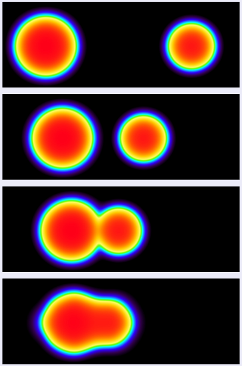


Restoration of symmetry in time-dependent calculations. Josephson effect study with the Gogny TDHFB calculation.

Guillaume SCAMPS

Center for computational sciences,
Tsukuba University

November 6th 2017

Collaboration : Y. Hashimoto, T. Nakatsukasa

Recent evolution of the mean-field dynamics

TDHF
No pairing correlations

TDHF+BCS
Simplified pairing correlations
Computational time ×1.5

TDHFB
Full pairing
Computational time ×1000

TDHF+BCS :

S. Ebata, T. Nakatsukusa, et al., Phys. Rev. C 82, 034306 (2010).
G. Scamps, D. Lacroix, Phys. Rev C 87, 014605 (2013).

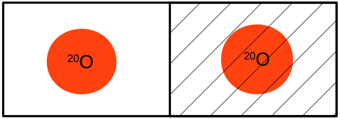
Time-dependent Hartree-Fock-Bogoliubov (TDHFB) :

I. Stetcu, A. Bulgac, P. Magierski, and K. J. Roche, Phys. Rev. C 84, 051309(R) (2011).
Y. Hashimoto, Phys. Rev. C 88, 034307 (2013).

Collision between two superfluid nuclei described with TDHFB with a Gogny force

HFB breaks the particle-number symmetry → qp-vacuum states have define gauge angles

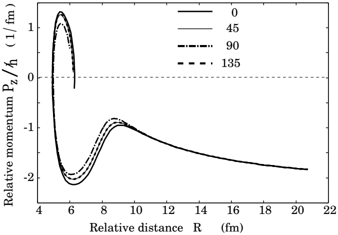
$^{20}O + ^{20}O$



at t=0, rotation of the gauge angle :

$$U \to e^{i\varphi} U \ \theta(z)$$
$$V \to e^{-i\varphi} V \ \theta(z)$$

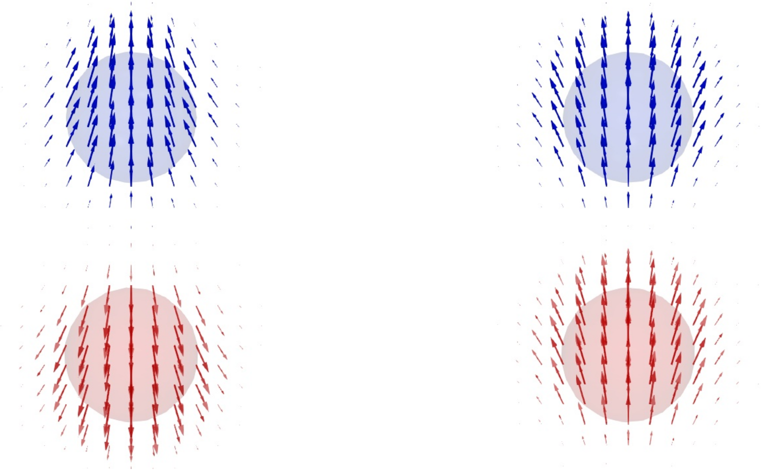
Results :



$E_{\rm cm} = 11.41$ MeV

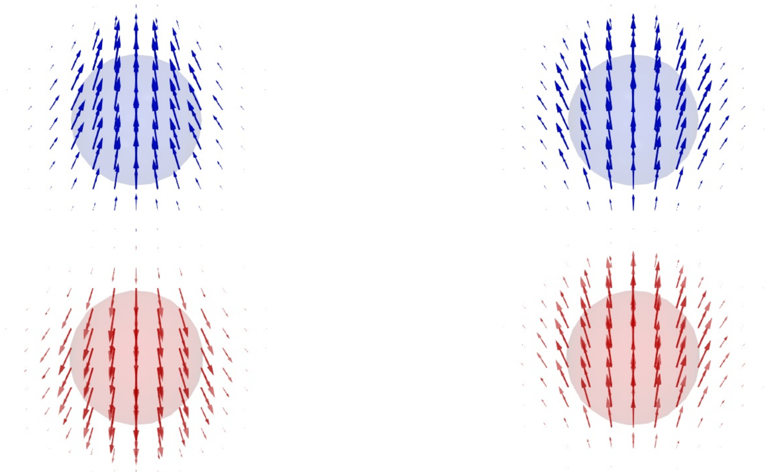
Y. Hashimoto, G. Scamps, Phys. Rev. C 94, 014610 (2016)

