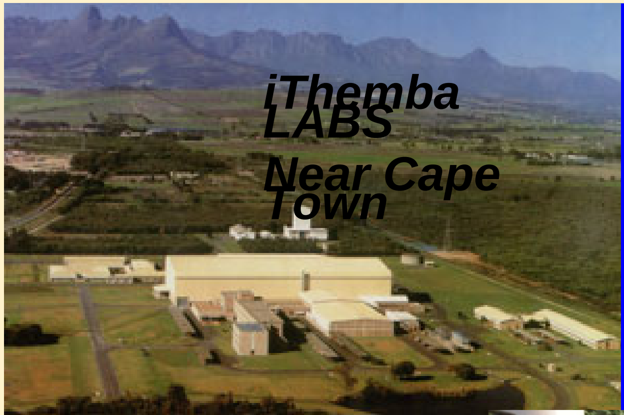
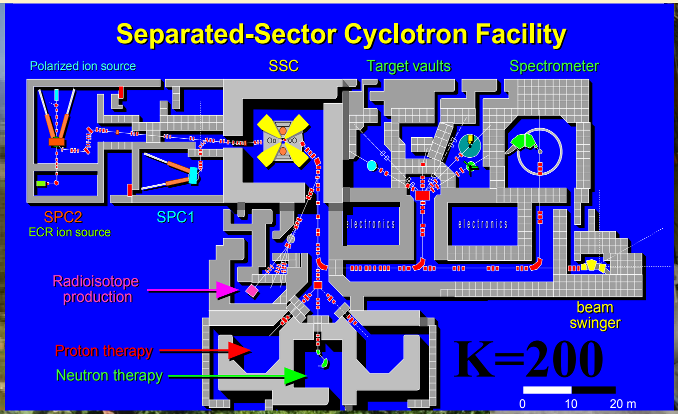
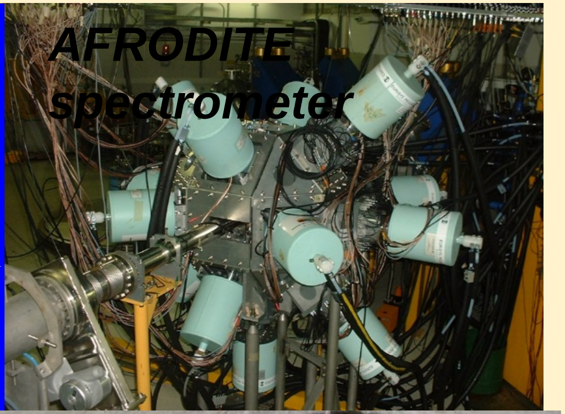
"Stiff" Deformed Nuclei and the β and ʐ Degrees of Freedom

John F Sharpey-Schafer

University of the Western Cape, Belleville, South Africa

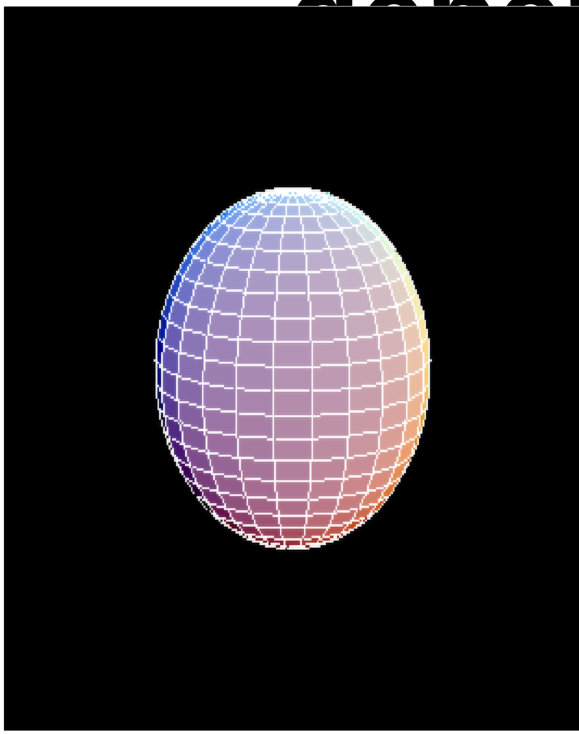






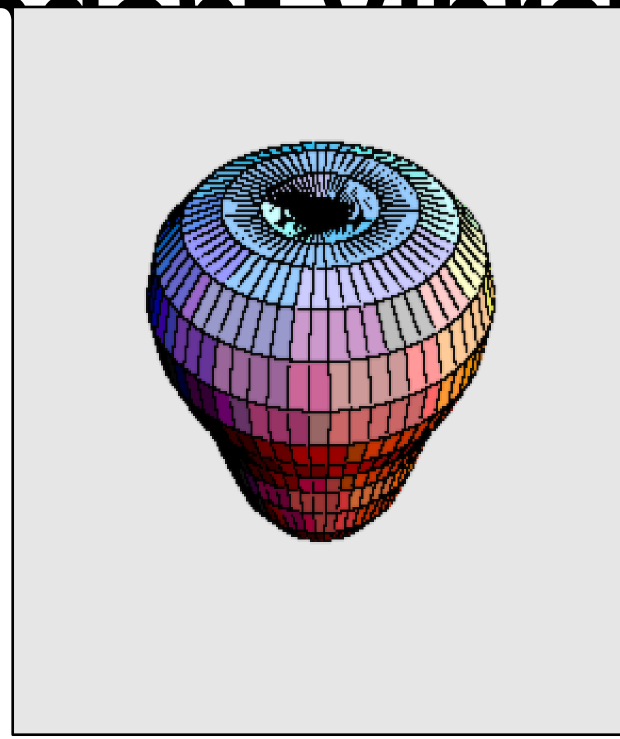
England. The only country to leave Europe twice in the one week.

Examples of collective time-dependent Vibrations



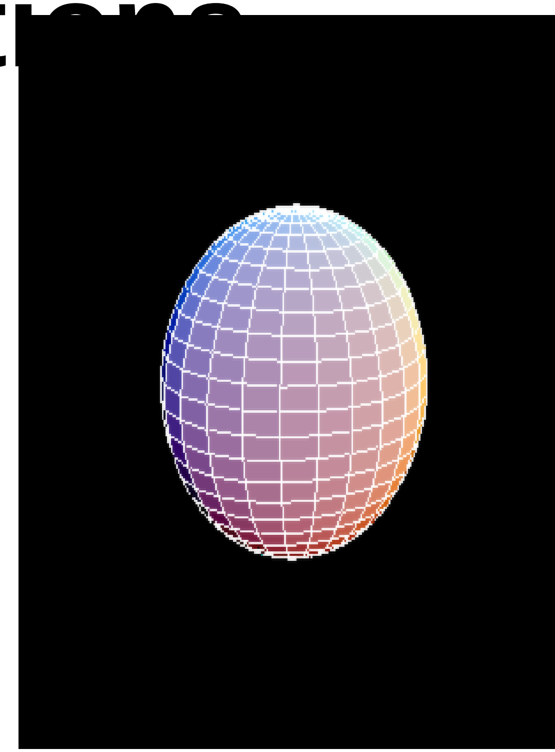
$\lambda = 2, 0$ $a_{2,0}$

Quadrupole β vibration



$\lambda = 3, 0$ $a_{3,0}$

Octupole vibration



$\lambda = 2, 2$ $a_{2,2}$

Quadrupole γ vibration

web-docs.gsi.de/~wolle/TELEKOLLEG/KERN/index-e.html

Classical Vibrations of a Liquid Drop

Lord Rayleigh (John William Strutt) Proc. Roy. Soc. 29,71 (1879) Appendix II Equ. 40, got:

(Frequency)2 = $\omega^2 = \dfrac{(\lambda-1)\lambda(\lambda+2)\gamma}{\rho R^3}$

for a superfluid incompressible liquid sphere.

Also See:
S Flügge, Ann Phys Lpz 431 (1941) 373

For a charged spherical nucleus this becomes:

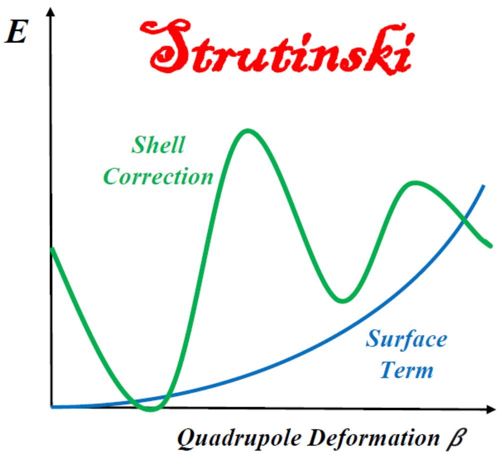
$$\omega^2 = \frac{(\lambda-1)\lambda(\lambda+2)}{3}\frac{C_s}{R_A^2 mA} - \frac{2(\lambda-1)\lambda}{(2\lambda+1)}\frac{e^2Z^2}{4\pi\epsilon_0 R_A^3 m_p A^2}$$

Cs ~ 18 MeV from the surface term in the Weiszaker Mass Formula and the second term has little effect for Z<80.

