# Nature of the near-degenerate bands in PRM: transverse wobbling bands?

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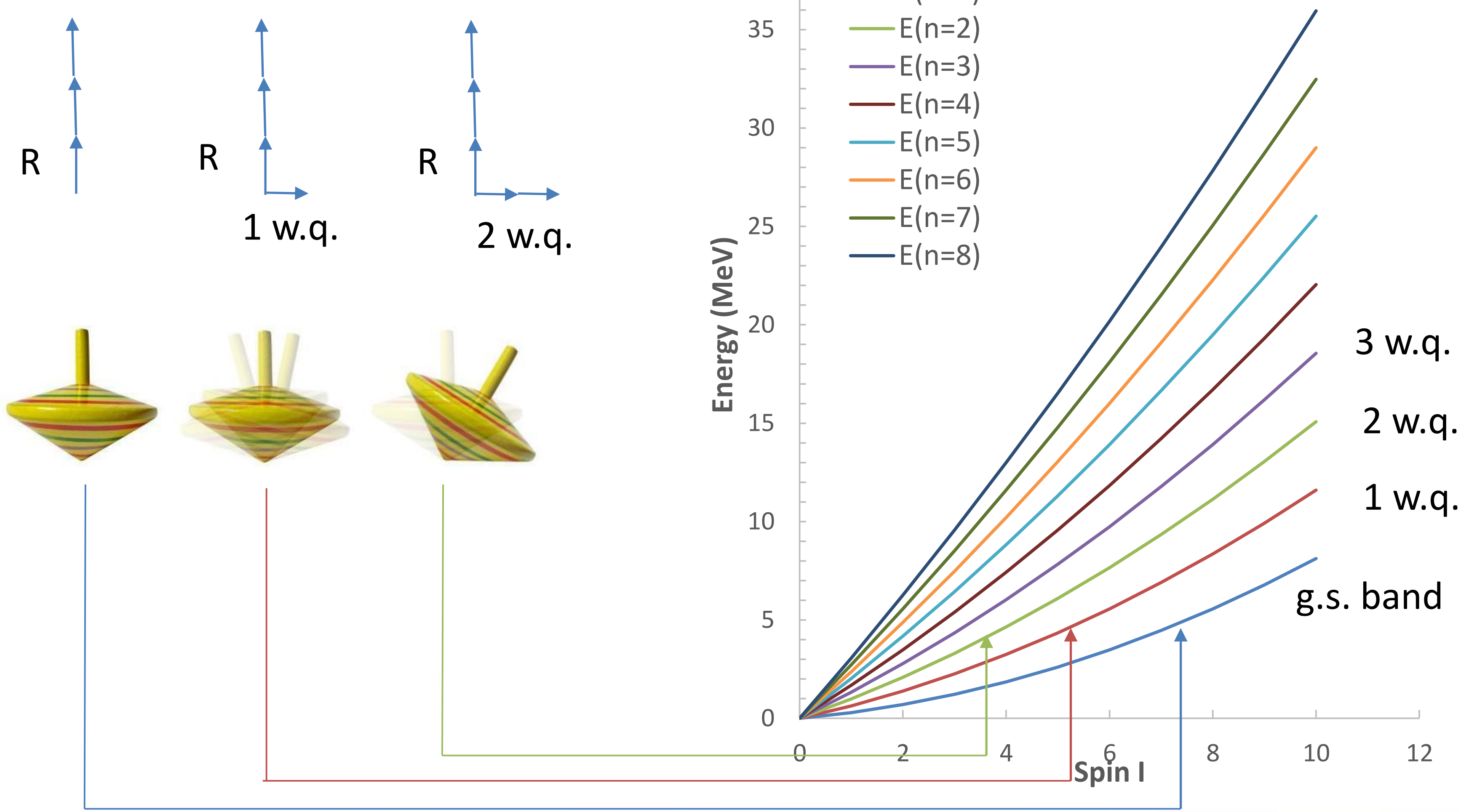
# Wobbling around the axis with largest MOI in triaxial even-even nuclei

Bohr & Mottelson

wobbling with A1 = 1, A2 = 4, A3 = 4

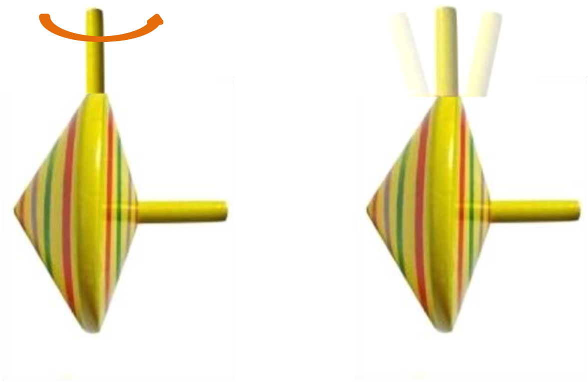



Approximation valid if $I_2^2 + I_3^2 \ll I^2$

good approximation **at high spins only**

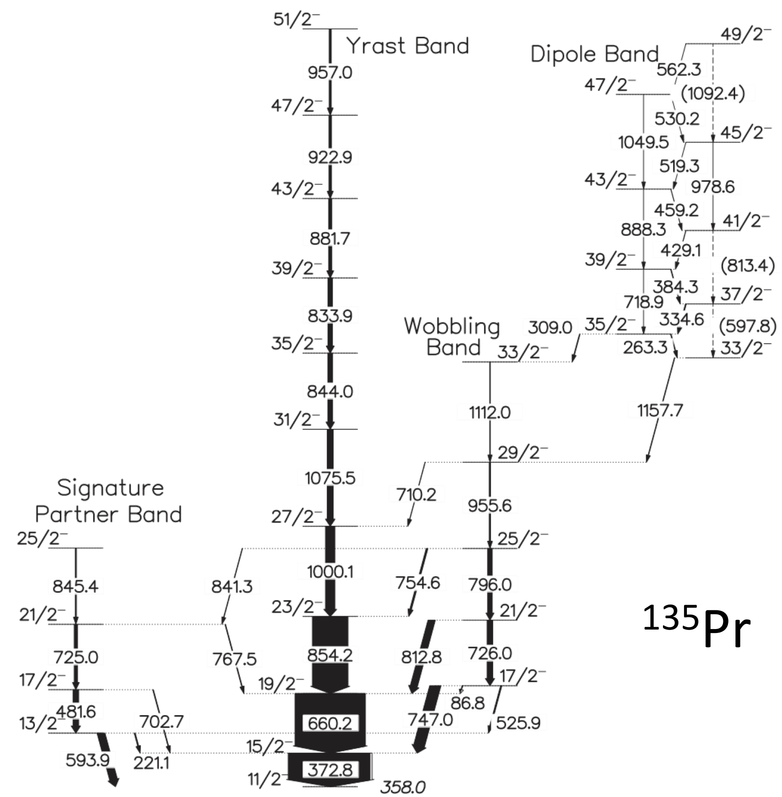
# Transverse wobbling - wobbling around an axis with medium MOI

