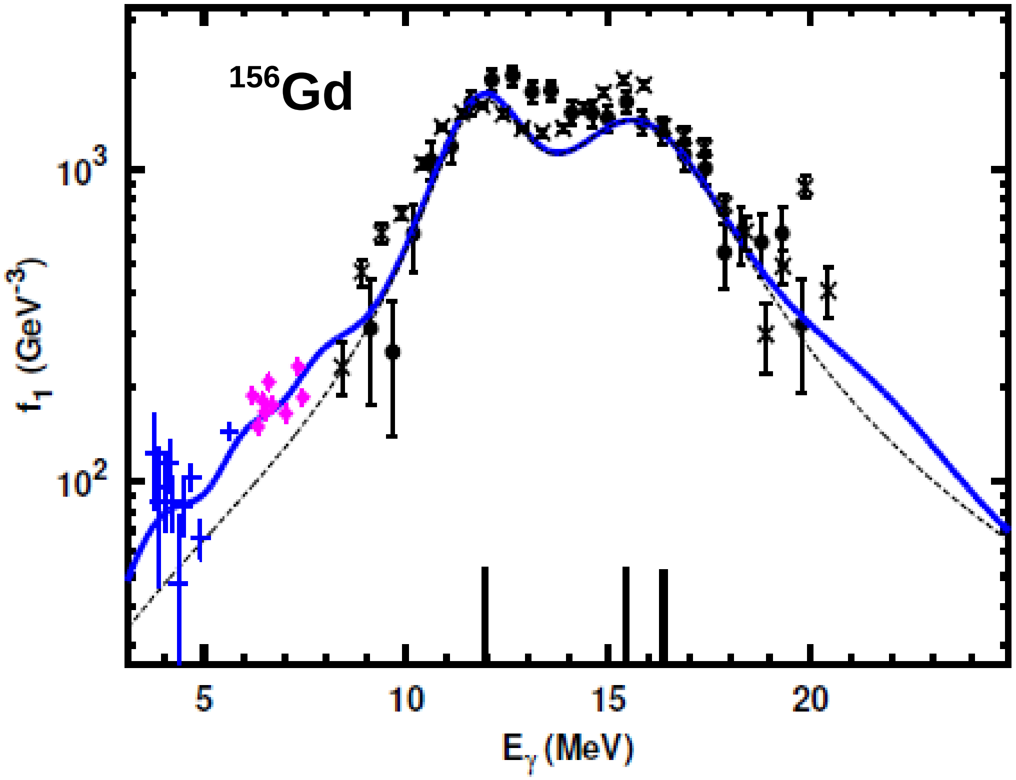


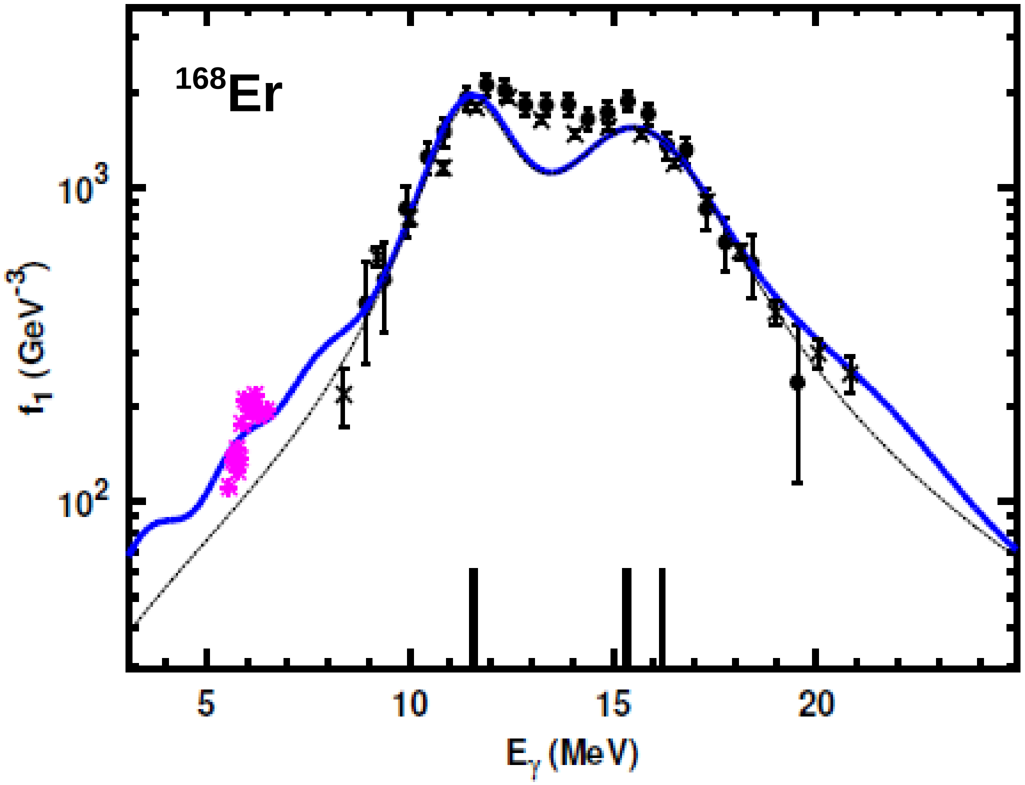


The dotted red curve shows the fit made by Plujko et al. ; it overpredicts the strength at low energy by a factor of ≈ 3.

