Triaxiality and single particle degrees of freedom

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Thank’s to my collaborators

Weichuan Li, Michigan State University, USA

Javid Sheikh, Gowar Bhat, Sheikh Jangir
Kashmir University, India

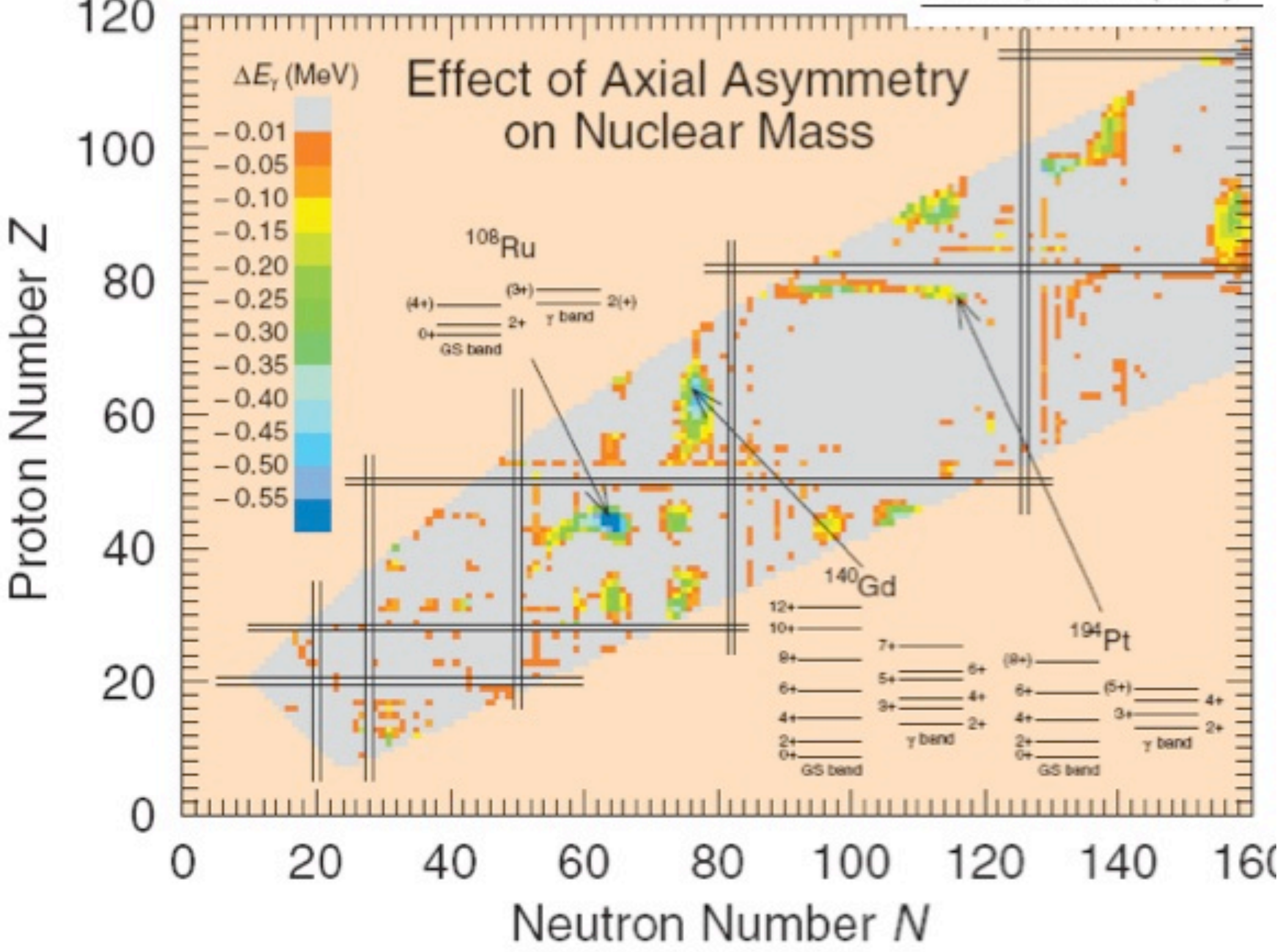
Quibo Chen, Peking University, China

Triaxiality at moderate Spin

Peter Möller,[1,*] Ragnar Bengtsson,[2] B. Gillis Carlsson,[2] Peter Olivius,[2] and Takatoshi Ichikawa[3]

PRL 97, 162502 (2006)

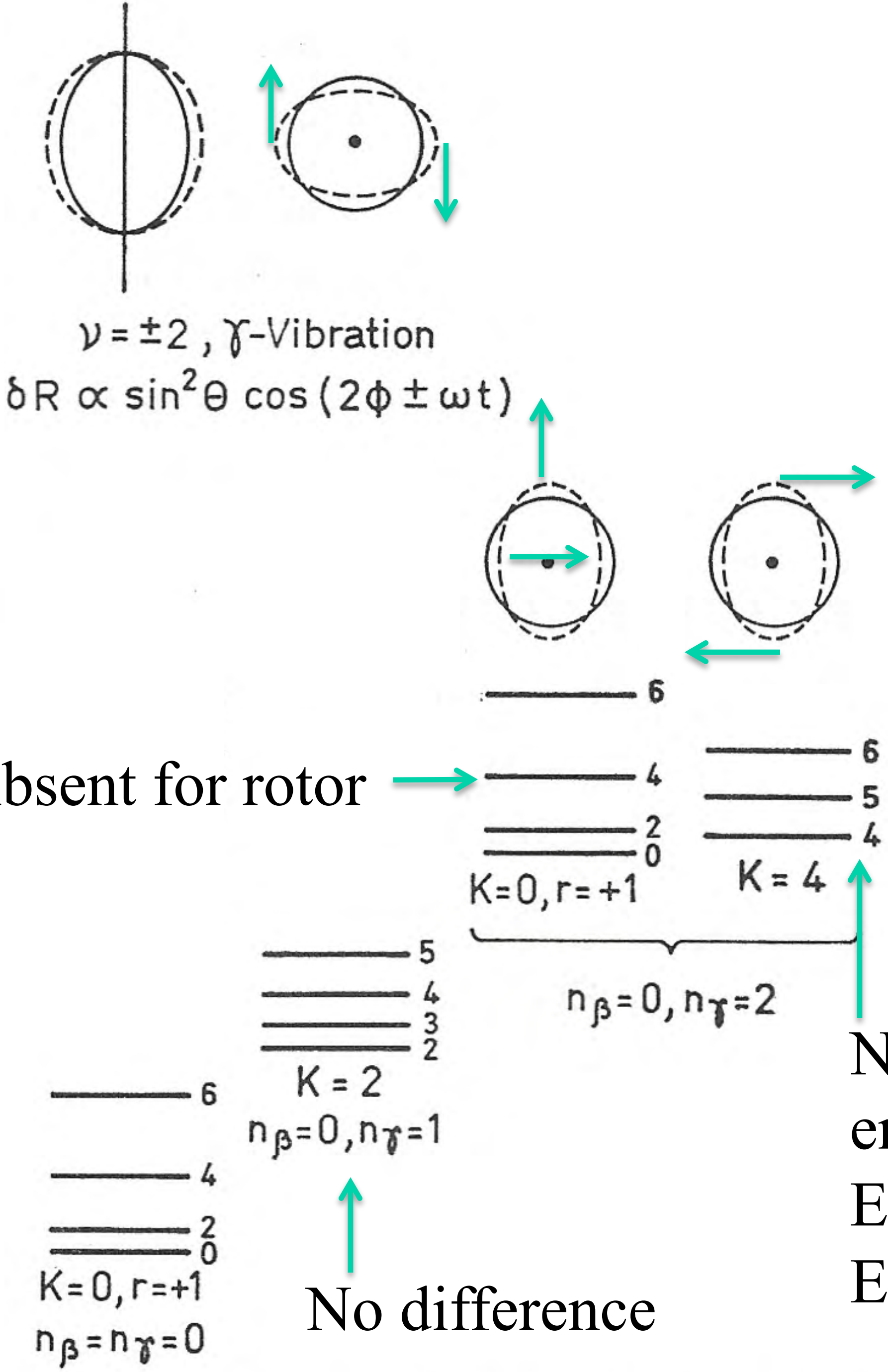
enhanced triaxiality $E(2_2^+) < E(4_1^+)$



states with $\langle N n_3 \Lambda \Omega | Y_{2\pm2} | N n_3 \Lambda \pm 2 \Omega \pm 2 \rangle$ near the Fermi surface

- Triaxiality rigid or dynamical?
- Is it the right question?
- Quasi gamma band in even-even nuclei
- Transverse wobbling

Triaxial rotor vs. γ vibrator

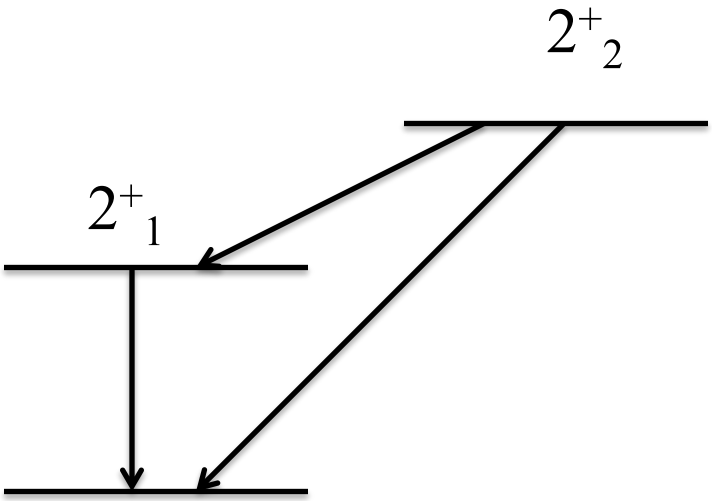


Absent for rotor

No difference

No difference
energy different
E(K=4)=2E(K=2) vibrator
E(K=4)>2E(K=2) rotor

Triaxial rotor description of the γ band

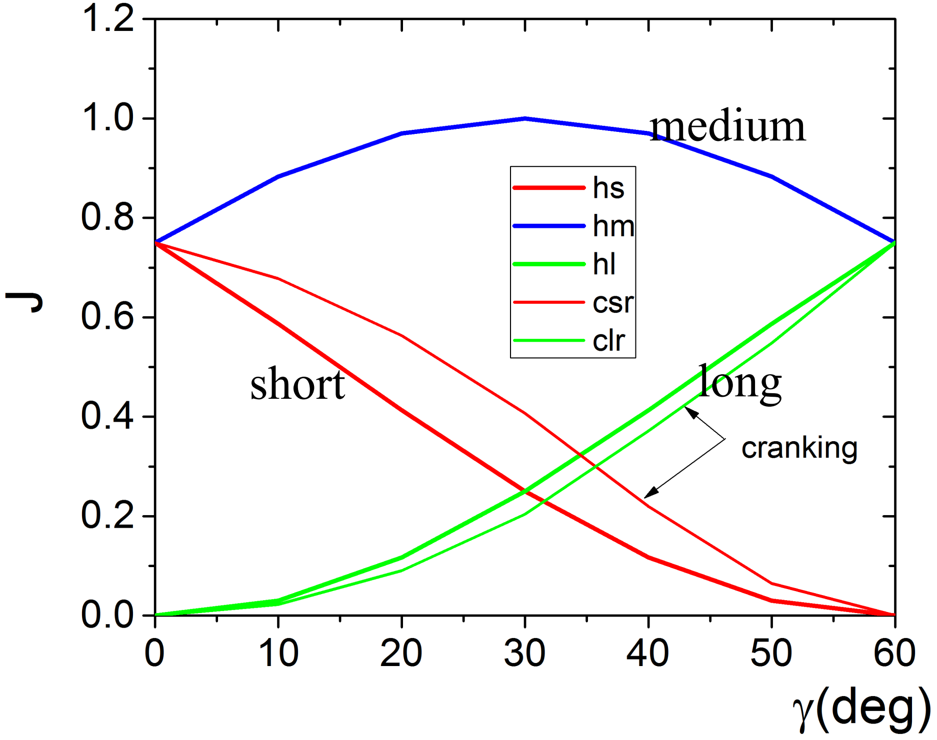


One rotational quant along
the unfavored axes

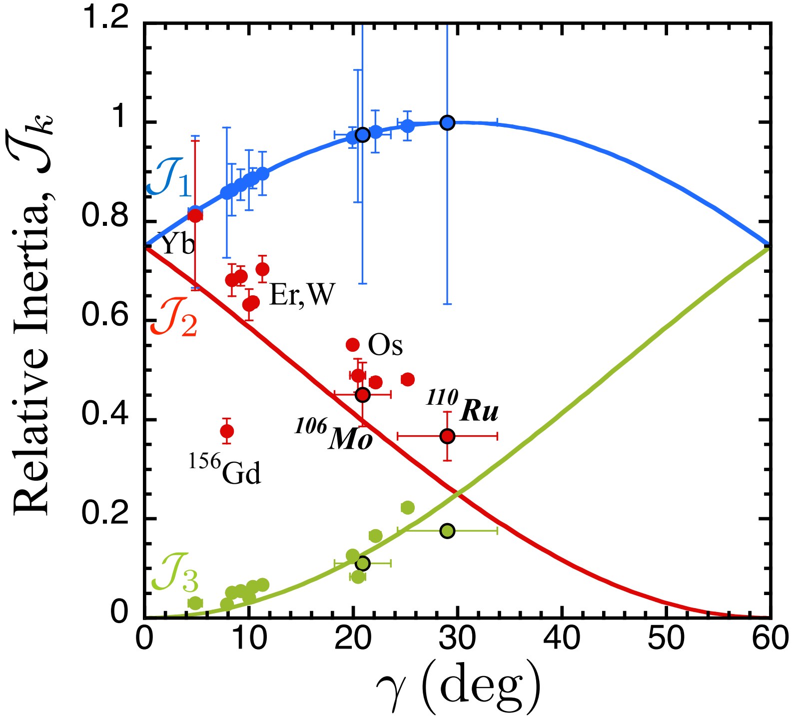
J.M. Allmond , J.L. Wood
PLB 767 (2017) 226

γ from COULEX
J_k from energy

calculated



experimental



cranking calculation Z=68 A168

$\varepsilon = 0.25 \quad \Delta_p = 1.1 MeV \quad \Delta_n = 1.0 MeV$

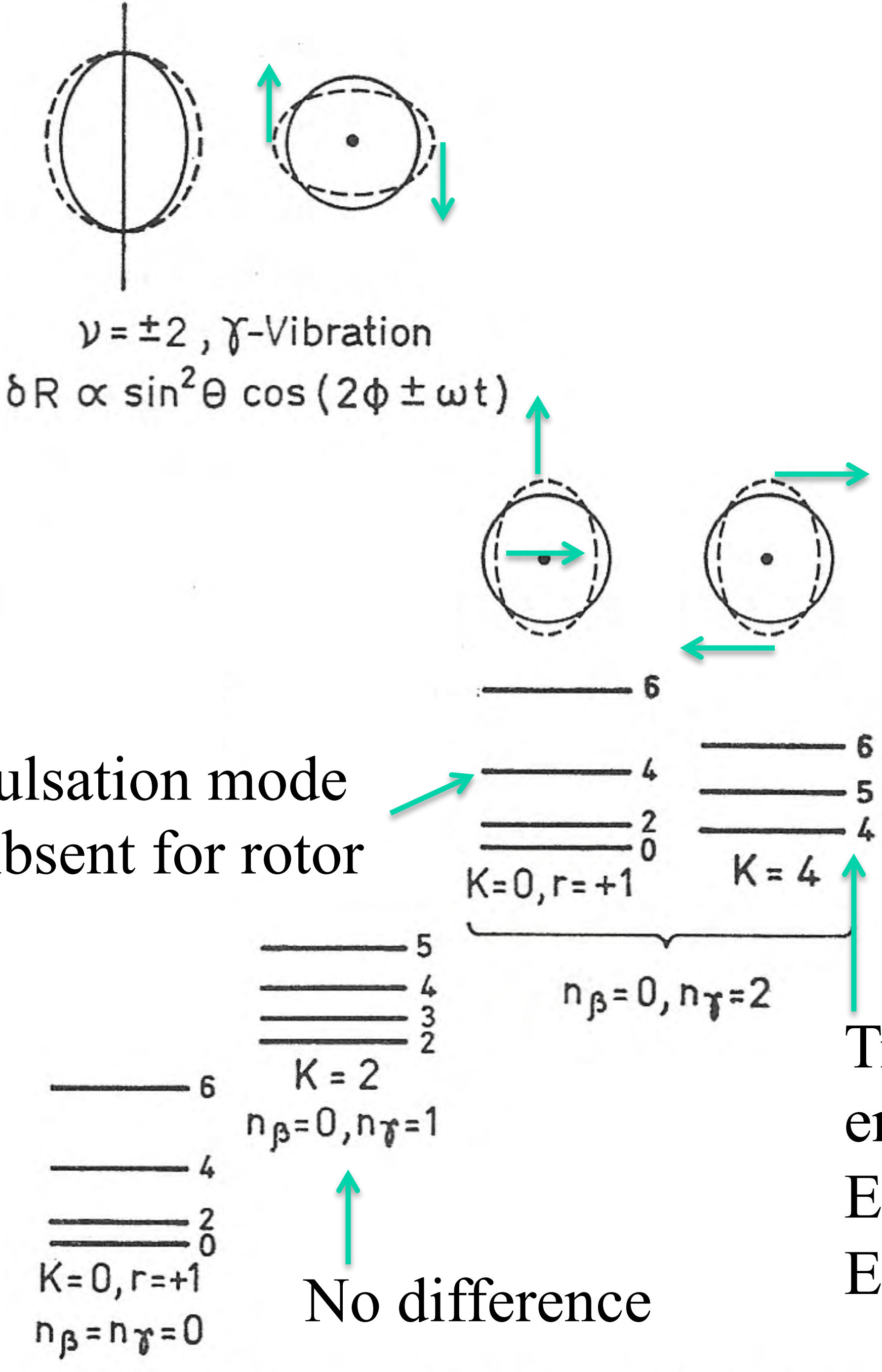
J.M. Allmond , J.L. Wood
PLB 767 (2017) 226