S. Frauendorf and F. Dönau, Phys. Rev. C 89, 014322 (2014).
J. T. Matta et al., Phys. Rev. Lett. 114 (2015) 082501



Where?
→ in odd nuclei
→ one qp with large spin, e.g. $h_{11/2}$
→ triaxial shape

How to identify it?
→ large mixing ratios on the linking transitions
→ decreasing wobbling energy



## Transverse wobbling - wobbling around an axis with medium MOI, 3-axis, $A_1 < A_3 < A_2$

S. Frauendorf and F. Dönau, Phys. Rev. C 89, 014322 (2014).

$H = A_3 (I_3 - j)^2 + A_1 I_1^2 + A_2 I_2^2 = A_3 (I - j)^2 + H'$

$H' = (A_1 - A_3') I_1^2 + (A_2 - A_3') I_2^2 \approx \alpha (n+1/2) + 1/2\beta(c^+c^+ + cc)$, where $A_3' = A_3(1-j/I)$

$E(n,I) = A_3 I(I+1) + (n+1/2)\hbar\omega$

$\hbar\omega = (\alpha^2 - \beta^2)^{1/2} = 2I[(A_1-A_3')(A_2-A_3')]^{1/2}$ → decreasing with I

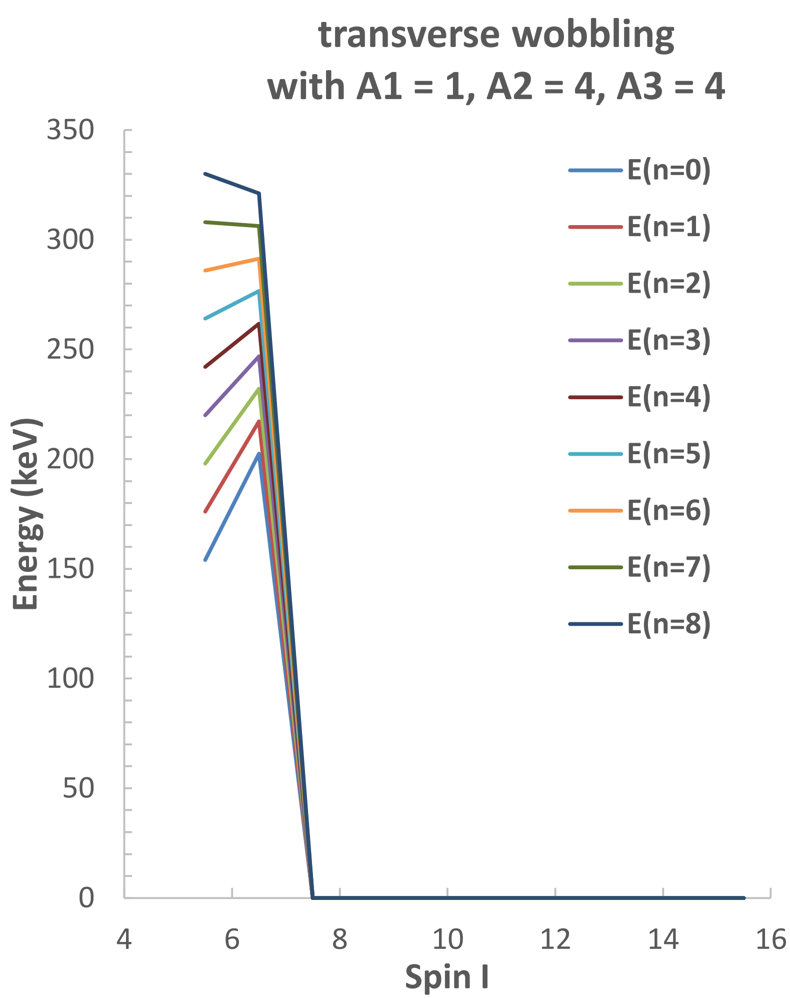
$$B(E2, n, I \rightarrow n, I \pm 2) = \frac{5}{16\pi} e^2 \frac{n}{I} Q_2{}^2$$