Quantizing using ***Eλ = ħω*** gives E20 = 2.4 MeV and E30 = 4.6 Me

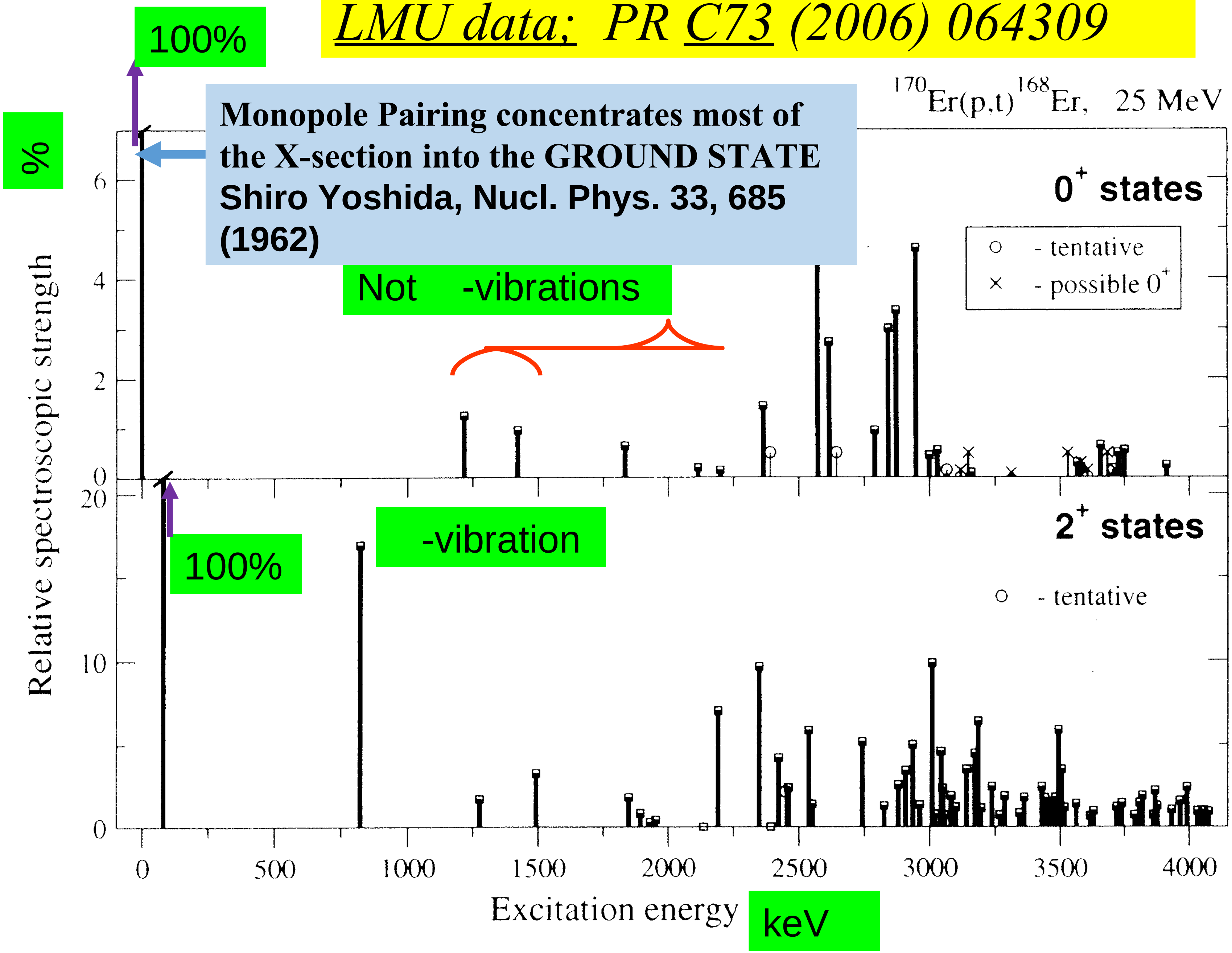
Moments-of-Inertia NOT Irrotational will also INCREASE ***Eλ***

Shell Corrections will have the effect of INCREASING ***Eλ***



LMU data; PR C73 (2006) 064309

Monopole Pairing concentrates most of the X-section into the GROUND STATE
Shiro Yoshida, Nucl. Phys. 33, 685 (1962)



UNEXPECTED STRONG PAIR CORRELATIONS IN EXCITED 0^+ STATES OF ACTINIDE NUCLEI*

J. V. Maher, J. R. Erskine, A. M. Friedman, J. P. Schiffer,† and R. H. Siemssen
Argonne National Laboratory, Argonne, Illinois 60439
(Received 1 June 1970)

The (p,t) reaction has been studied with 17-MeV protons on targets of Th^{230}, $U^{234,236,238}$, and $Pu^{242,244}$. The results indicate unexpectedly strong $l=0$ transitions to states at about 900-keV excitation. Their cross sections are approximately 15% of the ground-state transitions; this percentage does not change appreciably with neutron number. This result, together with other available evidence, seems to suggest a simple and rather stable collective mode which has not yet emerged from any theoretical calculations.

J V Maher et al.
PRL 25 (1970) 302

VOLUME 25, NUMBER 5 PHYSICAL REVIEW LETTERS 3 AUGUST 1970

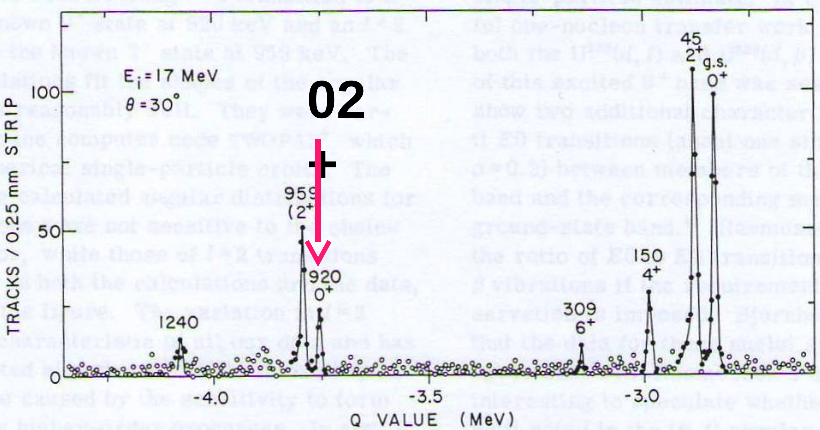


FIG. 1. Spectrum of tritons from the reaction $U^{238}(p,t)U^{236}$. The target was 35 $\mu g/cm^2$ of U^{238} evaporated onto a carbon foil. The peaks are labeled by the excitation energies (keV) and spins of the corresponding states in U^{236}.

238U(p,t)236 U
17 MeV
θ = 30º

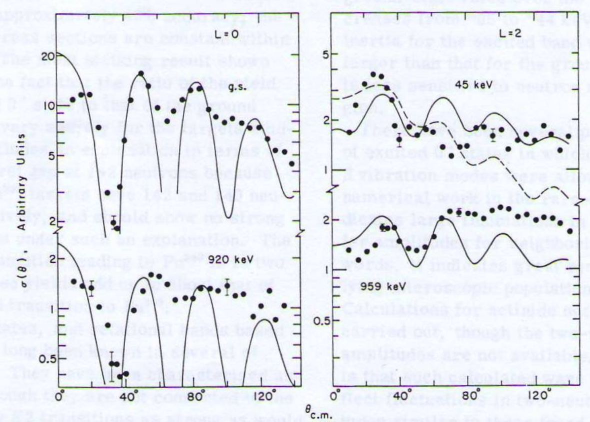


FIG. 2. Angular distributions for the reaction $U^{238}(p,t)U^{236}$. The relative yields for the various experimental data sets are correctly shown. The cross section at ~60° for the ground-state transition is 220 ±60 μb/sr. The DWBA curves were calculated with a spherical $3d_{5/2}$ form factor for the solid curves and $i_{11/2}$ for the dashed curve. Relative error bars are shown on a few representative points.

02+ ***NOT*** a -vibration
NOR a pairing vibration

Two Neutron Transfer to 154Gd (N=90)

Shahabuddin et al; NP **A340** (1980) 109

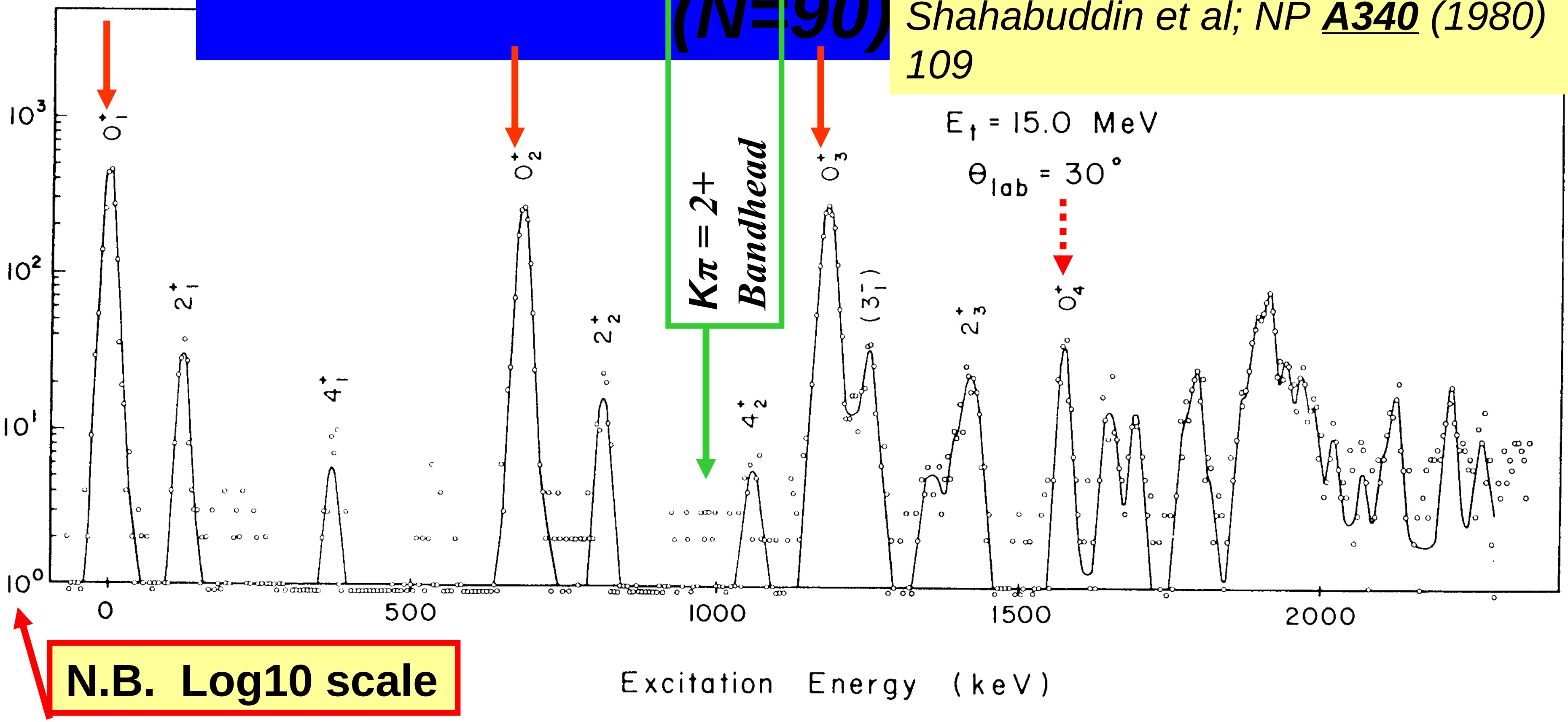


Fig. 1. A sample spectrum of the $^{152}Gd(t, p)^{154}Gd$ reaction at E_t = 15 MeV and θ_{lab} = 30°.