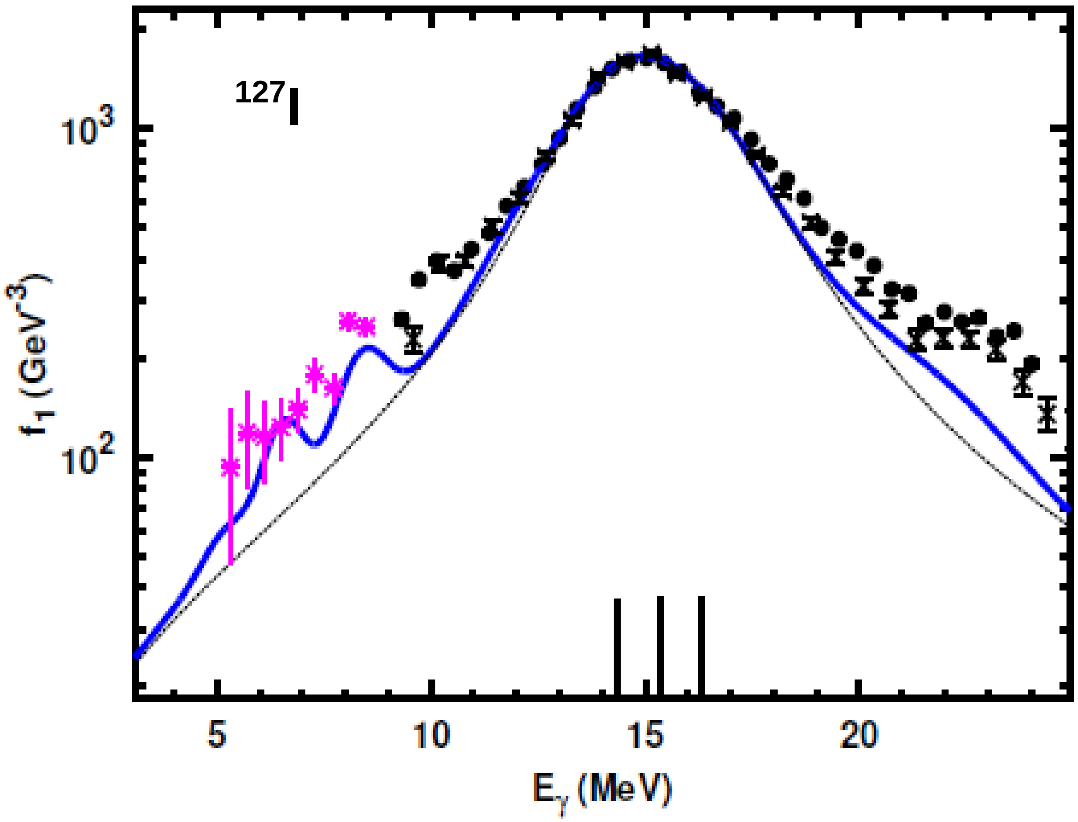
Data and TLO for these nuclei indicate: The top peak can be the smaller one, although it represents 2 components with equal integral but increased width.

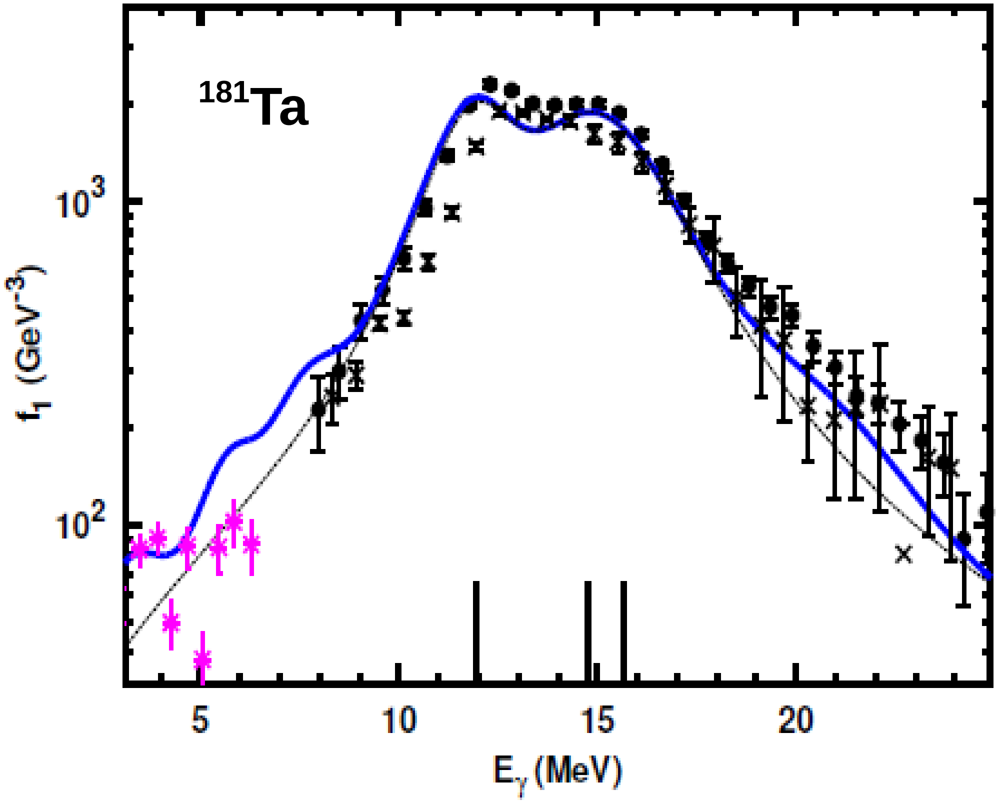
This has led to some confusion in older RIPL's.





Data for odd nuclei indicate: TLO can be applied as well.