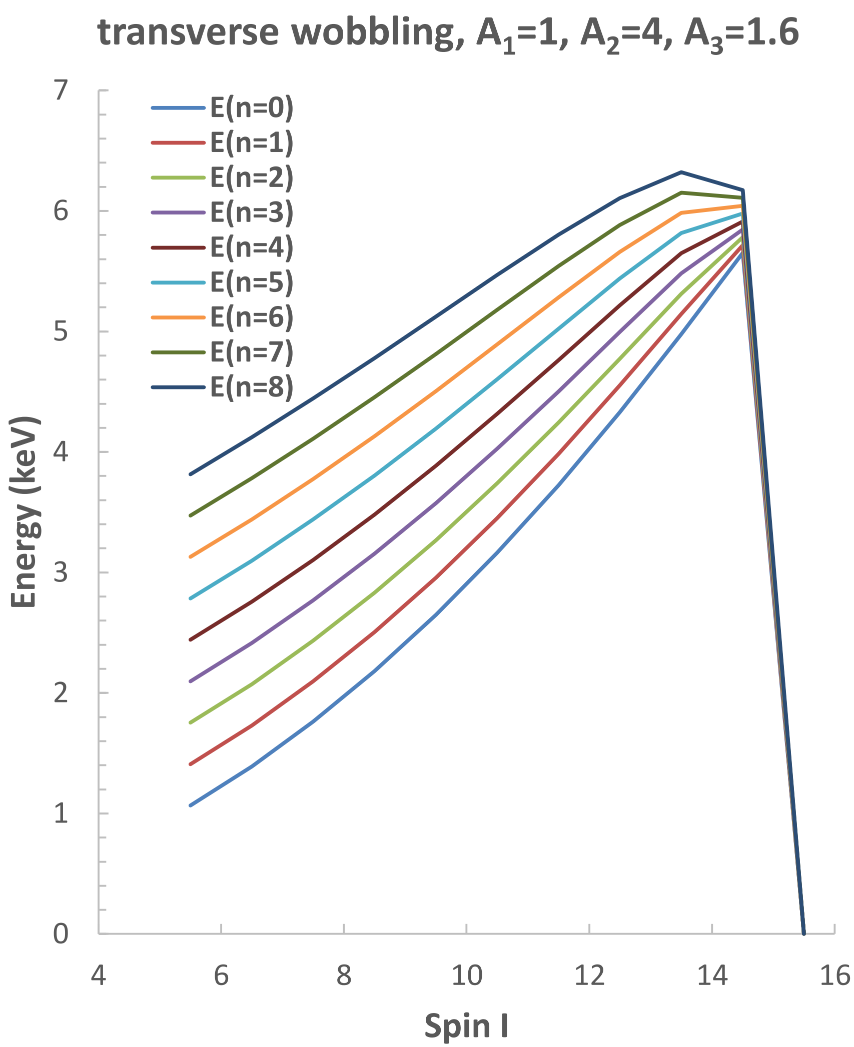
$$B(E2, n, I \rightarrow n-1, I-1) = \frac{5}{16\pi} e^2 \frac{n}{I} (\sqrt{3} Q_o x - \sqrt{2} Q_2 y)^2$$ → large

$$B(M1, n, I \rightarrow n-1, I-1) = \frac{3}{4\pi} \frac{n}{I} \left[j(g_j - g_R)x\right]^2;$$

Wobbling approximation is valid if:

1) frozen particle angular momentum
2) $A_1 - A_3' = A_1 - A_3(1 - j/I) > 0$ limit at $I_{max} < j A_3/(A_3-A_1)$
3) $I_1^2 + I_2^2 << I^2$ or $(2n+1) (A_2 + A_1 - 2A_3') /[(A_1-A_3')^{1/2}(A_2 - A_3')^{1/2}] << I$
   or $f = (2n+1) (A_2 + A_1 - 2A_3') /[(A_1-A_3')^{1/2}(A_2-A_3')^{1/2}] /I << 1$,




Irrotational MOI, $\gamma = 30^{o}$ → wobbling approximation crashes at $I_{max} < 15/2$

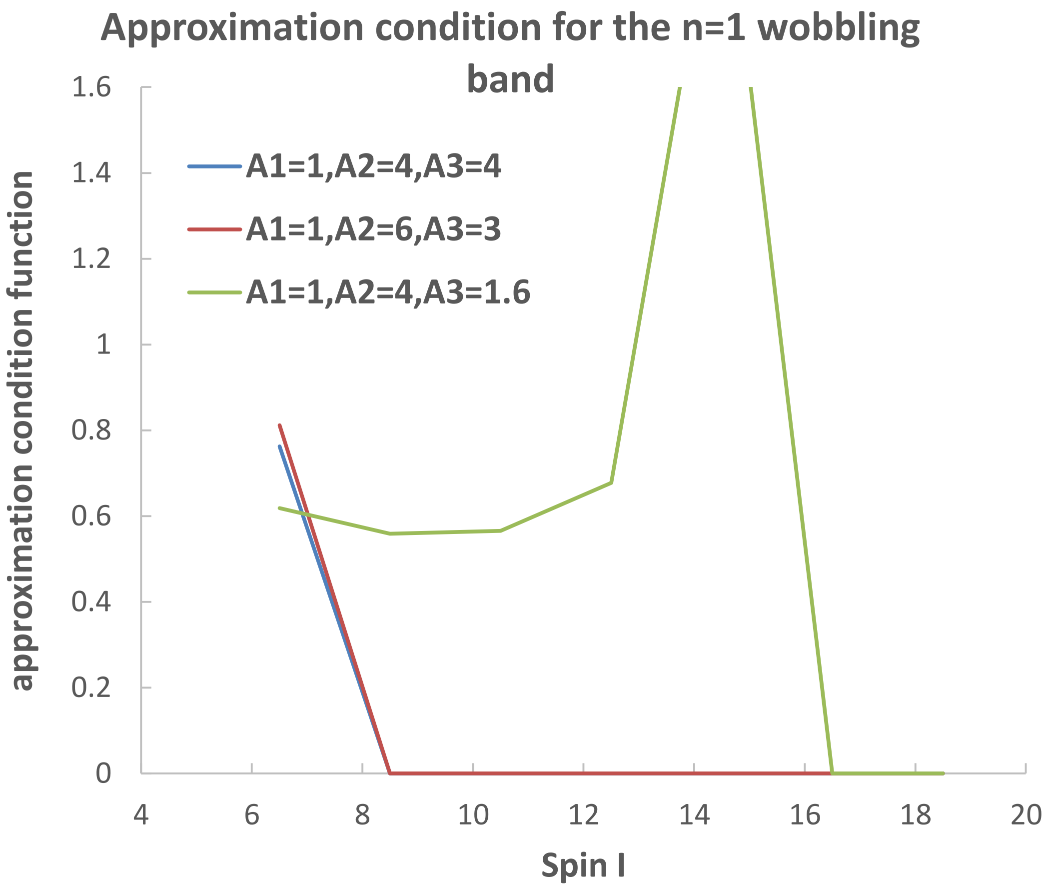
by an increase of the MOI along the 3-axis (by a factor of 2.5) $I_{max} < 31/2$