Triaxial rotor description of the γ band

γ from COULEX
J_k from energy

The three moments of inertia are irrotational flow-like:
Consequence of quantum coherence among Fermions

Triaxial rotor vs. γ vibrator



Pulsation mode
Absent for rotor

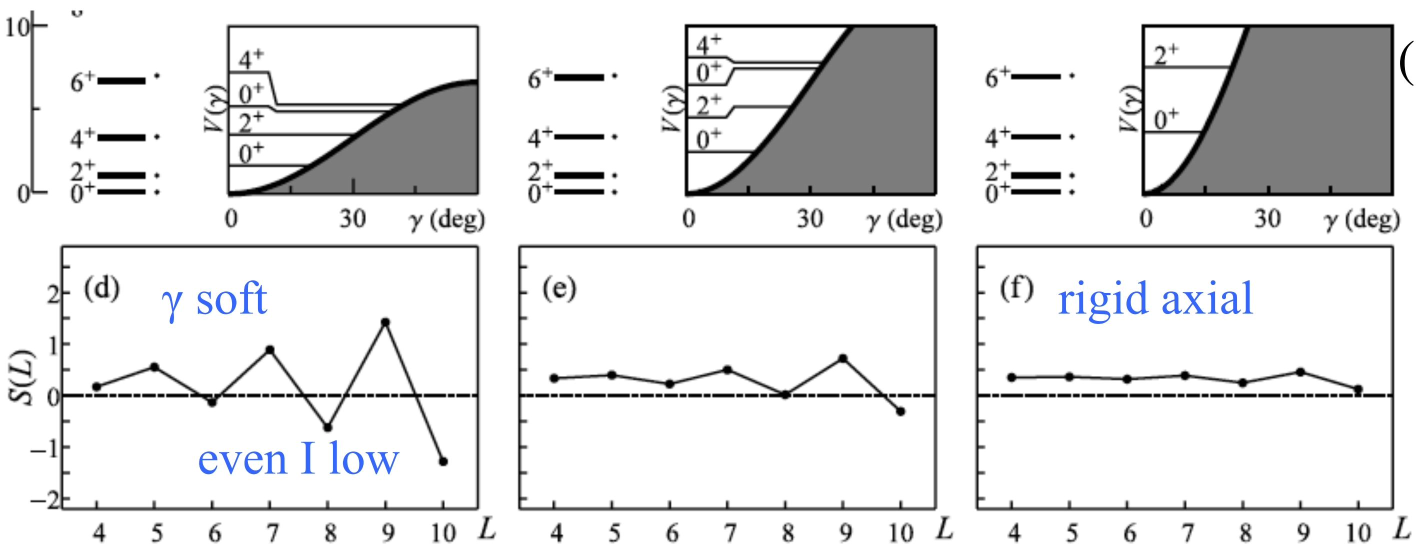
No difference

Travelling wave <->rigid rotation energy different
E(K=4)=2E(K=2) vibrator
E(K=4)>2E(K=2) rotor

Staggering of the γ band: soft vs. or rigid

Bohr-Hamiltonian with irrotational flow inertia

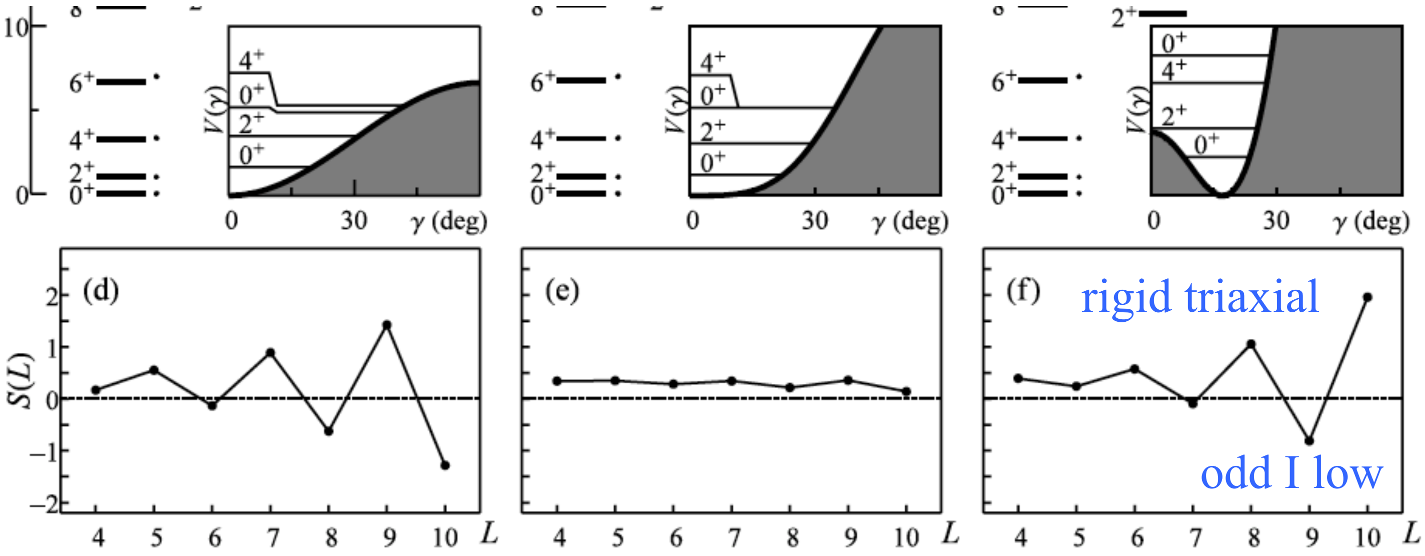
$$S(I) = E(I) - (E(I-1) + E(I+1))/2$$



γ soft

even I low

rigid axial



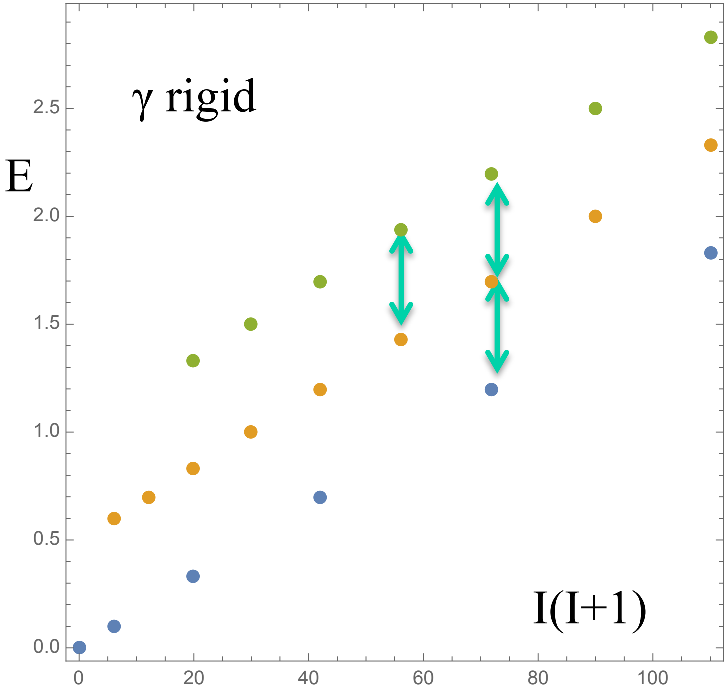
rigid triaxial

odd I low

M. A. Caprio
PRC **83**,
064309 (2011)

McCutchan et al.,
PRC 76
(2007) 024306.
IBM

Zamfir,Casten
PRL 87
(2001) 052503



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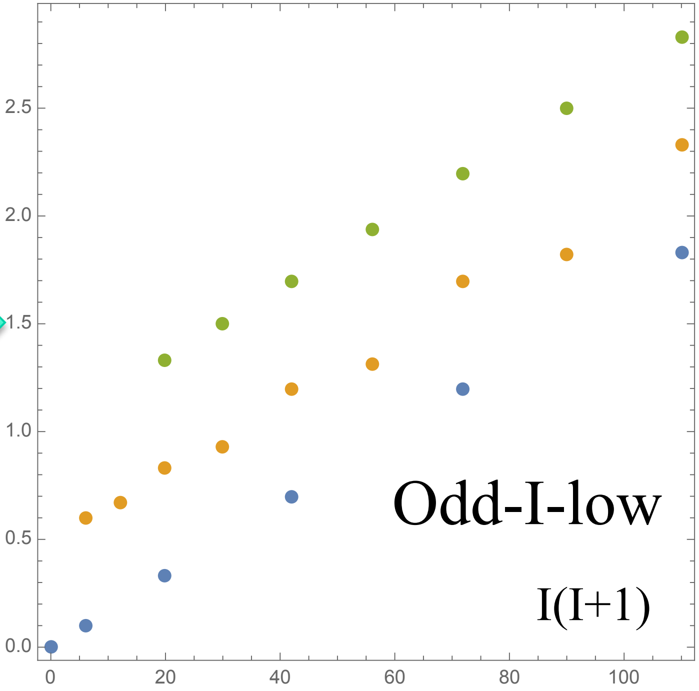
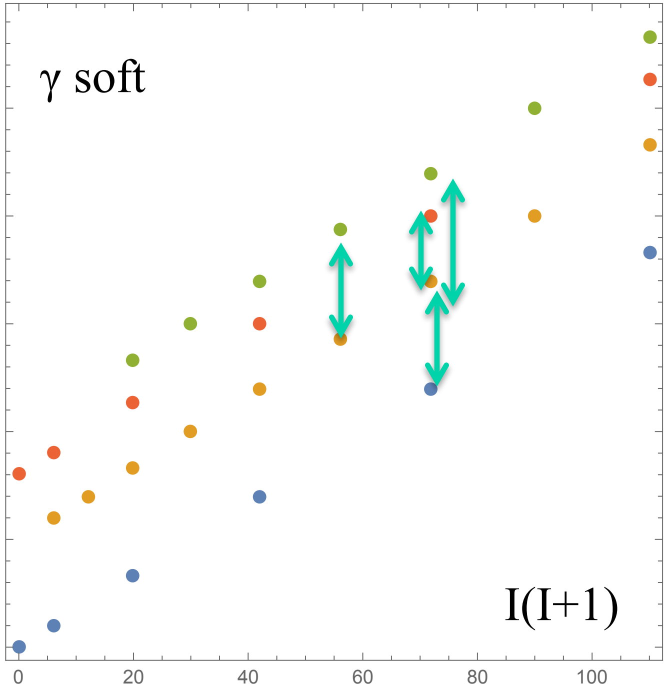
K=2

K=0





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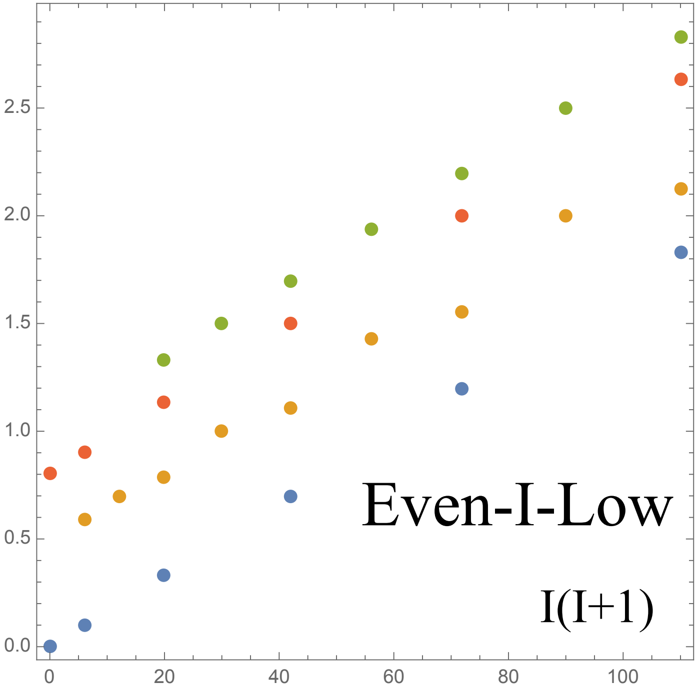
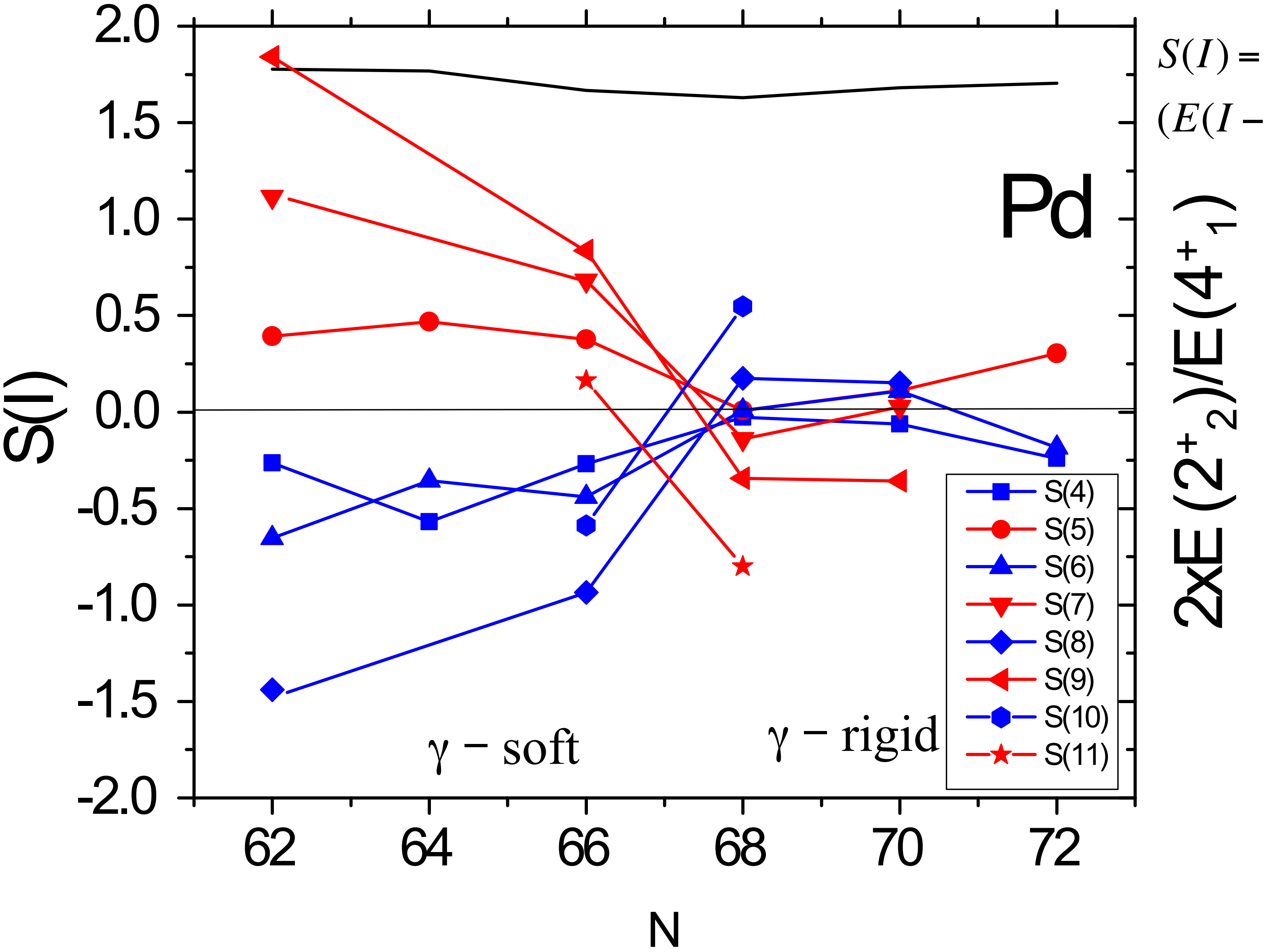
K=0
K=2