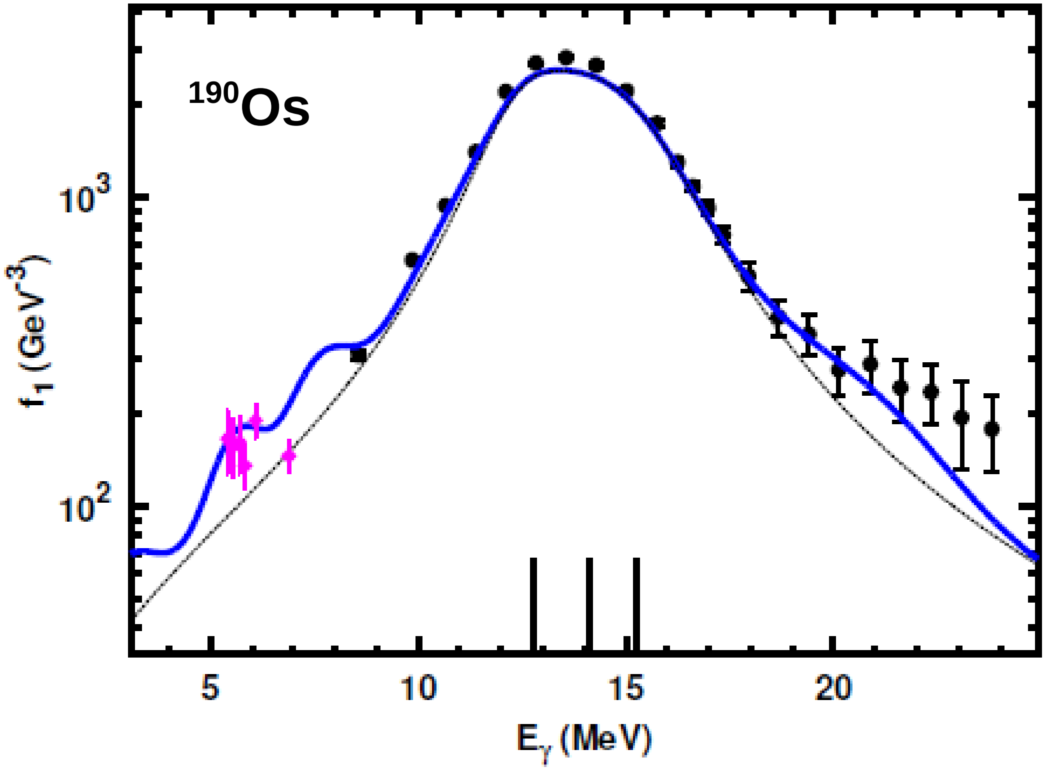


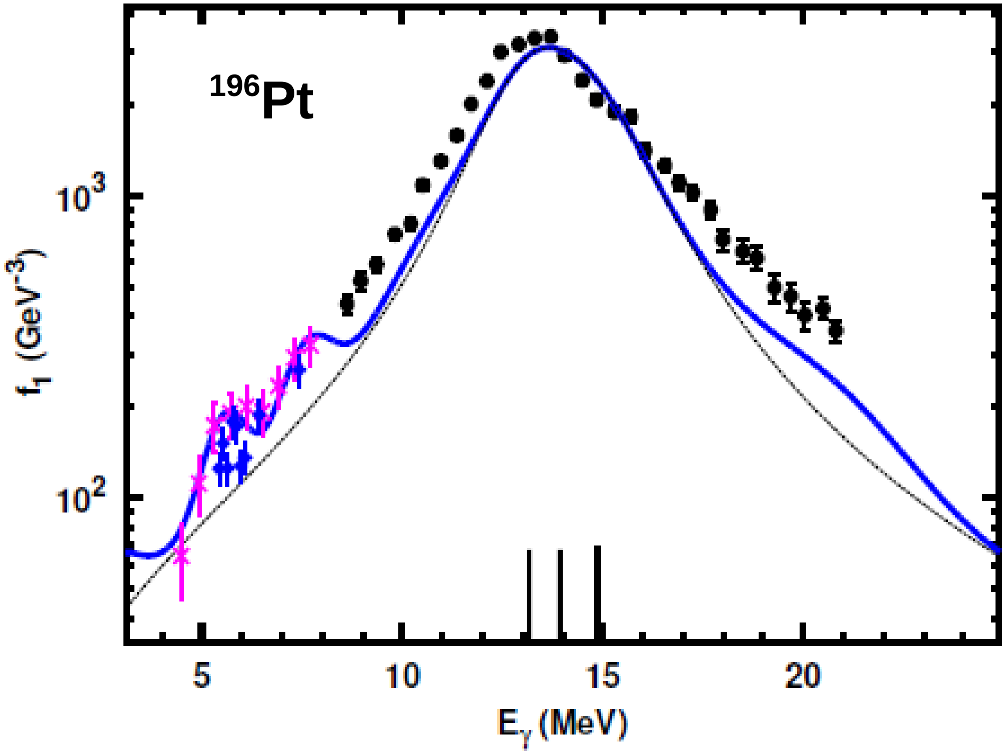
The strength observed corresponds to the cross section summed over a spin multiplet

with $m=\min(2\lambda+1,\ 2J_0+1)$.

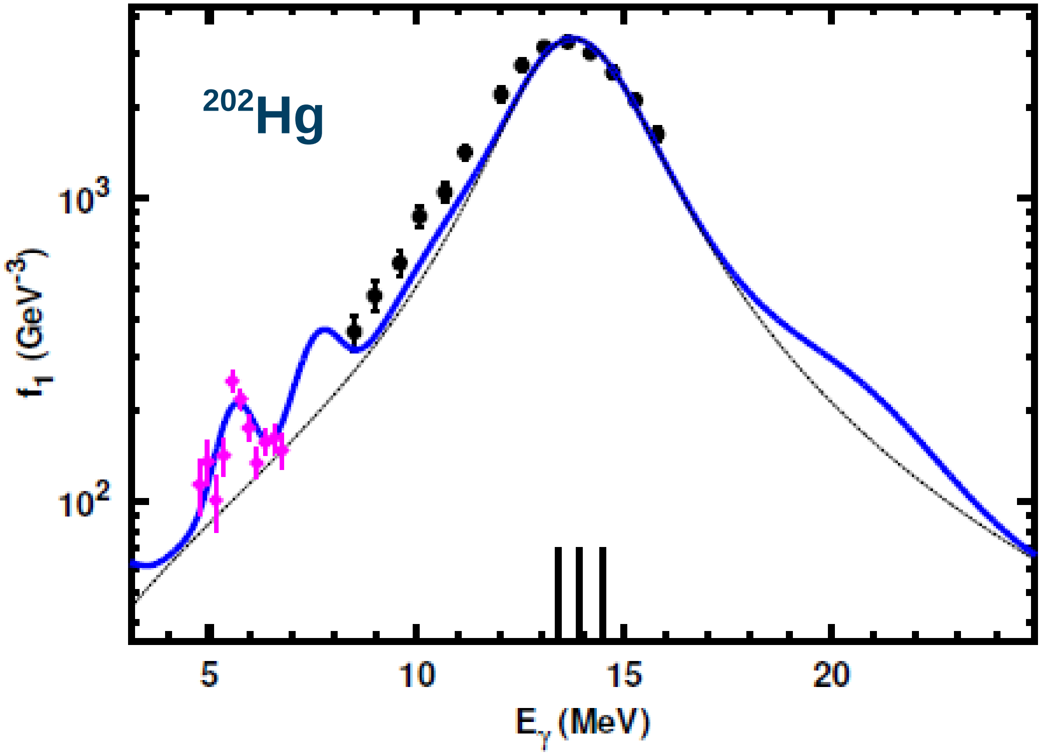
$$g_{eff} = \sum_{r=1,m} \frac{2J_r+1}{2J_0+1} = 2\lambda+1$$