S. Frauendorf and F. Dönau, Phys. Rev. C 89, 014322 (2014).

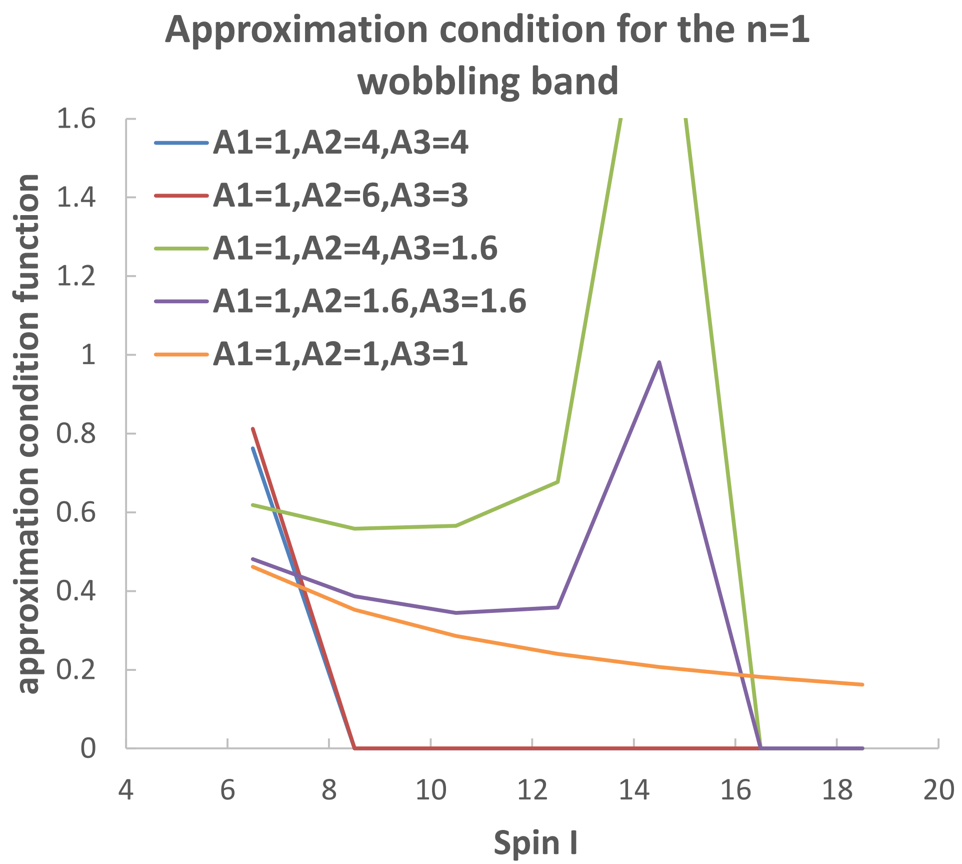
# Approximation condition for the harmonic wobbling description in PRM

$$f = (2n+1)\,(A_2 + A_1 - 2A_3')\,/[2(A_1-A_3')^{1/2}(A_2-A_3')^{1/2}]\,/I << 1$$



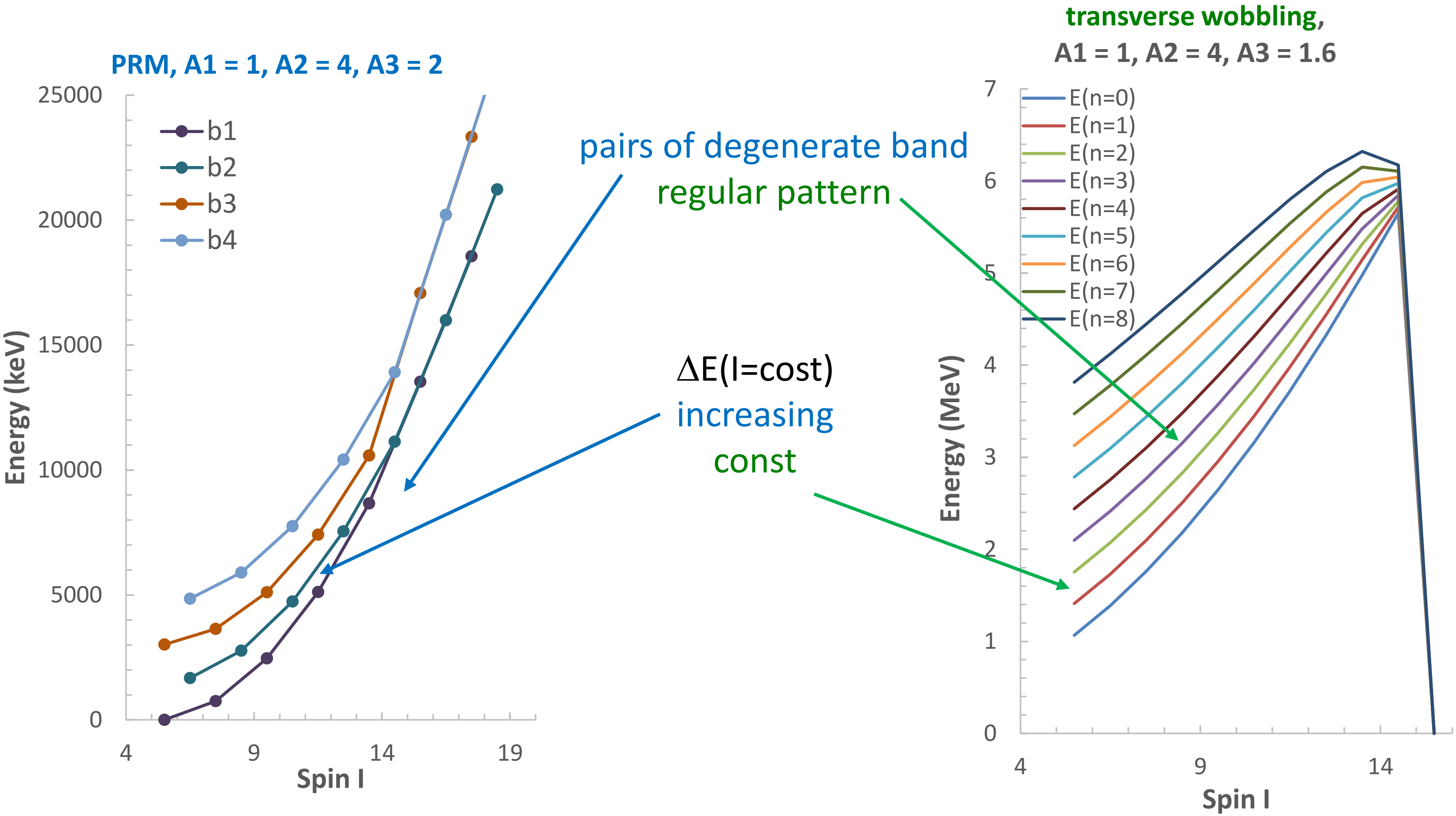
# Approximation condition for the harmonic wobbling description in PRM