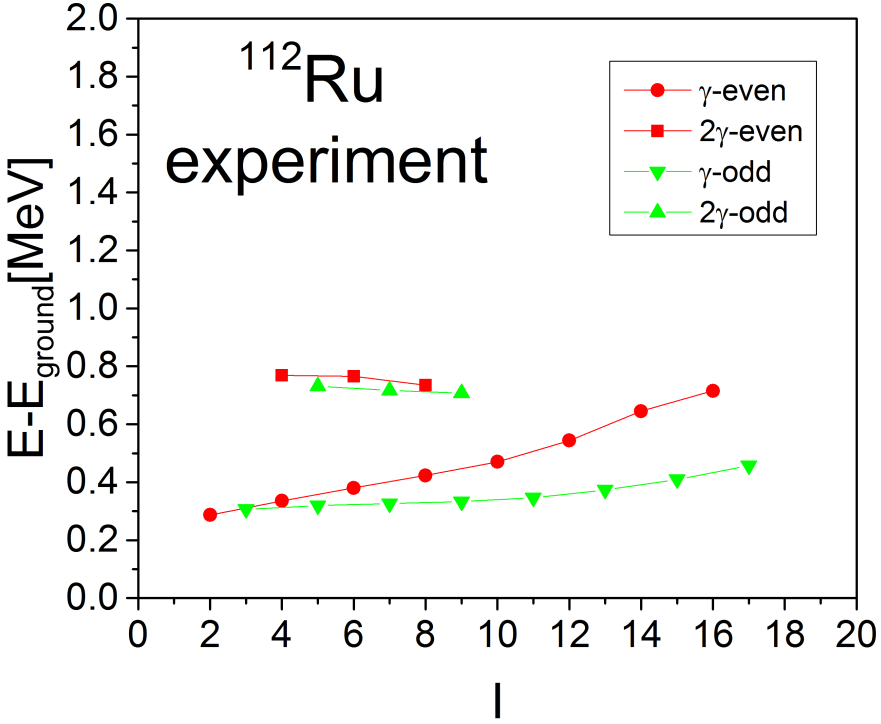
K=0

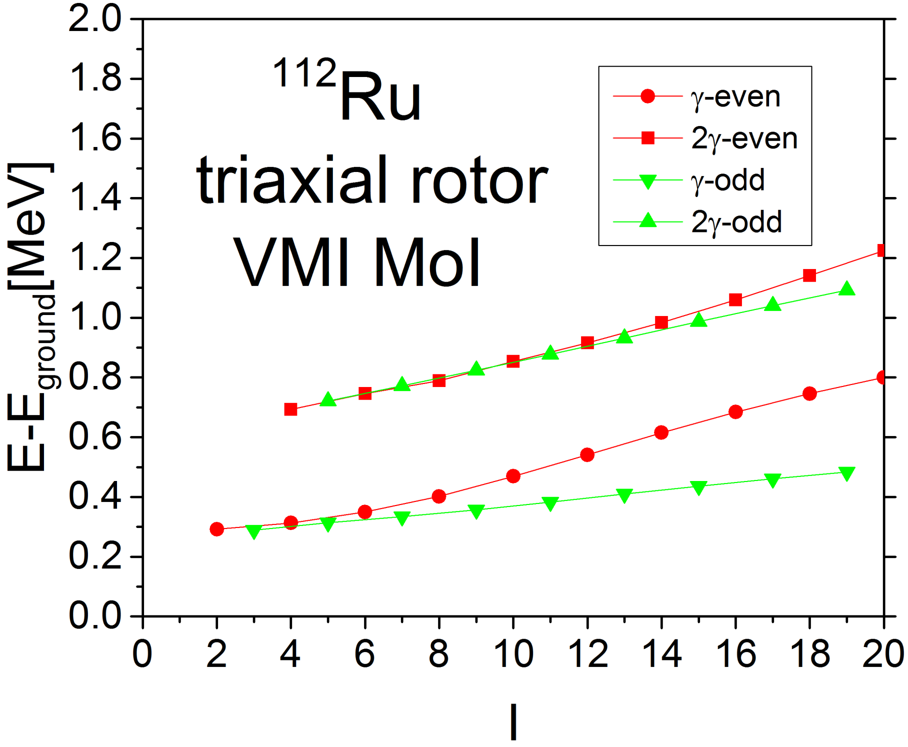




$$S(I) = E(I) - (E(I-1) + E(I+1))/2$$

S. Frauendorf, International Journal of Modern Physics E Vol. 24, No. 9 (2015) 1541001





rigid triaxaxial rotor

$$4 < E(4_3^+)/E(2_2^+) < 3.333$$

weak triaxiality strong

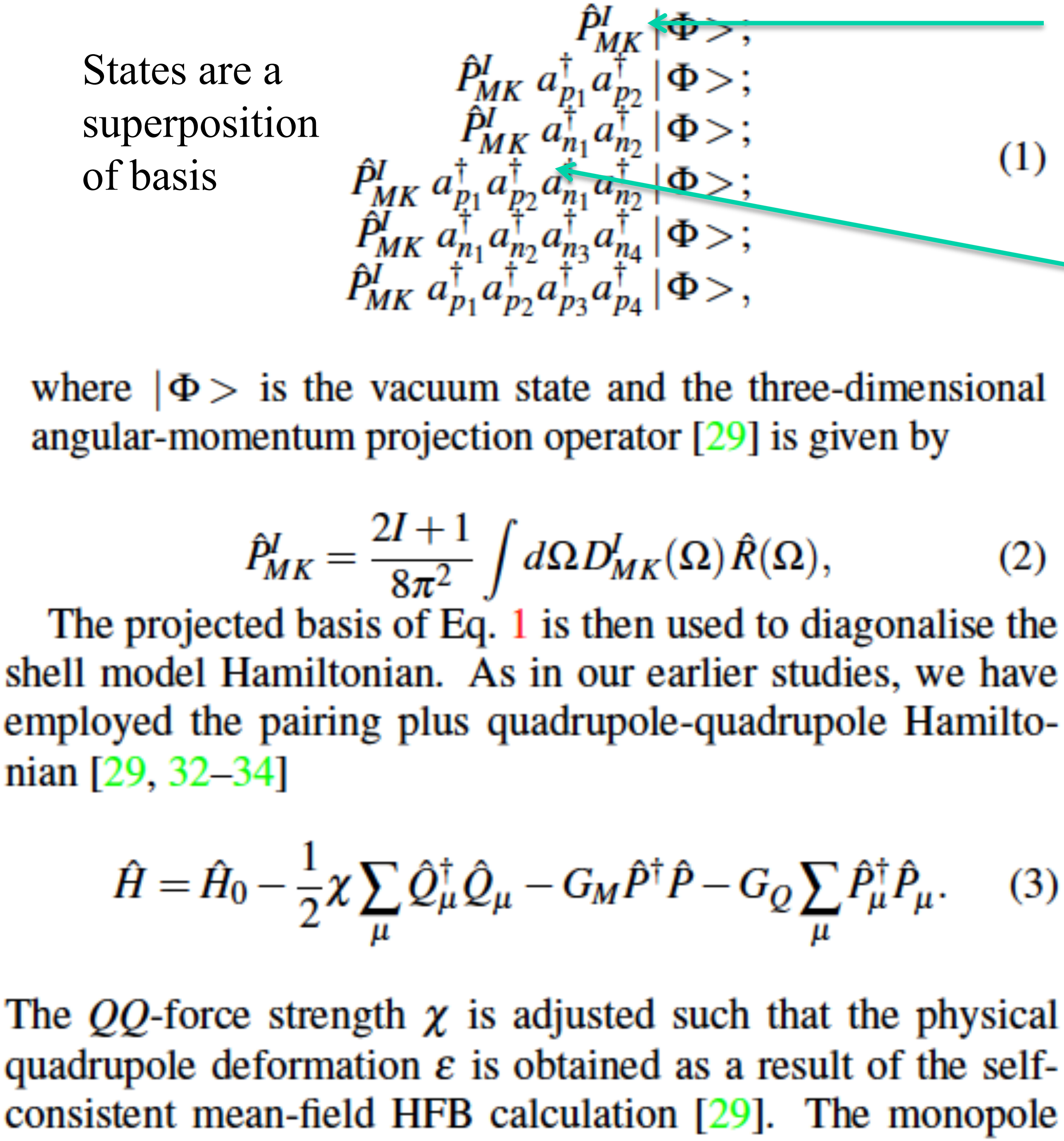
$$\exp: E(4_3^+)/E(2_2^+) = 2.8$$

$$\Theta_i = \Theta_{0i} + \Theta_{1i} I$$

I-dependence of
γ deformation

There should be some γ softness.

States are a superposition of basis



$$
\begin{aligned}
&\hat{P}^I_{MK}\,|\Phi>;\\
&\hat{P}^I_{MK}\,a^\dagger_{p_1}a^\dagger_{p_2}\,|\Phi>;\\
&\hat{P}^I_{MK}\,a^\dagger_{n_1}a^\dagger_{n_2}\,|\Phi>;\\
&\hat{P}^I_{MK}\,a^\dagger_{p_1}a^\dagger_{p_2}a^\dagger_{n_1}a^\dagger_{n_2}\,|\Phi>;\\
&\hat{P}^I_{MK}\,a^\dagger_{n_1}a^\dagger_{n_2}a^\dagger_{n_3}a^\dagger_{n_4}\,|\Phi>;\\
&\hat{P}^I_{MK}\,a^\dagger_{p_1}a^\dagger_{p_2}a^\dagger_{p_3}a^\dagger_{p_4}\,|\Phi>,
\end{aligned}
\qquad (1)
$$

Triaxial rotor dynamics

Single particle response

where $|\Phi>$ is the vacuum state and the three-dimensional angular-momentum projection operator [29] is given by

$$\hat{P}^I_{MK} = \frac{2I+1}{8\pi^2}\int d\Omega\, D^I_{MK}(\Omega)\,\hat{R}(\Omega), \qquad (2)$$

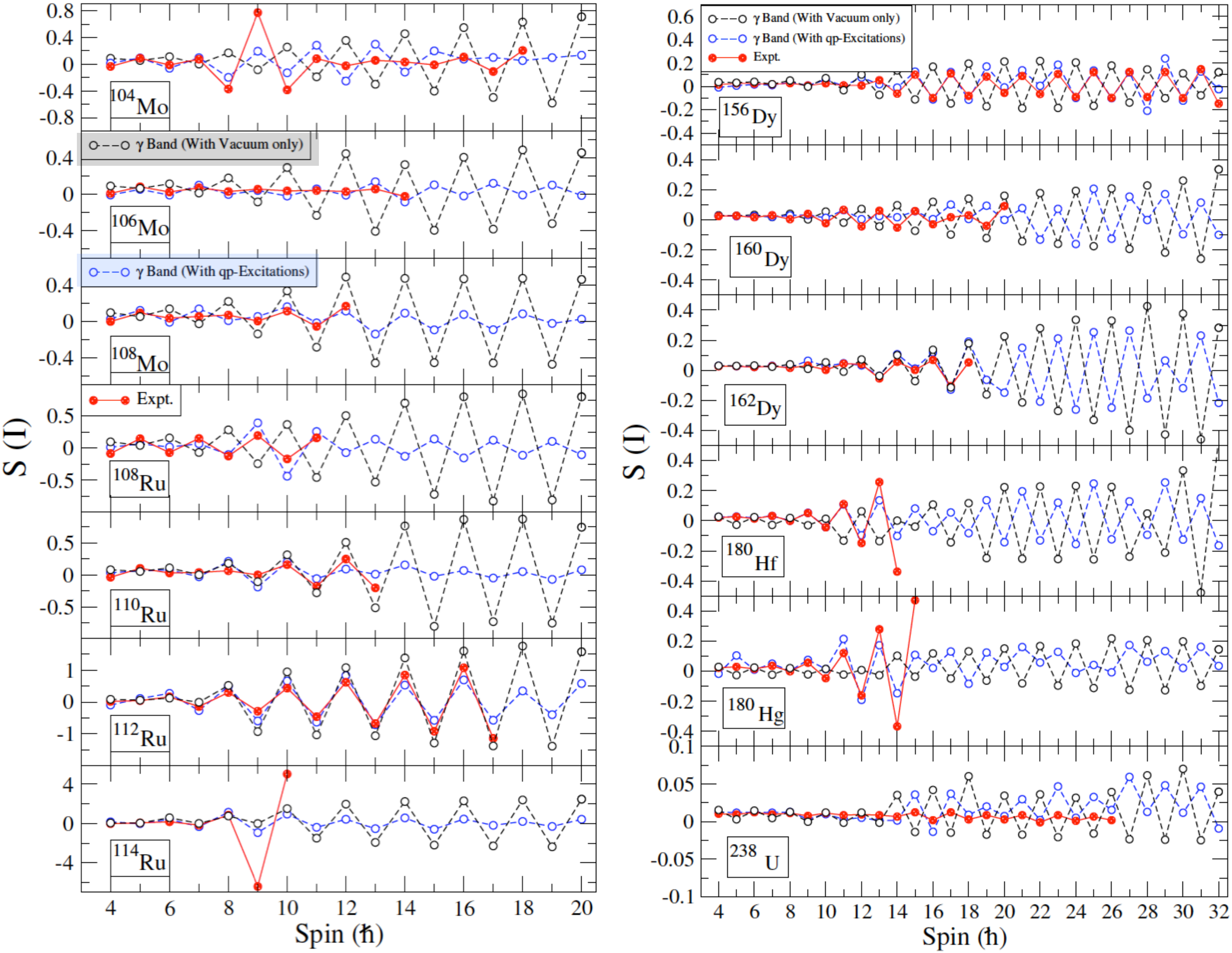
The projected basis of Eq. 1 is then used to diagonalise the shell model Hamiltonian. As in our earlier studies, we have employed the pairing plus quadrupole-quadrupole Hamiltonian [29, 32–34]

$$\hat{H} = \hat{H}_0 - \frac{1}{2}\chi\sum_\mu \hat{Q}^\dagger_\mu\hat{Q}_\mu - G_M\hat{P}^\dagger\hat{P} - G_Q\sum_\mu \hat{P}^\dagger_\mu\hat{P}_\mu. \qquad (3)$$

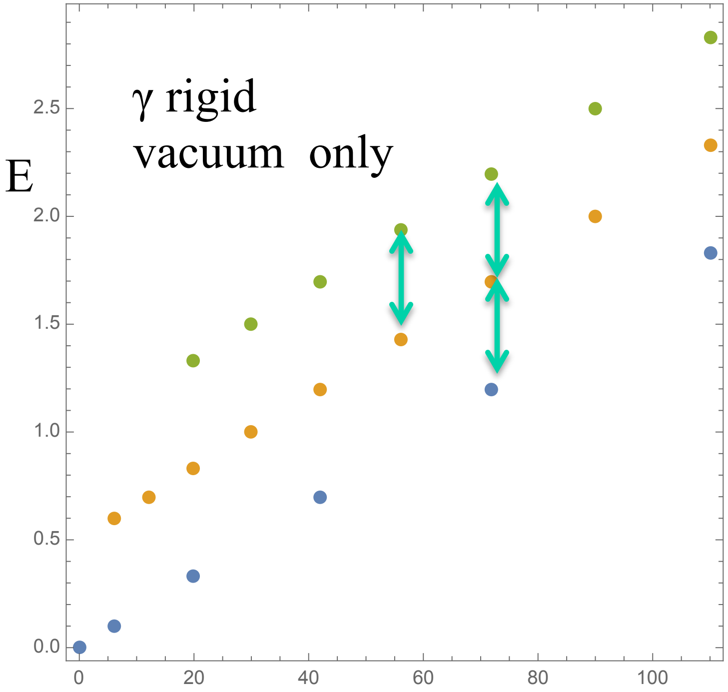
The *QQ*-force strength χ is adjusted such that the physical quadrupole deformation ε is obtained as a result of the self-consistent mean-field HFB calculation [29]. The monopole

Triaxial Projected Shell Model

The trixiality parameter is fixed!



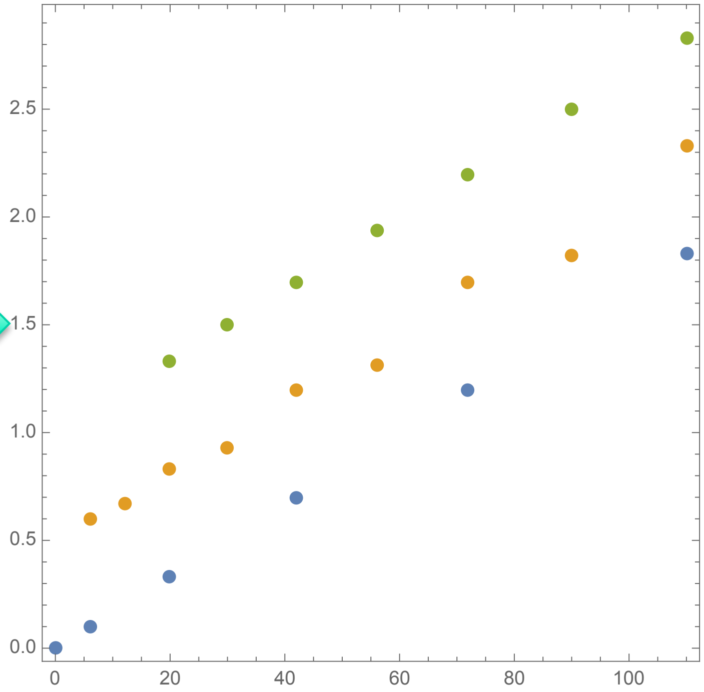
Bhat, Frauendorf, Sheikh, PRC to be published

