

Beyond the Standard Model: a noncommutative approach

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La Thuile, Moriond EW 09
08.03.2009

Overview

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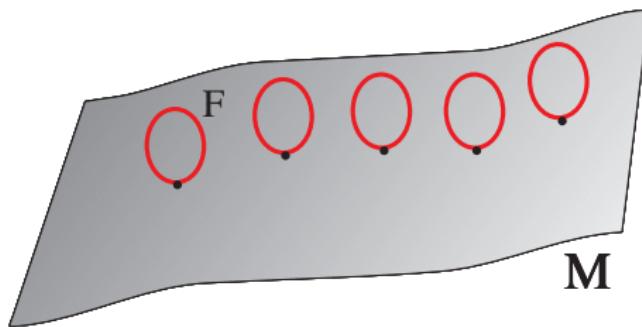
The Aim of Noncommutative Geometry:

Aim: To unify general relativity (GR) and the standard model of particle physics (SM) on the same geometrical level. This means to describe gravity and the electro-weak and strong forces as gravitational forces of a generalised “space-time”.

Idea: Find generalised “space-time” with symmetries of General Relativity and the Standard Model (SM).

Pos.Sol.: Try a special type of noncommutative geometry
(A. Connes)

Analogy: Noncomm. geometry \leftrightarrow Kaluza-Klein space

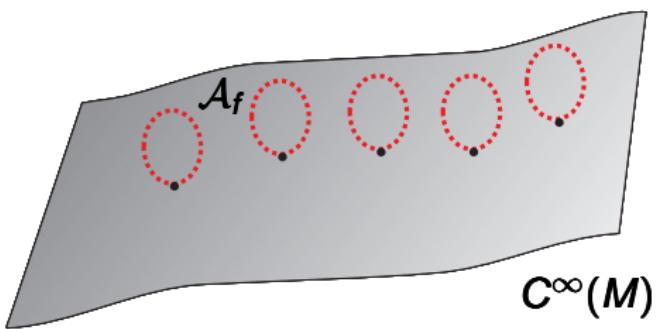


Idea: Spacetime \rightarrow Algebra

Compact extra dimension \rightarrow some "finite space"

