Mean Field Description of Exotic Nuclear Systems from Chiral NN and ΛN potentials

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Motivation I

selfconsistent mean-field calculated by **Hartree-Fock (HF)** or **Hartree-Fock-Bogoliubov (HFB)** methods traditionally use phenomenological **Skyrme** or **Gogny** interactions or are rooted in **relativistic mean-field**

attempts to calculate **HF mean field** from **realistic NN** interactions:

- not realistic **nuclear radii**, **single particle energies**

- necessity to add **phenomenologic**al terms (**density dependent** term or **contact 3-body** force)

E.g.:

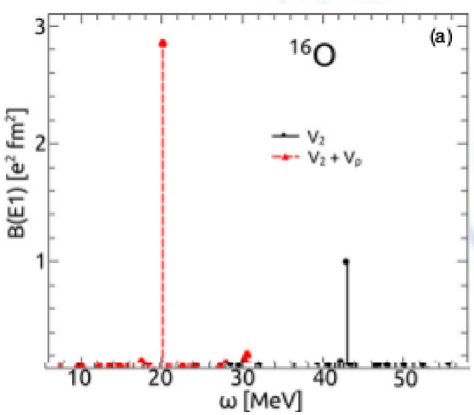
HFB+QRPA with (**Argonne V18+SRG)** NN potential + corrective **density dependent** term

H. Hergert, P. Papakonstantinou, R. Roth, **Phys. Rev. C** 83, 064317 (2011)

$$v_\rho = \frac{C_\rho}{6}(1+P_\sigma)\rho\left(\frac{\vec{r}_1+\vec{r}_2}{2}\right)\delta(\vec{r}_1-\vec{r}_2)$$

HFB+QTDA or **HFB+QRPA** with (**CD-Bonn+V**$_{\mathbf{lowk}}$) + corrective **density dependent** term

D. Bianco, F. Knapp, N. Lo Iudice, P. Vesely, F. Andreozzi, G. De Gregorio, A. Porrino, **J. Phys. G: Nucl. Part. Phys.** 41, 025109 (2014)



DD term **shrink gaps** between **major shells**

as results 1 **phonon** excitations (**RPA** or **TDA**) lower in energy

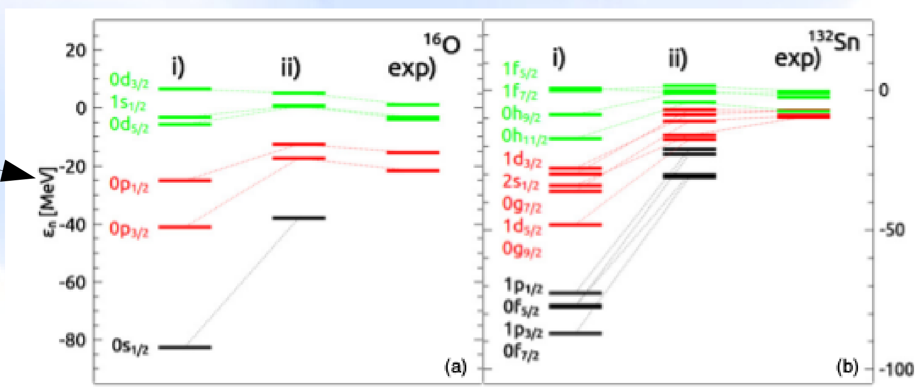


Figure 1. Neutron single-particle spectra in ^{16}O (a) and ^{132}Sn (b) with (i) V_2, (ii) V_2+V_ρ.

Motivation I

HF(B)+(Q)TDA or **HF(B)+(Q)RPA** usually just starting point for more sophisticated beyond mean-field calculations, e.g.:

HF+secondRPA with (**Argonne V18**+**SRG)** NN potential

H. Hergert, P. Papakonstantinou, R. Roth, **Phys. Rev. C** 83, 064317 (2011)

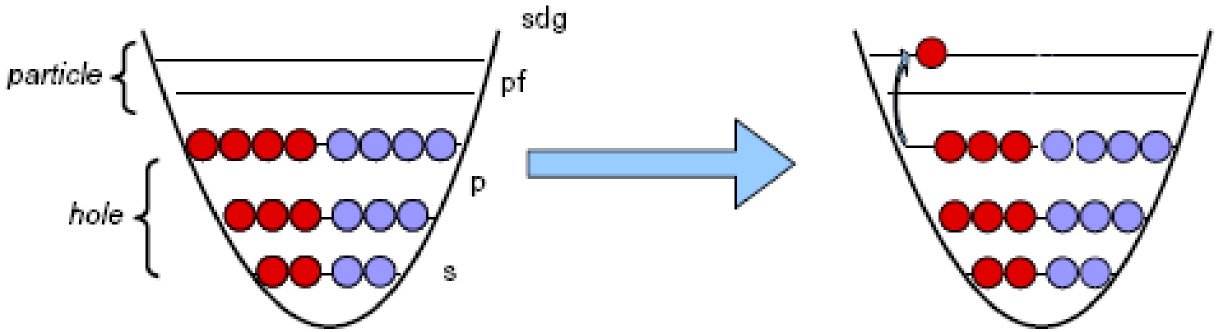
HF(B)+(Q)TDA+EMPM (**realistic NN**) + corrective **density dependent** term

D. Bianco, F. Knapp, N. Lo Iudice, F. Andreozzi, A. Porrino, **Phys. Rev. C85**, 014313 (2012)
G. De Gregorio, F. Knapp, N. Lo Iudice, P. Vesely, **Phys. Rev. C93**, 044314 (2016)
G. De Gregorio, F. Knapp, N. Lo Iudice, P. Vesely, **Phys. Rev. C94**, 061301(R) (2016)