Data for nuclei often assumed to be oblate

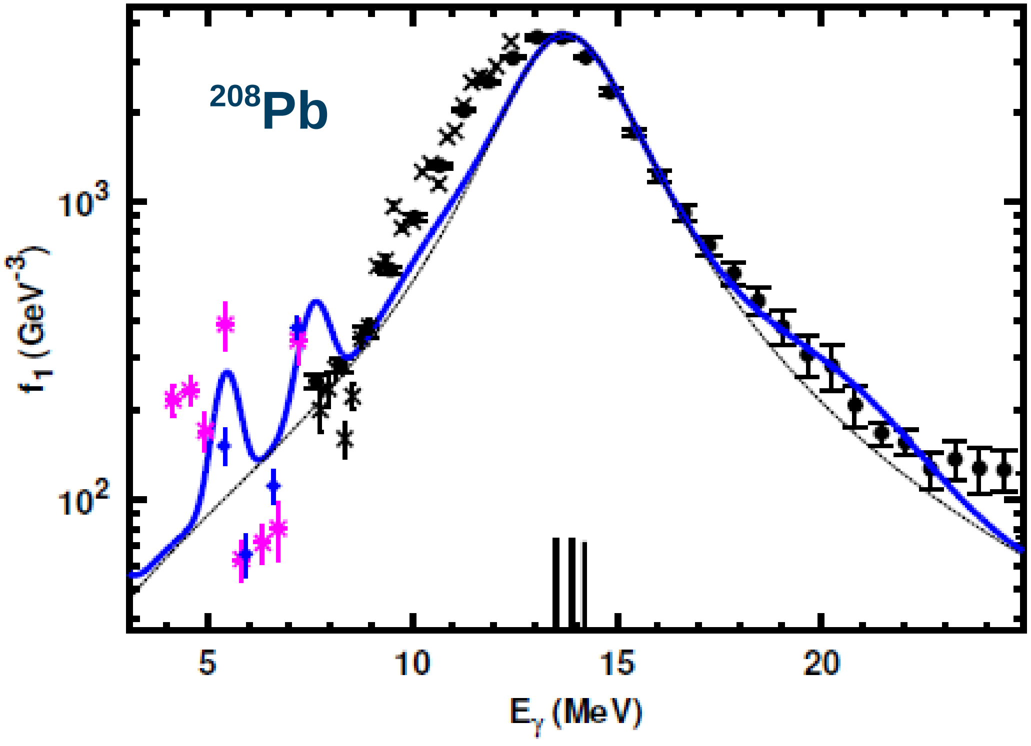




Data near shell closure

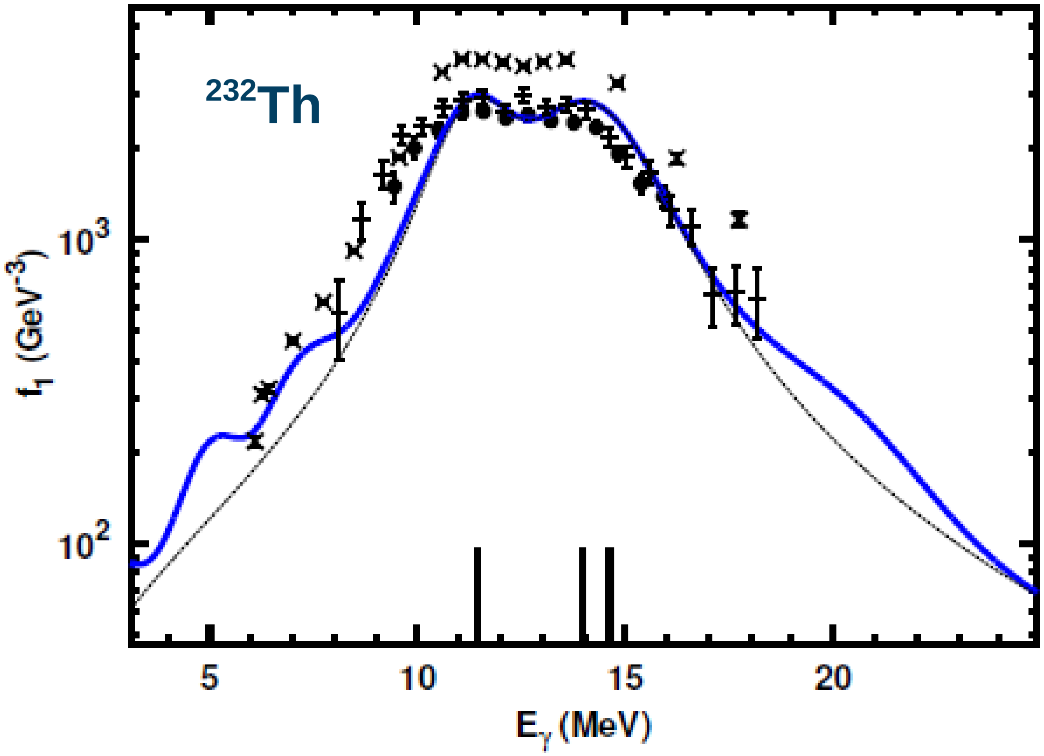


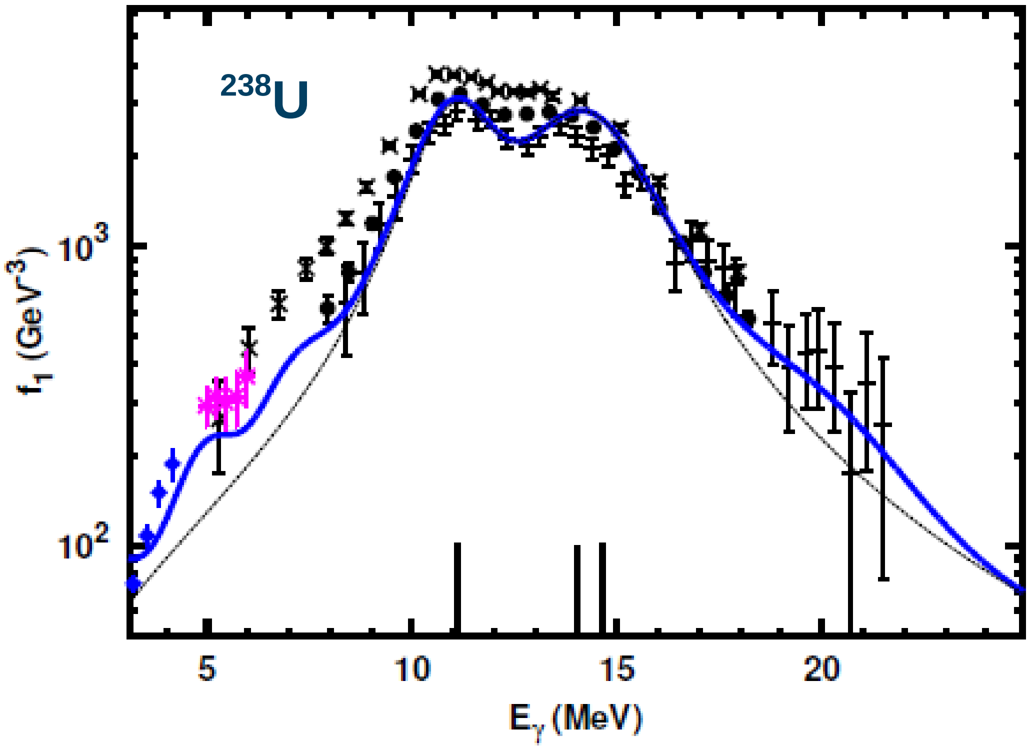
These old data for ^{202}Hg obtained at Urbana with low