Differential geometry \rightarrow spectral triple

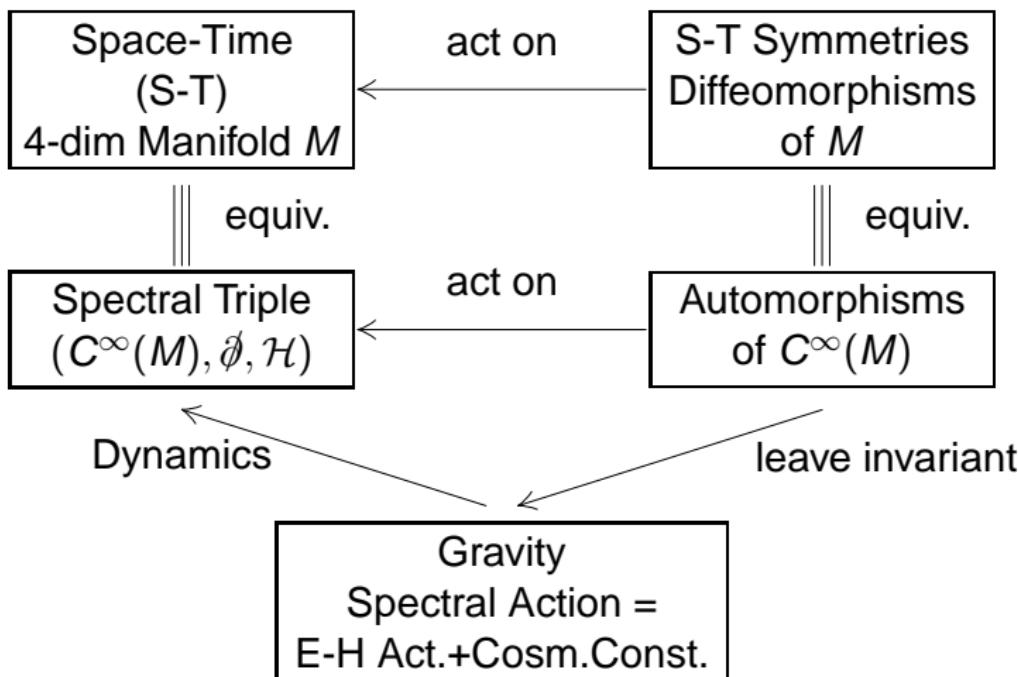
Noncommutative Geometry



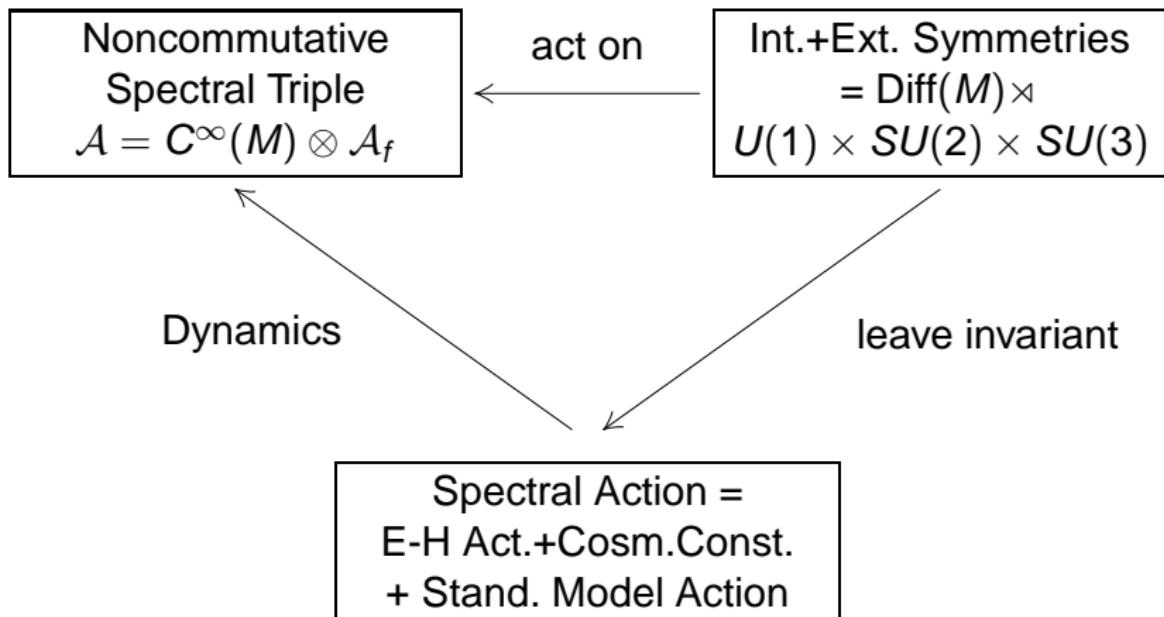
"finite space" $\rightarrow \mathcal{A}_f = M_1(\mathbb{K}) \oplus M_2(\mathbb{K}) \oplus \dots$

Kaluza-Klein space \rightarrow noncom. geometry, $\mathcal{A} = C^\infty(M) \otimes \mathcal{A}_f$

Euclidean space-time!



