Chiral basis for particle-rotor coupling model for odd-odd nuclei

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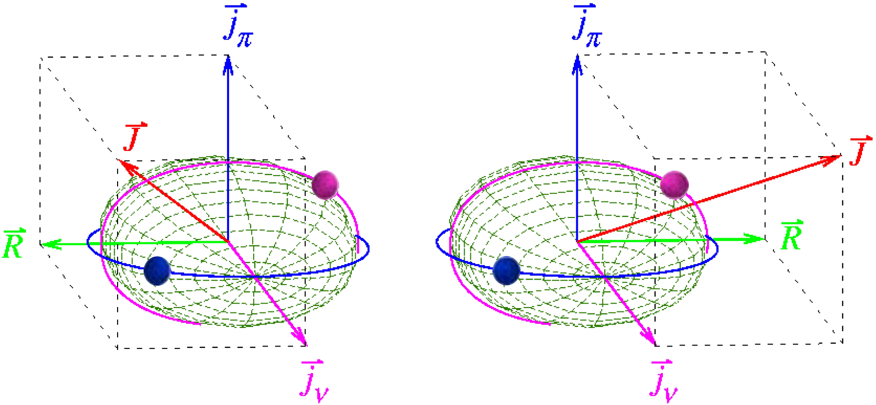
November 6, 2017

Chirality in atomic nuclei

Proposed by S. Frauendorf and J. Meng in Nucl. Phys. **A617**, 131 (1997).



Details of the derivation

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Invited Comment

Nuclear chirality, a model and the data

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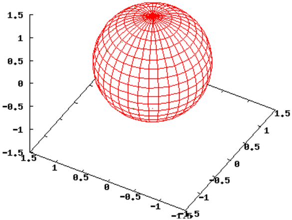
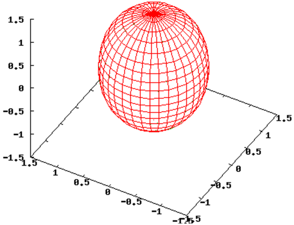
Abstract

In the last decade, the manifestation of chirality in atomic nuclei has become the subject of numerous experimental and theoretical studies. The common feature of current model calculations is that the chiral geometry of angular momentum coupling is extracted from expectation values of orientation operators, rather than being a starting point in construction of a model. However, using the particle–hole coupling model for triaxial odd–odd nuclei it is possible to construct a basis which contains right-handed, left-handed and planar states of angular momentum coupling. If this basis is used, the chirality is an explicit rather than an extracted feature as in any other models with non-chiral bases.

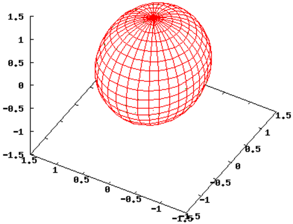
Shapes



spherical

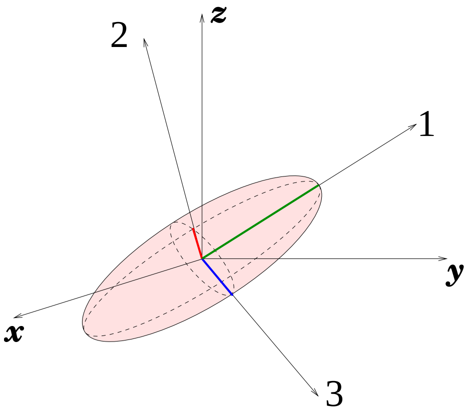


axial



triaxial

Laboratory/intrinsic reference frame



$$|RM\rangle = \sum_{K=-R}^{K=R} D^R_{MK}(\omega)\,|RK\rangle$$

R angular momentum

M z-projection of R

K 3-projection of R

$\omega = (\phi, \theta, \psi)$ Euler angles

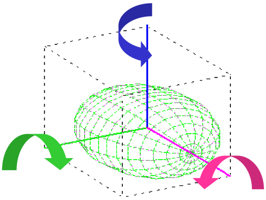
Hamiltonian for a rotor

$$H_R = \frac{\hbar^2}{2}\left(\frac{R_1^2}{J_1}+\frac{R_2^2}{J_2}+\frac{R_3^2}{J_3}\right)$$

R_i for i=1, 2, 3 are rotor's angular momentum components

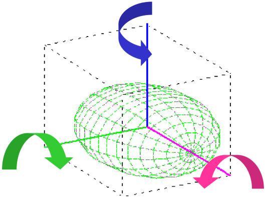
J_i for i=1, 2, 3 are rotor's principle-axes moments of inertia

D_2 symmetry of the Hamiltonian



$$\begin{aligned}\mathcal{R}_\kappa^2(\pi) &= \mathbb{1}, \quad \kappa = 1,2,3 \\ \left[R_k^2, \mathcal{R}_\kappa(\pi)\right] &= 0, \quad k = 1,2,3, \quad \kappa = 1,2,3 \\ [H_R, \mathcal{R}_\kappa(\pi)] &= 0, \quad \kappa = 1,2,3\end{aligned}$$

D_2 symmetric wave functions

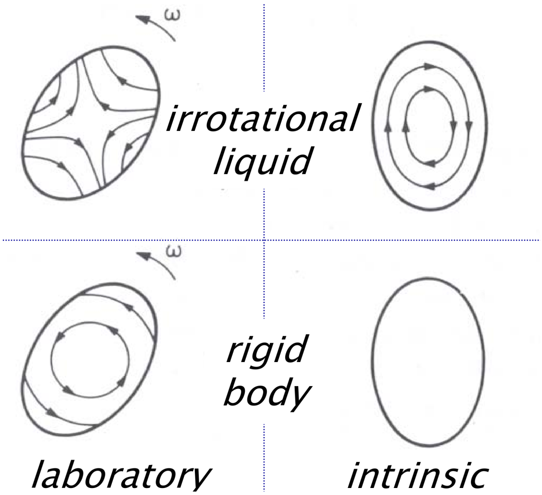


$$\hat{S} = \mathbb{1} + \mathcal{R}_1(\pi) + \mathcal{R}_2(\pi) + \mathcal{R}_3(\pi)$$

$$|RMK\rangle = \frac{1}{N}\hat{S} D^R_{MK}(\omega)$$

$$|RMK\rangle = \frac{1+(-1)^K}{2}\frac{1}{\sqrt{2(1+\delta_{K,0})}} \times \sqrt{\frac{2R+1}{8\pi^2}}\left(D^R_{MK}(\omega) + (-1)^R D^R_{M\overline{K}}(\omega)\right)$$