Equation of Motion Phonon Method (EMPM)

F. Andreozzi, F. Knapp, N. Lo Iudice, A. Porrino, J. Kvasil, **Phys. Rev. C75**, 044312 (2007)
F. Andreozzi, F. Knapp, N. Lo Iudice, A. Porrino, J. Kvasil, **Phys. Rev. C78**, 054308 (2008)

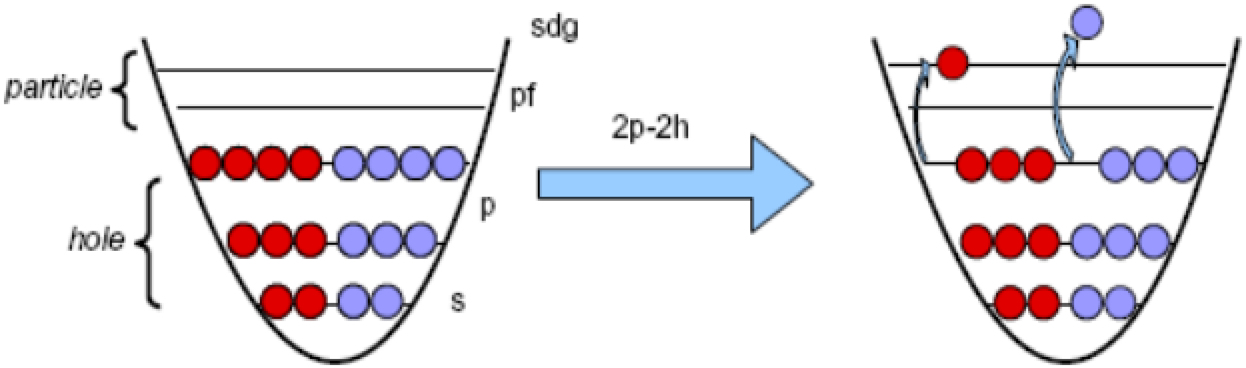
Equation of Motion Phonon Method



1ph excitations

$$O_\nu^\dagger = \sum_{ph} c_{ph}^\nu a_p^\dagger a_{\tilde{h}}$$

Tamm-Dancoff phonons



2ph excitations

and in general

“more“-**ph** excitations

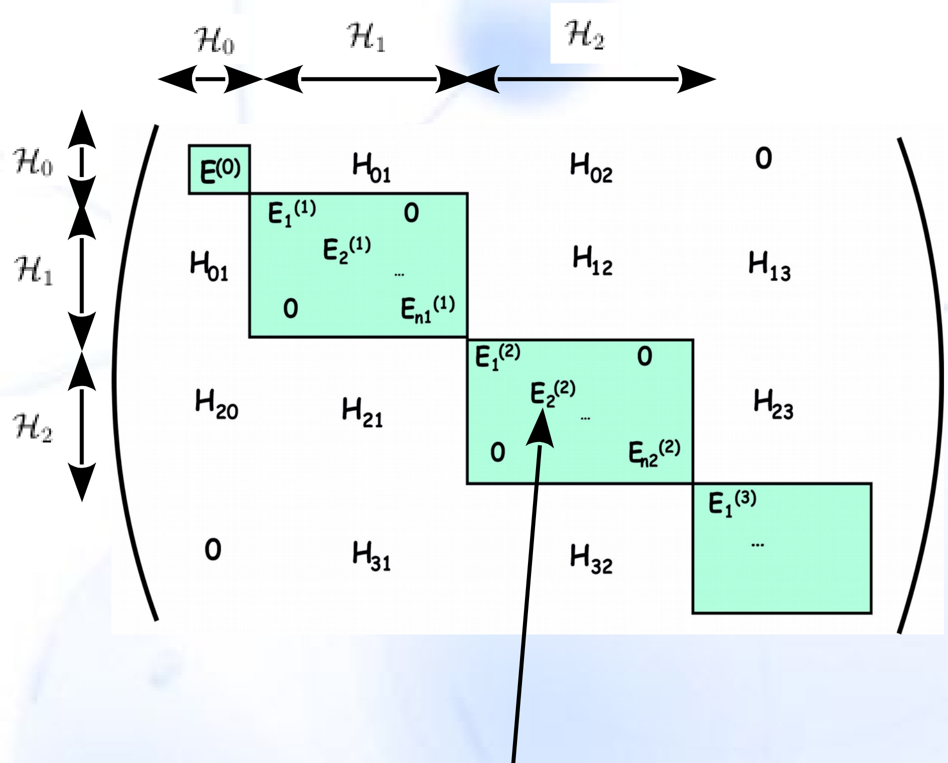
$$\mathcal{H} = \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \mathcal{H}_2 \oplus ... \oplus \mathcal{H}_n$$

Hilbert space – divided into separate n-phonon subspaces

$$\begin{aligned}
\mathcal{H}_0 &= \{|HF>\} \\
\mathcal{H}_1 &= \{O_{\nu_1}^\dagger|HF>\} \\
\mathcal{H}_2 &= \{O_{\nu_1}^\dagger O_{\nu_2}^\dagger|HF>\} \\
&. \\
&. \\
\mathcal{H}_n &= \{O_{\nu_1}^\dagger O_{\nu_2}^\dagger ... O_{\nu_n}^\dagger|HF>\}
\end{aligned}$$