Evolution of two TDHFB calculation at the vicinity of the barrier



$$\kappa(r,\uparrow,r,\downarrow) = |\kappa(r,\uparrow,r,\downarrow)|e^{2i\varphi(r)}$$

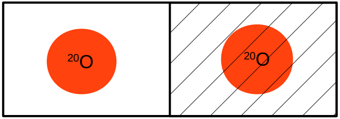
Evolution of two TDHFB calculation at the vicinity of the barrier



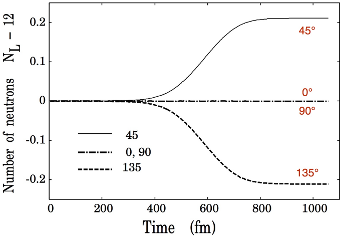
$\kappa(r,\uparrow,r,\downarrow) = |\kappa(r,\uparrow,r,\downarrow)| e^{2i\varphi(r)}$

Josephson transfer

$^{20}O+^{20}O$



Transfer



$$J_s \propto \sin(2\varphi) .$$

Josephson effect

The transfer of nucleons depends on the relative gauge angle.

Y. Hashimoto, G. Scamps, Phys. Rev. C 94, 014610 (2016)

Problem

Question

The relative gauge angle is not a parameter of the reaction. Does those results of TDHFB are spurious ?

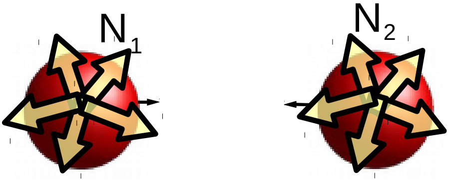
Projection method

We should restore the symmetry of the relative gauge angle

$$|\Psi(t=0)\rangle = \hat{P}_{N_L-N_R}(N_L - N_R)|\phi\rangle,$$

$$\hat{P}_{N_{\mathrm{L}}-N_{\mathrm{R}}}(N_{\mathrm{L}} - N_{\mathrm{R}}) = \frac{1}{2\pi}\int_0^{2\pi} e^{i\varphi\left[(\hat{N}_{\mathrm{L}}-\hat{N}_{\mathrm{R}})-(N_{\mathrm{L}}-N_{\mathrm{R}})\right]} d\varphi$$

We have to consider an evolution of a mixture of HFB states



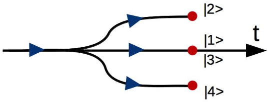
Approximation

Starting point

$$|\Psi(t=0)\rangle = \sum_{n=1}^{M} c_n |\phi_n(t=0)\rangle,$$

$$|\phi_n(t=0)\rangle = e^{i\frac{2n\pi}{M}\hat{N}_L}|\phi(t=0)\rangle.$$

Evolution



Assumption during the evolution

$$|\Psi(t)\rangle = \sum_{n=1}^{M} c_n |\phi_n(t)\rangle,$$

$|\phi_n(t)\rangle$ evolves with the TDHFB equation of motion

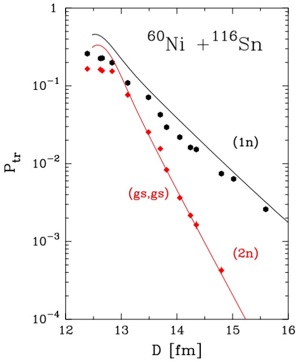
Observables

$$\mathcal{O} = \frac{\langle \Psi(t)|\hat{O}|\Psi(t)\rangle}{\langle \Psi(t)|\Psi(t)\rangle}$$

We assume an evolution of a set of TDHFB trajectories with fixed coefficients.