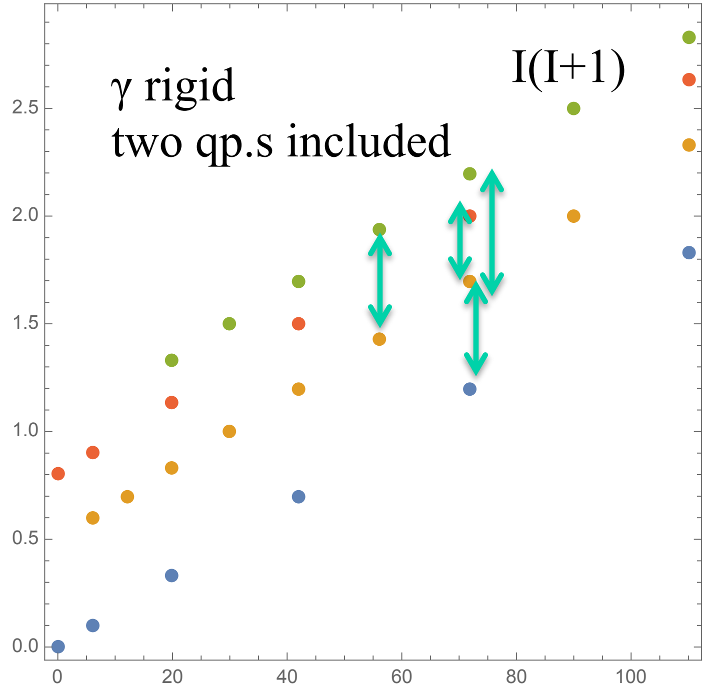


K=4

K=2

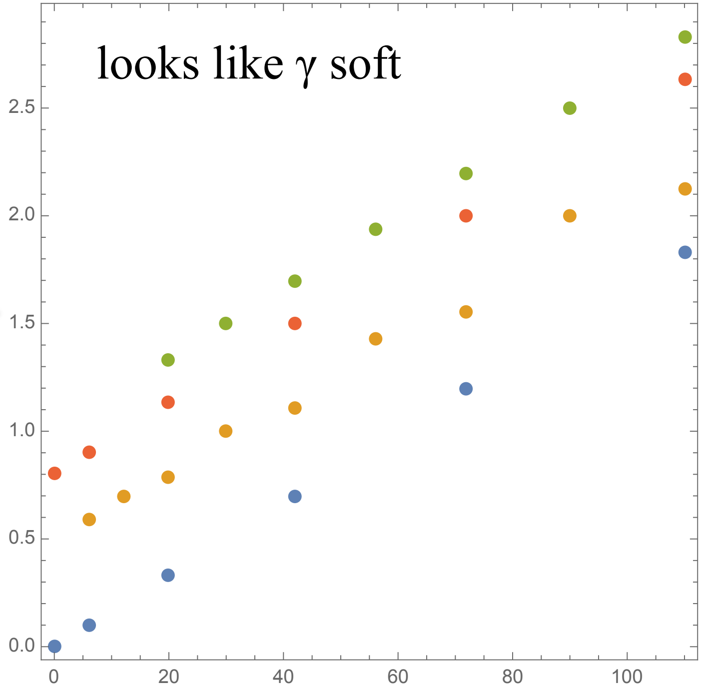
K=0

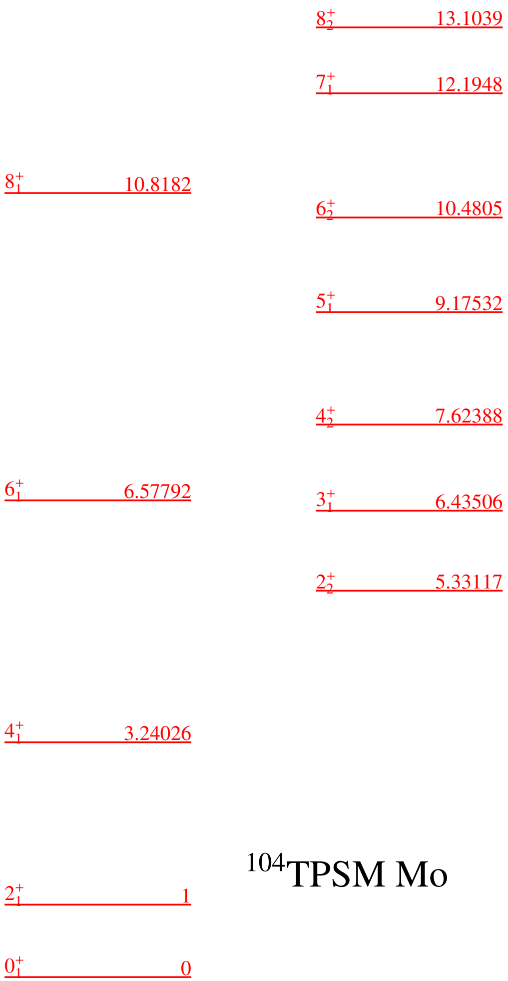




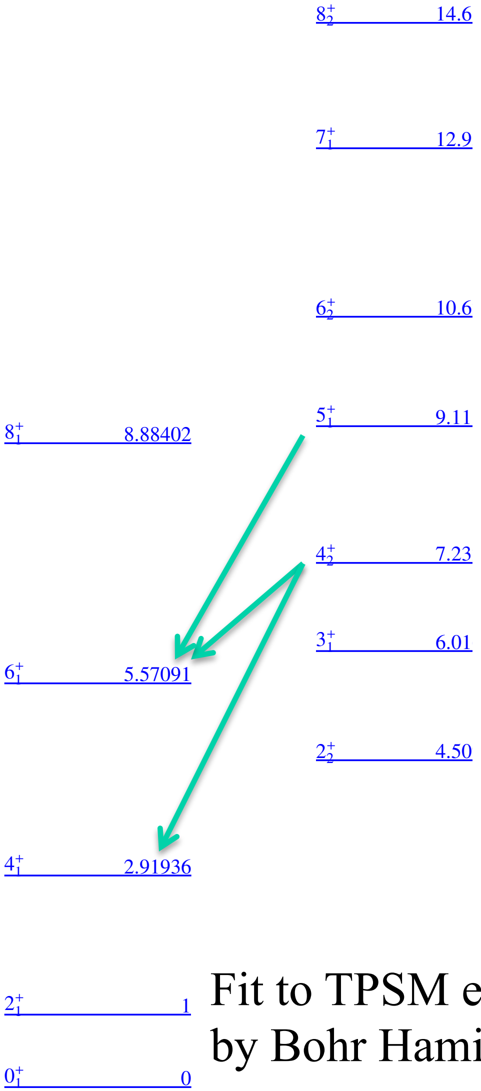
K=4
K=0
K=2

K=0



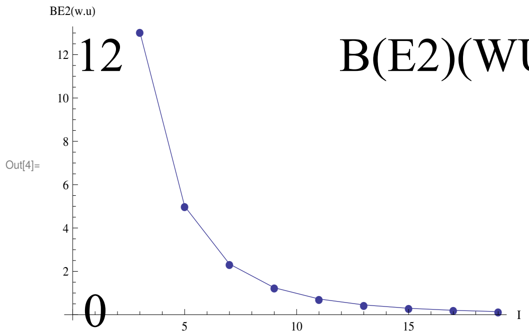


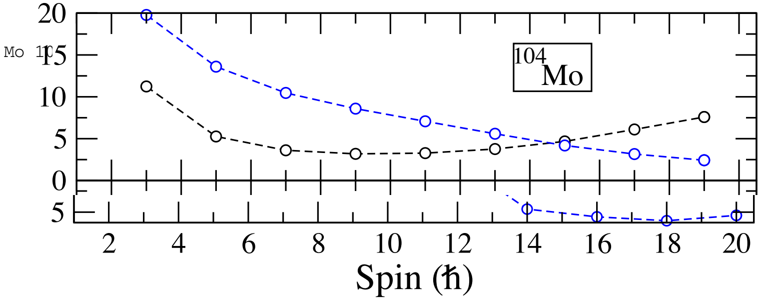
^{104}TPSM Mo



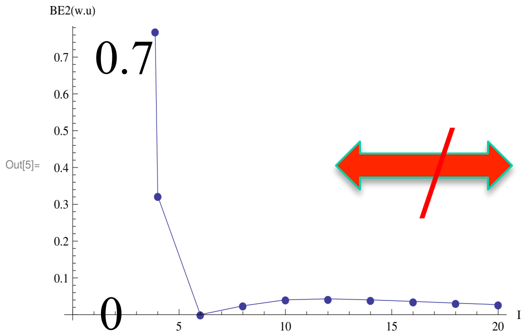
Fit to TPSM energies
by Bohr Hamiltonian

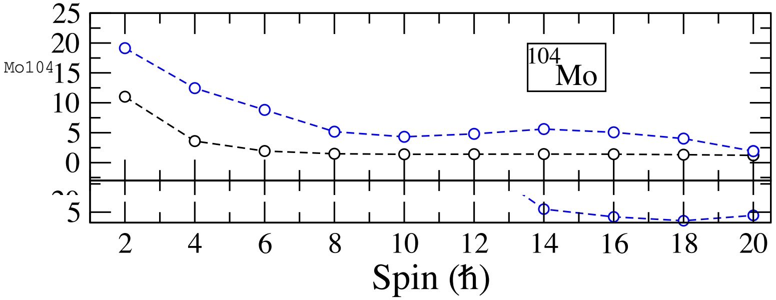
B(E2)(WU)



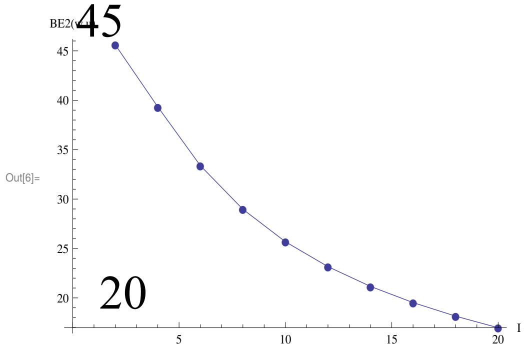


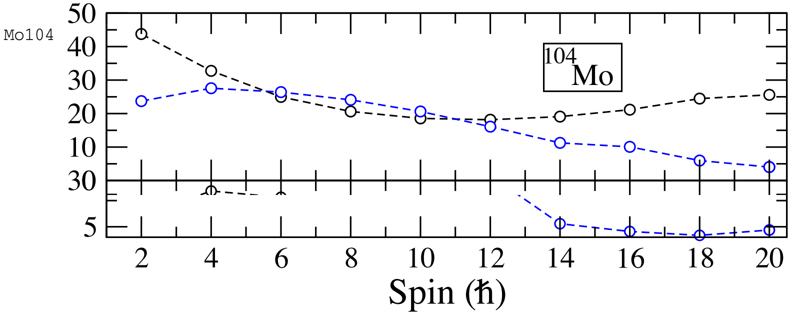
I->I-1





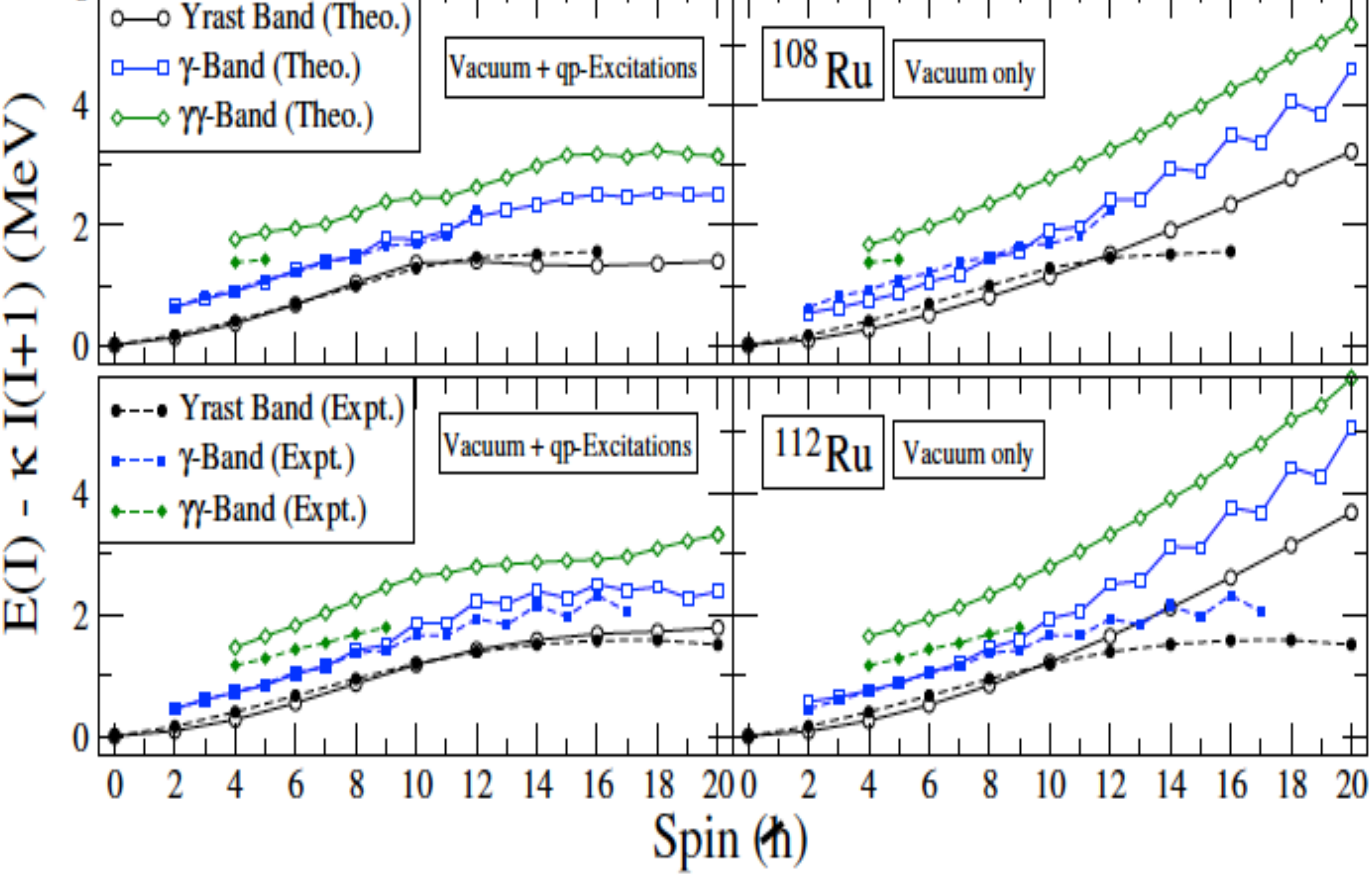
I->I-2





I->I

TPSM B(E2) from gamma to ground band



Both “best triaxial rotor” 112 and “softish” 108 reproduced with the same parameters, γγ-band too high with fixed γ deformation

Bhat, Frauendorf, Sheikh, PRC to be published

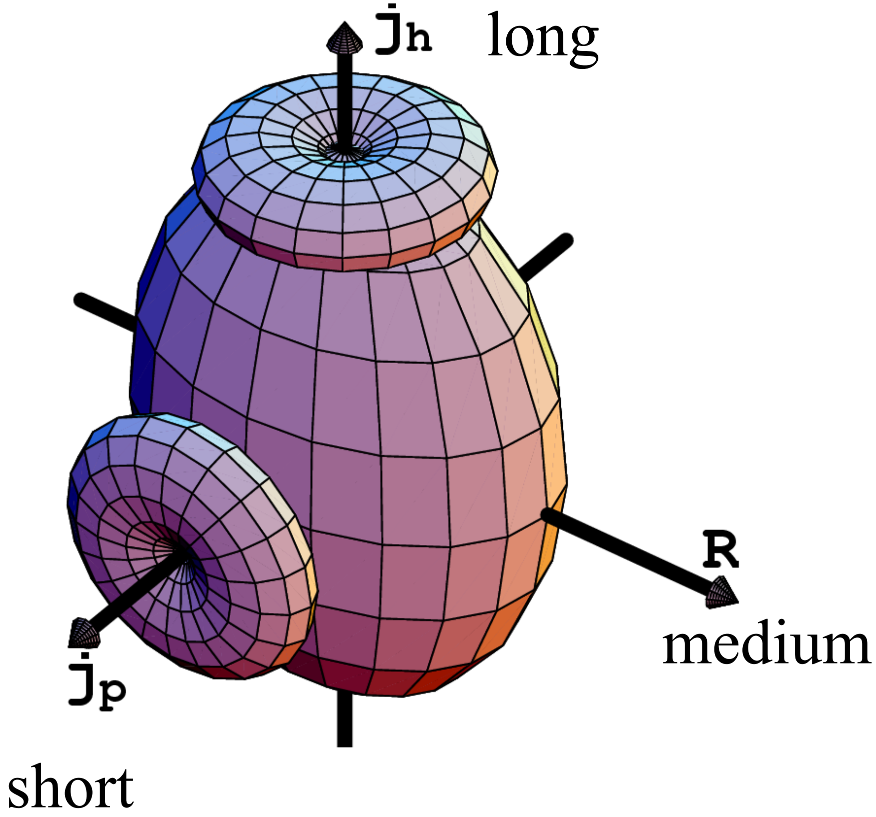
Even-even nuclei: γ rigid or soft?