Equation of Motion Phonon Method

$$\begin{aligned}\mathcal{H}_0 &= \{|HF>\}\\ \mathcal{H}_1 &= \{O^\dagger_{\nu_1}|HF>\}\\ \mathcal{H}_2 &= \{O^\dagger_{\nu_1}O^\dagger_{\nu_2}|HF>\}\\ &.\\ &.\\ \mathcal{H}_n &= \{O^\dagger_{\nu_1}O^\dagger_{\nu_2}...O^\dagger_{\nu_n}|HF>\}\end{aligned}$$



total **Hamiltonian** mixes configurations from different **Hilbert subspaces**

Equation of Motion (EoM) – recursive eq. to solve **eigen-energies** on each **i**-phonon subspace while knowing the (**i-1**)-phonon solution

$$< i,\beta_i|[\hat{H},O^\dagger_\nu]|i-1,\alpha_{i-1}> = (E^i_{\beta_i} - E^{i-1}_{\alpha_{i-1}}) < i,\beta_i|O^\dagger_\nu|i-1,\alpha_{i-1}>$$

non-diagonal blocks of **Hamiltonian** calculated from amplitudes $< i,\beta_i|O^\dagger_\nu|i-1,\alpha_{i-1}>$

Equation of Motion Phonon Method

eigen-value problem

we diagonalize the total **Hamiltonian**

$$E = \mathrm{diag}\begin{pmatrix} E^{(0)} & H_{01} & H_{02} & 0 \\ H_{01} & \begin{matrix} E_1^{(1)} & & 0 \\ & E_2^{(1)} & \\ & \ldots & \\ 0 & & E_{n1}^{(1)} \end{matrix} & H_{12} & H_{13} \\ H_{20} & H_{21} & \begin{matrix} E_1^{(2)} & & 0 \\ & E_2^{(2)} & \\ & \ldots & \\ 0 & & E_{n2}^{(2)} \end{matrix} & H_{23} \\ 0 & H_{31} & H_{32} & \begin{matrix} E_1^{(3)} \\ \ldots \end{matrix} \end{pmatrix}$$

correlations – wave functions of each state are **superpositions** of many configurations from different **Hilbert subspaces**

e.g.

$$|\Psi_{g.s.}> = C^{g.s.}_{HF}|HF> + \sum_{\nu_1} C^{g.s.}_{\nu_1}|i=1,\nu_1> + \sum_{\mu_2} C^{g.s.}_{\mu_2}|i=2,\mu_2> + \ldots$$

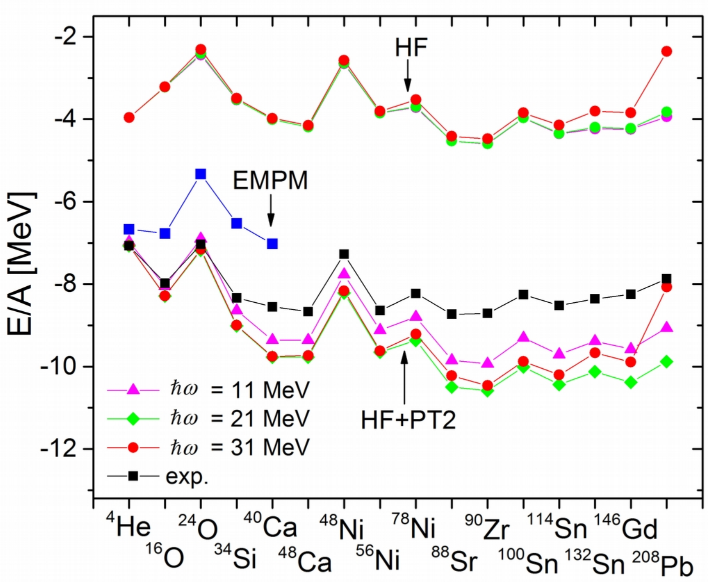
Equation of Motion Phonon Method

configurations beyond (**mean field** & **1 particle-hole** excitations) bring important effects:

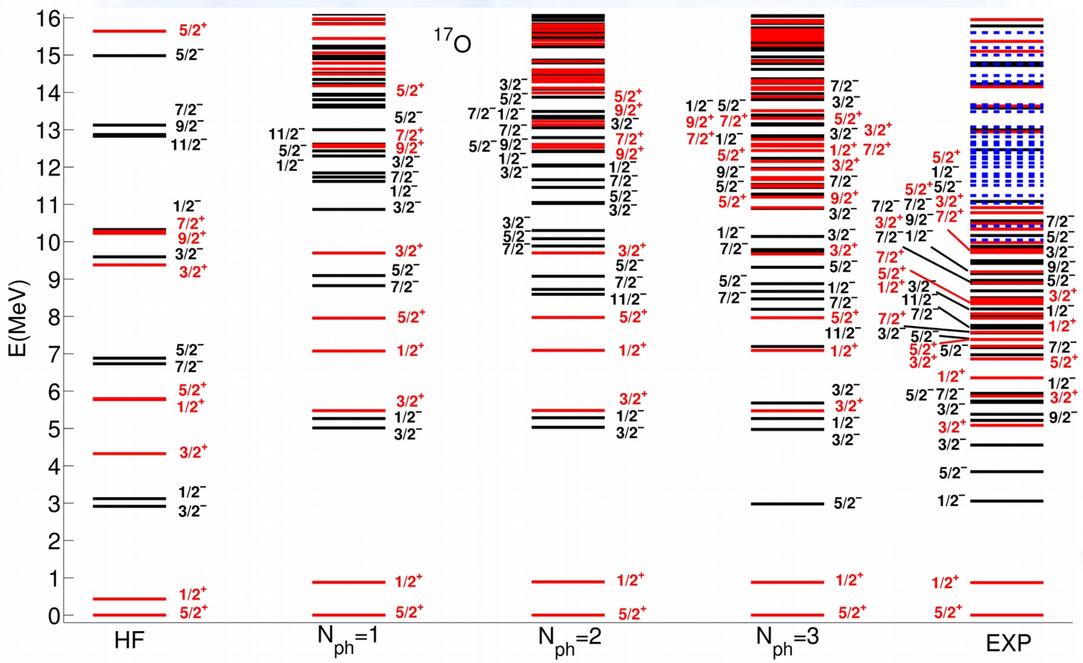
- **many-body correlations** into the **ground state** & **excitation states**
- modifies significantly **low lying spectrum** (more states, different structure)
- **multifragmentation** of giant resonances

etc.