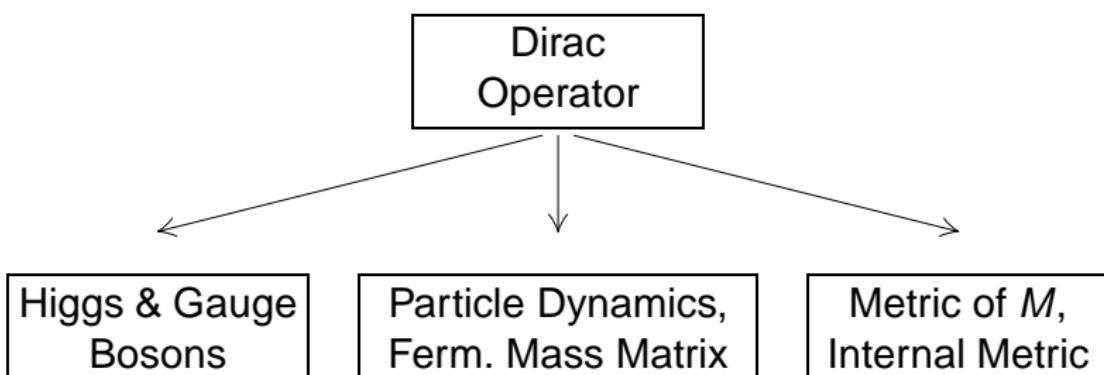
Noncommutative Standard Model (A.Chamseddine, A.Connes):



The noncommutative standard model automatically produces:

- The combined Einstein-Hilbert and standard model action
- A cosmological constant
- The Higgs boson with the correct quartic Higgs potential

The Dirac operator plays a multiple role:



The geometric setup imposes constraints:

- mathematical axioms
=> Restrictions on particle content
- symmetries of finite space
=> determines gauge group
- representation of matrix algebra
=> representation of gauge group
(only fundamental and adjoint representations)
- Dirac operator => allowed mass terms / Higgs fields

The Spectral Action (A. Connes, A. Chamseddine 1996):

$$(\Psi, \mathcal{D}\Psi) + S_{\mathcal{D}}(\Lambda) \quad \text{with } \Psi \in \mathcal{H}$$

- $(\Psi, \mathcal{D}\Psi)$ = fermionic action
 - includes Yukawa couplings
 - & fermion–gauge boson interactions
- $S_{\mathcal{D}}(\Lambda)$ = bosonic action
 - = # eigenvalues of \mathcal{D} up to cut-off Λ
 - = Einstein-Hilbert action + Cosm. Const.
 - + full bosonic SM action + constraints at Λ
- constraints => less free parameters than classical SM

The standard model (A. Chamseddine, A. Connes 1996 & J.-H. Jureit, T. Schücker, C.S. 2005):

- Discrete space: $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C}) \oplus \mathbb{C}$
- Symmetries of discrete space: $SU(2) \times U(3)$
- Hilbert space of minimal standard model fermion multiplets
- Dirac operator: ordinary Dirac op. + fermionic mass matrix (CKM/PMNS matrix)
- Majorana masses and SeeSaw mechanism for right-handed neutrinos (J. Barrett & A. Connes 2006)

Constraints on the SM parameters at the cut-off Λ :

$$g_2^2 = g_3^2 = \frac{Y_2^2}{H} \frac{\lambda}{24} = \frac{1}{4} Y_2$$

- g_2, g_3 : $SU(2)_w, SU(3)_c$ gauge couplings
- λ : quartic Higgs coupling
- Y_2 : sum of all Yukawa couplings squared
- H : sum of all Yukawa couplings to the fourth power

Consequences from the SM constraints:

Input:

- Big Desert
- $g_2(m_Z) = 0.6514$, $g_3(m_Z) = 1.221$
- renormalisation group equations

Output:

- $g_2^2(\Lambda) = g_3^2(\Lambda)$ at $\Lambda = 1.1 \times 10^{17}$ GeV
- $m_{Higgs} = 168.3 \pm 2.5$ GeV
- $m_{top} < 190$ GeV
- no 4th SM generation

Probably excluded by Tevatron!