- Bohr-Hamiltonian relates staggering of γ band with softness of triaxiality
- TSPM with fixed γ accounts for the different signs of staggering
- Energy staggering alone does not discriminate
- Differences in I->I-2 interband transitions
- More careful study of matrix elements from Coulex experiments on ^{104}Ru on the way
- Energy of γγ band indicates γ softness

Coupling of unpaired quasiparticles with the triaxial core

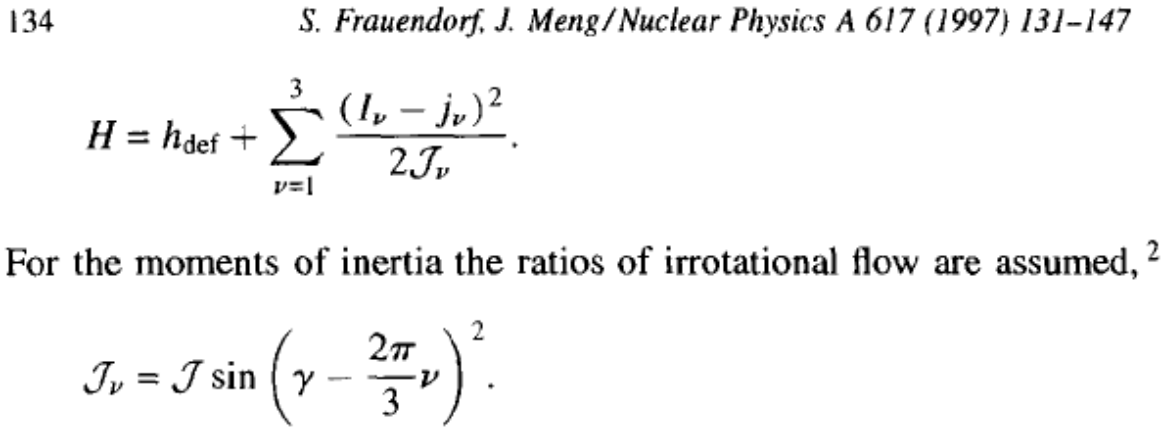
Combination of unpaired quasiparticles with Collective Triaxial Rotor

The players



$$\Theta_m > \Theta_s > \Theta_l$$

Multi-Quasiparticle+Triaxial Rotor Model



134 S. Frauendorf, J. Meng/Nuclear Physics A 617 (1997) 131–147

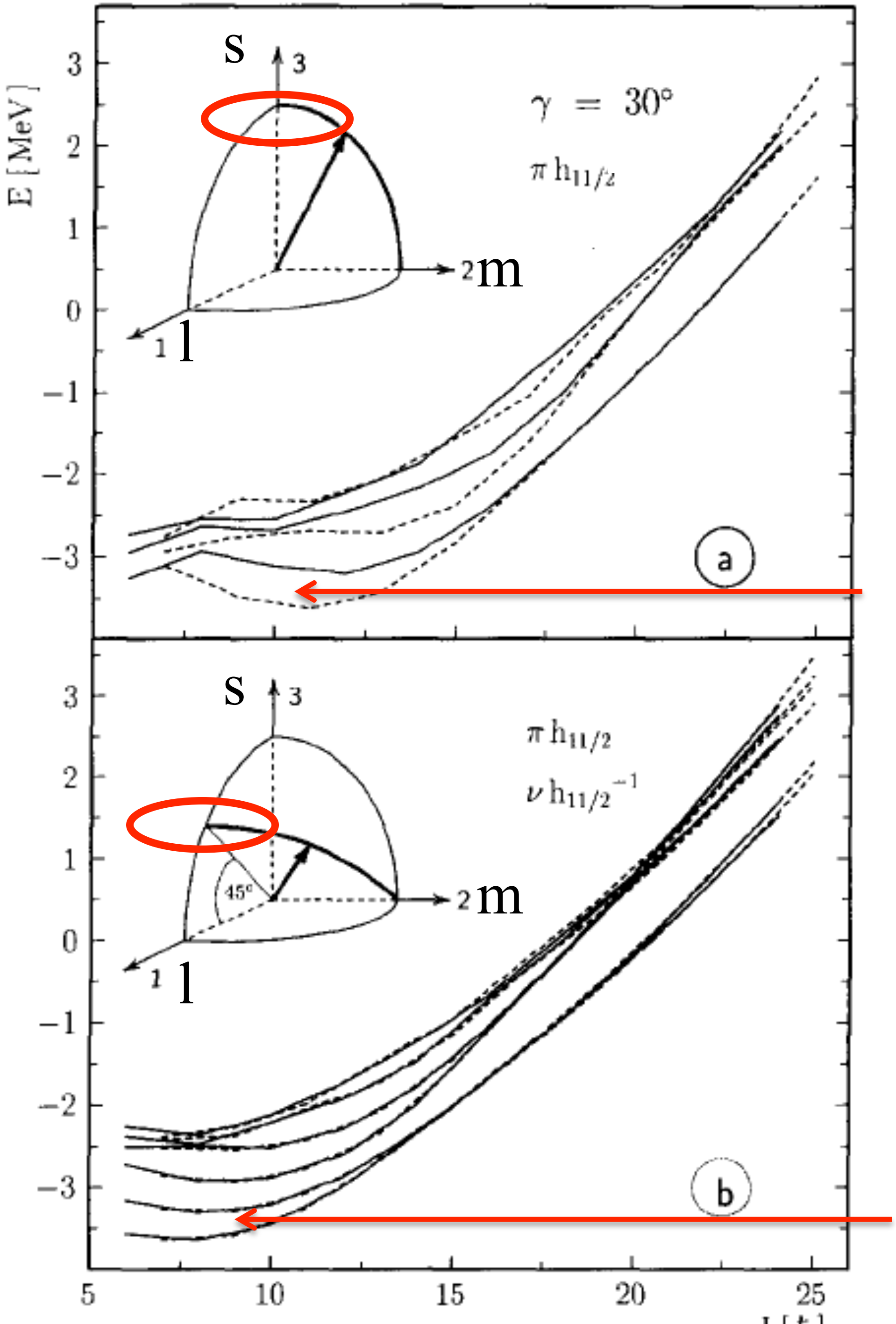
$$H = h_{\mathrm{def}} + \sum_{\nu=1}^{3} \frac{(I_\nu - j_\nu)^2}{2\mathcal{J}_\nu}.$$

For the moments of inertia the ratios of irrotational flow are assumed,[2]

$$\mathcal{J}_\nu = \mathcal{J} \sin\left(\gamma - \frac{2\pi}{3}\nu\right)^2.$$

or taken as adjustable
or taken from cranking calculations

Transverse Wobbling and Chirality come together



Transverse Wobbling

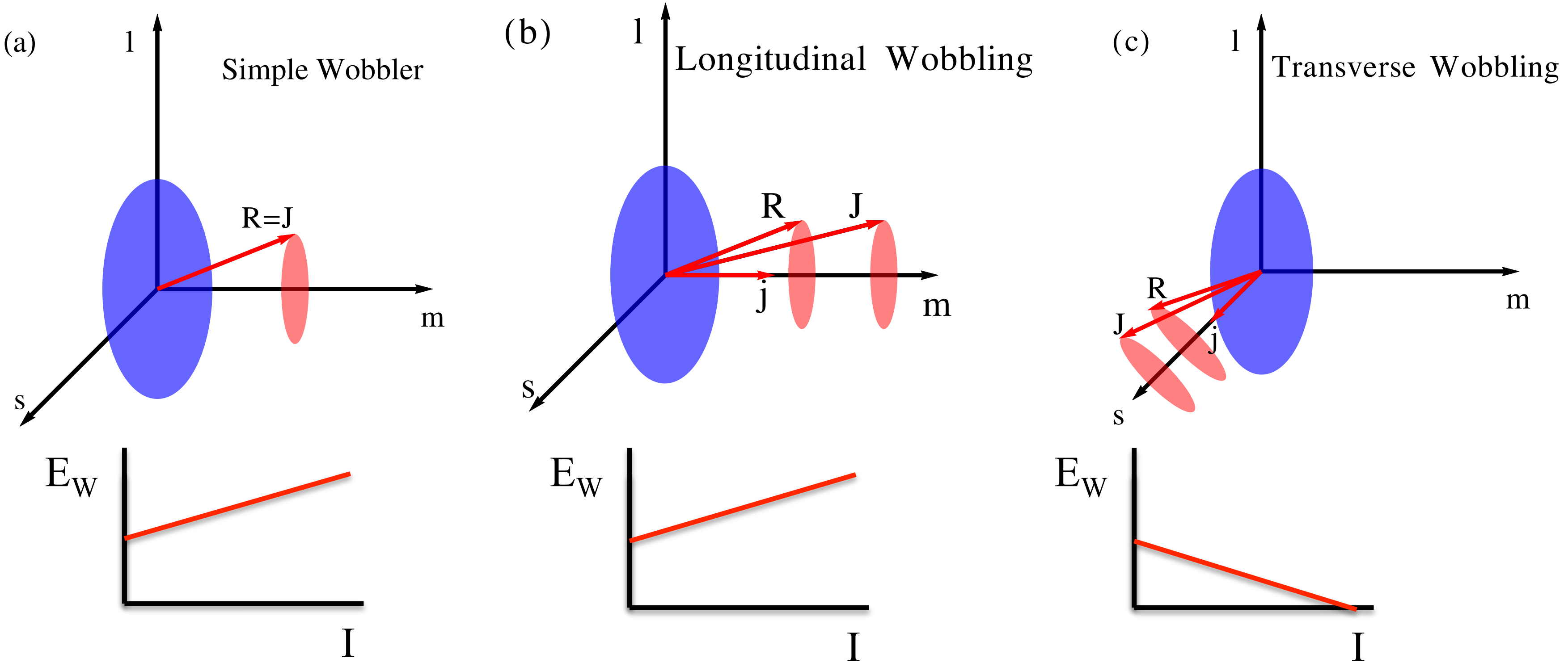
axis of max MoI: m
transverse W: j perp. m
longitudinal W: j par. m

$$\Theta_m > \Theta_s = \Theta_l, 4/1$$

Transverse Chiral Vibration

$$\Theta_m > \Theta_s > \Theta_l$$

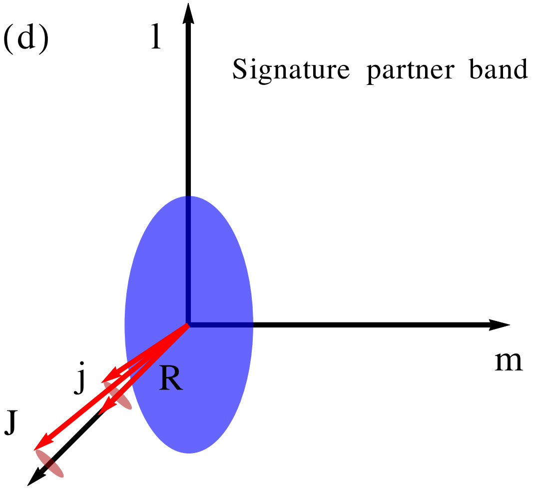
strong E2, I->I-1,



wobbling and signature partner bands

strong E2 I->I-1,

weak E2 I->I-1,



PRL **114,** 082501 (2015) PHYSICAL REVIEW LETTERS week ending 27 FEBRUARY 2015

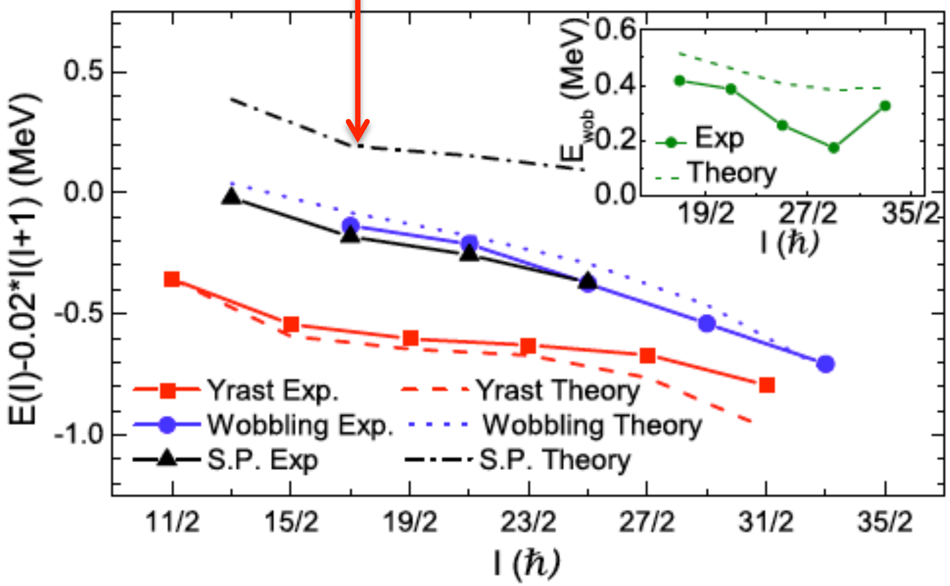
Too high!!

Transverse Wobbling in ^{135}Pr

$\pi h_{11/2} + TR$

$\Theta_m / \Theta_s / \Theta_l = 23/13/7$

J. T. Matta, U. Garg, W. Li, S. Frauendorf, A. D. Ayangeakaa,[†] D. Patel, and K. W. Schlax et al.
Physics Department, University of Notre Dame, Notre Dame, Indiana 46556, USA



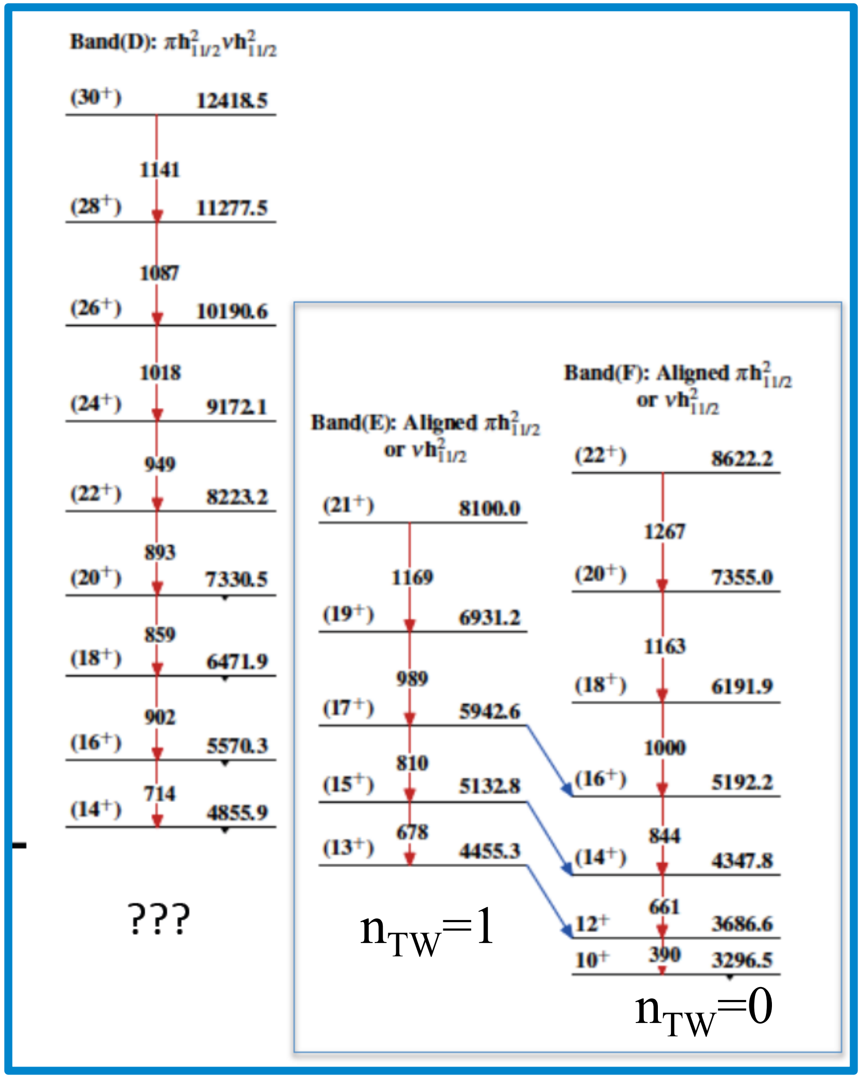
Signatures for TW:
- Wobbling energy decreases with I
- Strong collective E2 transitions between 1 and 0 phonon states

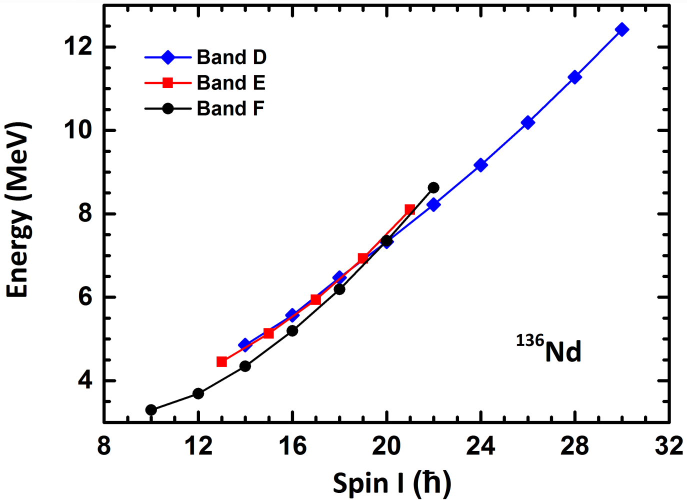
$B(E2_{out}) \sim 0.5 B(E2_{in}) = 50$WU

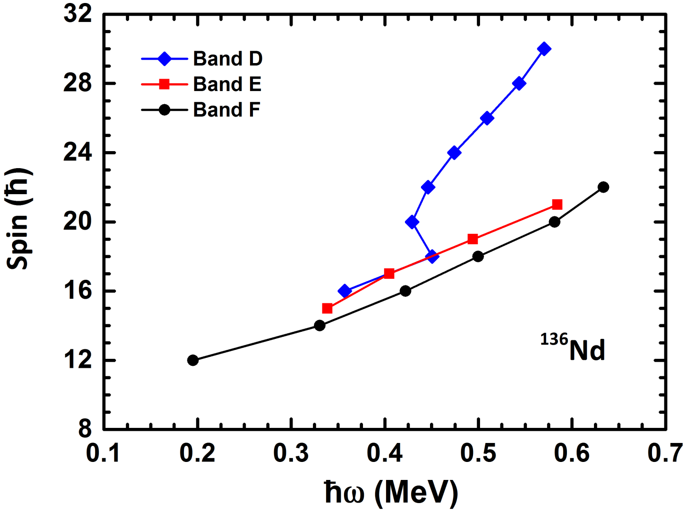
1 phonon -> 0 phonon transitions

						$\frac{B(M1_{out})}{B(E2_{in})} \left(\frac{\mu_N^2}{e^2b^2}\right)$		$\frac{B(E2_{out})}{B(E2_{in})}$	
Initial I^π	Final I^π	E_γ (keV)	δ	Asymmetry	E2 Fraction (%)	Experiment	QTR	Experiment	QTR
$\frac{17}{2}^-$	$\frac{15}{2}^-$	747.0	-1.24 ± 0.13	0.047 ± 0.012	60.6 ± 5.1	...	0.213	...	0.908
$\frac{21}{2}^-$	$\frac{19}{2}^-$	812.8	-1.54 ± 0.09	0.054 ± 0.034	70.3 ± 2.4	0.164 ± 0.014	0.107	0.843 ± 0.032	0.488
$\frac{25}{2}^-$	$\frac{23}{2}^-$	754.6	-2.38 ± 0.37	...	85.0 ± 4.0	0.035 ± 0.009	0.070	0.500 ± 0.025	0.290
$\frac{29}{2}^-$	$\frac{27}{2}^-$	710.2	...	...	...	$\leq 0.016 \pm 0.004$	0.056	$\geq 0.261 \pm 0.014$	0.191
$\frac{13}{2}^-$	$\frac{11}{2}^-$	593.9	-0.16 ± 0.04	-0.092 ± 0.023	2.5 ± 1.2	...	...	...	...