$$f = (2n+1)\,(A_2 + A_1 - 2A_3')\,/[2(A_1-A_3')^{1/2}(A_2-A_3')^{1/2}]\,/I \ll 1$$

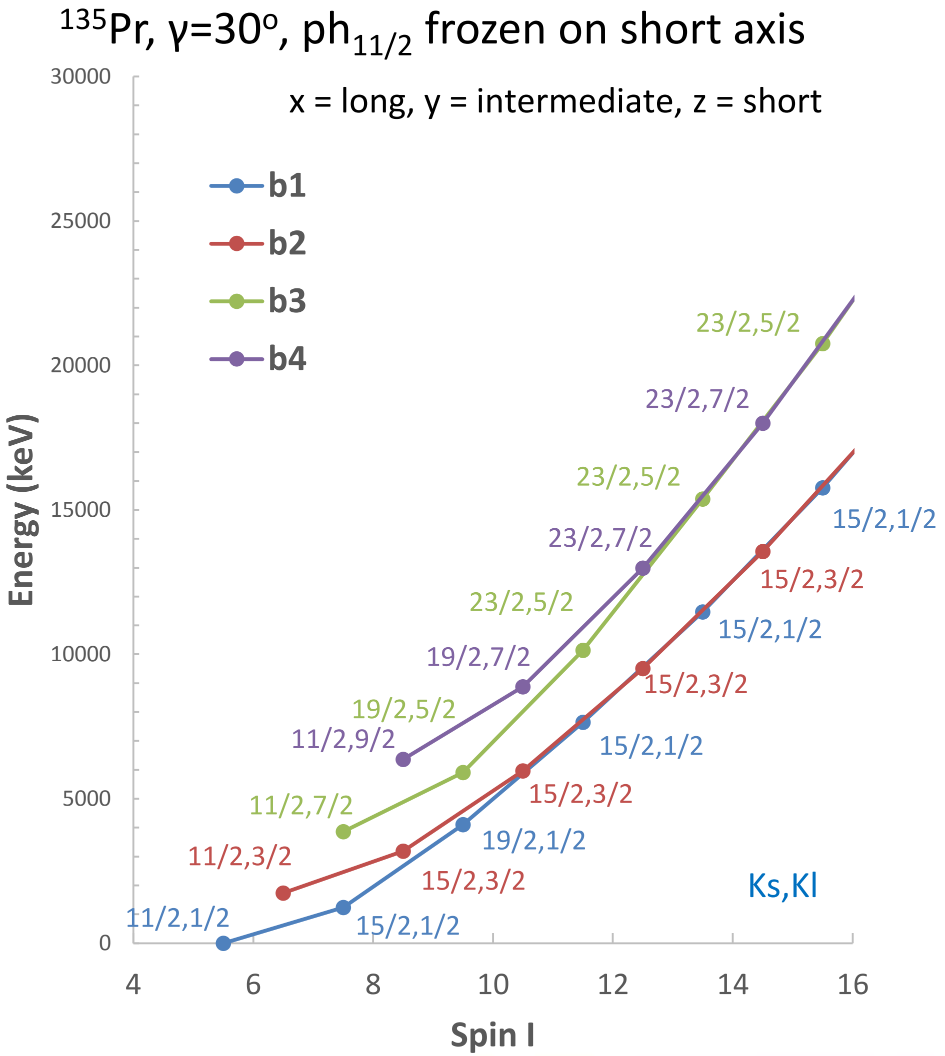



PRM ≈ transverse wobbling → **bad approximation!**

# PRM ← → wobbling
# excitation energy



# Particle – rotor model interpretation of the bands




→ "degeneracy" region

Ks = const, Kl=const, Ki ↗

deformation alignment

the two bands are signature partners

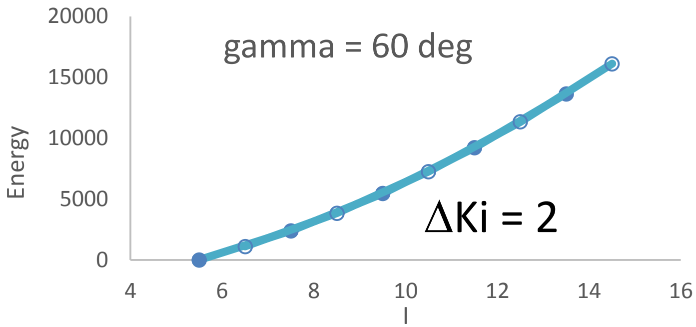
→ low-spin region

Ks ↗, Ki ↗

rotation alignment

the two bands are the favourite and unfavourite rotation aligned bands deformation alignment

