

SU(5) and A(4)

**Alfredo Urbano*, University of Salento
(Lecce, Italy)**

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*** with P.Ciafaloni, M.Picariello and E.Torrente-Lujan**

SU(5) and A(4): Motivations

- We try to set the **A(4)** Flavour Symmetry into a **SUSY** (with R-Parity) **GUT** based on **SU(5)**;
- We try to preserve the Renormalizability of the entire theory without losing the special role of the **A(4)** flavour symmetry;
- There are two “minimal” ways in which the last feature can be obtained..

About Flavour Symmetries..

Imposing the **A(4)** F.S. at **GUT** scale

A(4): Symmetry group
of the
Regular Tetrahedron,
“Plato’s Fire”

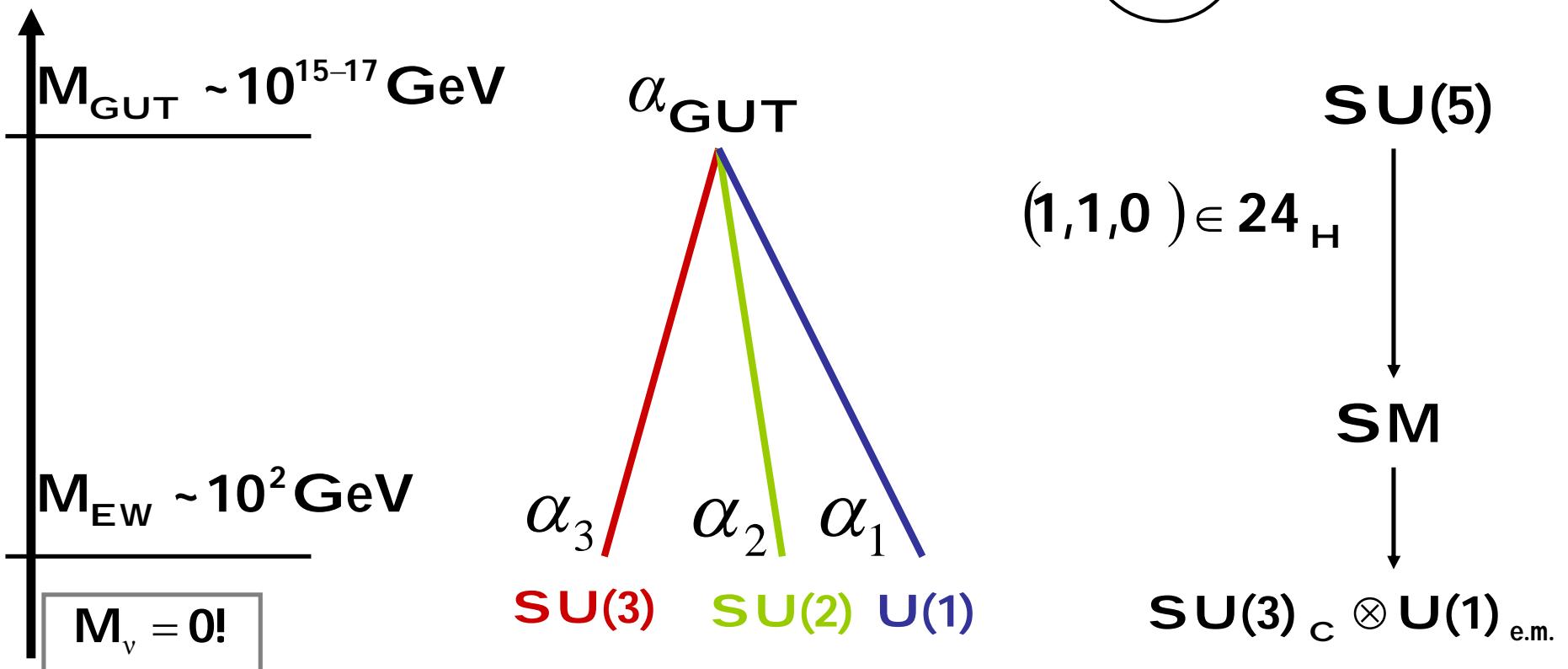
Harrison, Perkins, Scott;

PLB530,167

$$V_{\text{PMNS}} = \text{TBM} = \begin{pmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

$SU(5)$ as Unified Theory

Minimal $SU(5)$: $\left\{ \begin{array}{l} \text{Matter : } (10_T)_i, (\bar{5}_T)_i \\ \text{Higgs : } 24_H, 5_H, 45_H \end{array} \right.$ $M_D \neq M_E^T$



Renormalizable Adjoint SUSY $SU(5) \times A(4)$

Matter: $\left\{ \begin{array}{l} \left(\mathbf{10}_T\right)_{i=1,2,3} \\ \left(\overline{\mathbf{5}}_T\right)_{i=1,2,3} \\ \mathbf{24}_T \end{array} \right.$