Irrotational flow moments of inertia



A. Bohr and B Mottelson, *Nuclear Structure* vol. II

Quadrupole deformation

$$
\begin{aligned}
r(\beta,\gamma,\theta,\phi) &= r_0\left\{1+\beta\left[\cos\gamma\, Y_{2,0}(\theta,\phi)\right.\right. \\
&+ \left.\left.\frac{1}{\sqrt{2}}\sin\gamma\left(Y_{2,2}(\theta,\phi)+Y_{2,-2}(\theta,\phi)\right)\right]\right\}.
\end{aligned}
$$

$$
r_k = r_0\left(1+\sqrt{\frac{5}{4\pi}}\beta\cos(\gamma-k\cdot 120^\circ)\right), k=1,2,3.
$$

Irrotational flow moments of inertia

$$J_k = 4B\beta^2 \sin^2(\gamma - k \cdot 120^\circ)$$

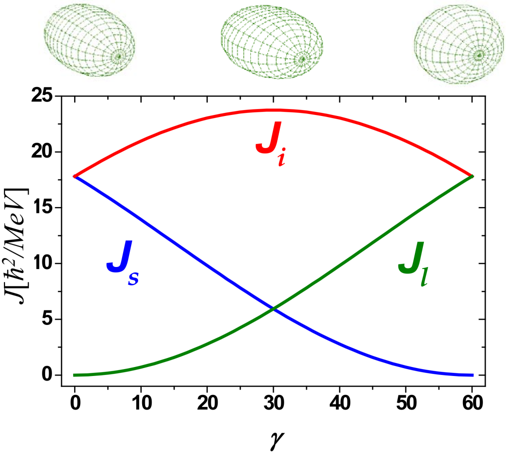
$$= J_0 \sin^2(\gamma - k \cdot 120^\circ)$$

$$k = 1, 2, 3$$

= long,

short,

intermediate



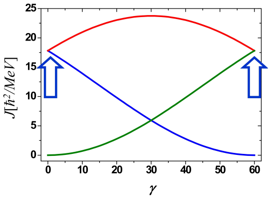
A. Bohr and B Mottelson, *Nuclear Structure* vol. II

Axial $\gamma = 0^\circ$ or $\gamma = 60^\circ$ quadrupole deformation

$$
\begin{aligned}
r(\beta,\gamma,\theta,\phi) &= r_0\left\{1+\beta Y_{2,0}(\theta,\phi)\right\} \\
&= r_0\left\{1+\sqrt{\frac{5}{16\pi}}\beta(3\cos^2\theta-1)\right\}
\end{aligned}
$$

$$
\begin{aligned}
r_1 = r_2 &= r_0\left\{1-\sqrt{\frac{5}{16\pi}}\beta\right\} \\
r_3 &= r_0\left\{1+2\sqrt{\frac{5}{16\pi}}\beta\right\}
\end{aligned}
$$

Axial rotation of an axial body

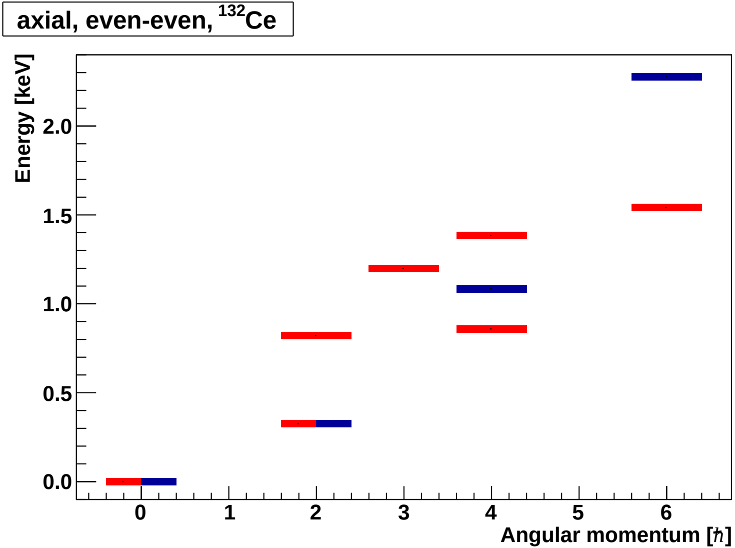


$$J_1 = J_2 = \frac{3}{4}J_0, \quad J_3 = 0$$

$$J_3 = 0 \Rightarrow R_3 = 0 \Rightarrow R^2 = R_1^2 + R_2^2 \Rightarrow K = 0$$

$$E_R = \frac{2\hbar^2}{3J_0}R(R+1) \text{ for even } R$$

Axial rotor fit to ^{132}Ce

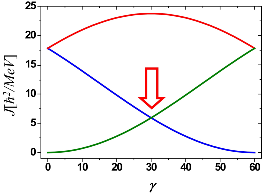


Triaxial $\gamma = 30^\circ$ or $\gamma = 90^\circ$ quadrupole deformation

$$
\begin{aligned}
r(\beta,\gamma,\theta,\phi) &= r_0\left\{1+\frac{\beta}{\sqrt{2}}\left[Y_{2,2}(\theta,\phi)+Y_{2,-2}(\theta,\phi)\right]\right\} \\
&= r_0\left\{1+\sqrt{\frac{15}{16\pi}}\beta\sin^2\theta\cos 2\phi\right\}
\end{aligned}
$$