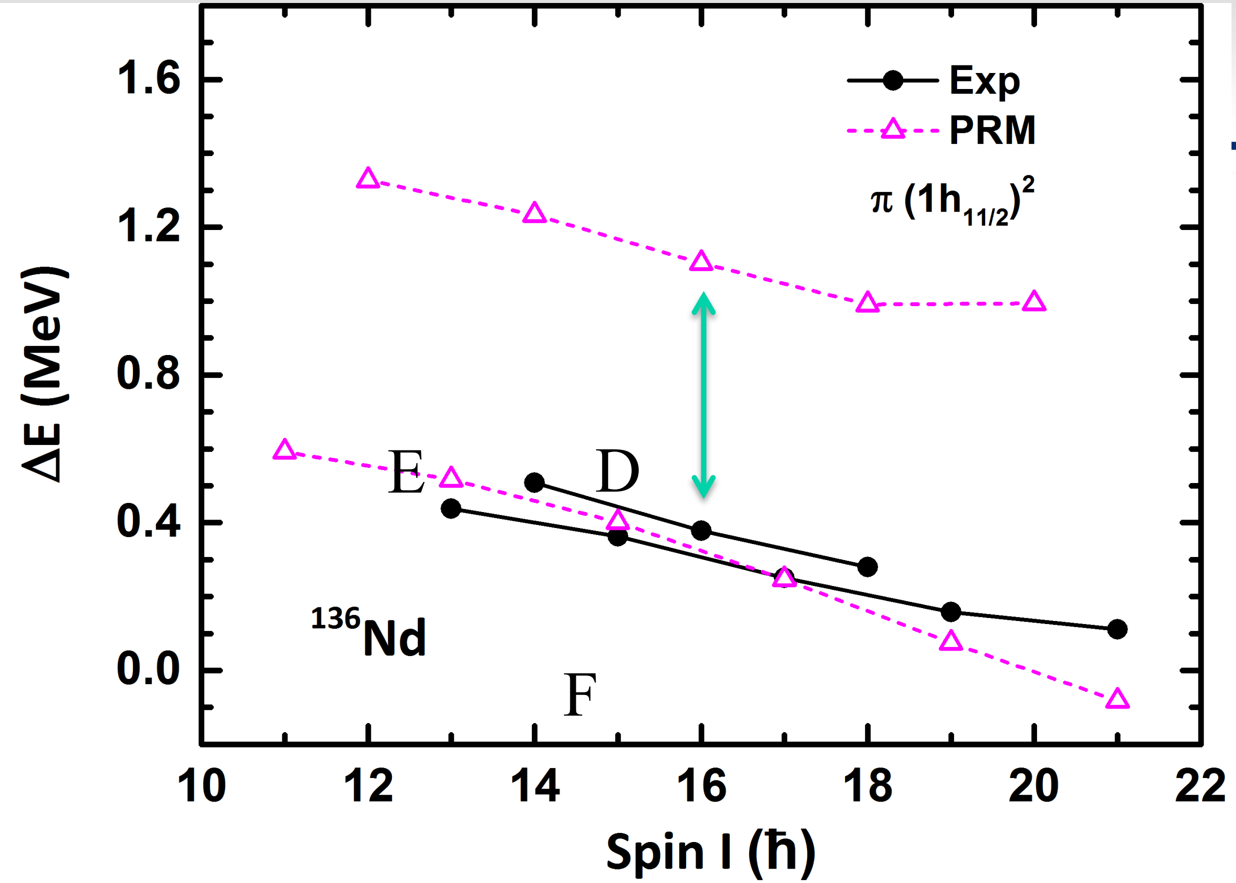
Experimental data







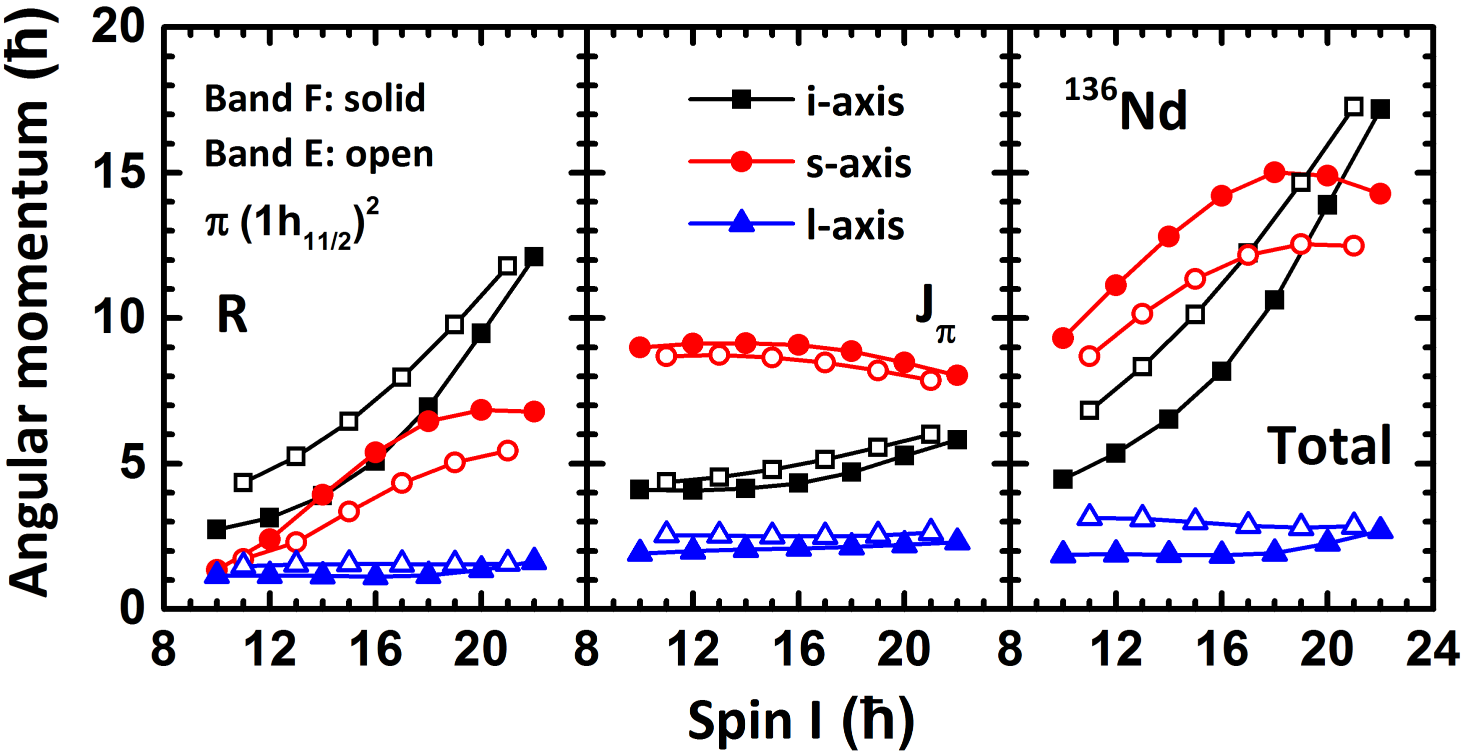
- **Backbending is seen for band D.**
- **Only the low spin part (I=14, 16, 18) might be the partner of band E.**



- **Decreasing trend of energy difference are reproduced, supporting the transverse wobbling interpretation.**

- **Energy differences between the bands E and F are well reproduced, while energy differences between D and F are overestimated about 0.8 MeV.**

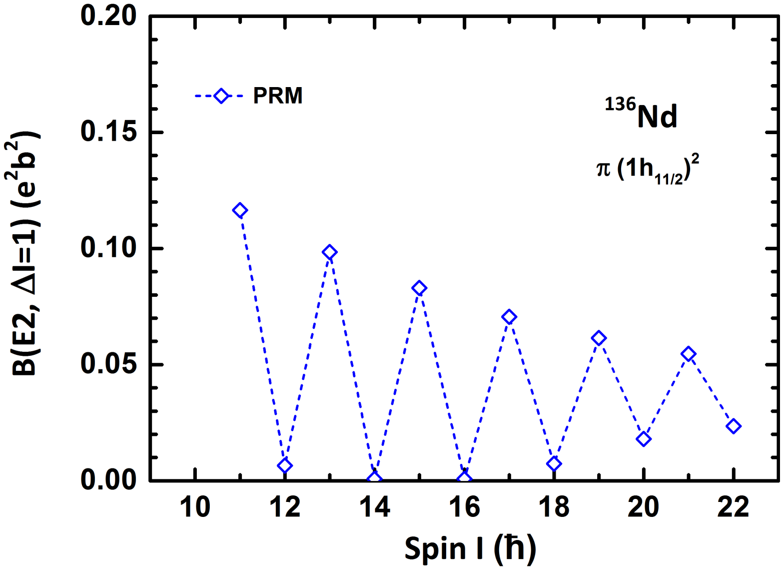
PRM: particle configuration

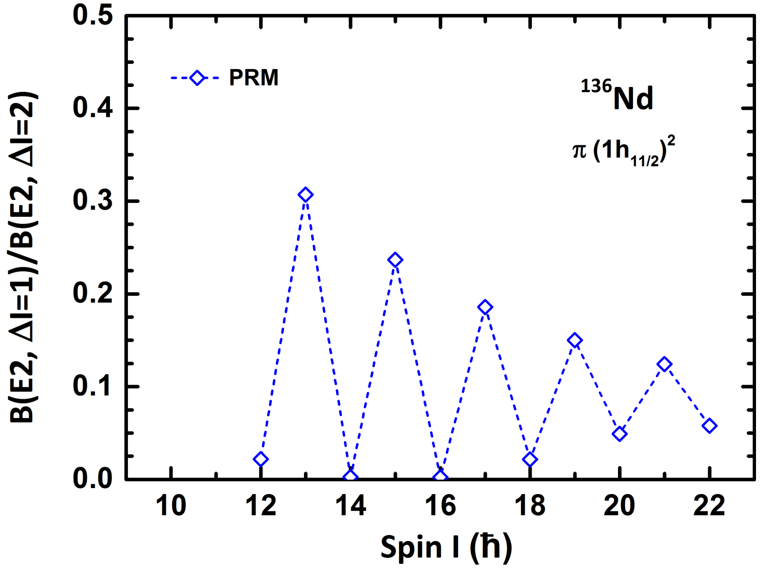


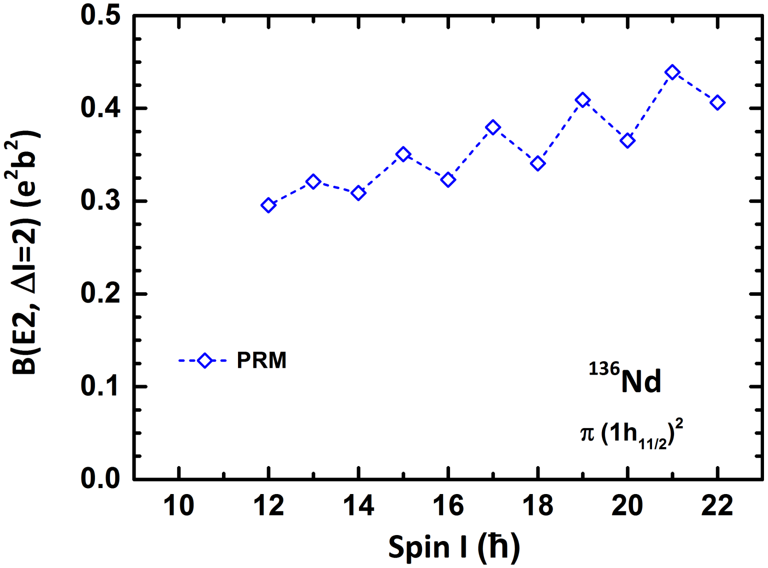
$$j_k = \sqrt{\langle j_k^2 \rangle}$$

- The angular momentum of proton particles that mainly align along short axis are indeed large.
- Total spin have large component along the short axis at the low spin region.

PRM: particle configuration

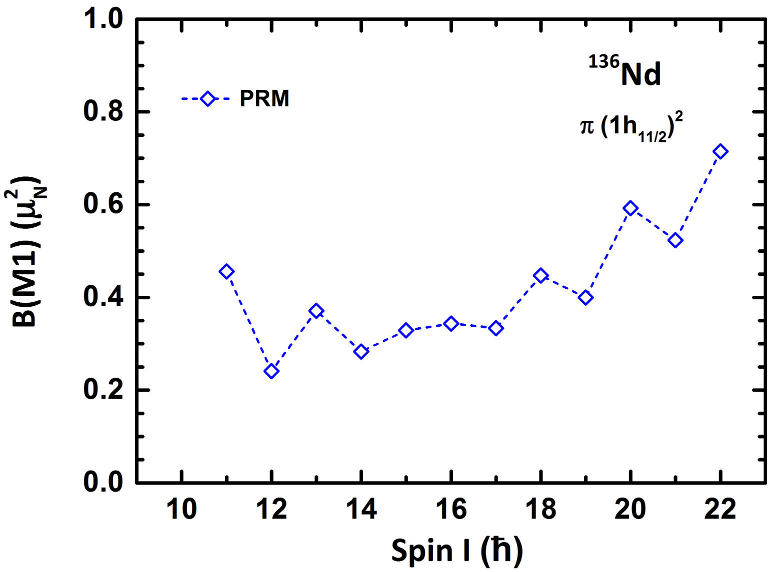


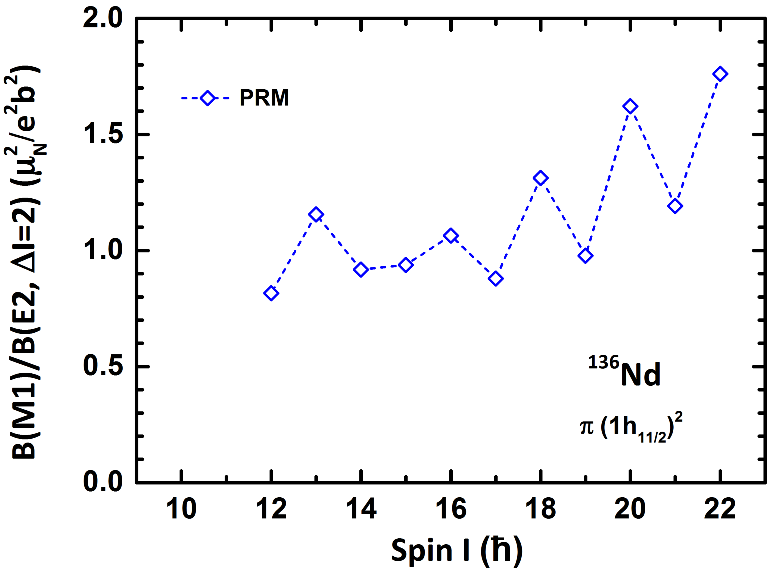


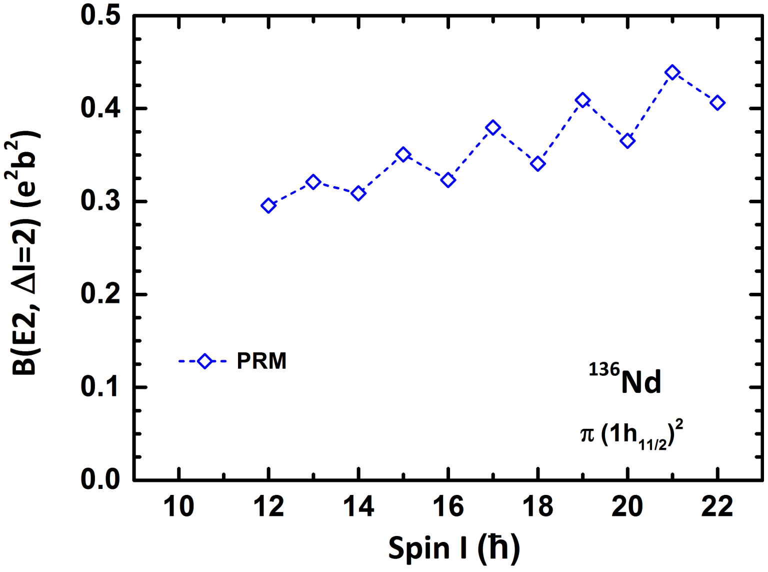


- **The magnitude of BE2 ratio of band E is comparable with that in 135Pr,**
- **B(E2 I->I-1)~(30-15)WU supporting the wobbling interpretation.**