Ki = Kl ↗



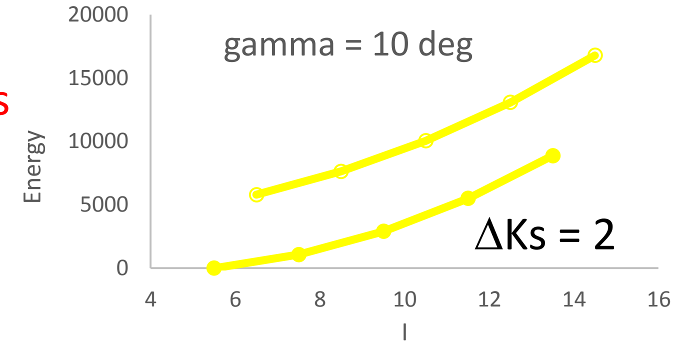

# Evolution of the rotation & deformation alignment with gamma

j frozen on the short axis

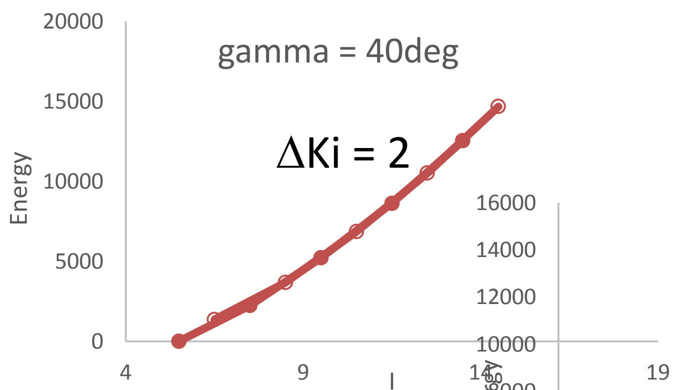
**competition between deformation alignment and rotation alignment**

gamma = 0 deg, Ki = Ks ↗

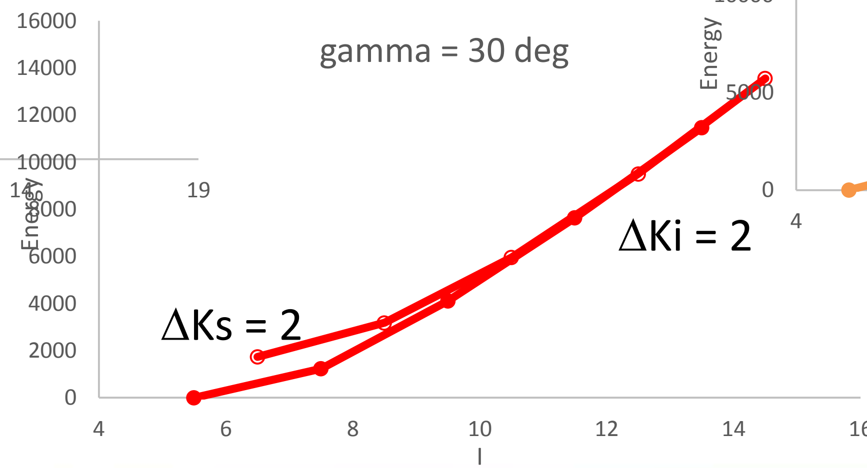
rotation alignment



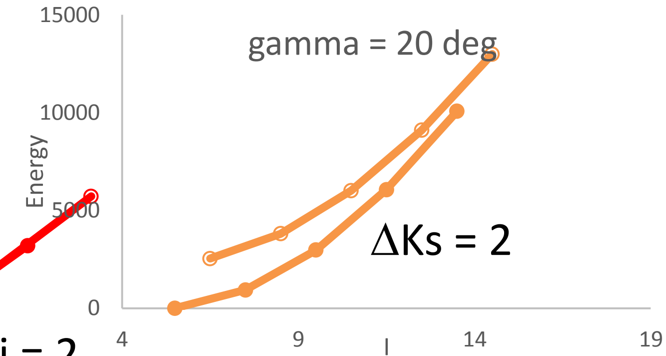

Ki ↗, Kl ↗, Ks ↗




Ki ↗, Kl = Ks ↗

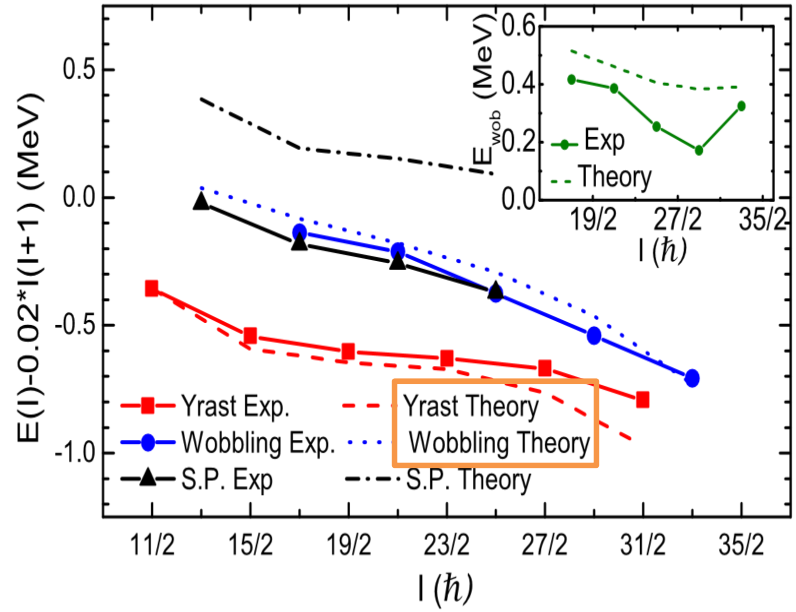



Ki ↗, Ks ↗, Kl ↗



Warning!