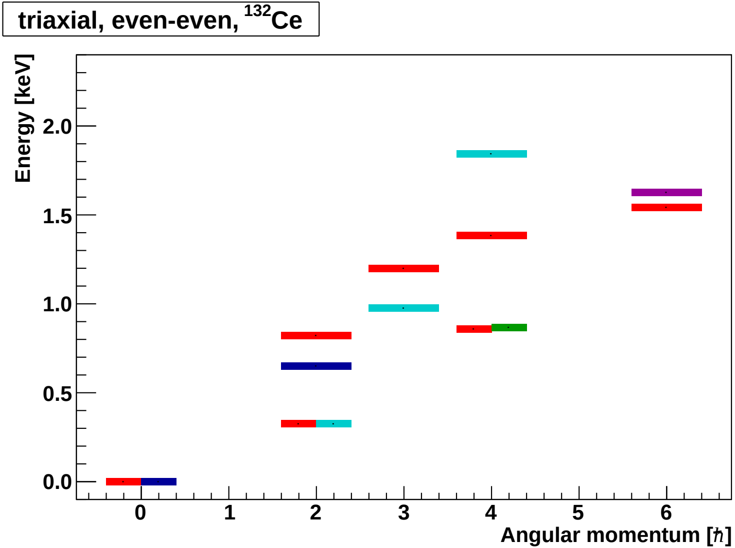
$$
\begin{aligned}
r_1 &= r_0\left\{1+\sqrt{\frac{15}{16\pi}}\beta\right\} \\
r_2 &= r_0\left\{1-\sqrt{\frac{15}{16\pi}}\beta\right\} \\
r_3 &= r_0
\end{aligned}
$$

Axial rotation of a triaxial body



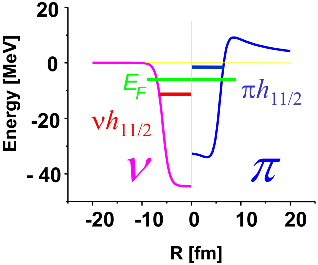
$$
\begin{aligned}
J_1 &= J_2 = \frac{1}{4}J_0, \;\; J_3 = J_0 \\
H_R &= \sum_{k=1}^{3} \frac{R_k^2 \hbar^2}{2J_k} = \frac{2\hbar^2}{J_0}((R_1^2 + R_2^2) + \frac{1}{4}R_3^2) = \\
& \frac{2\hbar^2}{J_0}((R^2 - R_3^2) + \frac{1}{4}R_3^2) = \frac{2\hbar^2}{J_0}(R^2 - \frac{3}{4}R_3^2) \\
E_{RK} &= \frac{2\hbar^2}{J_0}\left(R(R+1) - \frac{3}{4}K^2\right)
\end{aligned}
$$

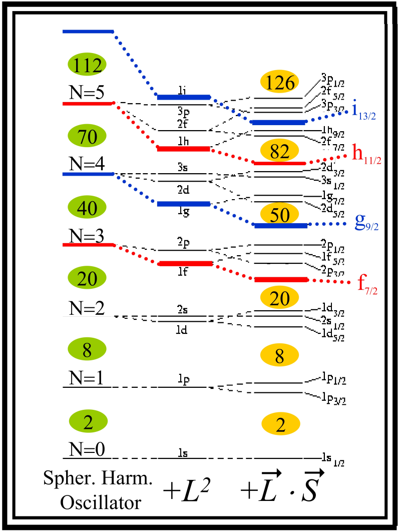
Trixial rotor fit to ^{132}Ce



Unique-parity orbitals in a spherical mean field

$$H_{SM} = V(r) + V_{LS}(r)\,\vec{L}\cdot\vec{S}$$





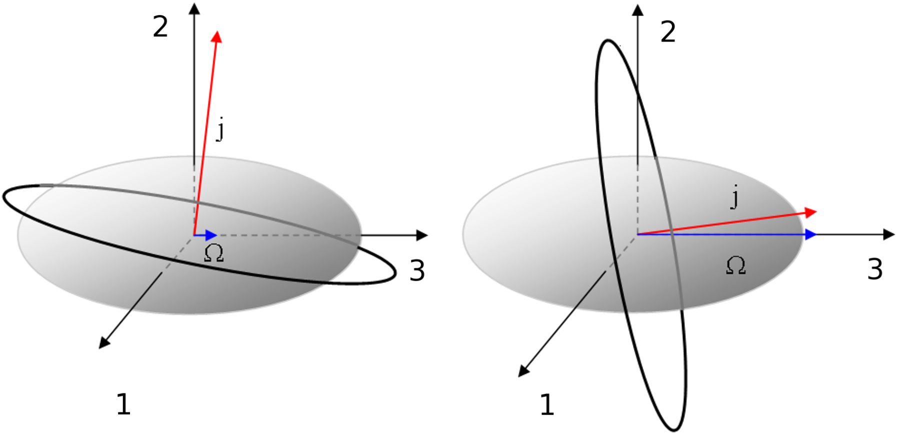
Deformed mean field

$$
\begin{aligned}
H_{sp} &= H_s + H_\beta \\
H_\beta(\beta,\gamma,r,\theta,\phi) &= -\kappa(r)\beta\left[\cos\gamma\, Y_{2,0}(\theta,\phi)\right. \\
&+ \left.\frac{1}{\sqrt{2}}\sin\gamma\left(Y_{2,2}(\theta,\phi)+Y_{2,-2}(\theta,\phi)\right)\right]
\end{aligned}
$$