energy resolution demonstrate a well localized enhancement near 5 MeV.



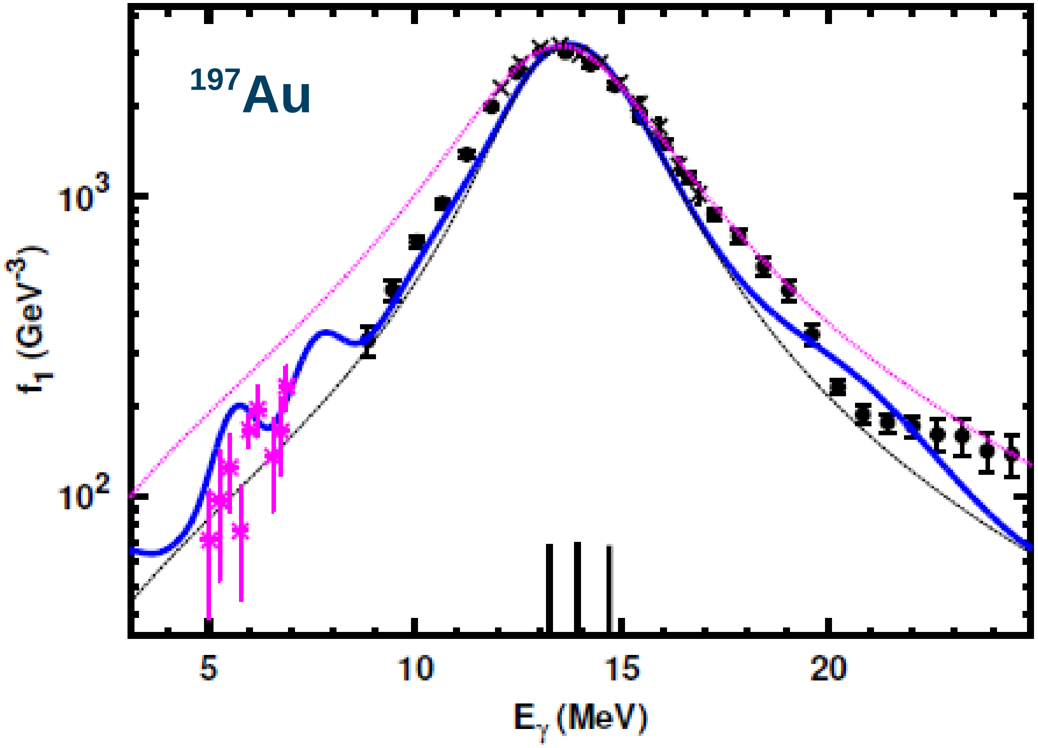
Data for ^{208}Pb show single peaks indicating Porter-Thomas fluctuations and at 5.2 MeV a strong one, identified with a neutron p-h-state.