correlated ground state **energies**:
G. De Gregorio, J. Herko, F. Knapp, N. Lo Iudice, P. Vesely, **Phys. Rev. C95**, 024306, (2017)



application of **EMPM** on the **odd** nuclear systems:
G. De Gregorio, F. Knapp, N. Lo Iudice, P. Vesely, **Phys. Rev. C94**, 061301(R) (2016)



Motivation II

within our approach **HF(B)+(Q)TDA+EMPM** → our **goal** is to **reduce** as much as possible free **parameters** of model (possibly only rely on **realistic nucleon forces**)

implementation of realistic **3-body NNN** interactions into our formalism

chiral **NN+NNN** interaction - χ **NNLO$_{\mathbf{sat}}$** (Ekström et al. **Phys. Rev. C** 91 (2015) 051301R)

HF+TDA formalism with **NNN** forces

$$\hat{H} = \hat{T}_N + \hat{V}^{NN} + \hat{V}^{NNN} - \hat{T}_{CM}$$

$$
\begin{aligned}
& t^{\mathrm{p(n)}}_{(n_i l_i j_i),(n_j l_j j_j)} \delta_{l_i l_j} \delta_{j_i j_j} \delta_{m_i m_j} \\
& + \sum_J \sum_{\substack{n_k l_k j_k \\ n_l l_l j_l}} V^{J,\mathrm{pp(nn)}}_{(n_i l_i j_i),(n_k l_k j_k),(n_j l_j j_j),(n_l l_l j_l)} \rho^{\mathrm{p(n)}}_{(n_l l_l j_l),(n_k l_k j_k)} \delta_{l_l l_k} \delta_{j_l j_k} \delta_{m_i m_j} \frac{(2J+1)}{(2j_i+1)} \\
& + \sum_J \sum_{\substack{n_k l_k j_k \\ n_l l_l j_l}} V^{J,\mathrm{pn(np)}}_{(n_i l_i j_i),(n_k l_k j_k),(n_j l_j j_j),(n_l l_l j_l)} \rho^{\mathrm{n(p)}}_{(n_l l_l j_l),(n_k l_k j_k)} \delta_{l_l l_k} \delta_{j_l j_k} \delta_{m_i m_j} \frac{(2J+1)}{(2j_i+1)} \\
& + \sum_{n_k l_k j_k m_k} \sum_{n_l l_l j_j m_l} \sum_{n_m l_m j_m m_m} \sum_{n_n l_n j_n m_n} \\
& \Big\{ \frac{1}{2} V^{\mathrm{ppp(nnn)}}_{n_i l_i j_i m_i, n_k l_k j_k m_k, n_l l_l j_l m_l, n_j l_j j_j m_j, n_m l_m j_m m_m, n_n l_n j_n m_n} \rho^{\mathrm{p(n)}}_{n_m l_m j_m m_m, n_k l_k j_k m_k} \rho^{\mathrm{p(n)}}_{n_n l_n j_n m_n, n_l l_l j_l m_l} \\
& + \frac{1}{2} V^{\mathrm{pnn(ppn)}}_{n_i l_i j_i m_i, n_k l_k j_k m_k, n_l l_l j_l m_l, n_j l_j j_j m_j, n_m l_m j_m m_m, n_n l_n j_n m_n} \rho^{\mathrm{n(p)}}_{n_m l_m j_m m_m, n_k l_k j_k m_k} \rho^{\mathrm{n(n)}}_{n_n l_n j_n m_n, n_l l_l j_l m_l} \\
& + V^{\mathrm{ppn(pnn)}}_{n_i l_i j_i m_i, n_k l_k j_k m_k, n_l l_l j_l m_l, n_j l_j j_j m_j, n_m l_m j_m m_m, n_n l_n j_n m_n} \rho^{\mathrm{p(n)}}_{n_m l_m j_m m_m, n_k l_k j_k m_k} \rho^{\mathrm{n(n)}}_{n_n l_n j_n m_n, n_l l_l j_l m_l} \Big\} \\
& = \varepsilon^{\mathrm{p(n)}}_i \delta_{ij}
\end{aligned}
$$

implementation of **3-body NNN** interaction **elements**

JT-coupled elements in HF code → decoupling into **m-scheme** “**on fly**“

TDA calculation:

2-body NN interaction **corrected** by the **normal order** contributions from **3-body NNN** interaction