HENCE Monopole Pairing is NOT Suff icient

Configuration Dependent Pairing

R. E. Griffin, A. D. Jackson and A. B. Volkov, Phys. Lett. 36B, 281 (1971).

Suggested that pp oo >> op

for Actinide Nuclei where 02+ states were observed in (p,t) that were not pairing- or β-vibrations.

Suppose there are *n prolate* and *n oblate* degenerate levels at the Fermi Surface

Assume that each pairing matrix element is the same for the same type *-a*

BUT the *prolate-oblate* matrix elements are very weak *–εa*

Then if the prolate *n*n* matrix is *A*, the oblate matrix is also *A*

Prolate

Oblate

The matrix for the total system is;

$$\begin{bmatrix} A & \varepsilon A \\ \varepsilon A & A \end{bmatrix}$$

(2n-

02+

01+

Then there are (2*n*-2) states with ZERO energy and 2 states with energies

$$E_{1,2} = -(1 \pm \varepsilon)\, na$$

I. Ragnarsson and R. A. Broglia, Nucl. Phys. A263, 315 (1976).
coined the term "pairing isomers" for these 02+ states

PHYSICAL REVIEW C VOLUME 15, NUMBER 6 JUNE 1977

N.B. Log10 Scale

Analysis of the (p,t) reaction on ^{158}Dy†

J. J. Kolata
Brookhaven National Laboratory, Upton, New York 11973

M. Oothoudt*
Princeton University, Princeton, New Jersey 08540

(Received 28 February 1977)

Jim Kolata and Mike Oothoudt
Phys. Rev. C15 (1977) 1947

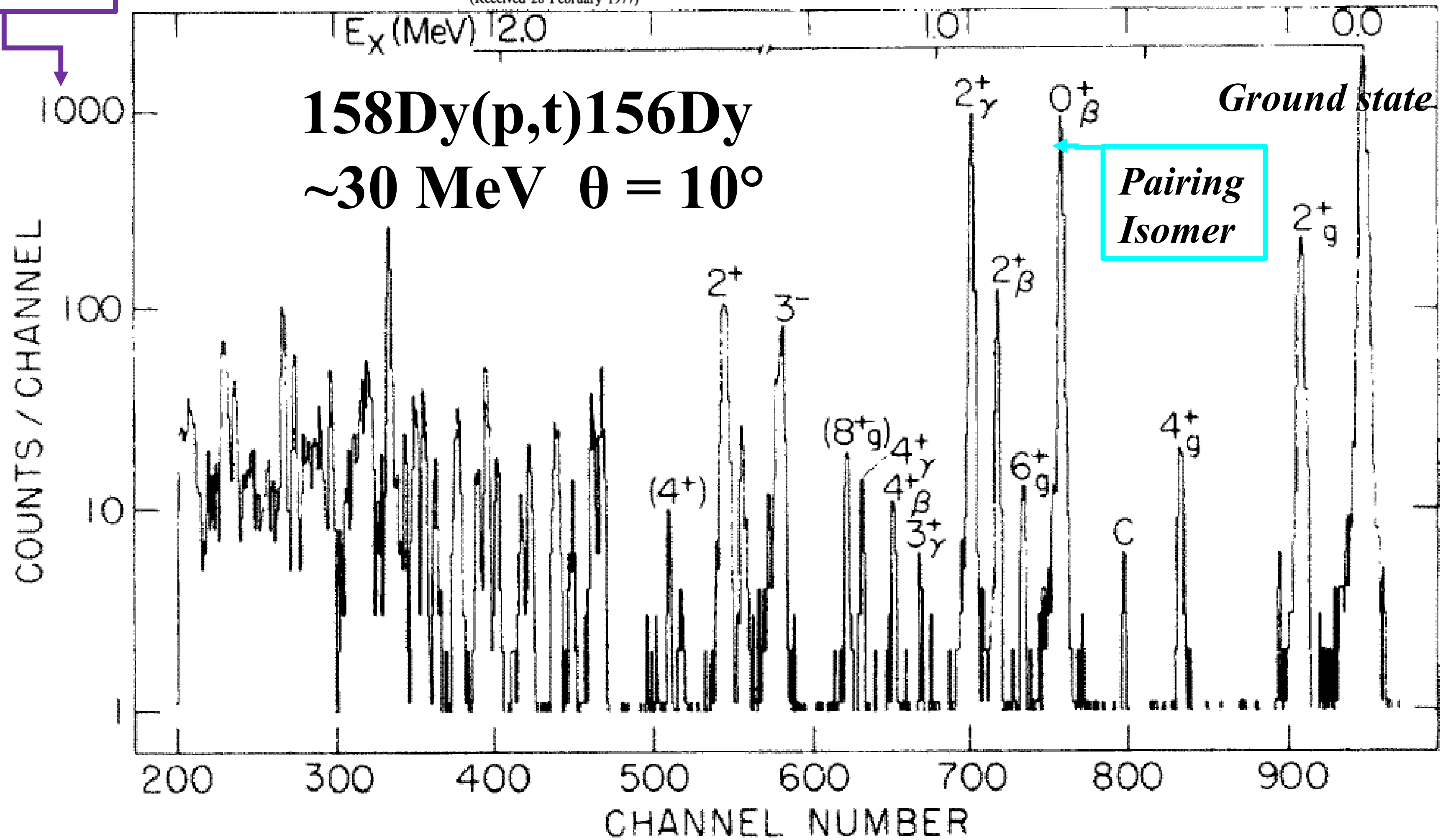
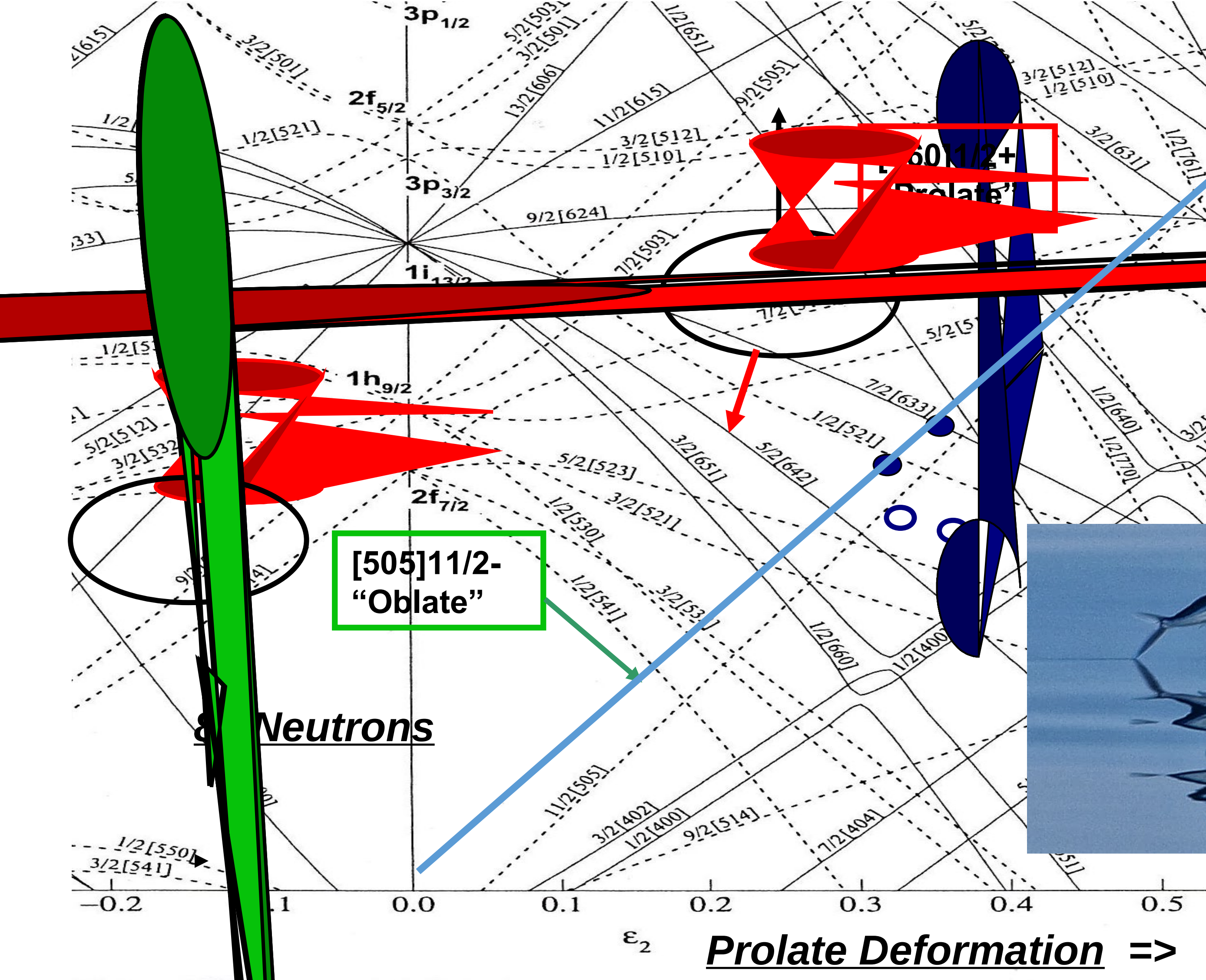


FIG. 1. Typical spectrum for the ^{158}Dy$(p,t)^{156}$Dy reaction at $E_p = 29.9$ MeV and $\theta_{lab} = 10°$. This is a com-



Configuration Dependent or Quadrupole Pairing; Assume $\Delta pp \approx \Delta oo >> \Delta op$

Experiments in mass 160 region using AFRODITE array of 8 or 9 clover detectors + LEP detectors

e.g.