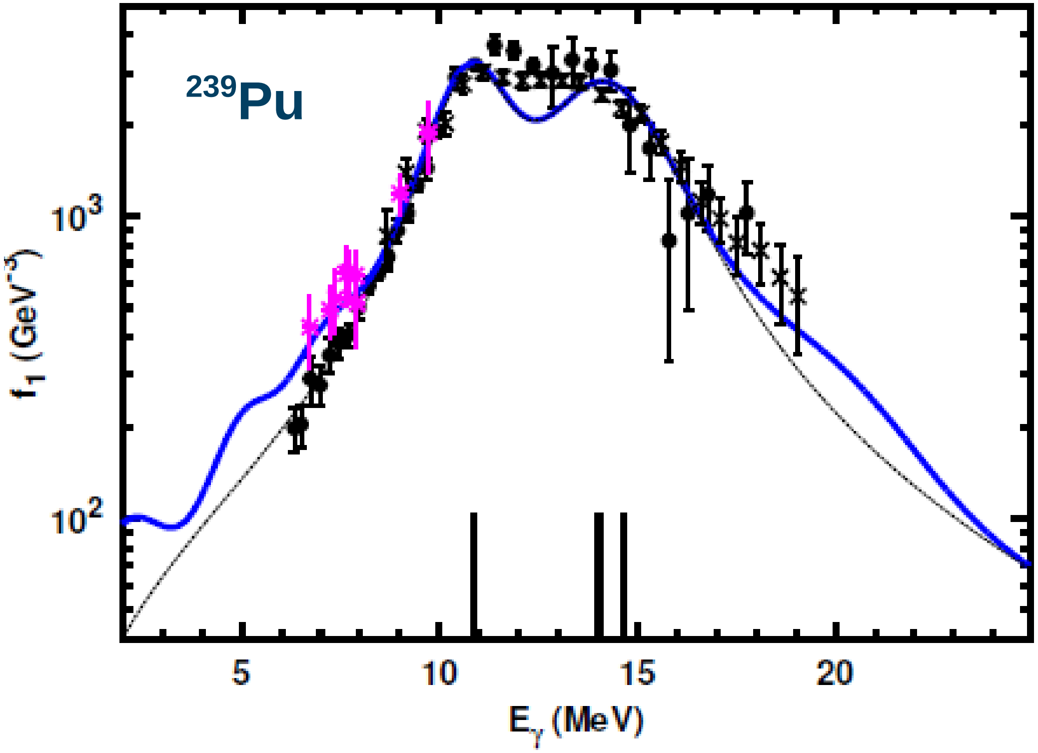
These actinide data show a clear disaccord between different experiments !





The agreement between experiment and is important with respect to the disagreeing data obtained at Livermore [**Caldwell et al., 1980*]. These cross sections for ^{232}Th and ^{238}U are exceptionally large in the sense, that an analysis with TLO indicates an overshoot of 30% as compared to the TRK sum.





The dotted red curve shows the local fit made by Plujko et al. ; it overpredicts the strength by a factor of more than 2.

Level densities: For atomic and molecular gases a ‘critical’ temperature was defined (with the Euler constant C=0.5772) $t_{pt} = \Delta_o\cdot eC/\pi = 0.567\cdot\Delta_0$, which we also use for nuclei.

For energies above E_{pt} the Fermi gas expression holds:

$$\omega_{qp}(E_x) = \frac{\sqrt{\pi}\cdot e^{\left(2\sqrt{\tilde{a}(E_x-E_{bs})}\right)}}{12\,\tilde{a}^{1/4}(E_x-E_{bs})^{5/4}}$$

The parameter ã relates energy and temperature of a Fermi gas; it is often (confusingly) called level density parameter and even used as a variable to be fitted.

We insert the nuclear matter value (with Fermi energy $\varepsilon_F = 37$ MeV)

$$\tilde{a} = \tilde{a}_{nm} = \frac{\pi^2 A}{4\varepsilon_F} \cong \frac{A}{15}$$

and derive the backshift energy E_{bs} by subtracting the mass M_{ld} given by a liquid drop formula from the measured mass M_{exp}: $E_{bs} = M_{exp} - M_{ld} + E_{co}$.

The backshift E_{bs} represents the energy between the Fermi gas zero and the gs of finite nuclei, it corrects for the nuclear binding. For $E_x < E_{pt} = \tilde{a}\cdot t_{pt}^{\,2} + E_{bs}$ we use a constant temperature (CTM) model:

$$\omega_{qp}(E_x) = \omega_{qp}(0)\exp\left(\frac{E_x}{T_{ct}}\right)$$

Nuclei are 3-dim, why not triaxial ?

Heavy nuclei may look like that,

only well deformed ones seem axial.

Triaxial object with $\gamma \approx 30°$

Conclusions:

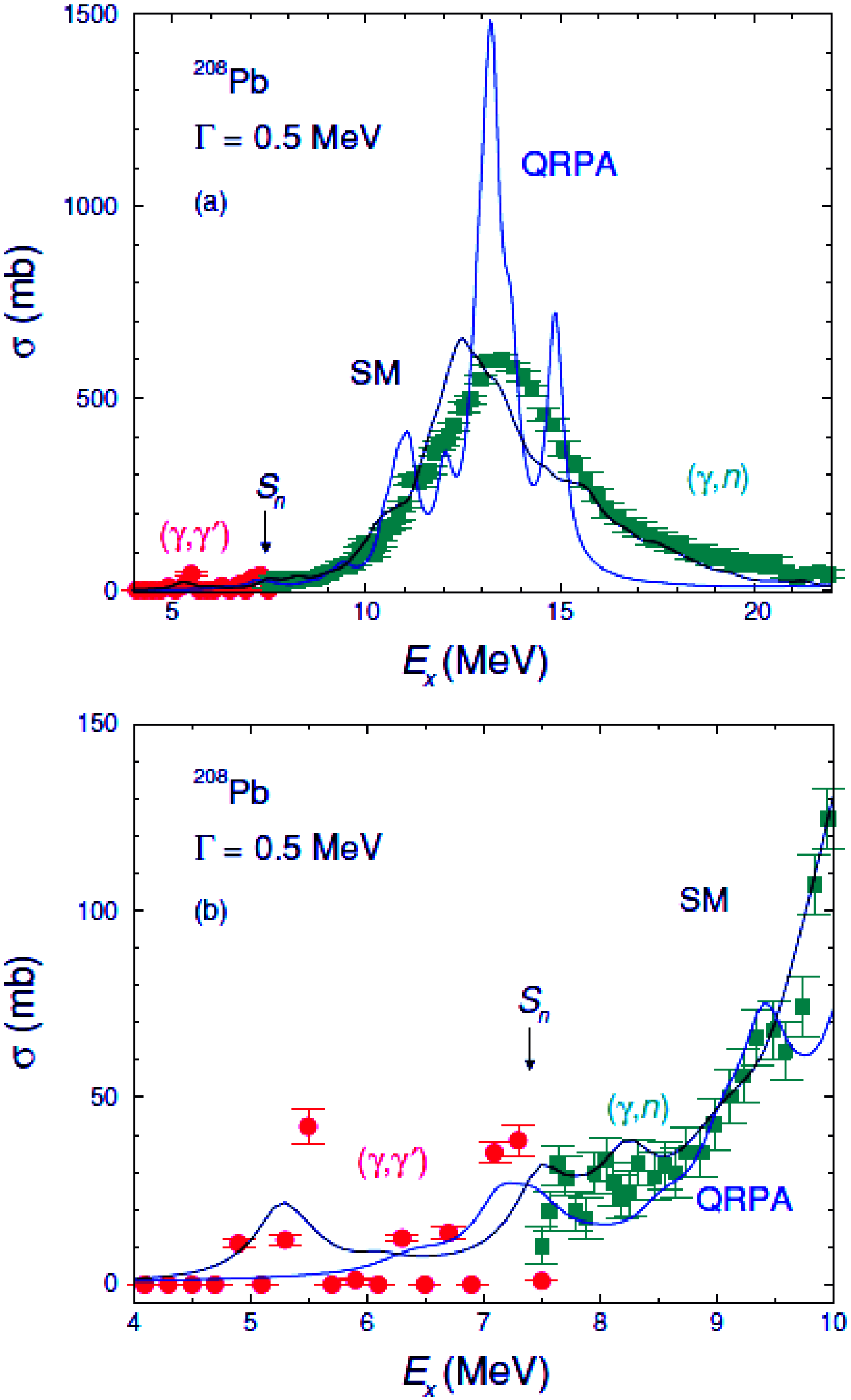
A symmetric configuration is not necessarily the one with lowest energy! (Jahn-Teller effect)