Single-j approximation for a deformed mean field

$$
\begin{aligned}
j_0 &= j_3, \\
j_{\pm 1} &= \mp \frac{1}{\sqrt{2}} \left(j_1 \pm i j_2 \right) \\
\left\langle j, \Omega \middle| Y_{2,0} \middle| j, \Omega' \right\rangle &= -\frac{1}{4}\sqrt{\frac{5}{4\pi}} \frac{\left\langle j, \Omega \middle| 3 j_0^2 - j^2 \middle| j, \Omega' \right\rangle}{j(j+1)} \\
\left\langle j, \Omega \middle| Y_{2,\pm 2} \middle| j, \Omega' \right\rangle &= -\frac{1}{4}\sqrt{\frac{5}{4\pi}} \sqrt{3} \frac{\left\langle j, \Omega \middle| j_{\pm 1}^2 \middle| j, \Omega' \right\rangle}{j(j+1)}
\end{aligned}
$$

Spin alignment through the mean-field deformation



Single-j approximation in a deformed mean field

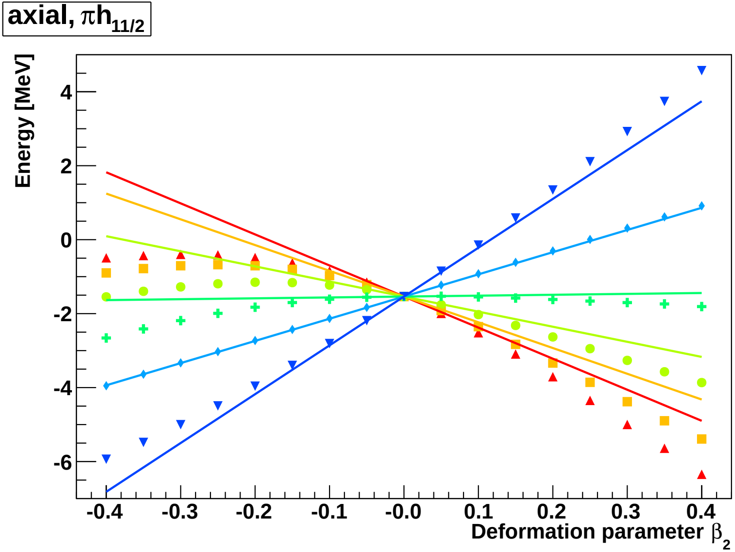
Axial deformation:

$$
\begin{aligned}
H_{sp} &= H_s + H_\beta = \\
&= H_s + \chi\beta(3j_0^2 - j^2)
\end{aligned}
$$

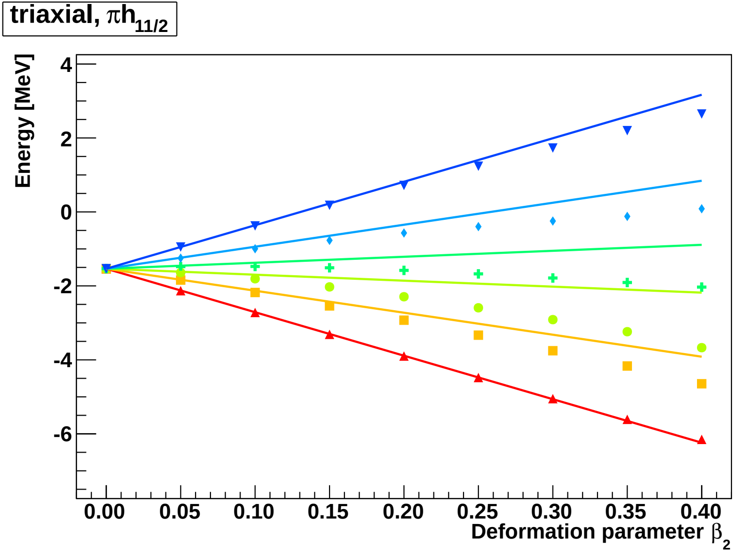
Triaxial deformation:

$$
\begin{aligned}
H_{sp} &= H_s + H_\beta = H_s + \chi\beta\sqrt{3}\left(j_{+1}^2 + j_{-1}^2\right) \\
&= H_s + \chi\beta\sqrt{3}\left(j_1^2 - j_2^2\right)
\end{aligned}
$$

Single-j approximation for $\pi h_{11/2}$ in axial potential



Single-j approximation for $\pi h_{11/2}$ in triaxial potential



Particle-rotor model

$$H = H_R + H_{sp} \text{ particle coupling}$$

$$H = H_R - H_{sp} \text{ hole coupling}$$

$$\vec{I} = \vec{R} + \vec{j}$$

Particle-rotor model

D_2 symmetric wave functions are

$$|IMKj\Omega\rangle = \frac{1+(-1)^{K-\Omega}}{2}\frac{1}{\sqrt{2}}\sqrt{\frac{2I+1}{8\pi^2}} \times$$

$$\left(D^I_{MK}(\omega)\,|j\Omega\rangle + (-1)^{I-j}D^I_{M\overline{K}}(\omega)\,|j\overline{\Omega}\rangle\right) =$$

$$\sqrt{\frac{2I+1}{16\pi^2}}\left(D^I_{MK}(\omega)\,|j\Omega\rangle + (-1)^{I-j}D^I_{M\overline{K}}(\omega)\,|j\overline{\Omega}\rangle\right),$$

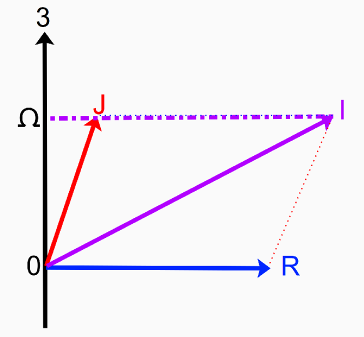
$$K_{\min} = {}^1\!/_2,\ K_{\max} = I,\ \Omega_{\min} = -j,\ \Omega_{\max} = j,$$