Standard Model Matter

Right-Handed Neutrinos
from $(1,3,0)$ & $(1,1,0)$

Higgs: $\left\{ \begin{array}{l} \mathbf{24}_H, \mathbf{5}_H, \overline{\mathbf{5}}_H \\ \mathbf{45}_H, \overline{\mathbf{45}}_H \end{array} \right.$

$M_D \neq M_E^T$

$SU(5)$ and $A(4)$: Renormalizable Adjoint $SU(5)$

Charged Leptons:

$$v_5, v_{\bar{5}} \gg v_{45}, v_{\bar{45}} \downarrow \\ m_\tau \gg m_\mu \gg m_e$$

$$\langle 5_H \rangle \sim \langle \bar{5}_H \rangle \sim \langle 1,0,0 \rangle$$

Alignment in Flavour Space

$$\langle \bar{45}_H \rangle \sim \langle v_{\bar{45}}, \delta v_{\bar{45}}, \delta \bar{v}_{\bar{45}} \rangle \quad \langle 45_H \rangle \sim \langle v_{45}, \delta v_{45}, \delta v_{45} \rangle$$

Neutrino Mass Matrix: $M_\nu = M_\nu^I + M_\nu^{III}$

$$\left(M^{typeI} \right)^T \frac{1}{M_1} M^{typeI} \quad \left(M^{typeIII} \right)^T \frac{1}{M_3} M^{typeIII}$$

SU(5) and A(4) with a Scalar Triplet

Matter Superfields: $\left\{ \begin{array}{l} \left(\begin{array}{c} \mathbf{10}_T \\ \bar{\mathbf{5}}_T \end{array} \right)_{i=1,2,3} \end{array} \right.$

Higgs Superfields: $\left\{ \begin{array}{l} \mathbf{24}_H, \mathbf{5}_H, \bar{\mathbf{5}}_H \\ \mathbf{45}_H, \bar{\mathbf{45}}_H \\ \mathbf{15}_H, \bar{\mathbf{15}}_H \end{array} \right.$

Neutrino Mass Matrix from (1,3,1)

A(4) charges and phenomenology..

SU(5)	24_H	5_H	$\bar{5}_H$	45_H	$\bar{45}_H$	15_H	$\bar{15}_H$
A(4)	1	$3+1$	$3+1$	$3+1$	$3+1$	$3+1'$	$3+1''$

Normal Hierarchy!

Neutrino Mass Matrix:

$$\begin{pmatrix} a & b & 0 \\ b & \omega a & 0 \\ 0 & 0 & \omega^2 a \end{pmatrix}$$

$$\omega = \exp\left(\frac{2\pi i}{3}\right)$$

$$\begin{aligned} \langle 5_H \rangle &\sim \langle \bar{5}_H \rangle \sim (1, 1, 1) \\ \langle 45_H \rangle &\sim \langle \bar{45}_H \rangle \sim (1, 1, 1) \\ \langle 15_H \rangle &\sim (0, 0, 1) \end{aligned}$$

In Flavour Space, it follows directly from the minimization of the scalar potential..!

A(4) charges and phenomenology..

$$\left\{ \begin{array}{l} \mathbf{U}_\omega^T \mathbf{M}_f \mathbf{U}_\omega = \mathbf{M}_f^{\text{diag}} \\ \mathbf{V}^T \mathbf{M}_v \mathbf{V} = \mathbf{M}_v^{\text{diag}} \end{array} \right. \rightarrow \begin{aligned} \mathbf{U}_\omega &= \frac{1}{\sqrt{3}} \begin{pmatrix} \omega & \omega^2 & 1 \\ \omega^2 & \omega & 1 \\ 1 & 1 & 1 \end{pmatrix} \\ \mathbf{V} &= \begin{pmatrix} \omega/\sqrt{2} & 0 & -i\omega/\sqrt{2} \\ \omega^2/\sqrt{2} & 0 & i\omega^2/\sqrt{2} \\ 0 & 1 & 0 \end{pmatrix} \end{aligned}$$

At LO; off-diagonal elements generated through small perturbations in vacuum alignments..

$$\mathbf{U}_{\text{CKM}} = \mathbf{Id} \quad \mathbf{V}_{\text{PMNS}} = \mathbf{T} \mathbf{B} \mathbf{M} = \begin{pmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

$$\mathbf{U}_U^T \mathbf{U}_D \quad \mathbf{U}_E^T \mathbf{V}$$

SU(5) and A(4): Conclusions..

- In the **SUSY Adjoint case (hybrid type I + type III Seesaw)** we are just able to fit the experimental values for masses and mixing, losing the nice features of **A(4)** as a flavour group;
- Generating neutrino masses with a scalar triplet, we obtain the correct phenomenology of mixing in a **Renormalizable GUT theory**;
- As usual hierarchy between masses escapes from a simple **A(4)** analysis.