PRM: particle configuration

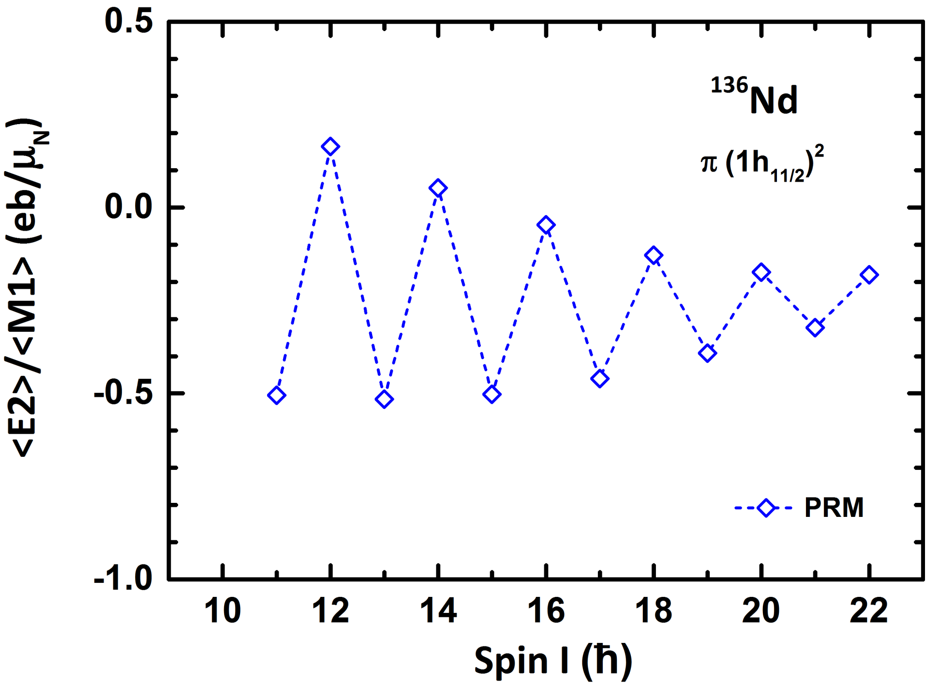






- **The magnitude of B(M1)/ B(E2) ratio ~10 times larger than that in 135Pr.**

PRM: particle configuration

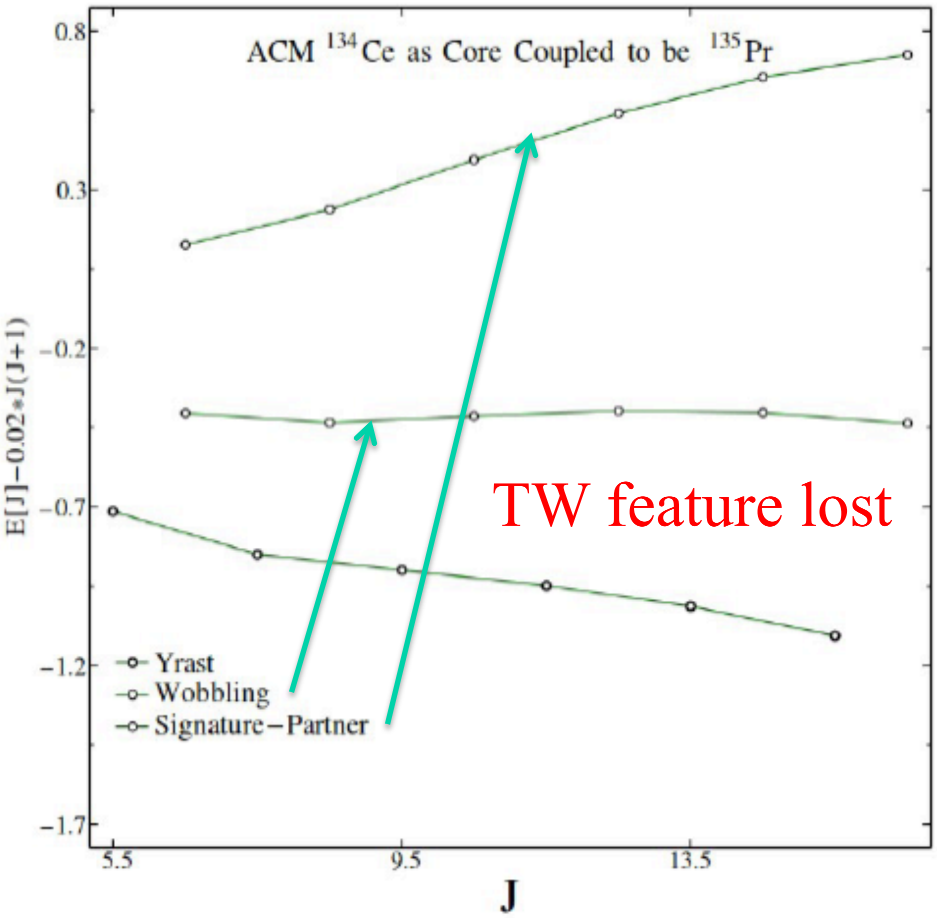


$$B(E2/M1, I \rightarrow I-1) = \langle E2/M1 \rangle^2$$

- **The magnitude of this value is smaller than in 135Pr (from -1.25 to -0.50).**

^{135}Pr : ACM ^{134}Ce as core

Weichuan Li



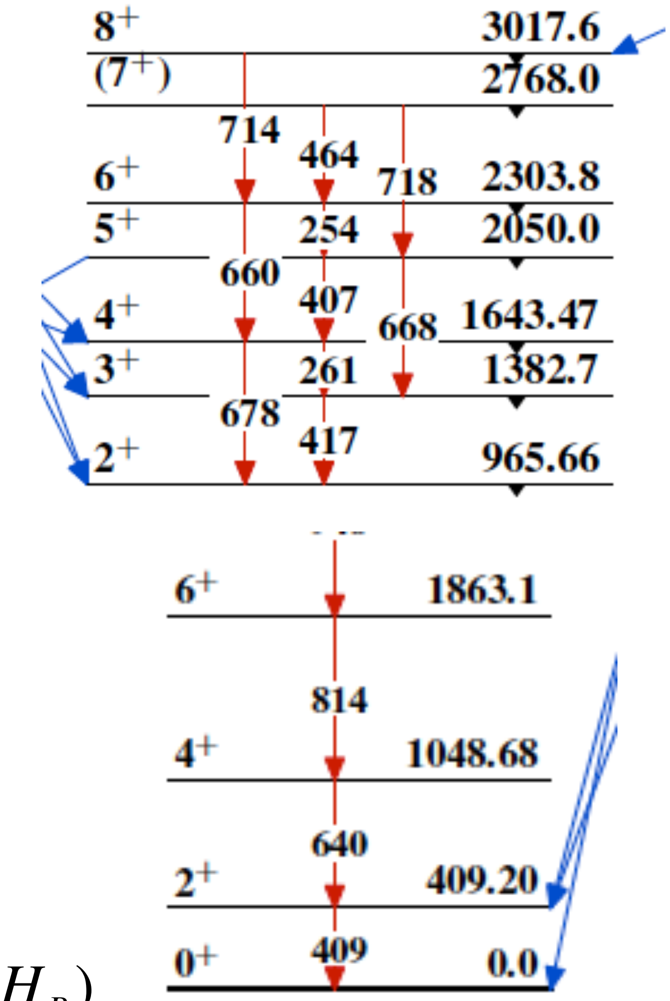
TW feature lost

coupling to a "γ soft core" (Bohr Hamiltonian H_B)

by quasiparticle coupling model

$$H = h_{sph} + \kappa[qQ]_0 + \Delta P - \lambda N + H_B$$

Band(D): Quasi γ-band



Fit to 4 parameter Bohr Hamiltonian

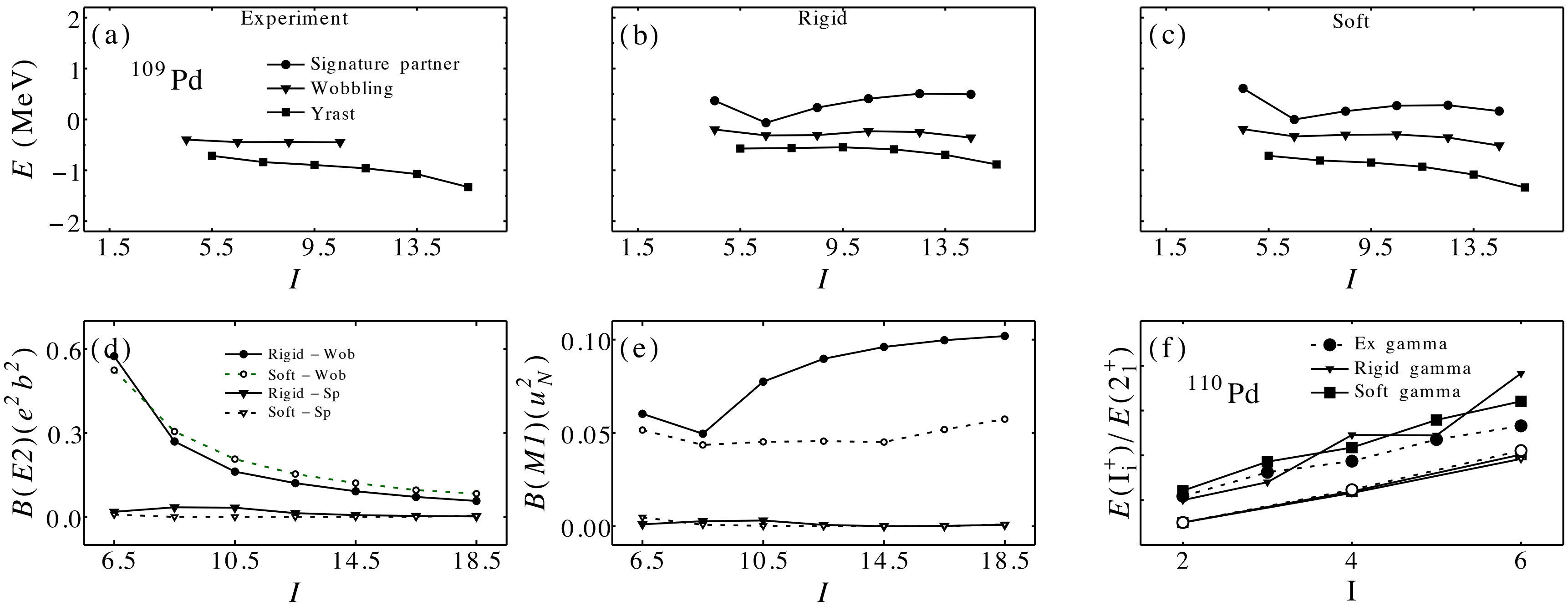
Odd-A nuclei: γ rigid or soft?

- Transverse wobbling appears only for stiff core
- Holds also for appearance of chiral pairs
- Does the presence of odd quasiparticles make the triaxial core stiff?
- The core is not soft as suggested by the collective Bohr Hamiltonian but stiff as assumed by the TSPM

Odd-A nuclei: γ rigid or soft?

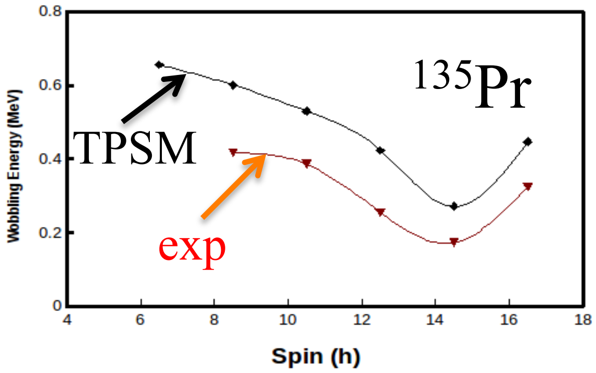
- Not the right question
- Maybe better question:

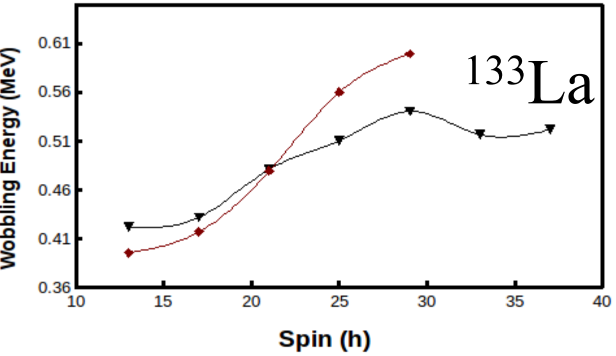
  How coherent is the γ excitation mode?
- Pure collective models are have a very limited range of application
- Strong coupling to the quasiparticle degrees of freedom is crucial
- QP+TR and TPSM are only the first steps.
- Relation to the adiabatic microscopic BH

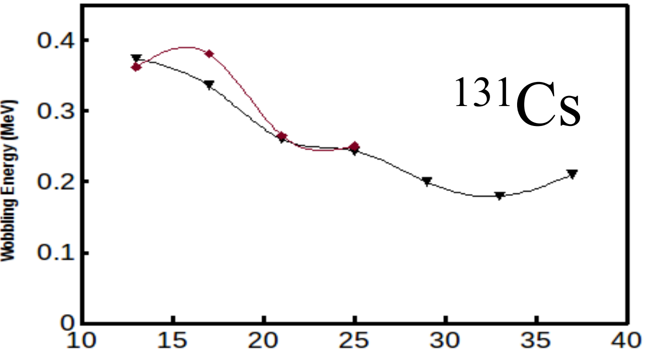

(a)
Experiment
^{109}Pd
Signature partner
Wobbling
Yrast
E (MeV)
2
1
0
−1
−2
1.5
5.5
9.5
13.5
I
(b)
Rigid
1.5
5.5
9.5
13.5
I
(c)
Soft
1.5
5.5
9.5
13.5
I
(d)
Rigid – Wob
Soft – Wob
Rigid – Sp
Soft – Sp
$B(E2)(e^2b^2)$
0.6
0.3
0.0
6.5
10.5
14.5
18.5
I
(e)
$B(M1)(u_N^2)$
0.10
0.05
0.00
6.5
10.5
14.5
18.5
I
(f)
^{110}Pd
Ex gamma
Rigid gamma
Soft gamma
$E(I_i^+)/E(2_1^+)$
2
4
6
I

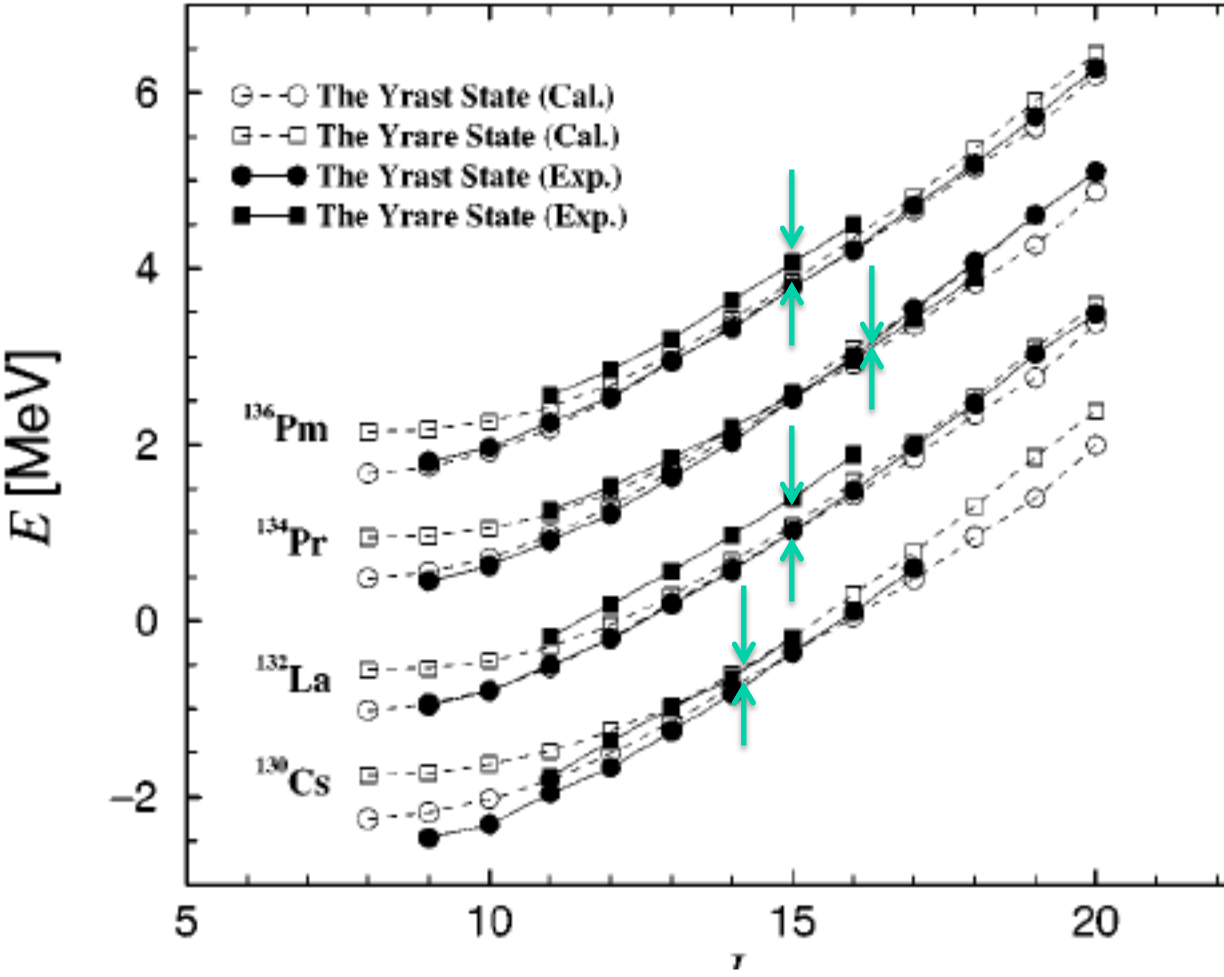
Problem with the QPTR model

Z-dependence of the TW and TCV vibrational frequencies not reproduced. TPSM OK. Pauli Principle core-qp?









J. Peng et al. PHYSICAL REVIEW C **68**, 044324 (2003

4th order Rotor Hamiltonian

- Low energy dynamics is dominated orientation degrees of freedom
- The adiabatic approximation good when RPA energy is ’small’

The simplest Hamiltonian that can describe our tilted system is a forth order rotor Hamiltonian.

$$H_I = c_1 I_1^2 + c_2 I_2^2 + c_3 I_3^2 + c_4 I_1^4 + c_5 I_3^4 + \frac{c_6}{2}(I_1^2 I_3^2 + I_3^2 I_1^2)$$

where the forth order terms mimic the tilted mean field solution.