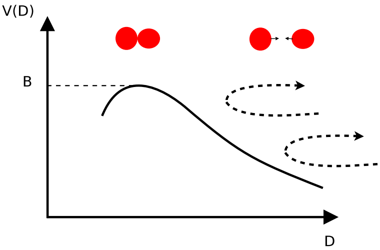
Multi-nucleon transfer in the sub-barrier regime

Motivations :



Montanari et al., PRL 113 (2014)

Transfer under the barrier



Calculation of the transfer probabilities

Projection method for TDHF

$$P_{\rm L}(N) = \langle \Psi(t)|\ \hat{P}_{\rm L}(N)|\Psi(t)\rangle$$

$$\hat{P}_{\rm L}(N) = \frac{1}{2\pi}\int_0^{2\pi} e^{i\varphi(\hat{N}_{\rm L}-N)}d\varphi$$

C. Simenel, PRL 105 (2010).

Projection method with pairing

$$P_{\rm L}(N) = \frac{\langle \Psi(t)|\hat{P}_{\rm L}(N)\hat{P}(N_{\rm tot})|\Psi(t)\rangle}{\langle \Psi(t)|\hat{P}(N_{\rm tot})|\Psi(t)\rangle}$$

G. Scamps, D. Lacroix, PRC 87, (2013).

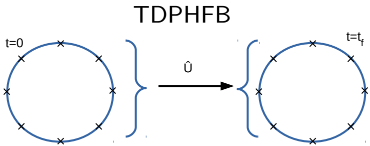
*only when one of the fragment is superfluid

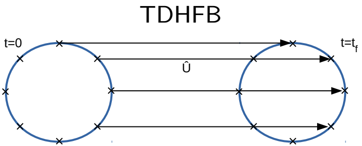
Triple projection method

Triple projection method

$$P_{\mathrm{L}}(N,t) = \frac{\langle \Psi(t)| \hat{P}_{\mathrm{L}}(N) |\Psi(t)\rangle}{\langle \Psi(t)|\Psi(t)\rangle}$$

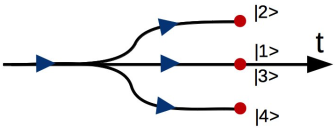
$$|\Psi(t)\rangle = \frac{1}{(2\pi^2)} \int_0^{2\pi} d\varphi_1 \int_0^{2\pi} d\varphi_2 e^{i\varphi_1(\hat{N}-N_{\mathrm{tot}})} \hat{U}^{\mathrm{TDHFB}}(t_0,t) e^{i\varphi_2(\hat{N}_{\mathrm{L}}-N_{\mathrm{L}})} |\phi(t=0)\rangle$$





Triple projection : Pfaffian method

Evolution



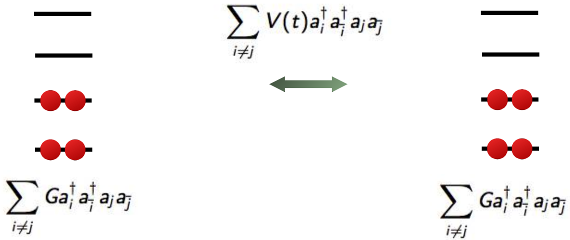
Overlap

G. F. Bertsch and L. M. Robledo, PRL 108, 042505 (2012)

$$\langle w|\mathcal{R}|w'\rangle = (-1)^n \frac{\det C^* \det C'}{\prod_\alpha^n v_\alpha v'_\alpha} \mathrm{pf}\begin{bmatrix} V^T U & V^T R^T V'^* \\ -V'^\dagger R V & U'^\dagger V'^* \end{bmatrix},$$

Optimized Pfaffian calculation : M. Wimmer, ACM Trans. Math Softw. 38, 30 (2012).