For most heavy nuclei several experimental facts indicate broken axial symmetry :

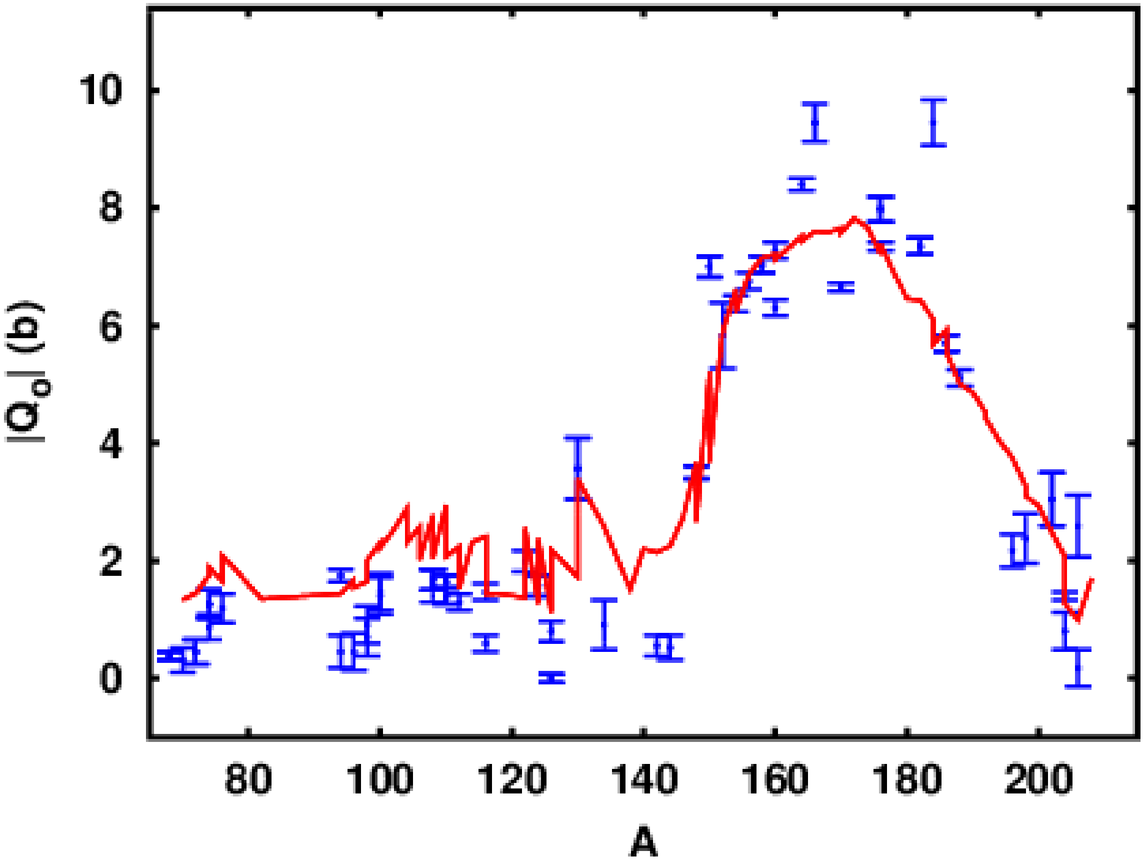
1. *Level densities predicted on absolute scale with $\tilde{a} = \pi^2 \cdot A/4 \cdot \varepsilon_F$*
2. *Multiple Coulomb excitation analysed via <u>rotation - invariants</u>*
3. *Various other spectroscopic data, esp. for odd and odd-odd nuclei*
4. *Split of the giant dipole resonance indicates triaxiality – with β, γ from HFB-GCM*
5. *Neutron capture cross sections are well described for $70 < A < 240$ with*