This Hamiltonian has the energy surface

$$E_J = c_1 J_1^2 + c_2 J_2^2 + c_3 J_3^2 + c_4 J_1^4 + c_5 J_3^4 + c_6 J_1^2 J_3^2 + c_0$$

$$E_J = c_1 (J^2 - J_3^2)\cos^2\varphi + c_2 (J^2 - J_3^2)\sin^2\varphi + c_3 J_3^2 +$$

$$c_4 (J^2 - J_3^2)^2 \cos^4\varphi + c_5 J_3^4 + c_6 J_3^2 (J^2 - J_3^2)\cos^2\varphi$$

at a constant I which we used to fit the c_i parameters to the energy surface calculated with the full TAC Hamiltonian.

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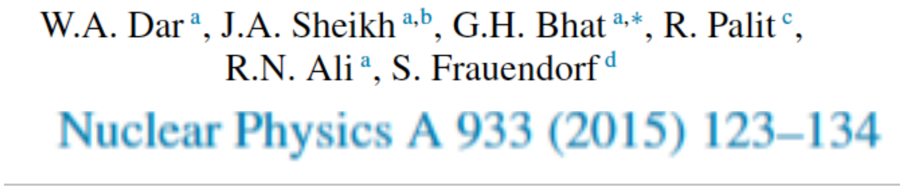
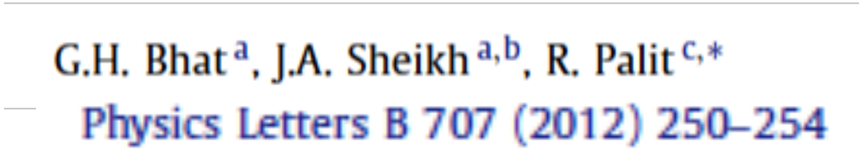
Triaxial Projected Shell Model TPSM


W.A. Dar[a], J.A. Sheikh[a,b], G.H. Bhat[a,*], R. Palit[c],
R.N. Ali[a], S. Frauendorf[d]
Nuclear Physics A 933 (2015) 123–134

o-o A≈100


G.H. Bhat[a], J.A. Sheikh[a,b], R. Palit[c,*]
Physics Letters B 707 (2012) 250–254

^{128}Cs


G.H. Bhat[a], J.A. Sheikh[a,b], R. Palit[c,*], SF

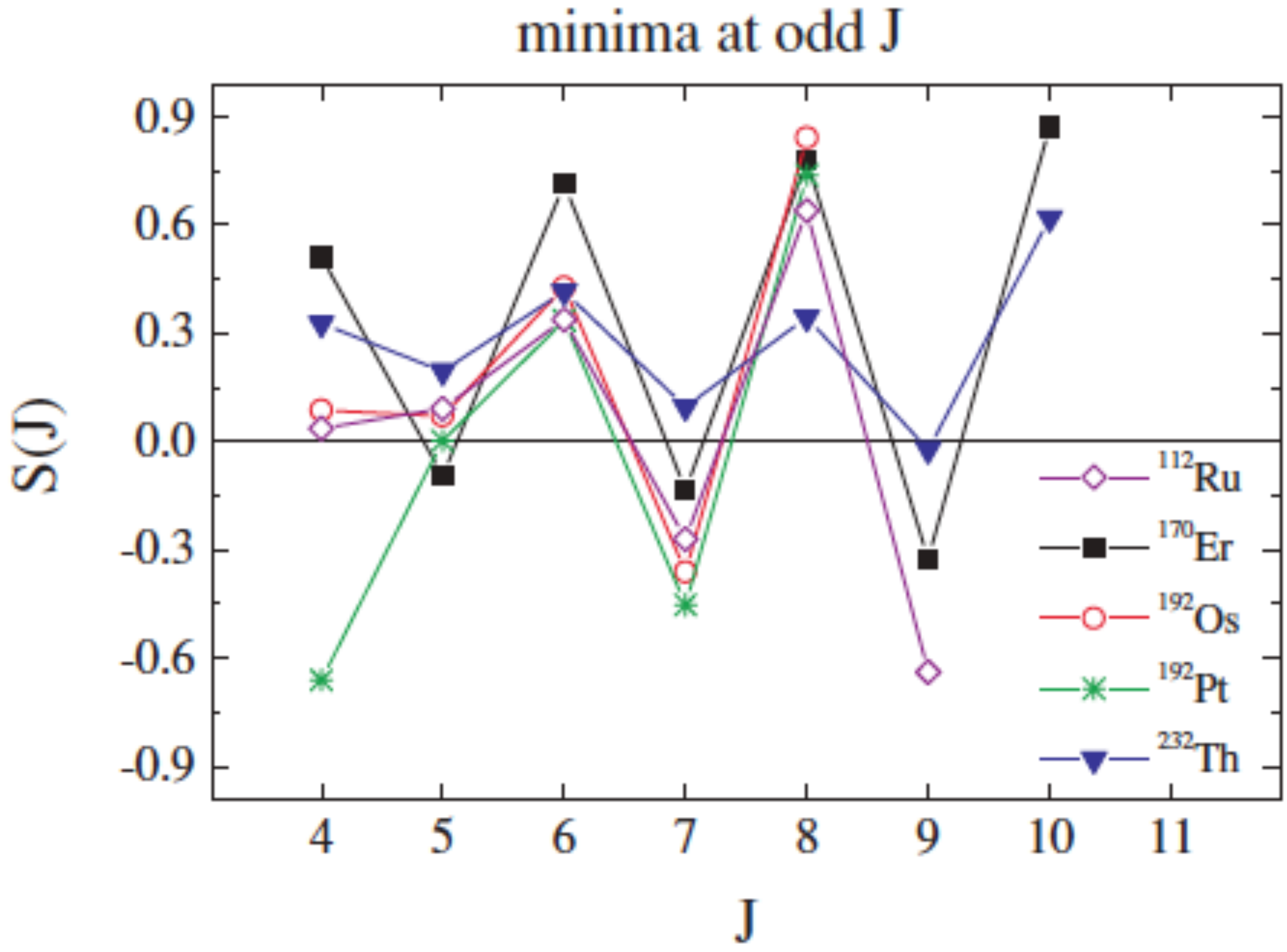
o-e A≈135 TW

Microscopic (no phenomenological core)

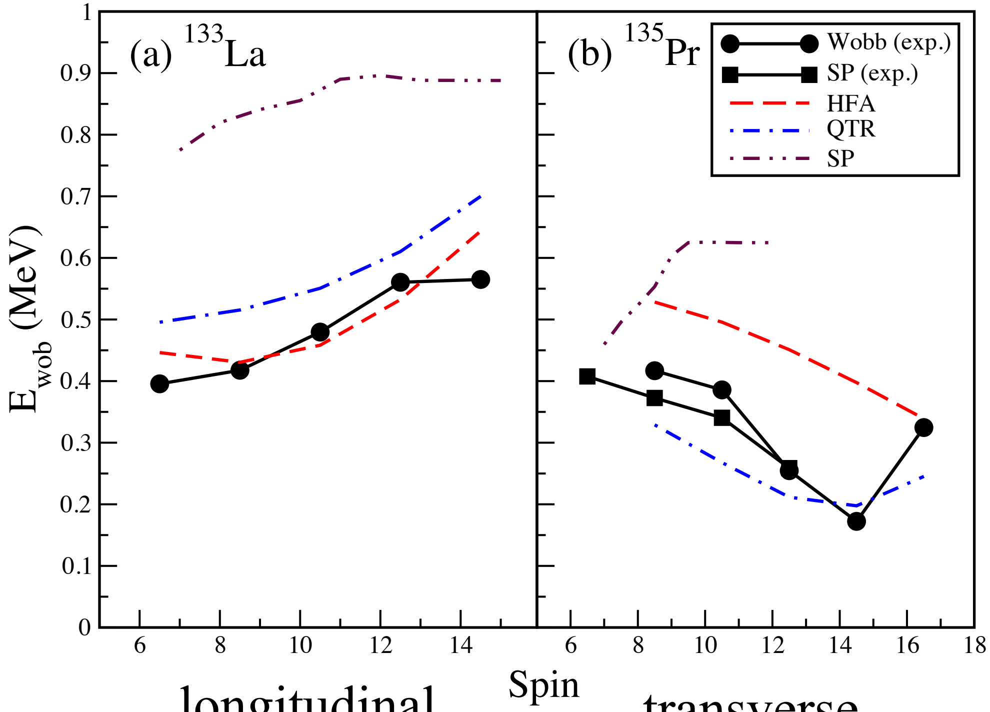
Non-adiabatic

Large amplitude

Fixed shape



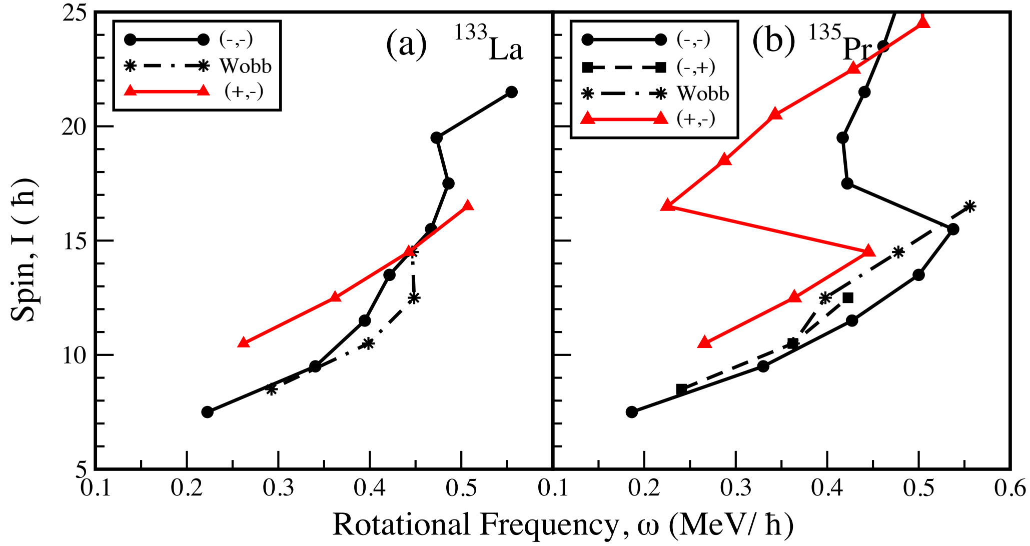
S. Frauendorf, International Journal of Modern Physics E Vol. 24, No. 9 (2015) 1541001



longitudinal

transverse

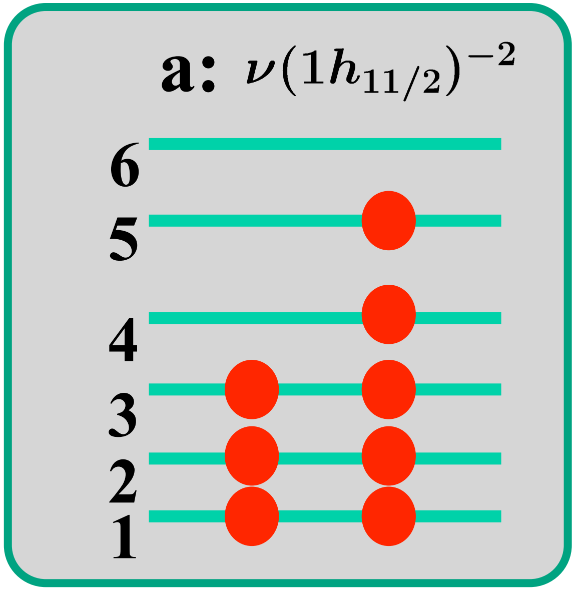
What’s going on??

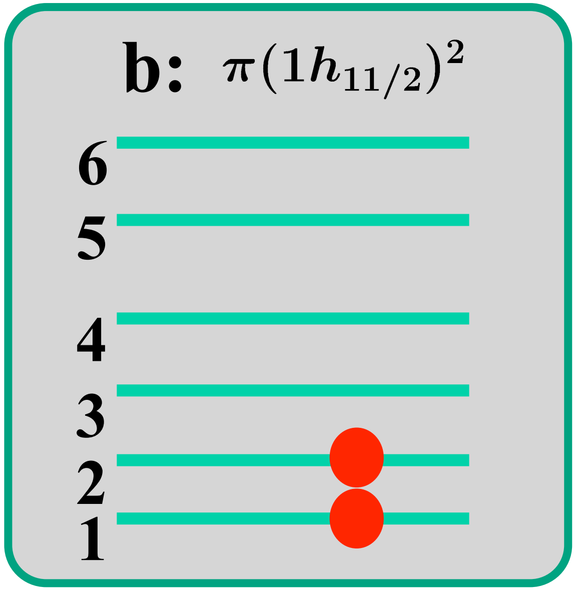


CDFT calculations

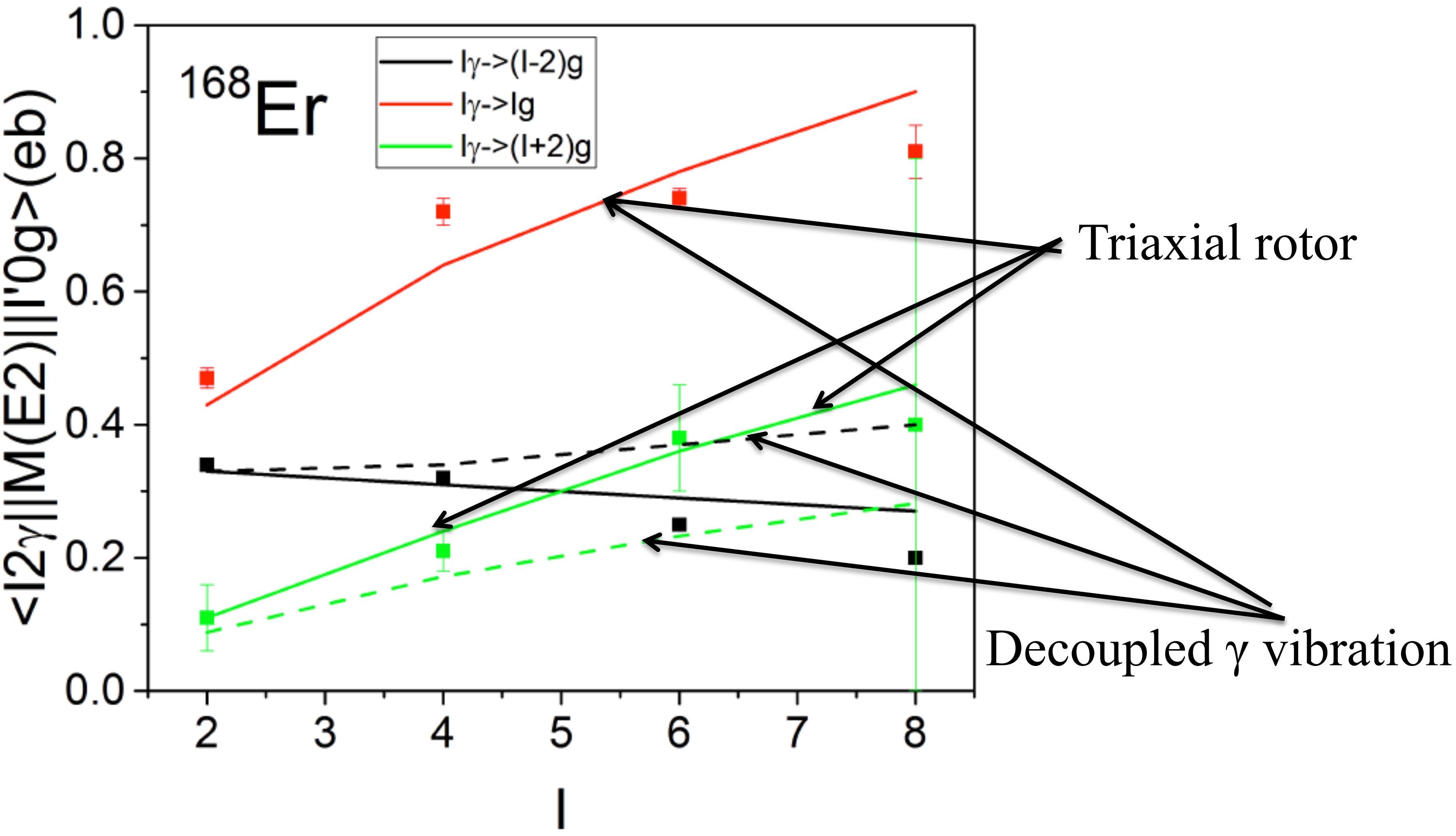
- **Numerical details:**
  - **Interaction parameter: PC-PK1;**
  - **Harmonic oscillator shells: Nf = 12;**
  - **Paring correlations are neglected.**

State	E_x	(β,γ)	Valence configuration	Unpaired configuration	Parity
A	0.00	$(0.14, 60.0^\circ)$	$\pi(gd)^{10}\otimes\nu(1h_{11/2})^{-4}$		+
a	0.68	$(0.18, 23.4^\circ)$	$\pi(gd)^{10}\otimes\nu(1h_{11/2})^{-4}$	$\nu(1h_{11/2})^{-2}$	+
b	1.03	$(0.21, 18.1^\circ)$	$\pi(gd)^{8}(1h_{11/2})^{2}\otimes\nu(1h_{11/2})^{-4}$	$\pi(1h_{11/2})^{2}$	+





Coupling between γ mode and rotation is very well Described by triaxial rotor (Q_2=7eb)



Triaxial rotor

Decoupled γ vibration