Test on toy model



Simple model

$$V(t) = V_0 exp(-\alpha t^2)$$

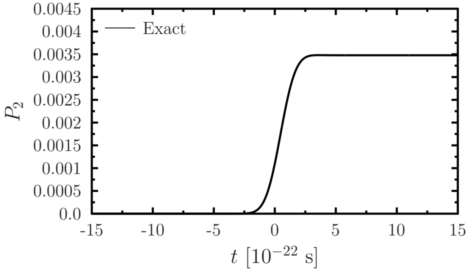
Exact solution

Time-dependent
Multi-configuration method

K. Dietrich, Phys. Let. B 32, 6 (1970).

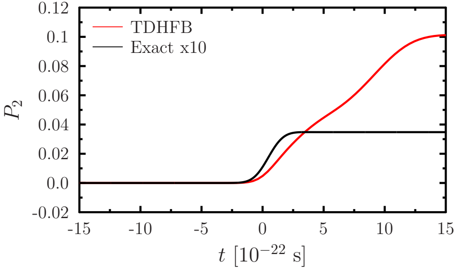
Toy model

Result



Toy model

Result



Conclusion

Spurious result with the TDHFB evolution

Stationary condition

Assumption during the evolution

$$|\Psi(t)\rangle = \sum_{n=1}^{M} c_n |\phi_n(t)\rangle,$$

$|\phi_n(t)\rangle$ evolves with the TDHFB equation of motion

Stationary condition

Without interaction :

$$\langle \phi_n(t+dt)|\phi_n(t)\rangle \simeq 0$$

Important point

We need to impose stationary condition in the TDHFB equation of motion

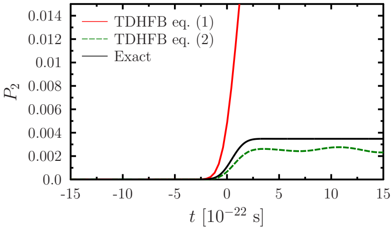
Modification of the equation of motion

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} U(t) \\ V(t) \end{pmatrix} = \mathcal{H}\begin{pmatrix} U(t) \\ V(t) \end{pmatrix},$$

$$\mathcal{H} = \begin{pmatrix} h & \Delta \\ -\Delta^* & -h^* \end{pmatrix} \qquad (1)$$

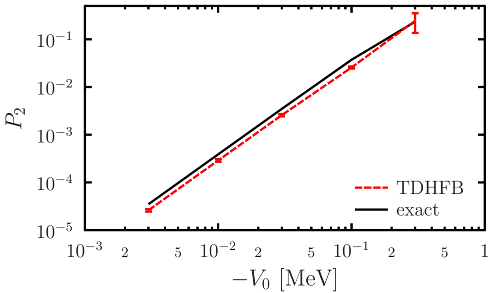
$$\mathcal{H} = \begin{pmatrix} h - \delta\lambda_{\rm L,R}(t) - \epsilon_k(t) & \Delta \\ -\Delta^* & -h^* + \delta\lambda_{\rm L,R}(t) - \epsilon_k(t) \end{pmatrix} \qquad (2)$$

$\delta\lambda_{\rm L,R}(t) = \lambda_{\rm L,R}(t) - \lambda_{\rm L,R}(t=0)$



Comparison

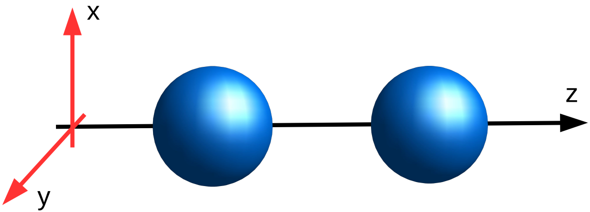
Results



Conclusion

The method works well in the toy model. We can expect a predictive power in realistic calculations.