If wobbling is claimed
→ use the wobbling equations for the energy and the transition probabilities!




| Initial $I^\pi$ | Final $I^\pi$ | $E_\gamma$ (keV) | $\delta$ | Asymmetry | E2 Fraction (%) | $\frac{B(M1_{out})}{B(E2_{in})}\left(\frac{\mu_N^2}{e^2b^2}\right)$ Experiment | QTR | $\frac{B(E2_{out})}{B(E2_{in})}$ Experiment | QTR |
|---|---|---|---|---|---|---|---|---|---|
| $\frac{17}{2}^-$ | $\frac{15}{2}^-$ | 747.0 | $-1.24 \pm 0.13$ | $0.047 \pm 0.012$ | $60.6 \pm 5.1$ | ... | 0.213 | ... | 0.908 |
| $\frac{21}{2}^-$ | $\frac{19}{2}^-$ | 812.8 | $-1.54 \pm 0.09$ | $0.054 \pm 0.034$ | $70.3 \pm 2.$ | $.164 \pm 0.014$ | 0.107 | $0.843 \pm 0.032$ | 0.488 |
| $\frac{25}{2}^-$ | $\frac{23}{2}^-$ | 754.6 | $-2.38 \pm 0.37$ | ... | $85.0 \pm 4.0$ | $0.035 \pm 0.009$ | 0.070 | $0.500 \pm 0.025$ | 0.290 |
| $\frac{29}{2}^-$ | $\frac{27}{2}^-$ | 710.2 | ... | ... | ... | $\leq 0.016 \pm 0.004$ | 0.056 | $\geq 0.261 \pm 0.014$ | 0.191 |
| $\frac{13}{2}^-$ | $\frac{11}{2}^-$ | 593.9 | $-0.16 \pm 0.04$ | $-0.092 \pm 0.023$ | $2.5 \pm 1.2$ | ... | ... | ... | ... |

J. T. Matta et al., Phys. Rev. Lett. 114 (2015) 082501

Good agreement with PRM **does not support wobbling, but the PRM interpretation**, i.e. competition of rotation & deformation alignment

# Summary

The wobbling approximation in PRM is a **bad approximation**,
i.e. it neglects terms that are not negligible.

Wobbling approximation and PRM describe different physics

PRM interpretation → competition of rotation and deformation aligned bands

To test the wobbling interpretation use the wobbling equations!

The experimental data in $^{135}$Pr is well described by PRM!
thus it is not transverse wobbling,
but evolution of rotation alignment towards deformation alignment!

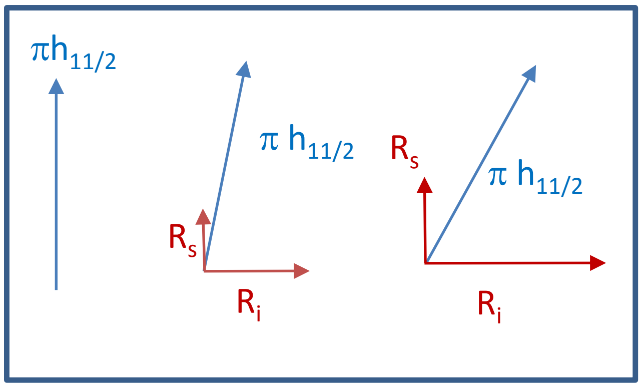
## Standard PRM

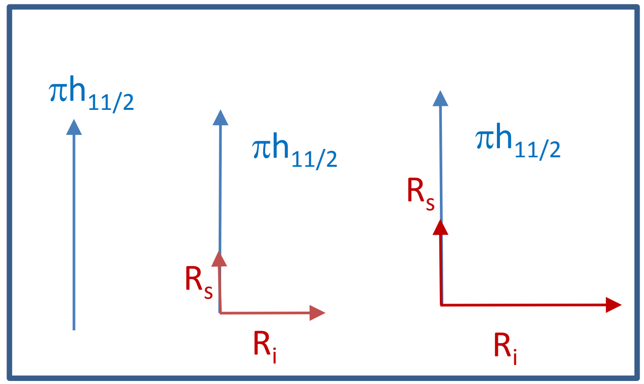
- ➢ $\pi h_{11/2}$ (free )
- $j_\pi$ on s-axis,
- max rotation along intermediate axis
- $j_\pi$ aligns

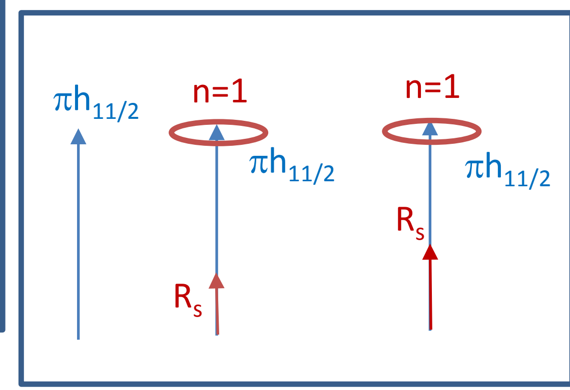
## Approximation 1 → ? Approximation 2

- ➢ $\pi h_{11/2}$ (frozen)
- $j_\pi$ on s-axis,
- max rotation along intermediate axis
- $j_\pi$ does not aligns

- ➢ rotations along the l- and i-axes are replaced by wobbling quantum excitations




**Wobbling approximation is valid if:**

1) $A_1 - A_3' = A_1 - A_3(1 - j/I) > 0$

since $A_1 < A_3 < A_2$ this is ok for $I < jA_3/(A_3-A_1)$, limit at $I_{max} < j\,A_3/(A_3-A_1)$

2) $I_1^2 + I_2^2 << I^2$ or $(2n+1)\,(A_2 + A_1 - 2A_3')\,/[(A_1-A_3')^{1/2}(A_2 - A_3')^{1/2}] << I$

or $f = (2n+1)\,(A_2 + A_1 - 2A_3')\,/[(A_1-A_3')^{1/2}(A_2-A_3')^{1/2}]\,/I << 1$, i.e. low I