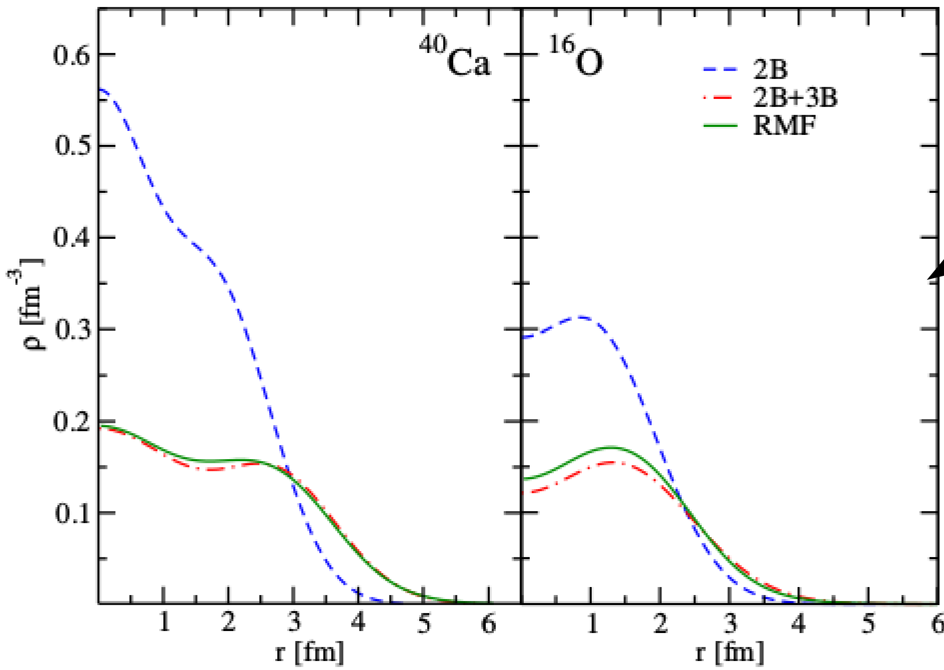
NNN force

NN+NNN interaction - χ **NNLO$_{sat}$** (Ekström et al. **Phys. Rev. C** 91 (2015) 051301R)

test calculations in **minimal configuration space**
but most of **qualitative** effect from **NNN** already there!

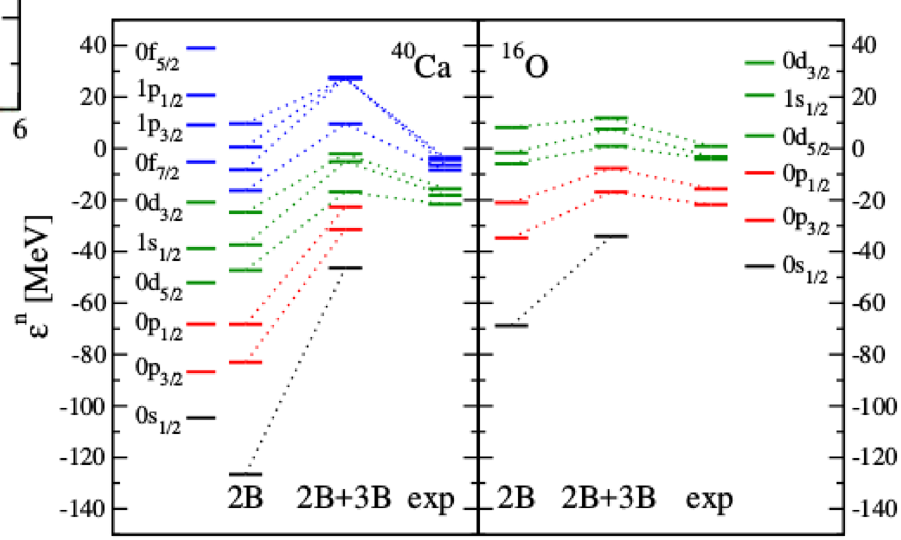
HO basis $N = (2n + l)$

hω = 20 MeV **N$_{max}$** up to 4



nuclear **radial density**

NNN interaction shrinks gaps between **major shells**



charged radii

	r_{ch} [fm]		
^{A}X	2B	2B+3B	exp
^{40}Ca	2.58	3.18	3.48
^{16}O	2.23	2.67	2.70

HF energy

	E_{HF}/A [MeV]		
^{A}X	2B	2B+3B	exp
^{40}Ca	-11.65	-0.60	-8.55
^{16}O	-7.31	-2.19	-7.98

NNN force

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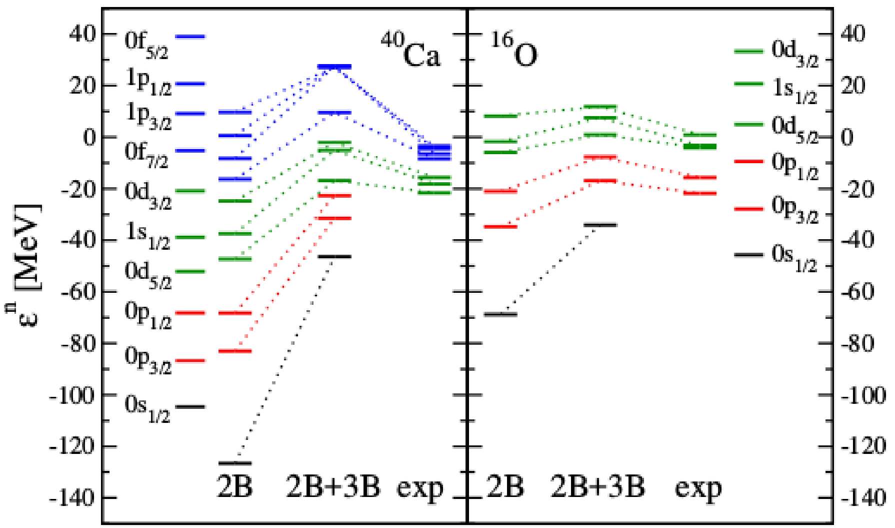
test calculations in **minimal configuration space**
but most of **qualitative** effect from **NNN** already there!