$K - \Omega$ even

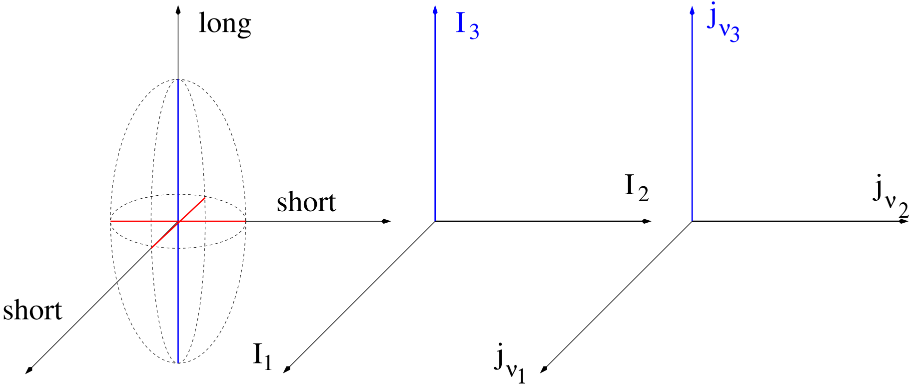
Particle-axial-rotor model

$$J_3 \quad = \quad 0 \Rightarrow R_3 = 0 \Rightarrow I_3 = j_3 \Rightarrow K = \Omega$$

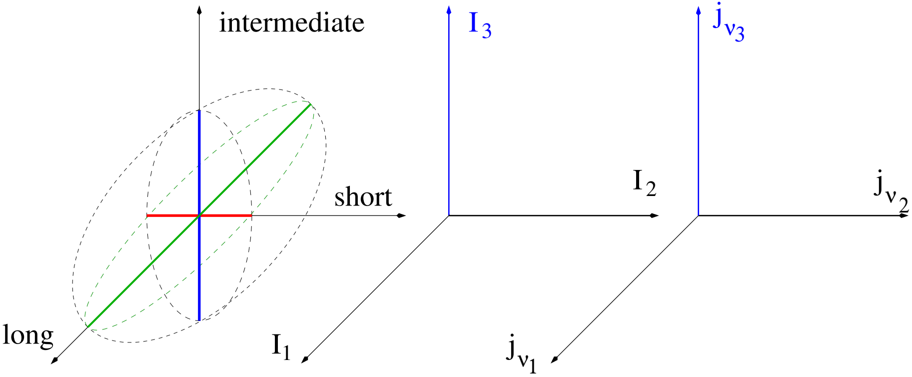
$$|IMj\Omega\rangle \quad = \quad \sqrt{\frac{2I+1}{16\pi^2}}\left(D^I_{M\Omega}(\omega)\,|j\Omega\rangle + (-1)^{I-j} D^I_{M\overline{\Omega}}(\omega)\,|j\overline{\Omega}\rangle\right)$$



Standard axial $\gamma = 0^\circ$ particle-rotor basis



Standard triaxial $\gamma = 90^\circ$ particle-rotor basis



Hole-triaxial-rotor model

$$H = H_d + H_p$$

$$H_d = \frac{2\hbar^2}{J_0}\left(I^2 + j^2 - \frac{3}{4}I_0^2\right) - \sqrt{3}\chi\beta j_1^2$$

H_d diagonal in the new basis

$$H_p = \frac{2\hbar^2}{J_0}\left(2I_{+1}j_{-1} + 2I_{-1}j_{+1} - \frac{1}{2}I_0 j_0 - \frac{3}{4}j_0^2\right) + \sqrt{3}\chi\beta j_2^2$$

H_p perturbing eigen states of H_p

Hole-triaxial-rotor model

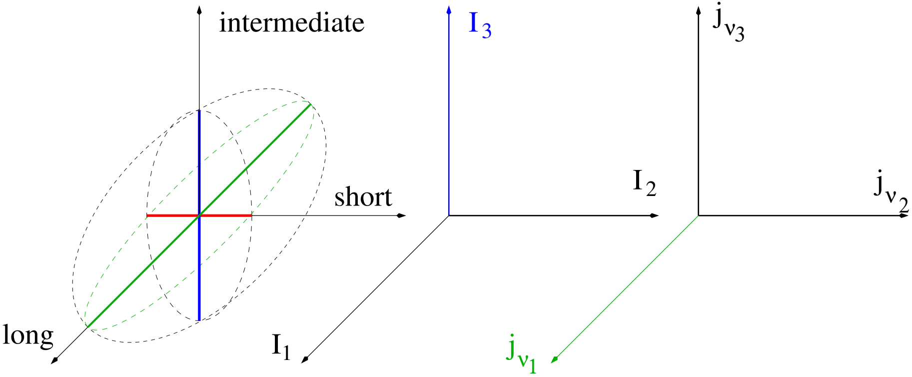
The new basis results from a $\pi/2$ rotation of standard single-hole states around the 2nd (short) axis

$$|IMKj\kappa\rangle = \sum_{\Omega} d^{j}_{\kappa\Omega}\left(-\frac{\pi}{2}\right)|IMKj\Omega\rangle$$

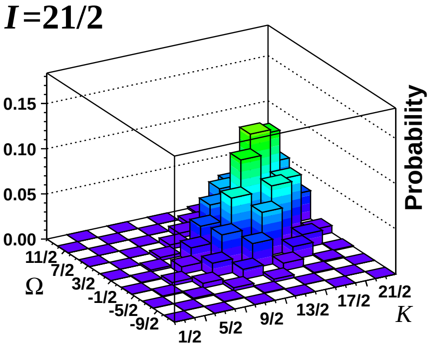
$$\begin{aligned}|IMKj\kappa\rangle &= \frac{1}{2}\sqrt{\frac{2I+1}{16\pi^2}}\Big(D^{I}_{MK}(\omega)\,|j\kappa\rangle \\ &+ (-1)^{I-\kappa}D^{I}_{M\overline{K}}(\omega)\,|j\kappa\rangle \\ &+ (-1)^{j+K}D^{I}_{MK}(\omega)\,|j\overline{\kappa}\rangle \\ &+ (-1)^{I+j-\kappa-K}D^{I}_{M\overline{K}}(\omega)\,|j\overline{\kappa}\rangle\Big), \\ & K>0,\ \kappa>0,\end{aligned}$$