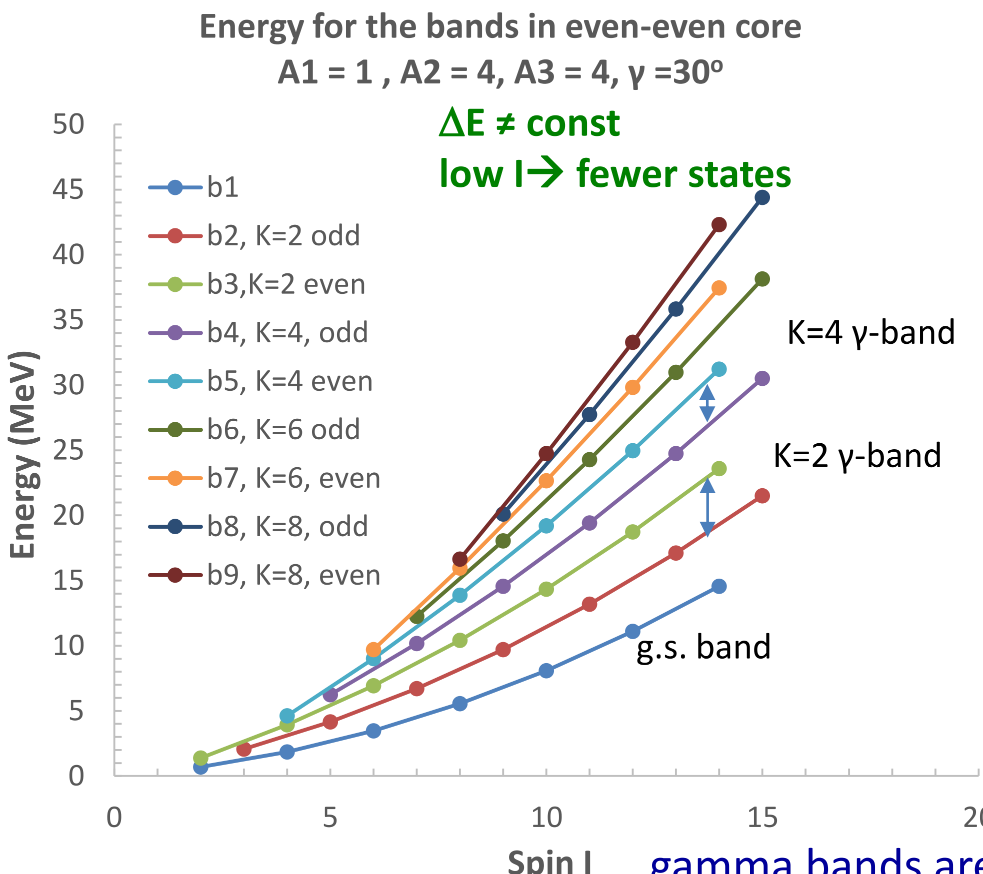


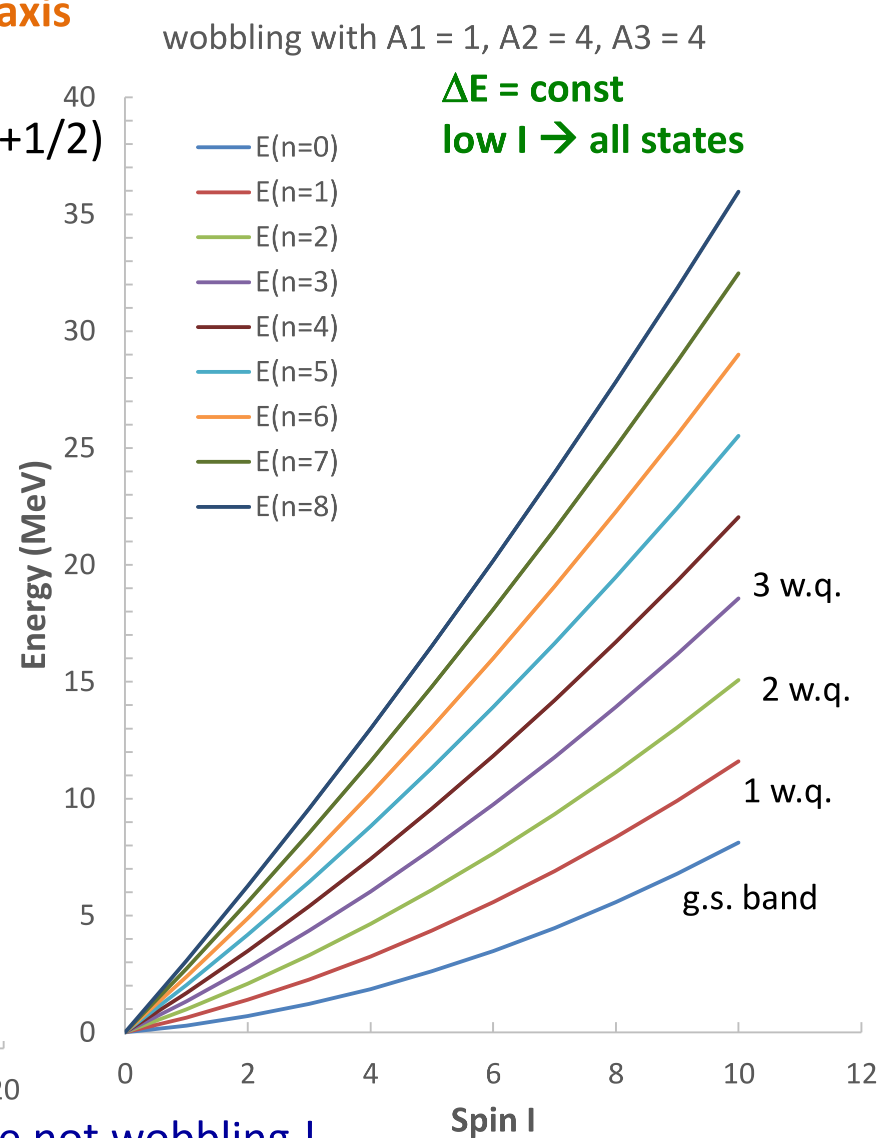
S. Frauendorf and F. Dönau, Phys. Rev. C 89, 014322 (2014).

# Wobbling around the axis with largest MOI – 1-axis

$$H = A_1 I_1^2 + A_2 I_2^2 + A_3 I_3^2 = A_1 I^2 + H' \approx A_1 I^2 + \hbar\omega\,(n+1/2)$$

Bohr & Mottelson




gamma bands are not wobbling !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Approximation valid if $I_2^2 + I_3^2 \ll I^2$

good approximation **at high spins only**


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