152Sm(∈,2n)154Gd

152Sm(∈,4n)152Gd

147Sm(16¬ ,4n)160Yb

144Sm(17¬ ,3n)158Yb

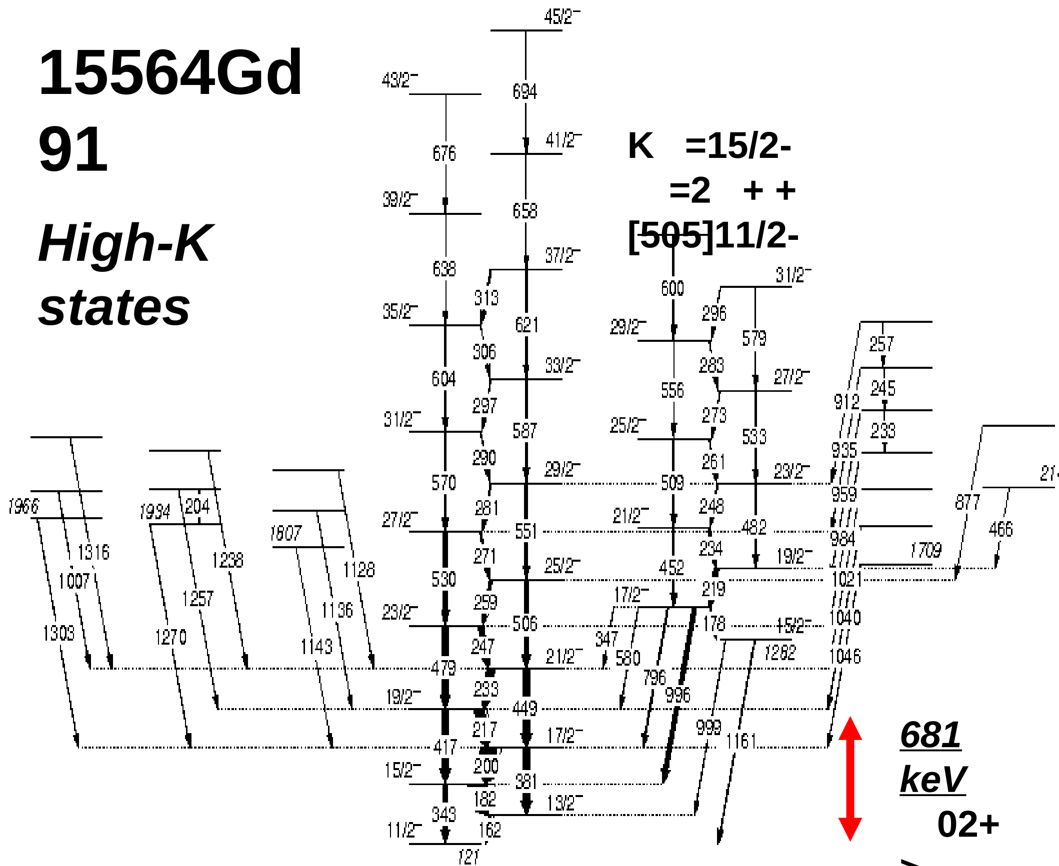
152Sm(12C,4n)160Er

148Sm(12C,4n)156Er

Typical statistics
of 1 or 2 Gevent
of B2B coincidences

15564Gd 91

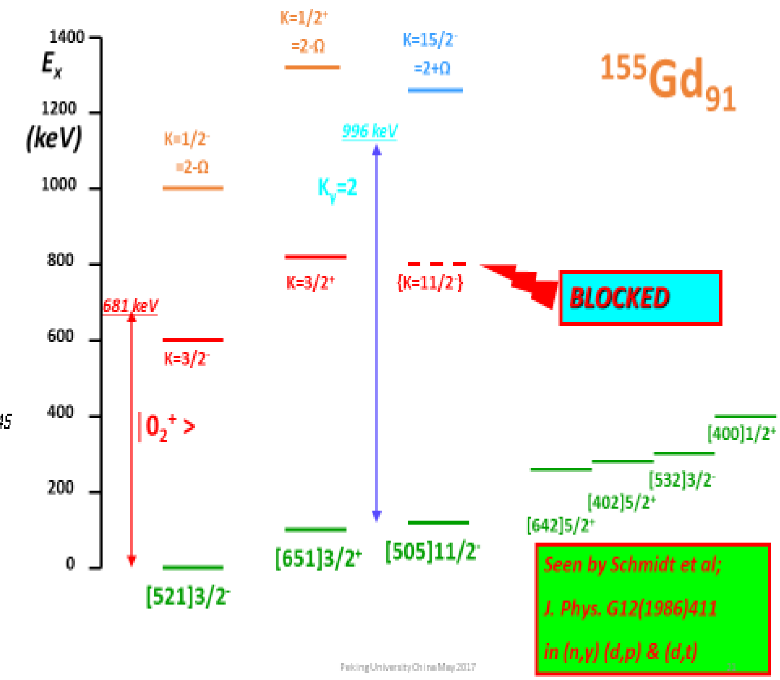
High-K states



K =15/2-
=2 + +
[505]11/2-

681 keV
02+ >

[505]11/2-

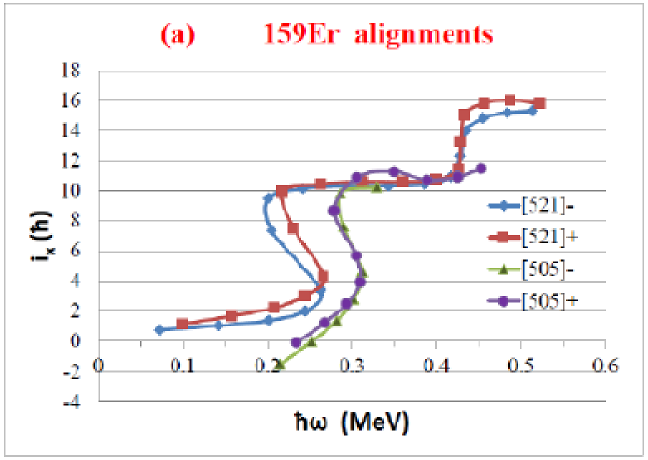


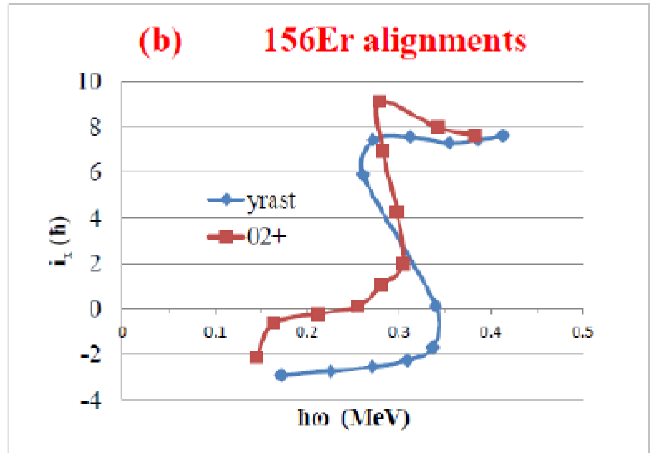
$\varepsilon \approx 0.05$??

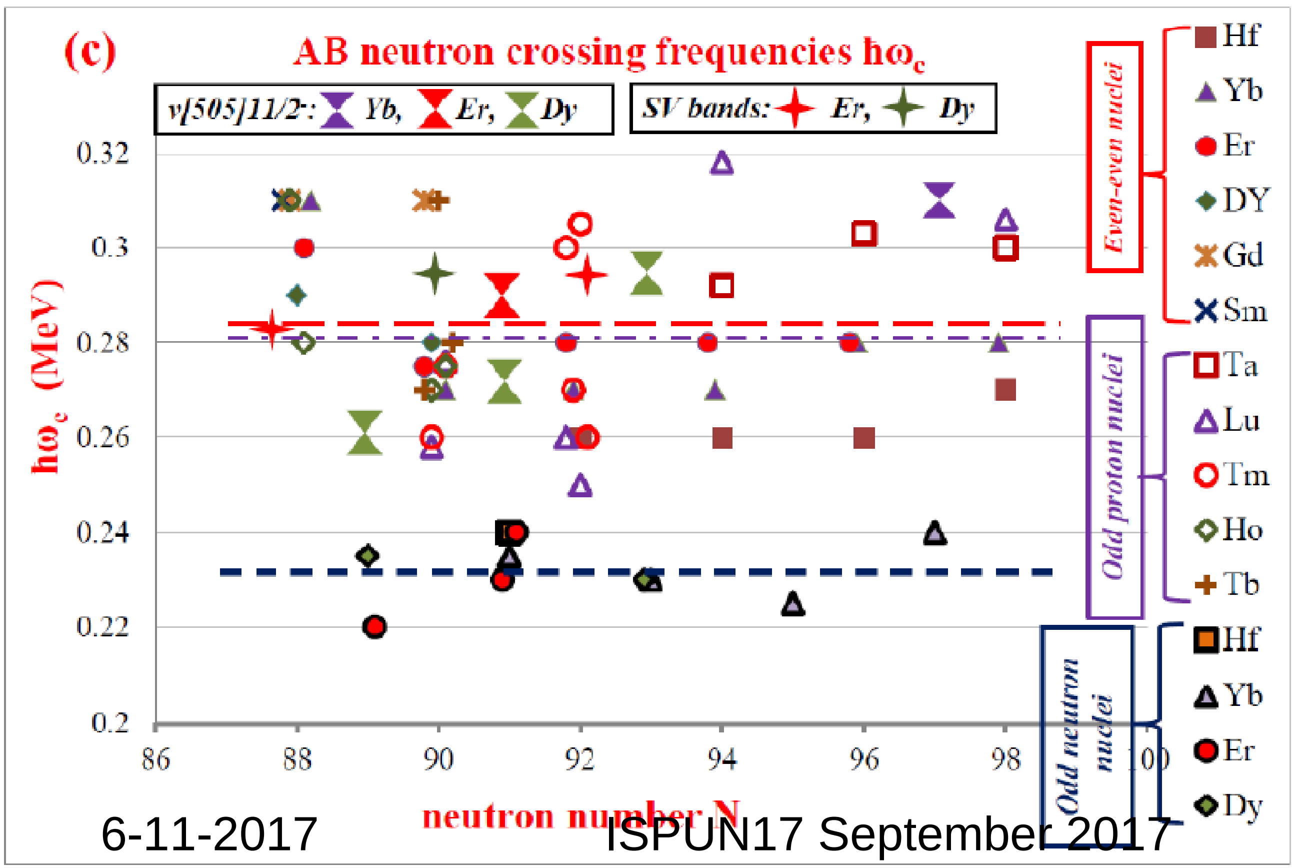
What you really need is
SPLIT MONOPOLE PAIRING
so that

$$-H_{pairing} = G_{p-p}\dot{P}_p^{\dagger}\dot{P}_p + G_{o-o}\dot{P}_o^{\dagger}\dot{P}_o + \varepsilon G_{p-p}\dot{P}_{po}^{\dagger}\dot{P}_{po}$$