TDHFB with Gogny interaction



- x and y direction : Harmonic oscillator basis $n_x + n_y \leq 4$
- z direction : Lagrange mesh nz$=$ 46
- $N_{\rm base} = 2760$
- In comparison, a full cartesian mesh is about 100 000 degrees of freedom

Cost of the calculation : one collision done in one day with 20 CPUs

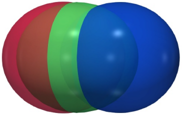
Dispersion of the trajectories

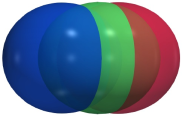
Evolution of the set of TDHFB trajectories

Dispersion of the trajectories

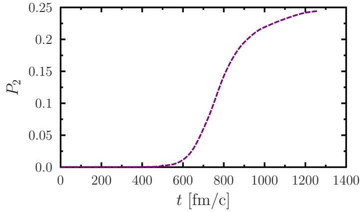
Evolution of the set of TDHFB trajectories

Effect of the dispersion of the trajectories

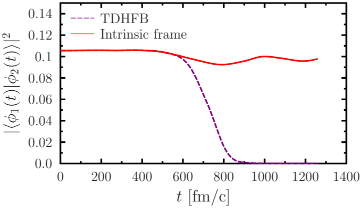




Non stable results

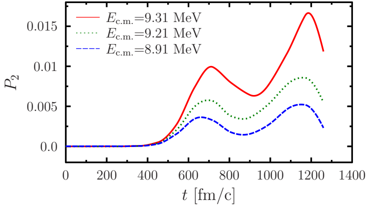


Overlap

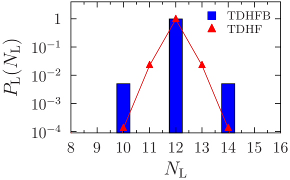


$^{20}O + {}^{20}O$

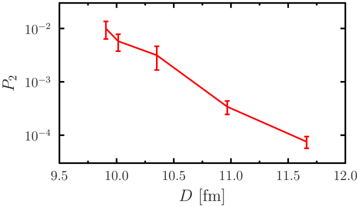
Results



Comparison with TDHF



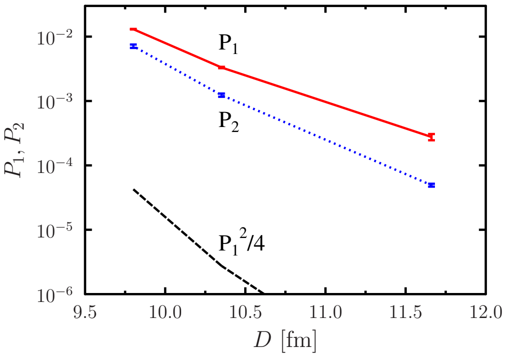
Pair transfer as a function of the distance of closest approach



Important point

This calculation predicts no individual transfer.

Asymmetric reaction $^{14}O + ^{20}O$



Expected value for uncorrelated pair $P_2 = P_1^2/4$ (K. Hagino, G. Scamps, PRC 92 (2015)).

Important point

Large enhancement factor.

Summary and outlook

Summary

- Restoration of the symmetry for TDHFB
- Evolution of a set of HFB states
- Prescription to obtain a correct behavior of the probabilities
- Method tested on a toy model and applied on realistic calculation

Prospects

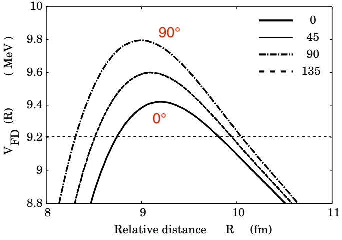
- Study of larger systems for comparison with experimental data
- Development of a theory to make the self-consistent evolution of a projected state

G. Scamps, and Y. Hashimoto, PRC 96, 031602(R) (2017).

Thank you

Nucleus-Nucleus potential

Frozen potential



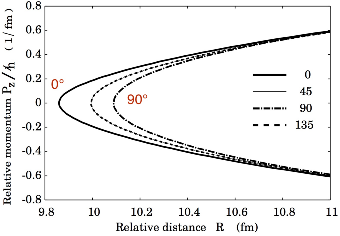
$$U \to e^{i\varphi} U \, \theta(z)$$

$$V \to e^{-i\varphi} V \, \theta(z)$$

$$\rho = V^* V^t \to \rho$$

$$\kappa = V^* U^t \to \kappa e^{2i\varphi}$$

Trajectory



$$\kappa = \kappa^{(l)} + \kappa^{(r)} e^{2i\varphi}$$

$$E_{\rm pair} = \frac{1}{4} \sum_{\alpha\beta\gamma\delta} \overline{v}_{\alpha\beta\gamma\delta} \kappa^*_{\alpha\beta} \kappa_{\gamma\delta}$$