κ projection of j on the long axis

New ("chiral") hole-rotor basis



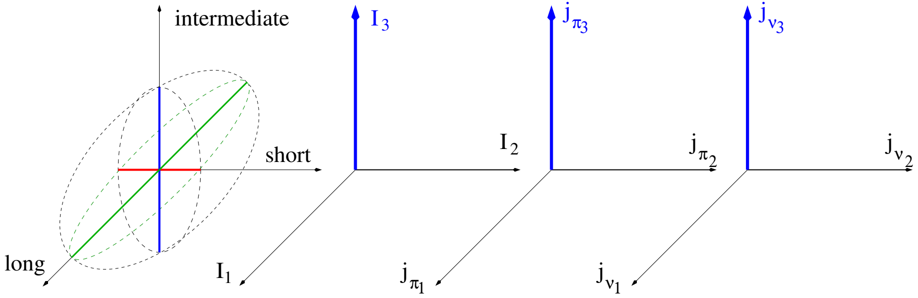
Wave function in the standard and in the chiral basis



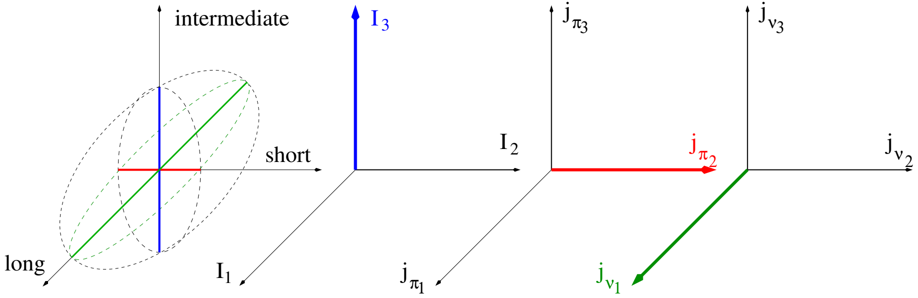
standard

chiral

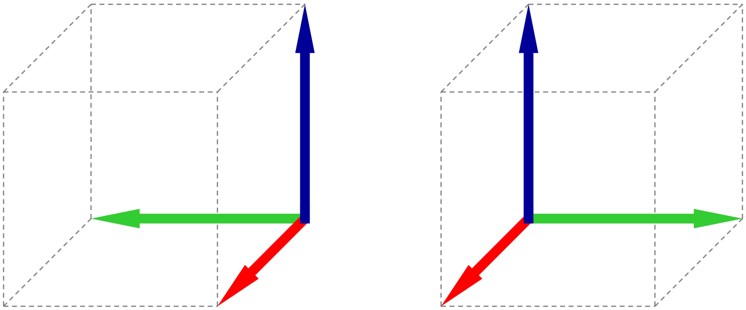
Standard particle-hole-rotor basis



Chiral particle-hole-rotor basis



Orthogonal coupling of three vectors



Particle-hole-triaxial-rotor model

The chiral basis results from a $^{\pi}/_{2}$ rotation of standard single-particle proton/neutron states around the 1st (long) axis and a $^{\pi}/_{2}$ rotation of standard single-particle neutron/proton states around the 2nd (short) axis

$$|j\kappa\rangle = \sum_{\Omega} d^{j}_{\kappa\Omega}\left(-^{\pi}/_{2}\right)|j\Omega\rangle$$

$$|j\varkappa\rangle = \sum_{\Omega} (-i)^{\varkappa-\Omega} d^{j}_{\varkappa\Omega}\left(-^{\pi}/_{2}\right)|j\Omega\rangle$$

$\kappa/\varkappa$ projection of j_{ν}/j_{π} on the long/short axis.

Correlations built into the basis prior to diagonalization.

Particle-hole-triaxial-rotor model chiral wave functions

$$
\begin{aligned}
|IMKj_\pi \varkappa_\pi j_\nu \kappa_\nu\rangle \; = \; & \frac{1}{2}\sqrt{\frac{2I+1}{8\pi^2}}\Big(D^I_{MK}(\omega)\,|j_\pi \varkappa_\pi\rangle\,|j_\nu \kappa_\nu\rangle \\
& + (-1)^{I+j_\pi+\kappa_\nu} D^I_{M\overline{K}}(\omega)\,|j_\pi \overline{\varkappa}_\pi\rangle\,|j_\nu \kappa_\nu\rangle \\
& + (-1)^{I-j_\nu+K-\varkappa_\pi-\kappa_\nu} D^I_{M\overline{K}}(\omega)\,|j_\pi \varkappa_\pi\rangle\,|j_\nu \overline{\kappa}_\nu\rangle \\
& + (-1)^{j_\pi+j_\nu+K-\varkappa_\pi} D^I_{MK}(\omega)\,|j_\pi \overline{\varkappa}_\pi\rangle\,|j_\nu \overline{\kappa}_\nu\rangle\Big)
\end{aligned}
$$

Particle-hole-triaxial-rotor model chiral wave functions

$$
\begin{aligned}
\left|IMKj_\pi \varkappa_\pi j_\nu \kappa_\nu\right\rangle &= \frac{1}{2}\sqrt{\frac{2I+1}{8\pi^2}}\Big(D^I_{MK}(\omega)\left|j_\pi \varkappa_\pi\right\rangle\left|j_\nu \kappa_\nu\right\rangle \\
&+ (-1)^{I+j_\pi+\kappa_\nu} D^I_{M\overline{K}}(\omega)\left|j_\pi \overline{\varkappa}_\pi\right\rangle\left|j_\nu \kappa_\nu\right\rangle \\
&+ (-1)^{I-j_\nu+K-\varkappa_\pi-\kappa_\nu} D^I_{M\overline{K}}(\omega)\left|j_\pi \varkappa_\pi\right\rangle\left|j_\nu \overline{\kappa}_\nu\right\rangle \\
&+ (-1)^{j_\pi+j_\nu+K-\varkappa_\pi} D^I_{MK}(\omega)\left|j_\pi \overline{\varkappa}_\pi\right\rangle\left|j_\nu \overline{\kappa}_\nu\right\rangle\Big)
\end{aligned}
$$