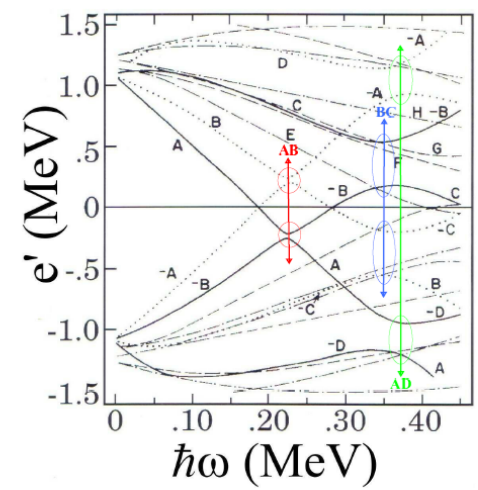
i13/2 neutron AB alignments from N =88 to 98 and Z = 62 to 72



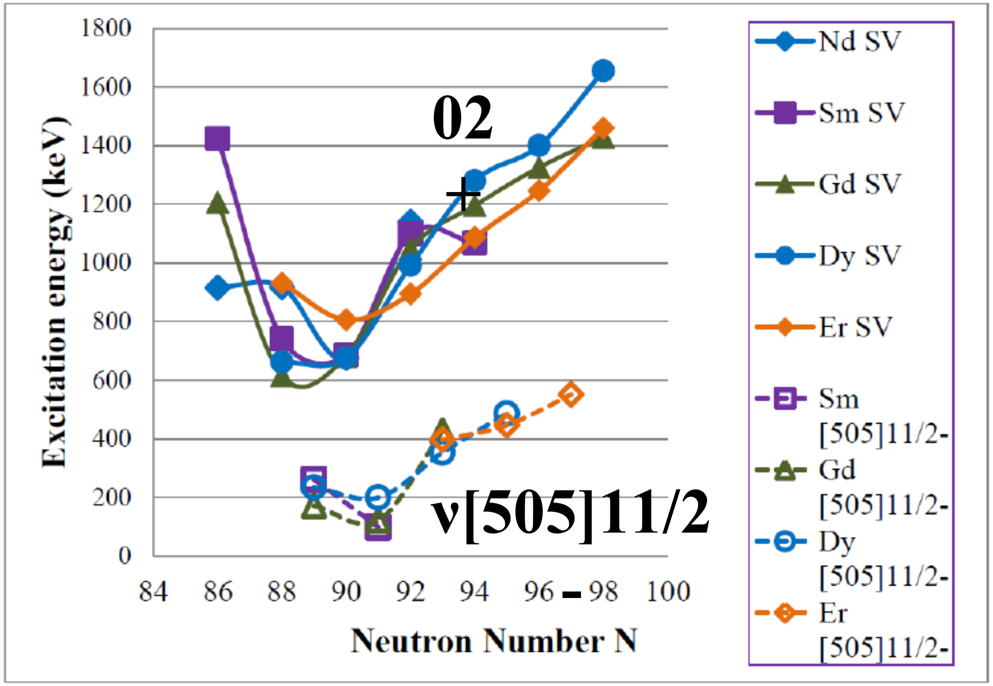


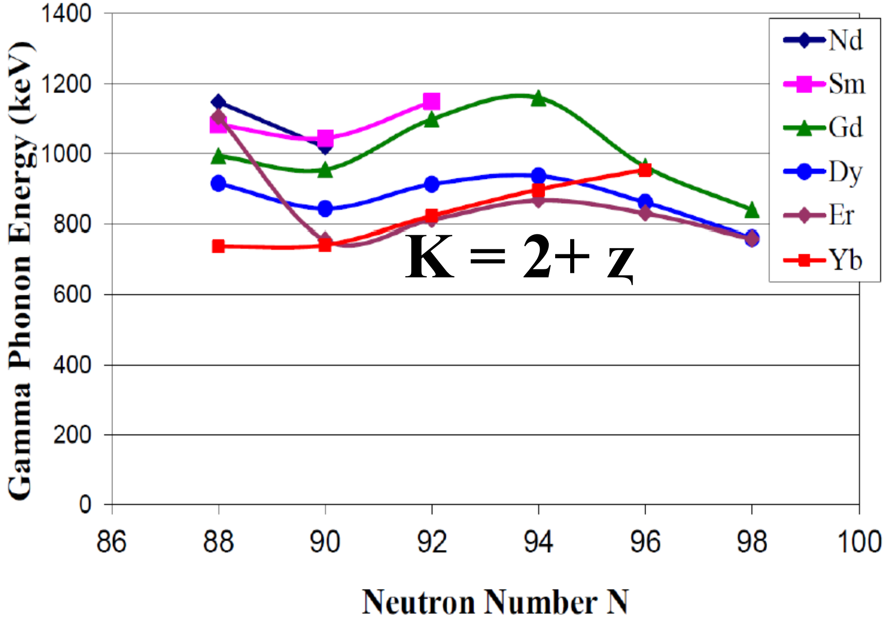


See:
Jerry Garrett et al.
PL B118 (1982) 297



Cranked Shell Model Routhians *e'*



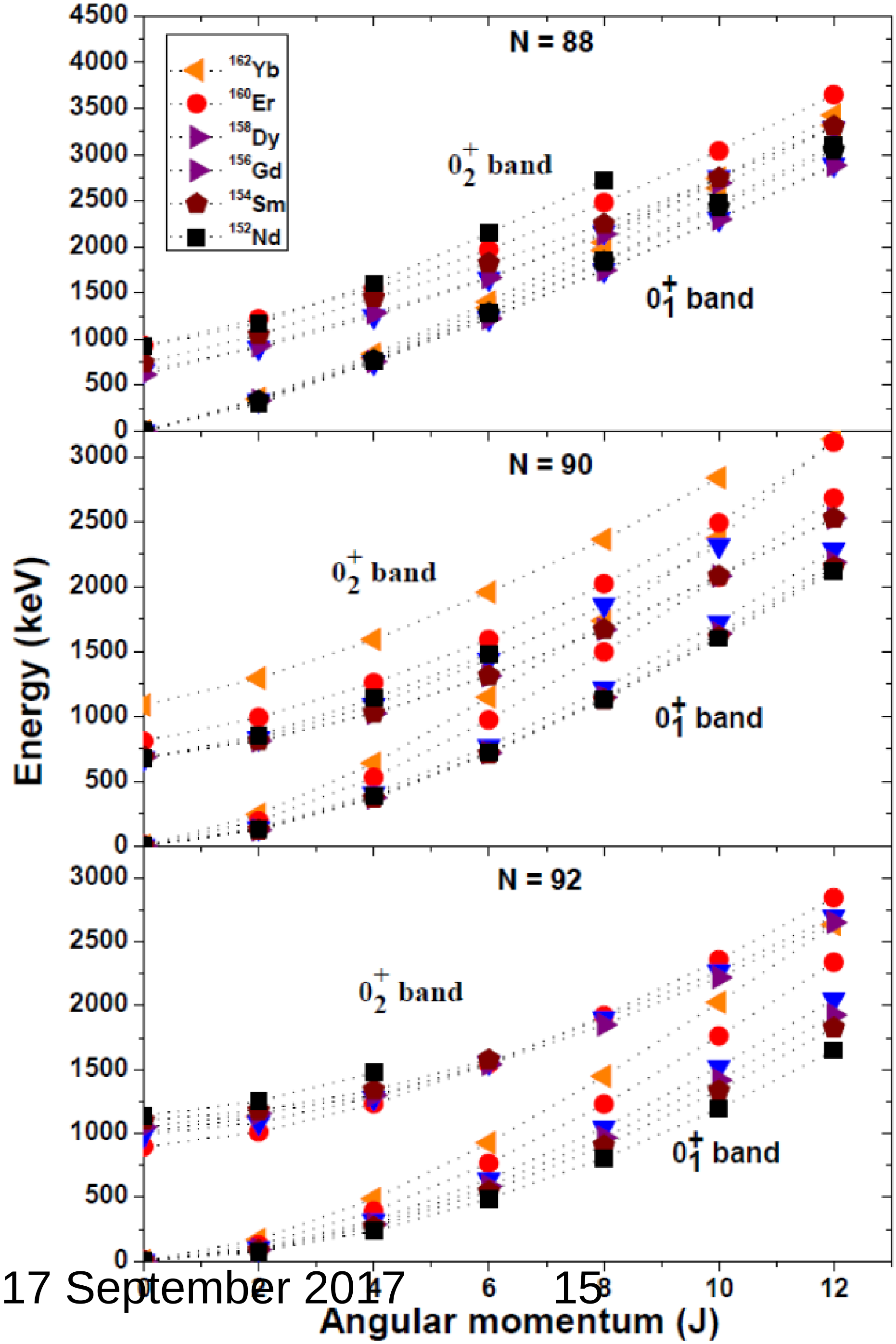


Excitation Energies of 02+ band-heads and ν[505]11/2- isomers

Excitation Energies of K = 2+ ɀ band-heads

Systematics of the Energies of the 01+ ground state band and the 02+ first excited band in even-even N = 88, 90, 92 nuclei.