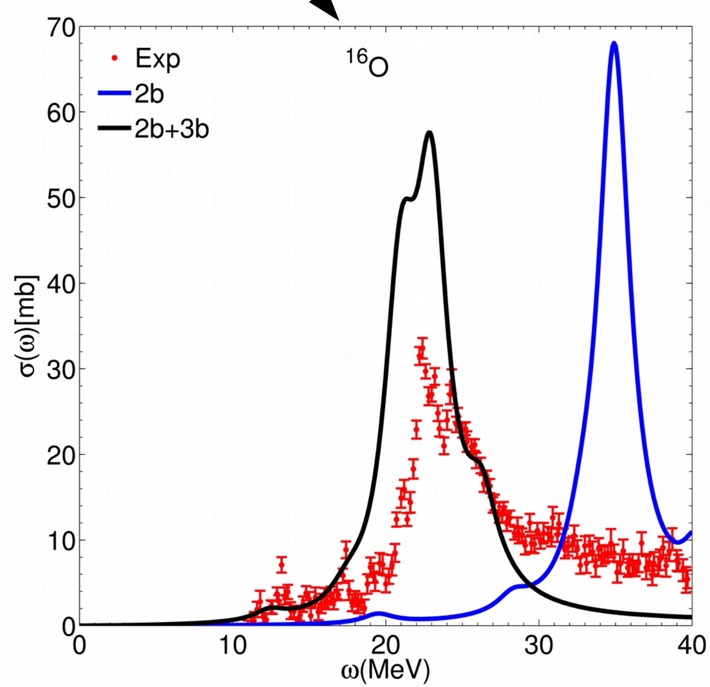
HO basis $N = (2n + l)$

hω = 20 MeV **N$_{max}$** up to 4

NNN interaction shrinks gaps between **major shells** →
important for correct description of **giant resonance**



TDA calculation of **photoabsorption** cross section – shrinked s.p. spectra shifts giant resonance down in energy



charged radii

	r_{ch} [fm]		
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HF energy

	E_{HF}/A [MeV]		
^AX	2B	2B+3B	exp
^{40}Ca	-11.65	-0.60	-8.55
^{16}O	-7.31	-2.19	-7.98

Motivation III

application of approach **HF(B)+(Q)TDA+EMPM** on exotic nuclear systems → **single Λ hypernuclei**

$$\hat{H} = \hat{T}_N + \hat{T}_\Lambda + \hat{V}^{NN} + \hat{V}^{NNN} + \hat{V}^{\Lambda N} + \xcancel{\hat{V}^{\Lambda NN}} - \hat{T}_{CM}$$

NN+NNN interaction - χ **NNLO$_{sat}$** (Ekström et al. **Phys. Rev. C** 91 (2015) 051301R)

ΛN interaction - χ **LO** (H. Polinder, J. Haidenbauer, U. Meissner, **Nucl. Phys. A** 779 (2006) 244)
cut-off λ = 550 MeV

so far implemented:
extension of **HF+TDA** formalism on **hypernuclei** → **proton-neutron-Λ HF** + **ΛN TDA**

(replacement of the **nucleon** by **Λ**)

formalism derived also for 3-body **ΛNN** forces – but these forces **not present** yet (only **leading order ΛN** interaction used) … alternatively **ΛNN** may appear indirectly as **SRG induced** from **ΛN**

main effect of **3-body NNN** force on **single particle energies** of **Λ**:

Λ interacts with **nuclear core** via **ΛN** interaction

NNN force modifies **nuclear core** (**distribution of density**, **s.p. energies** of nucleons)

modification of **nuclear core** modifies **single particle energies** of **Λ**

NNN force – effect on hypernuclei

$$\hat{H} = \hat{T}_N + \hat{T}_\Lambda + \hat{V}^{NN} + \hat{V}^{NNN} + \hat{V}^{\Lambda N} + \cancel{\hat{V}^{\Lambda NN}} - \hat{T}_{CM}$$

NN+NNN interaction - χ **NNLO**$_{sat}$ (Ekström et al. **Phys. Rev. C** 91 (2015) 051301R)

ΛN interaction - χ **LO** (H. Polinder, J. Haidenbauer, U. Meissner, **Nucl. Phys. A** 779 (2006) 244)

cut-off λ = 550 MeV

HO basis

N = (2n + l)

N$_{max}$ up to 4

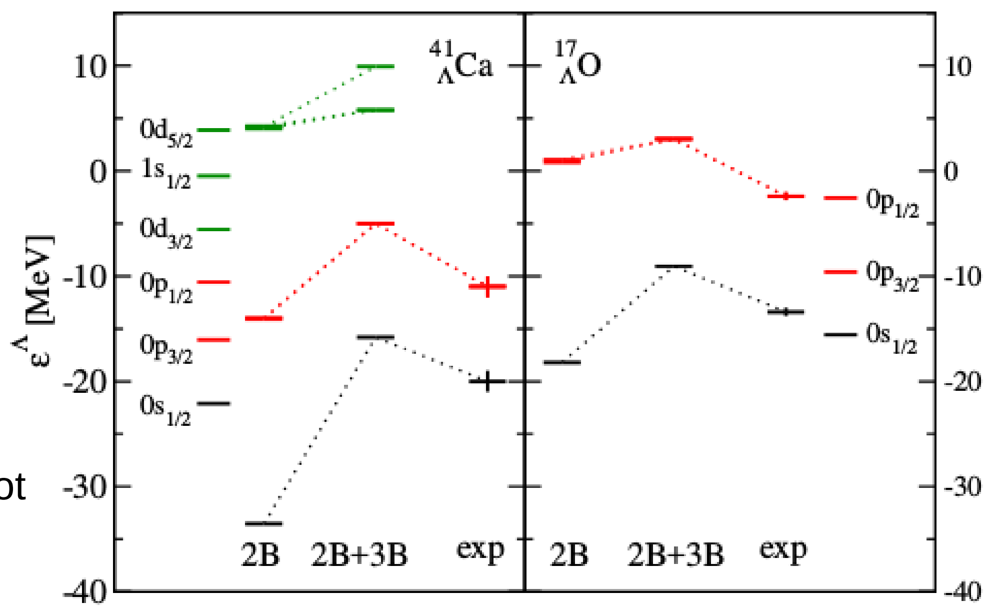
ħω = 20 MeV

only **qualitative study** → we need to enlarge configuration space

sd-shell in **hypercarbon** not realistic due to small space

s.p. Λ energies:

	ε^Λ [MeV]					
	$^{41}_{\Lambda}$Ca			$^{17}_{\Lambda}$O		
s.-p. level	2B	2B+3B	exp	2B	2B+3B	exp
$0s_{1/2}$	-33.561	-15.820	-20.0 ± 1.0	-18.203	-9.055	-13.5 ± 0.4
$0p_{3/2}$	-14.095	-5.016	-11.0 ± 1.0	1.076	3.090	-2.4 ± 0.4
$0p_{1/2}$	-13.958	-4.987	-11.0 ± 1.0	0.805	3.005	-2.4 ± 0.4



main effect:

NNN force **shrinks gaps** between major shells also for **Λ** !

relative energies between **s-** & **p- shells realistic** but **absolute scale wrong**

Problems:

- **wrong order** spin-orbit partners **0p**$_{3/2}$ **& 0p**$_{1/2}$ in **hyperoxygen**

- **strong dependence** on **cut-off** of **ΛN** force

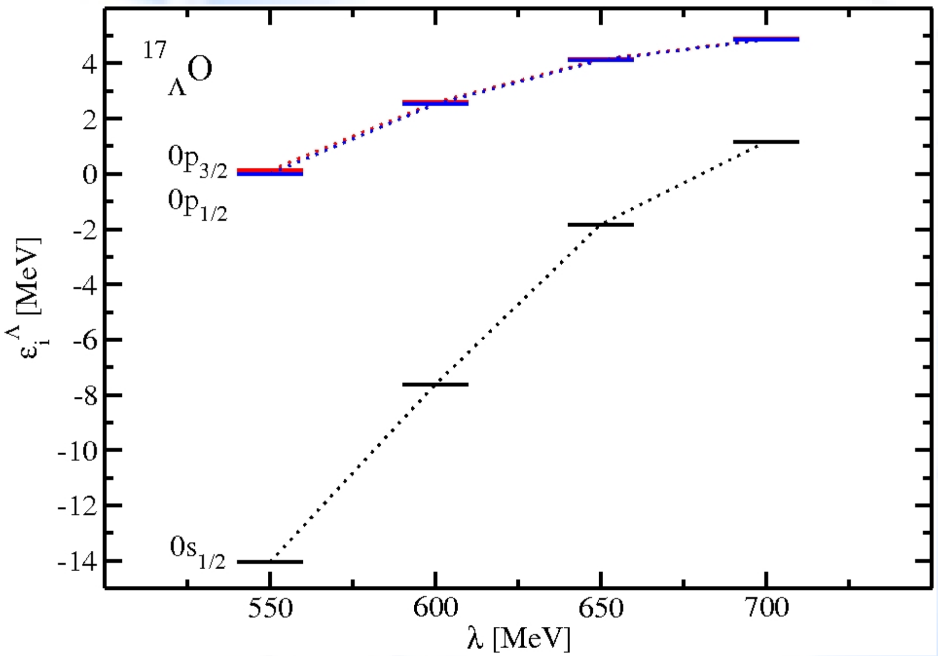
NNN force – effect on hypernuclei

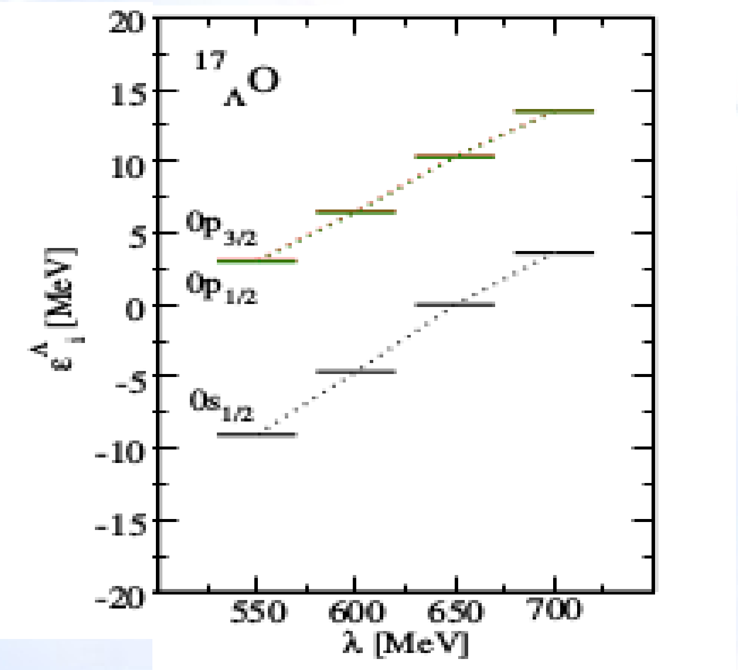
$$\widehat{H} = \widehat{T}_N + \widehat{T}_\Lambda + \widehat{V}^{NN} + \widehat{V}^{NNN} + \widehat{V}^{\Lambda N} + \widehat{V}^{\Lambda NN} - \widehat{T}_{CM}$$

NN+NNN interaction - χ **NNLO$_{sat}$** (Ekström et al. **Phys. Rev. C** 91 (2015) 051301R)