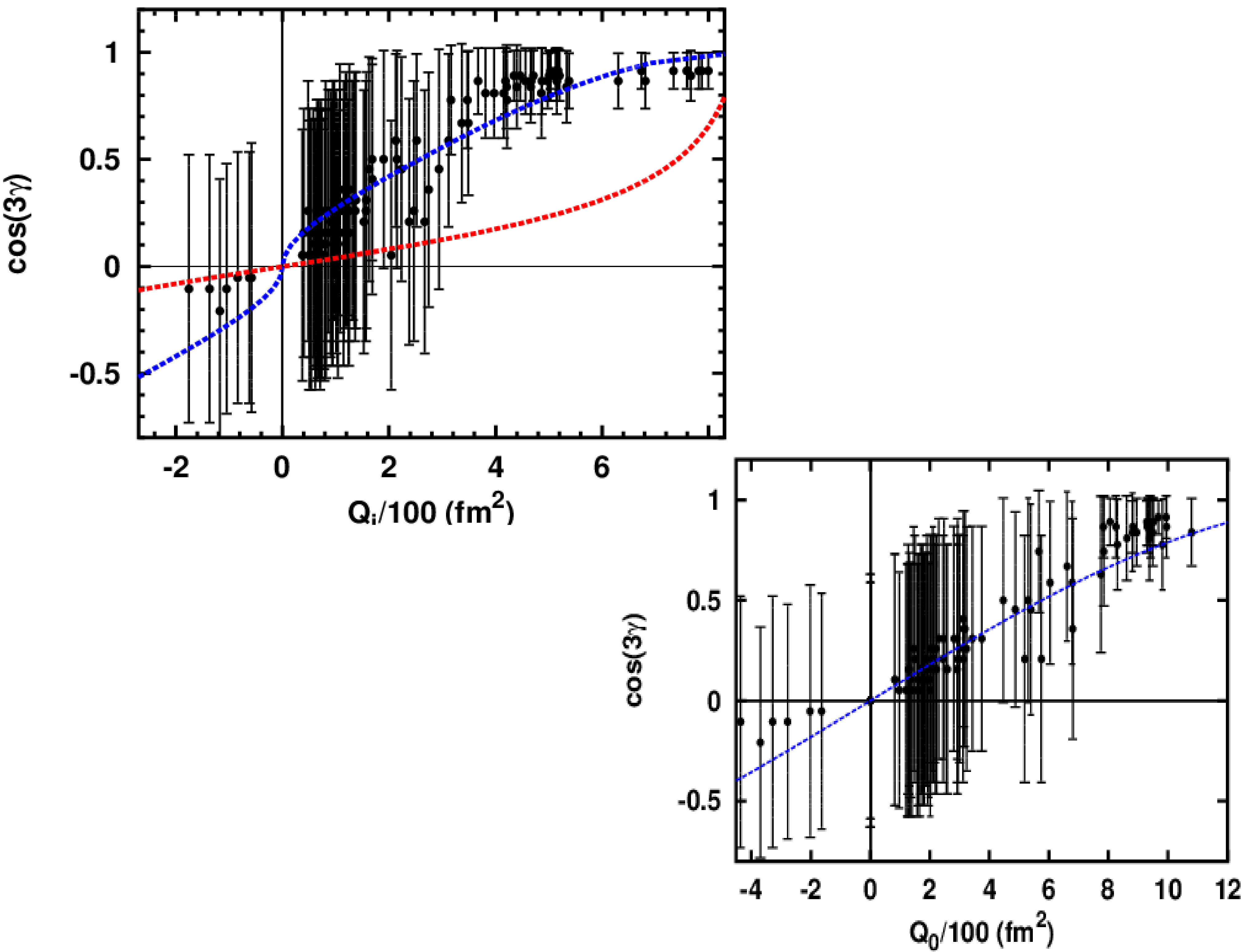
   only one (global) fit parameter for spreading widths.

HFB calculations with mapping onto a 5-dim. collective quadrupole Hamiltonian (GCM) predict triaxiality



^{208}Pb
Γ = 0.5 MeV
(a)
QRPA
SM
S_n
(γ,γ')
(γ,n)
σ (mb)
E_x (MeV)
0
500
1000
1500
5
10
15
20
^{208}Pb
Γ = 0.5 MeV
(b)
SM
S_n
(γ,n)
(γ,γ')
QRPA
σ (mb)
E_x (MeV)
0
50
100
150
4
5
6
7
8
9
10


|Q_o| (b)
A
0
2
4
6
8
10
80
100
120
140
160
180
200


cos(3γ)
1
0.5
0
-0.5
-2
0
2
4
6
Q_i/100 (fm^2)
cos(3γ)
1
0.5
0
-0.5
-4
-2
0
2
4
6
8
10
12
Q_0/100 (fm^2)

Experimental information on axial symmetry breaking in heavy nuclei

and microscopic calculations

Eckart Grosse

Technische Universität Dresden

Level densities - n-tof data

Spectroscopy of odd nuclei – energies and rates

Quadrupole observables in even nuclei

Splitting of giant dipole resonances

Photon strength and n-capture

HFB and RPA

Rotation invariants

Rotational enhancement of nuclear level density vs. symmetry class

The intrinsic ***quasi-particle state density*** *in a finite nucleus* $\omega_{qp}(E_x)$ *is* ***not*** *yet the observable density of nuclear levels with* ***well defined spin*** $\rho(E_x, J = I_{rot}+j, \pi)$.

To fix J the underlying collective symmetry has to be introduced:

small J limit

1. *spherical* ⇒ *only q-p states* $\rho(E_x, J, \pi) \rightarrow \frac{2J+1}{2\cdot\sqrt{8\pi}\,\sigma^3}\,\omega_{qp}(E_x)$
2. *axial* ⇒ *q-p states & rotation ⊥ axis* $\rho \rightarrow \frac{2J+1}{2\cdot\sqrt{8\pi}\,\sigma}\,\omega_{qp}(E_x)$
3. ***non-axial*** *(triax)* ⇒ *q-p states & rotation about any axis* $\rho \rightarrow \frac{2J+1}{2\cdot 4}\,\omega_{qp}(E_x)$
4. *no reflection symmetry* ⇒ *q-p states & octupole deform.* $\rho \rightarrow \frac{2J+1}{2}\,\omega_{qp}(E_x)$

1 parity

Thomas-Fermi Model ➡ $\sigma^2 = \frac{\tilde{a}\cdot t}{11}A^{2/3} \approx \frac{A^{5/3}}{143}\cdot t$

H.Bethe, Rev. Mod. Phys. 9 (1937) 69; S.Bjørnholm, A.Bohr, B.Mottelson; Rochester conf. 1974, IAEA-SM-I74/205