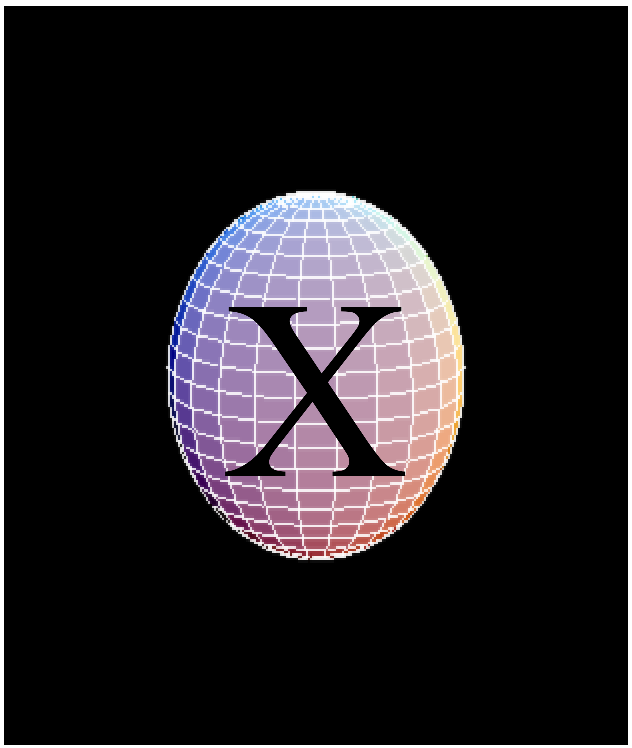
Tshepo Dinoko et al. to be published

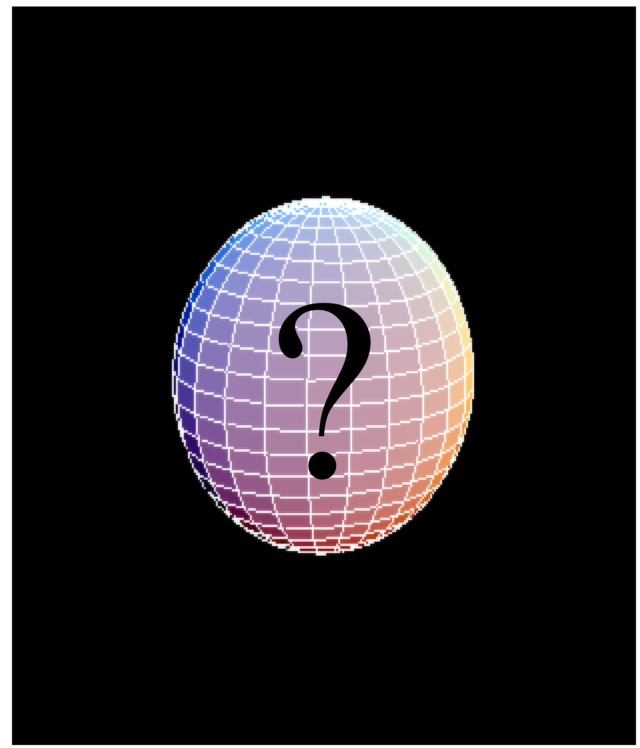


In complete ***CONTRADICTION*** to the predictions of the Interacting Boson Model of Arima and Iachello

***If* “□ bands” are NOT time-dependent shape vibrations**

What about K = 2+ “□ bands” ??





First Approach !! **; ALIGNMENTS**

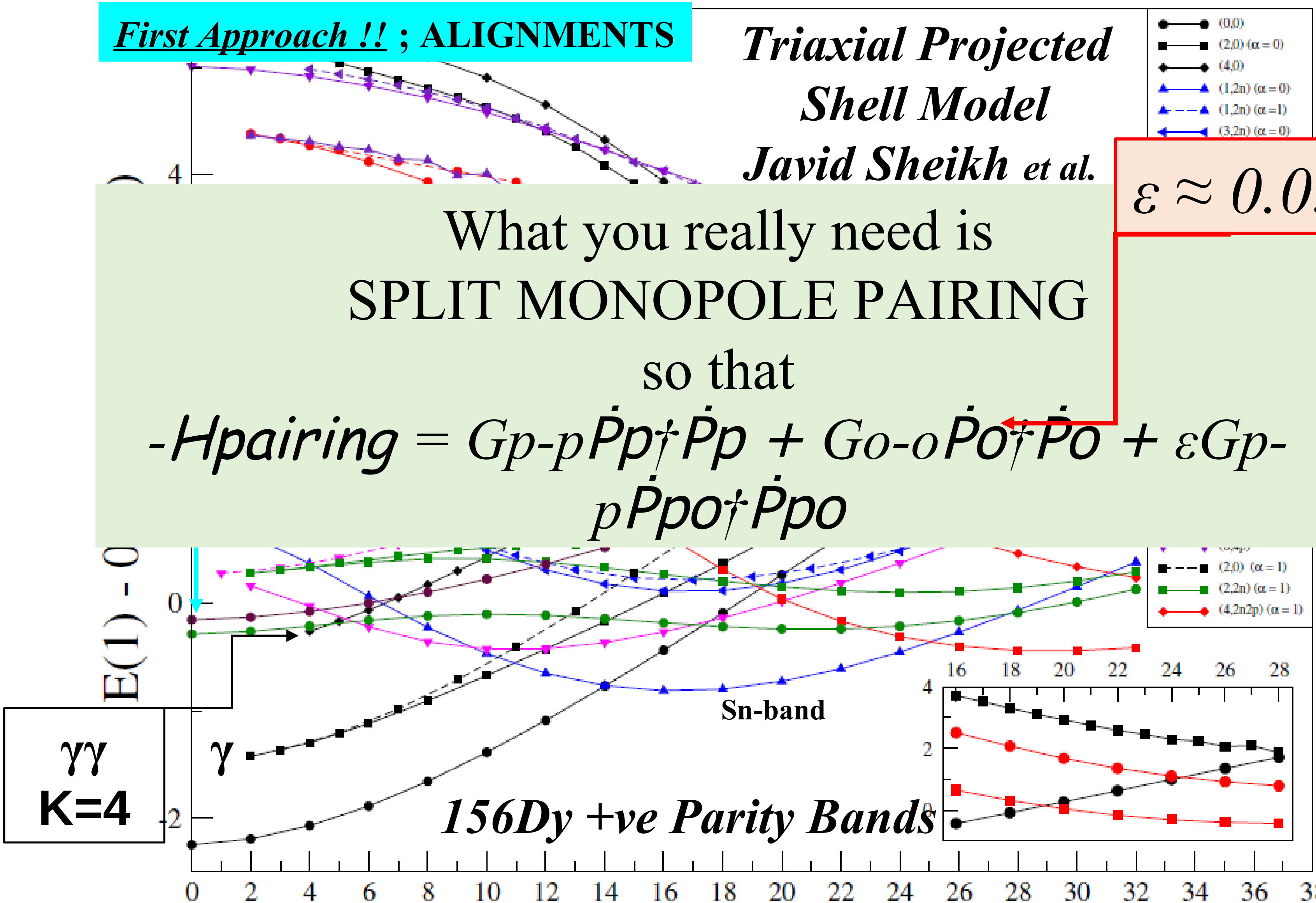
Triaxial Projected Shell Model

***Javid Sheikh** et al.*

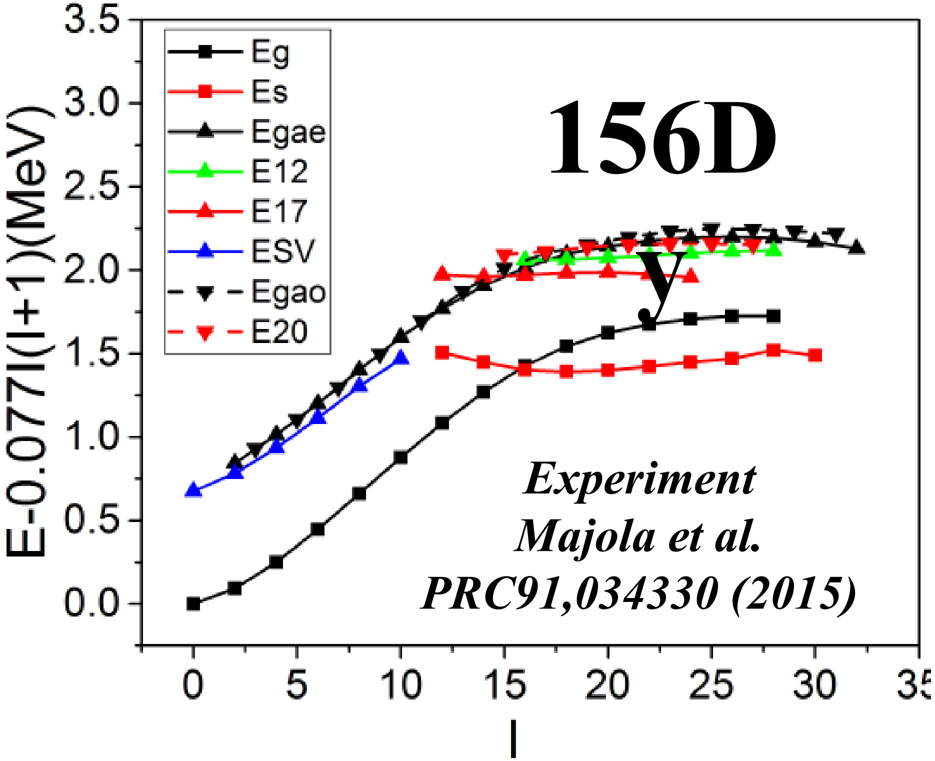
$\varepsilon \approx 0.05$??

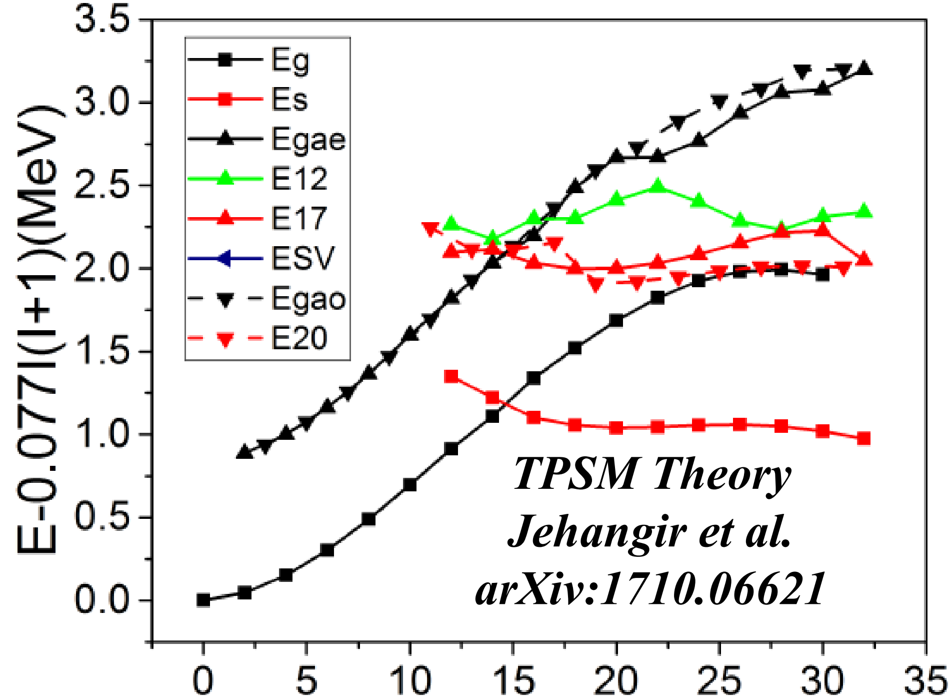
What you really need is
SPLIT MONOPOLE PAIRING
so that

$$-H_{pairing} = G_{p-p}\dot{P}_p^\dagger \dot{P}_p + G_{o-o}\dot{P}_o^\dagger \dot{P}_o + \varepsilon G_{p-p}\dot{P}_{po}^\dagger \dot{P}_{po}$$