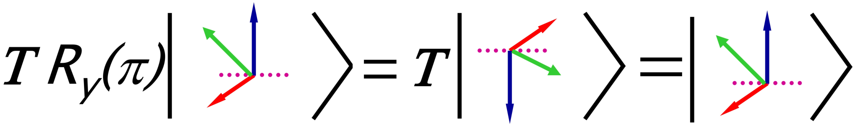
$K = 0$ or $\varkappa = \pm 1/2$ or $\kappa = 1/2$: planar states

$K > 0,\ \varkappa \in [3/2, j], \kappa \in [3/2, j]$: right-handed states

$K > 0,\ \varkappa \in [-j, -3/2], \kappa \in [3/2, j]$: left-handed states

Planar states

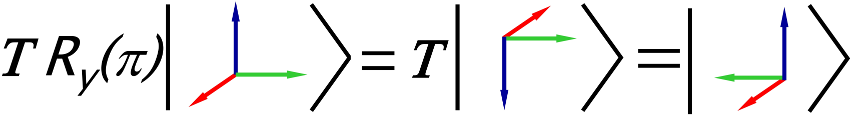
$$O\,|I\,P\rangle = T\,R_y(\pi)\,|I\,P\rangle = |I\,P\rangle\,.$$



Chiral states

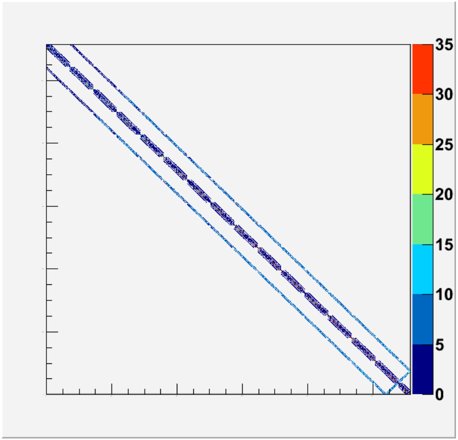
$$O\,|I\,R\rangle = T\,R_y(\pi)\,|I\,R\rangle = |I\,L\rangle\,,$$