Beyond SM: the general strategy (bottom-up approach)

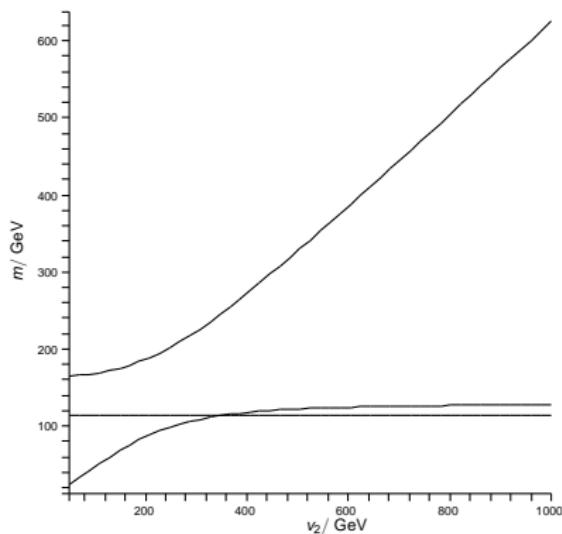
- Find finite geometry that has SM as sub-model (tricky)
=> particle content, gauge group & representation
- (Make sure everything is anomaly free)
- Compute the spectral action => constraints on parameters
- Determine the cut-off scale Λ with suitable sub-set of the constraints
- Use renorm. group equations to obtain low energy values of interesting parameters
- Check with experiment!

SM + $U(1)_X$ scalar field + $U(1)_X$ fermion singlet (C.S. 2009):

- Discrete space: $\mathbb{C} \oplus M_2(\mathbb{C}) \oplus M_3(\mathbb{C}) \oplus \mathbb{C} \oplus \mathbb{C} \oplus \mathbb{C}$
- Gauge group: $U(1)_Y \times SU(2)_w \times SU(3)_c \times U(1)_X$
- New fermions: $U(1)_X$ -vector singlets (X -particles)
neutral w.r.t SM gauge group , $M_X \sim \Lambda$
- New scalar: $U(1)_X$ singlet φ , neutral w.r.t SM gauge group
- $\mathcal{L}_{scalar} = -\mu_1^2 |H|^2 + \frac{\lambda_1}{6} |H|^4 - \mu_2^2 |\varphi|^2 + \frac{\lambda_2}{6} |\varphi|^4 + \frac{\lambda_3}{3} |H|^2 |\varphi|^2$
- $U(1)_Y \times SU(2)_w \times SU(3)_c \times U(1)_X \rightarrow U(1)_{el.} \times SU(3)_c$
- $\mathcal{L}_{ferm+gauge} = \bar{X}_L M_X X_R + g_{\nu, X} \bar{\nu}_R \varphi X_L + h.c. + 1/g_4^2 F_X^{\mu\nu} F_{X,\mu\nu}$

The constraints at Λ :

- only top-quark & ν_τ
- $g_2 = g_3$
 $\Rightarrow \Lambda = 1.1 \times 10^{17} \text{ GeV}$
- $g_2^2 = \frac{\lambda_1}{24} \frac{(3g_t^2 + g_\nu^2)^2}{3g_t^4 + g_\nu^4}$
- $g_2^2 = \frac{\lambda_2}{24}$
- $g_2^2 = \frac{\lambda_3}{24} \frac{3g_t^2 + g_\nu^2}{g_\nu^2}$
- $g_2^2 = \frac{1}{4} (3g_t^2 + g_\nu^2)$
- free parameters: $|\langle \varphi \rangle|, g_4$



Mass EVs of scalar fields for
 $v_2 = \sqrt{2} |\langle \varphi \rangle|,$
 $\sqrt{2} |\langle H \rangle| = 246 \text{ GeV}, g_4(m_Z) = 0.3$

To-do-List:

- Models beyond the Standard Model
(LHC signature and/or dark matter?)
- Mechanisms for Neutrino masses
(Dirac/Majorana masses or something different?)
- Renorm. group flow for all couplings in the spectral action
(Exact Renorm Groups, M. Reuter et al. ?)
- Spectral triples with Lorentzian signature
(A. Rennie, M. Paschke, R. Verch,...)
- Action principle in Lorentzian signature
(Local index formula? J. Tolksdorf et al.)

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