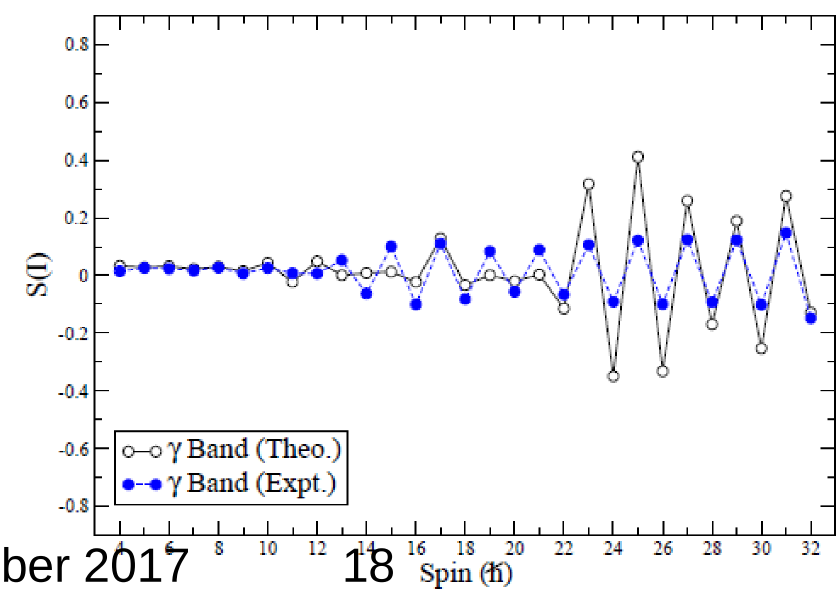
Jehangir et al., arXiv:1710.06621v1
Submitted to PRC



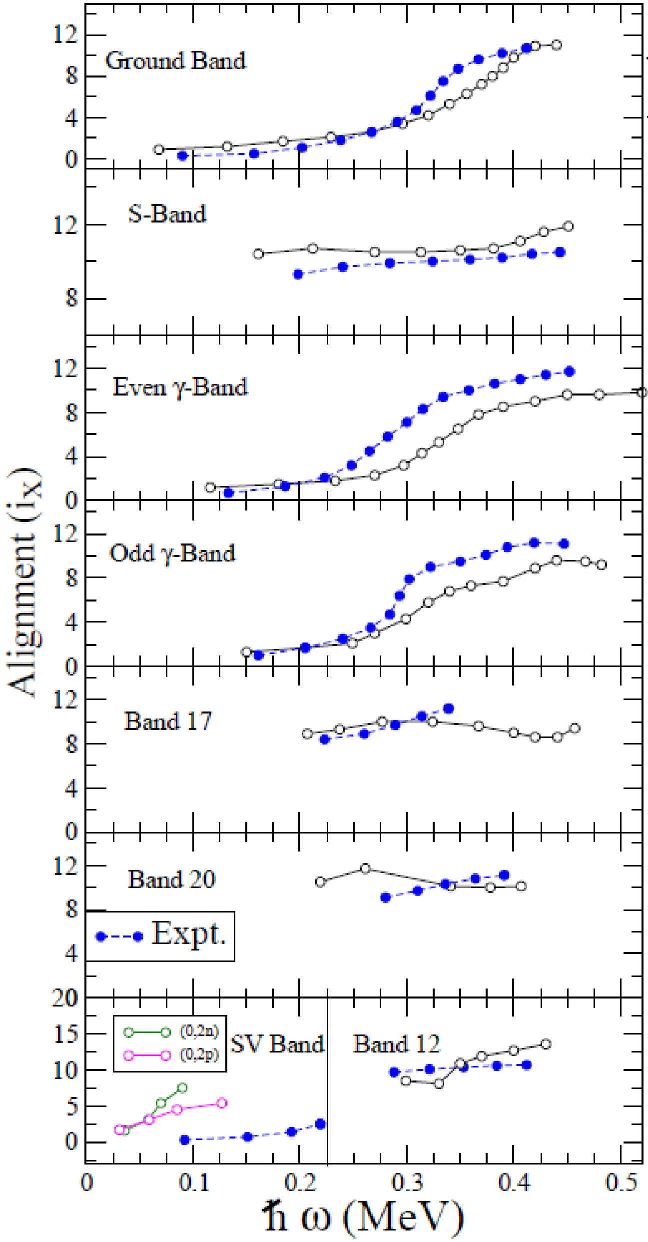


Staggering Parameter

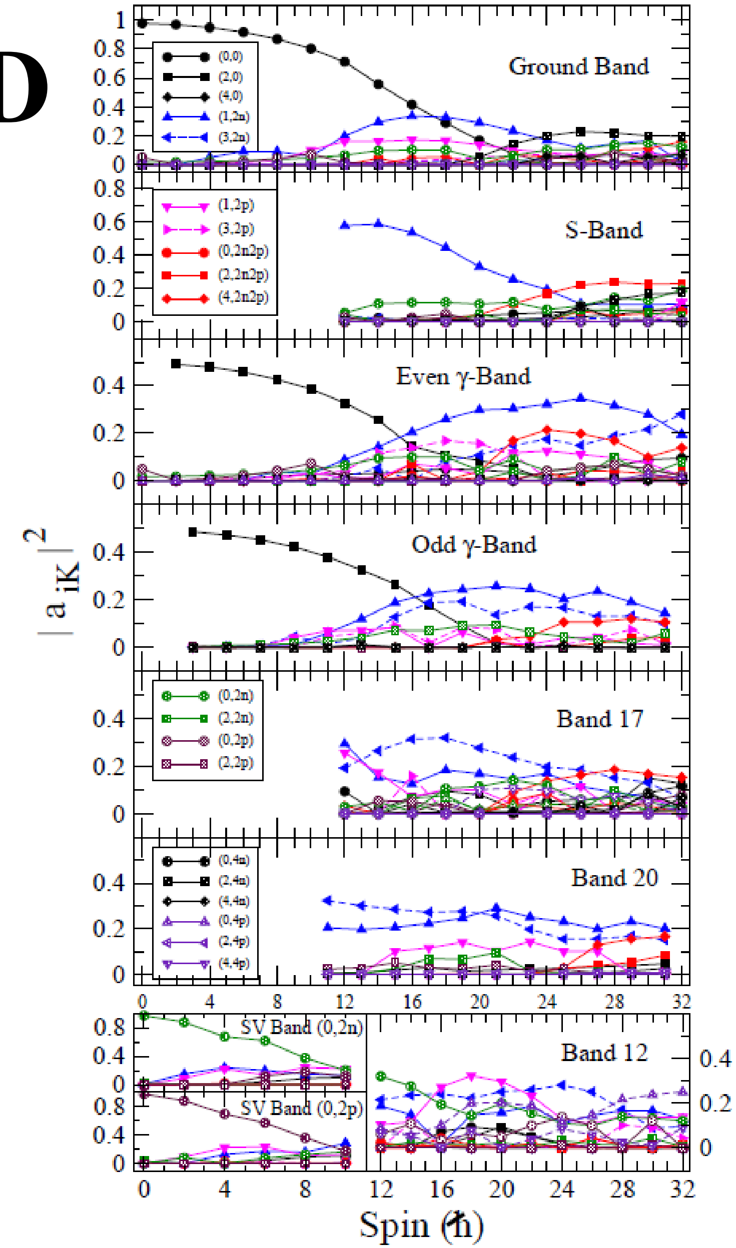
$$S(I) = \frac{[E(I)-E(I-1)]-[E(I-1)-E(I-2)]}{E(2_1^+)}$$



156Dy



Ground Band
S-Band
Even γ-Band
Odd γ-Band
Band 17
Band 20
Expt.
SV Band
Band 12
(0,2n)
(0,2p)
Alignment (ix)
ħ ω (MeV)



Ground Band
S-Band
Even γ-Band
Odd γ-Band
Band 17
Band 20
SV Band (0,2n)
SV Band (0,2p)
Band 12
(0,0)
(2,0)
(4,0)
(1,2n)
(3,2n)
(1,2p)
(3,2p)
(0,2n2p)
(2,2n2p)
(4,2n2p)
(0,2n)
(2,2n)
(0,2p)
(2,2p)
(0,4n)
(2,4n)
(4,4n)
(0,4p)
(2,4p)
(4,4p)
|a iK|^2
Spin (ħ)

Second Approach !!

5-D Collective Hamiltonian (5DCH) + PC-PK1 Covarient Density Functional Theory (CDFT)

Return to the Bohr Hamiltonian: (i.e. is NOT microscopic !! → NO alignments)

$$\hat{H} = \hat{T}_{\mathrm{vib}} + \hat{T}_{\mathrm{rot}} + V_{\mathrm{coll}} \qquad \hat{T}_{\mathrm{rot}} = \frac{1}{2}\sum_{k=1}^{3}\frac{\hat{J}_k^2}{\mathcal{I}_k}$$

$$\hat{T}_{\mathrm{vib}} = -\frac{\hbar^2}{2\sqrt{wr}}\left\{\frac{1}{\beta^4}\left[\frac{\partial}{\partial\beta}\sqrt{\frac{r}{w}}\beta^4 B_{\gamma\gamma}\frac{\partial}{\partial\beta} - \frac{\partial}{\partial\beta}\sqrt{\frac{r}{w}}\beta^3 B_{\beta\gamma}\frac{\partial}{\partial\gamma}\right] + \frac{1}{\beta\sin 3\gamma}\left[-\frac{\partial}{\partial\gamma}\sqrt{\frac{r}{w}}\sin 3\gamma B_{\beta\gamma}\frac{\partial}{\partial\beta} + \frac{1}{\beta}\frac{\partial}{\partial\gamma}\sqrt{\frac{r}{w}}\sin 3\gamma B_{\beta\beta}\frac{\partial}{\partial\gamma}\right]\right\}$$

Compare with

The differential Bohr equation with

$$H = -\frac{\hbar^2}{2B}\left[\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} + \frac{1}{\beta^2\sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma\frac{\partial}{\partial\gamma} - \frac{1}{4\beta^2}\sum_{\kappa}\frac{Q_\kappa^2}{\sin^2(\gamma - \frac{2}{3}\pi\kappa)}\right] + V(\beta,\gamma)$$

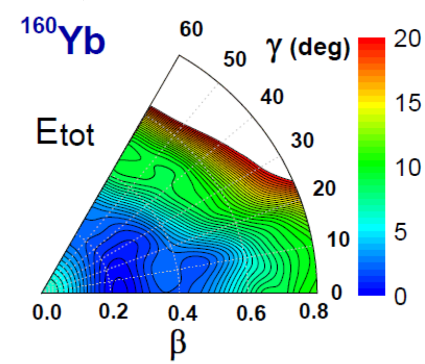
Vibration

Mass parameter

Rotation

How do you get Vcoll = V(β,γ) ? Relativistic Mean Field "Density Functionals"

T. Niksic et al PRC 79, 034303 (2009), Z.P. Li et al., PRC 79, 054301 (2009)



How do you get the inertial parameters?