ΛN interaction - χ **LO** (H. Polinder, J. Haidenbauer, U. Meissner, **Nucl. Phys. A** 779 (2006) 244)

cut-off λ = 550 MeV





NN interaction - χ **NNLO$_{opt}$**

ΛN interaction - χ **LO**

NN+NNN interaction - χ **NNLO$_{sat}$**

ΛN interaction - χ **LO**

presence of **NNN** interaction makes **dependence** of **s.p. Λ energies** on **cut-off** of **ΛN** force almost **linear** → relative energy between **0s** & **0p** shells very stable with respect of **cut-off**

NNN force – effect on hypernuclei

$$\hat{H} = \hat{T}_N + \hat{T}_\Lambda + \hat{V}^{NN} + \hat{V}^{NNN} + \hat{V}^{\Lambda N} + \cancel{\hat{V}^{\Lambda NN}} - \hat{T}_{CM}$$

NN+NNN interaction - χ **NNLO$_{sat}$** (Ekström et al. **Phys. Rev. C** 91 (2015) 051301R)

ΛN interaction - χ **LO** (H. Polinder, J. Haidenbauer, U. Meissner, **Nucl. Phys. A** 779 (2006) 244)

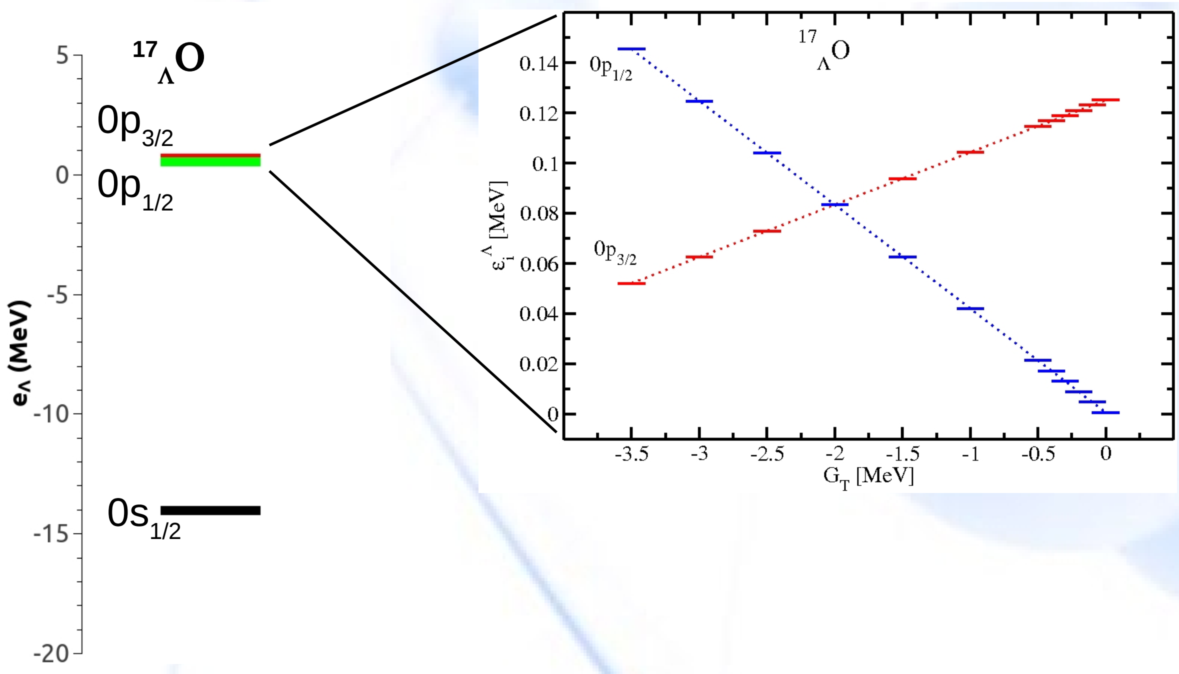
cut-off λ = 550 MeV

to solve problems – highly desirable to improve **ΛN** interaction: χ **NLO ΛN** interaction

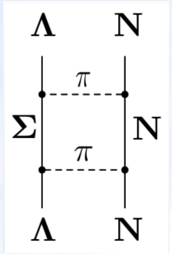
χ **NLO ΛN** includes the **tensor term** which may solve problem of **0p$_{3/2}$** & **0p$_{1/2}$**

documented by **toy model calculation** with purely **phenomenological tensor** term

$$\hat{V}_T = G_T \hat{S}_{12} \quad \text{where} \quad \hat{S}_{12} \equiv \frac{3}{r^2}(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2$$



also necessity to **implement Λ–Σ mixing**



Outlook

next goals:

repeat **HF + TDA** calculations with **NNN** interaction in bigger configuration space (convergence)

repeated applications of **EMPM** in nuclei (**ground state correlations, low lying spectra**, etc.) with **normal ordered** 2-body terms from **NNN** force

extension of **EMPM** formalism on **hypernuclei**

improvment of interaction: **χ NLO ΛN** force, **Λ-Σ** mixing, **ΛNN** interaction

possibly calculations of electro**production** of **hypernuclei**

^{40}Ca (e,e‘ K^{+}) $^{40}{}_{\Lambda}$K

^{48}Ca (e,e‘ K^{+}) $^{48}{}_{\Lambda}$K

List of Collaborators

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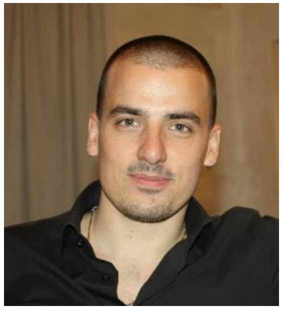


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Thank you!!