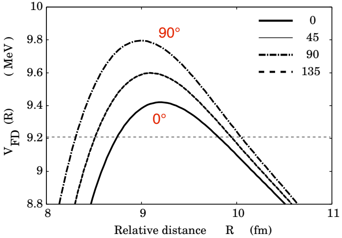
$$\Delta E \propto sin(\varphi)^2$$

Important point

The Nucleus-nucleus potential depends on the relative gauge angle

Nucleus-Nucleus potential

Frozen potential



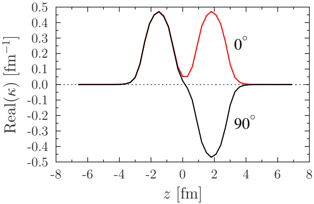
$$U \to e^{i\varphi} U \, \theta(z)$$

$$V \to e^{-i\varphi} V \, \theta(z)$$

$$\rho = V^* V^t \to \rho$$

$$\kappa = V^* U^t \to \kappa e^{2i\varphi}$$

κ density



$$\kappa = \kappa^{(l)} + \kappa^{(r)} e^{2i\varphi}$$

$$E_{\rm pair} = \frac{1}{4} \sum_{\alpha\beta\gamma\delta} \overline{v}_{\alpha\beta\gamma\delta} \kappa^*_{\alpha\beta} \kappa_{\gamma\delta}$$

$$\Delta E \propto sin(\varphi)^2$$

Important point

The Nucleus-nucleus potential depends on the relative gauge angle

Triple projection : Pfaffian method

G. F. Bertsch and L. M. Robledo, PRL 108, 042505 (2012)

$$\langle w|\mathcal{R}|w'\rangle = (-1)^n \frac{\det C^* \det C'}{\prod_\alpha^n v_\alpha v'_\alpha} \mathrm{pf}\begin{bmatrix} V^T U & V^T R^T V'^* \\ -V'^\dagger R V & U'^\dagger V'^* \end{bmatrix},$$

$$P_{\mathcal{B}}(N,t) = \frac{1}{\mathcal{N}} \frac{1}{(2\pi)^4} \iiiint_0^{2\pi} e^{i(\varphi_1-\varphi_4)N_i - i\varphi_2 N - i\varphi_3 N_{\mathrm{tot}}} \langle \Psi(\varphi_1,t)| e^{i\varphi_2 \hat{N}_{\mathcal{B}}} e^{i\varphi_3 \hat{N}_{\mathrm{tot}}} |\Psi(\varphi_4,t)\rangle d\varphi_1 d\varphi_2 d\varphi_3 d\varphi_4$$

$$\mathcal{N} = \frac{1}{(2\pi)^3} \iiint_0^{2\pi} e^{i(\varphi_1-\varphi_4)N_i - i\varphi_3 N_{\mathrm{tot}}} \langle \Psi(\varphi_1,t)| e^{i\varphi_3 \hat{N}_{\mathrm{tot}}} |\Psi(\varphi_4,t)\rangle d\varphi_1 d\varphi_3 d\varphi_4 .$$

$$\langle \Psi(\varphi_1,t)| e^{i\varphi_2 \hat{N}_{\mathcal{B}}} e^{i\varphi_3 \hat{N}_{\mathrm{tot}}} |\Psi(\varphi_4,t)\rangle = (-1)^n \frac{\det C^* \det C'}{\prod_\alpha^n v_\alpha v'_\alpha} \mathrm{pf}\begin{bmatrix} V^T U & V^T e^{i\varphi_3}(1+\Theta(z)e^{i\varphi_2})V'^* \\ -V'^\dagger e^{i\varphi_3}(1+\Theta(z)e^{i\varphi_2})V & U'^\dagger V'^* \end{bmatrix}.$$

Optimized Pfaffian calculation : M. Wimmer, ACM Trans. Math Softw. 38, 30 (2012).