$$O\,|I\,L\rangle = T\,R_y(\pi)\,|I\,L\rangle = |I\,R\rangle\,.$$

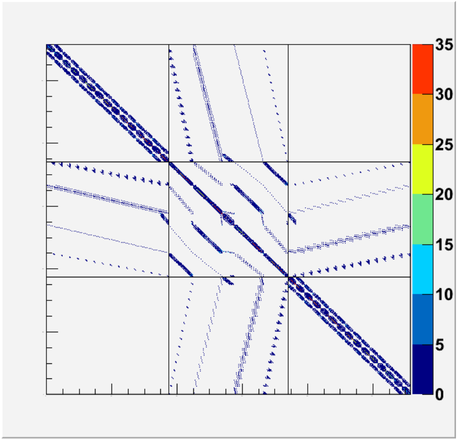


The Hamiltonian

standard

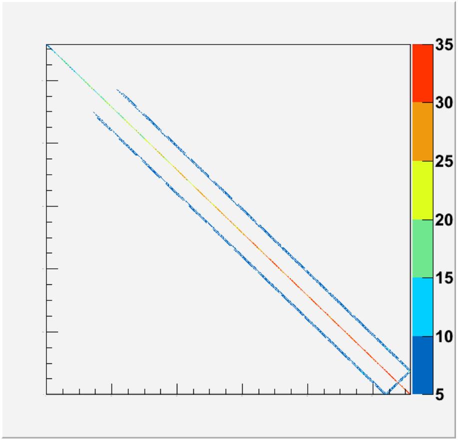


chiral

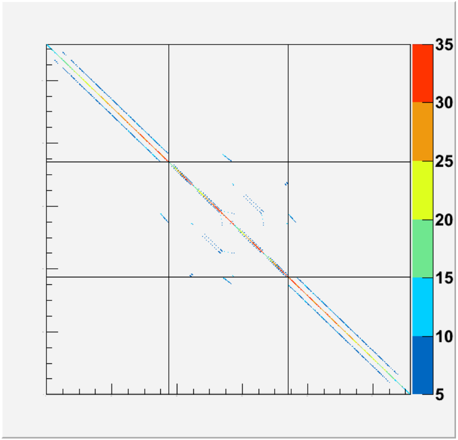


The Hamiltonian for ME > 5 MeV

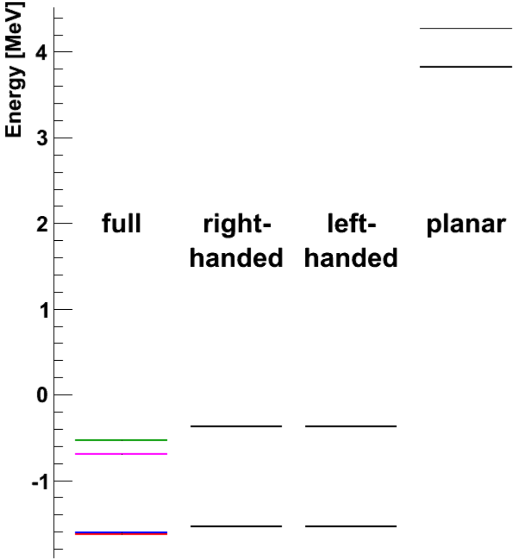
standard



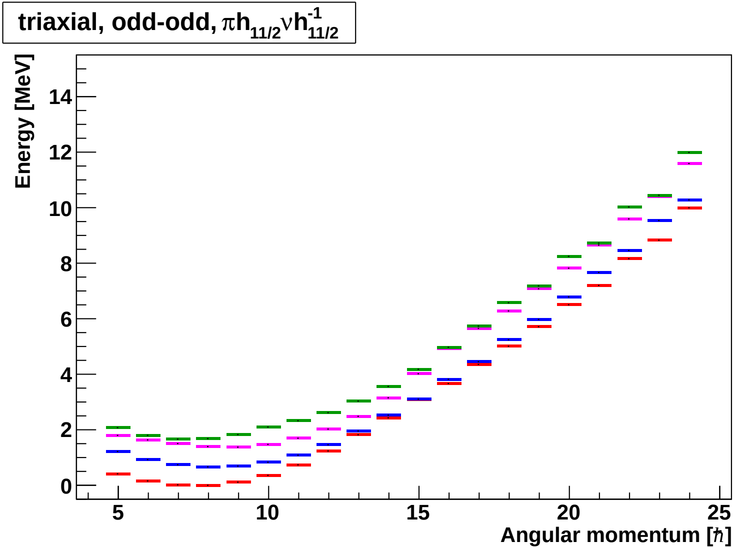
chiral



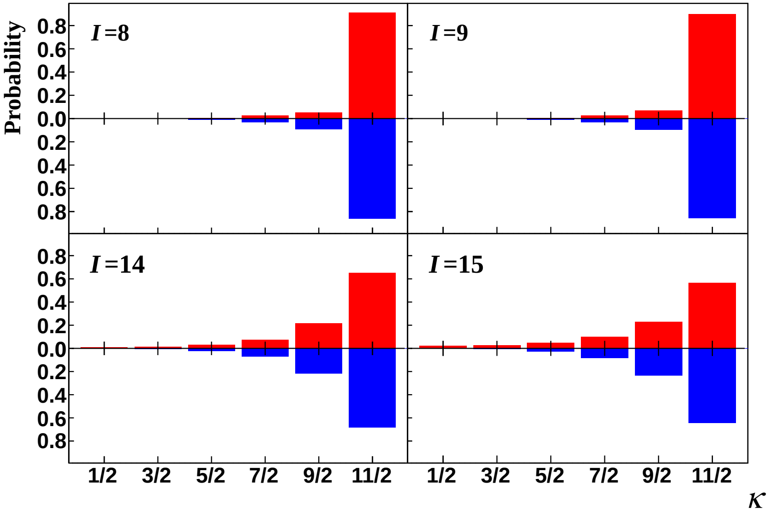
Eigen states of the Hamiltonian



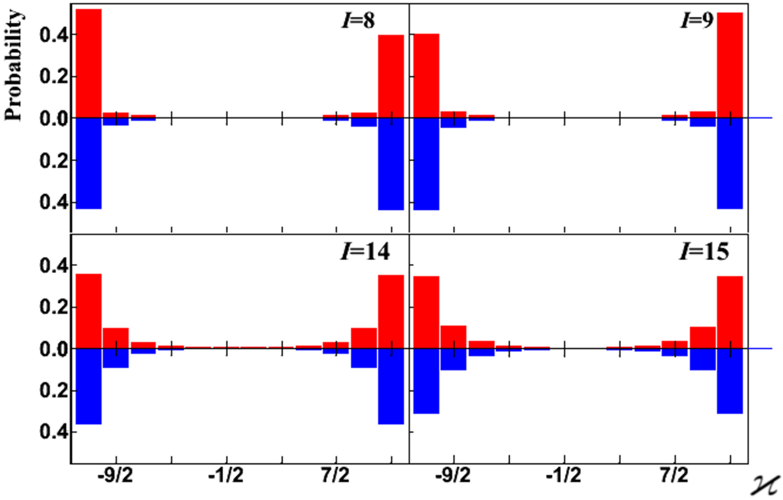
$\pi h_{11/2} \nu h_{11/2}^{-1}$ particle-hole-triaxial-rotor model



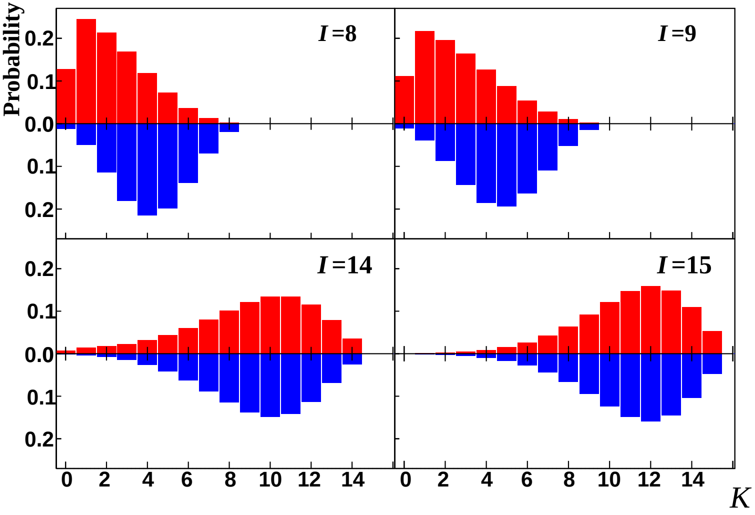
$\pi h_{11/2} \nu h_{11/2}^{-1}$ particle-hole-triaxial-rotor model



$\pi h_{11/2} \nu h_{11/2}^{-1}$ particle-hole-triaxial-rotor model



$\pi h_{11/2} \nu h_{11/2}^{-1}$ particle-hole-triaxial-rotor model



Conclusions

- A new basis is proposed for the triaxial particle-rotor model.
- It is applicable to particle/hole coupling in odd- and odd-odd nuclei.
- High-j particle and high-j hole coupling to the triaxial core results in basis states which are left-handed, right-handed, and planar.
- Chiral basis diagonalize a significant fraction of the model Hamiltonian providing efficient expansion for wave functions of final states.
- Calculations confirm doubling of states arising from orthogonal coupling of angular momentum vectors.
- The wave function expanded in the left-handed, right-handed, and planar states will be used to identify and investigate observables sensitive to chirality in angular momentum coupling.