Moment of Inertia: Inglis-Belyaev formula

can be improved by using Thouless-Valatin moments-of-inerti

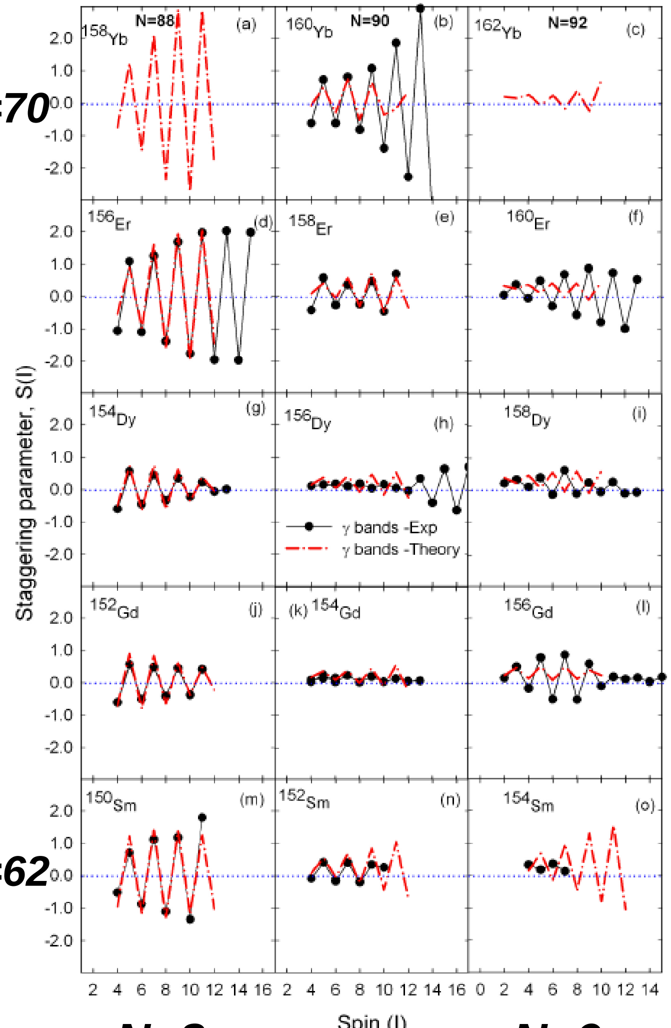
Z.P. Li. et al PRC 86, 034334 (2012) but too time consuming !!

Hence; renormalize to 21+ energies

Signature splitting in the γ band

$$S(I) = \frac{[E(I) - E(I-1)] - [E(I-1) - E(I-2)]}{E(2_1^+)}$$

Z=70

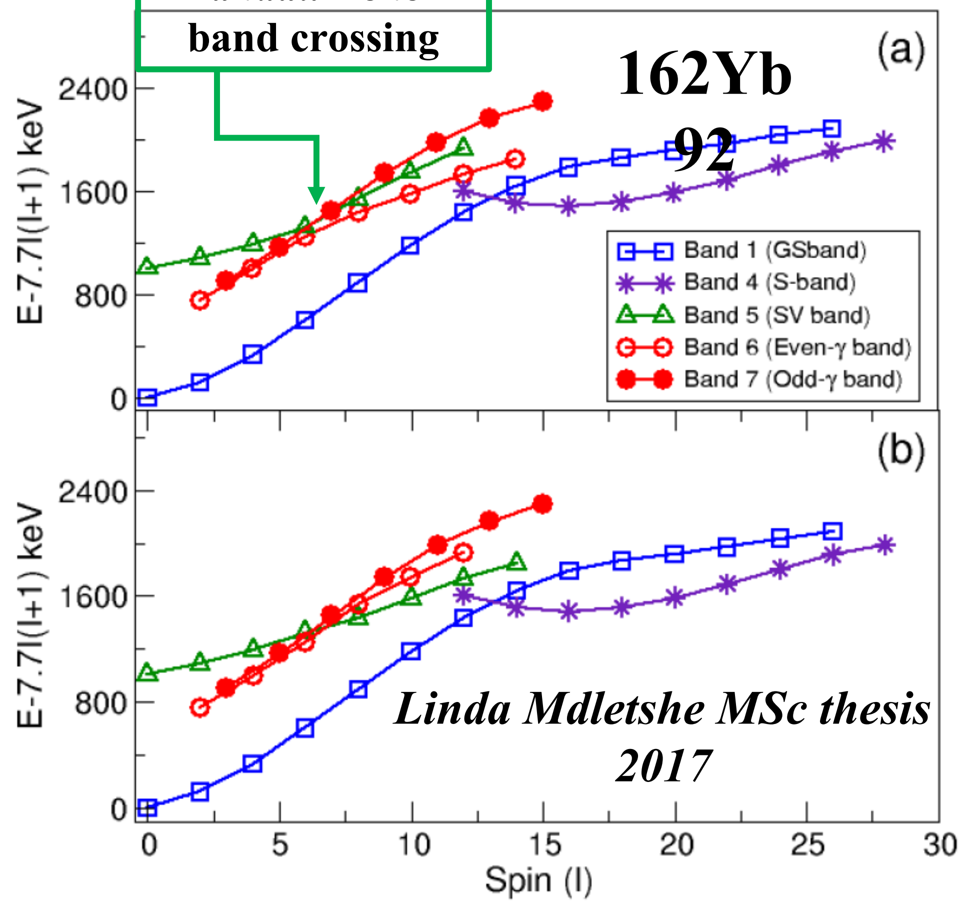


Z=62

N=88 N=90 N=92

Z. Shi, B.Y. Song, Z.P. Li, S.Q. Zhang
5DCH-CDFT

"Landau-Zener" band crossing



Linda Mdletshe MSc thesis 2017

__ALSO:__ fits to Moments-of Inertia, E(0)s, E(2)s and branching ratios

Much More but Finally

1. **Low-lying Time-dependent Quadrupole Vibrations are not found in the Pairing Gap.**
2. **Rotations arise from breaking spherical symmetry, *K*□ = *2* +bands arise from breaking axial symmetry.**
3. **Classical Considerations Suggest that Time-dependent Octupole Vibrations are Even Higher in Energy.**
4. **We do not have to worry about Rotation-Vibration coupling.**
5. **Etc.**

What are we left with ??

Excitation Energies in Nuclei represented by a Mean Field Potential depend on the NUCLEAR SHAPE and the existence of these DEGREES of FREEDOM

V(r,□,□,Y3,□ …)

THE LEVEL SCHEME BUILDERS

¶ Rob Bark
*Suzan Bvumbi
¶ Tshepo Dinoko
#Tshfiwa Madiba
†Siyabonga Majola
#Poke Mashita
†Linda Mdletshe
*Lumkile Msebi
#John Sharpey-Schafer
§ Maciej Stankiewicz
*George Zimba
from
¶iThemba LABS
*University of Johannesburg
#University of the Western Cape
†University of Zululand
§University of Cape Town

TRIAXIAL PROJECTED SHELL MODELLERS

¶Sheikh Jehangir
¶G. H. Bhat
¶Javid Sheikh
*Stefan Frauendorf
#P. A. Ganai
from
¶University of Kashmir
*University of Notre Dame
#Nat. Inst. Tech., Srinagar

RELATIVISTIC MEAN FIELD & COLLECTIVE MODELLERS

¶Zhi Shi
*Zhipan Li
#Jie Meng
*Bangyan Song
*Chunyan Song
#Jiangjing Yao
#Shuangquan Zhang
from
¶Beihang University
*University of South West China
#Peking University

Plus Many Megathanks to the Crews at;
AFRODITE, Jyvaskyla and Gammasphere

AND many enlightening & fruitful discussions with;
Mitch Allmond, Paul Garrett, Daryl Hartley, Pete Jones, Rauno Julin, David Kulp, Elena Lawrie, Lee Riedinger, David Rowe, John Wood, Sven Åberg et al.

It is important to realize that it is not possible to distinguish between a triaxial rotor and a harmonic γ-vibration, as long as one considers only the first excited (K=2) band. In both cases an angular-momentum of $J_3 = 2\hbar$ is generated by a γ-deformed shape rotating about the 3-axis. To differentiate one has to take the next (K=4) band into consideration. The K - dependence of ω_γ is indicated by the level distance $\hbar\omega_\gamma = (E_\gamma(K) - E_\gamma(K-2))/2$. In the case of a vibration, ω_γ is the same for the $K = 2$ and $K = 4$ bands and deformation parameter $\gamma(K = 4) = \sqrt{2}\gamma(K = 2)$, which means $E_\gamma(K = 4) = 2E_\gamma(K = 2)$. In the case of the rigid triaxial rotor, $\gamma(K = 4) = \gamma(K = 2)$ and $\omega_\gamma(K = 4) = 2\omega_\gamma(K = 2)$, which means $E_\gamma(K = 4) = 4E_\gamma(K = 2)$. This explains why the TPSM approach [21] which operates with a fixed γ deformation describes the first excitations of γ vibrational type in ^{156}Dy, which has an axial shape at the moderate spins of interest in this communication.

Stefan Frauendorf