Non-adiabatic instability in coupled dark sectors

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It has been recently pointed out that coupled dark matter-dark energy systems suffer from non-adiabatic instabilities at early times and large scales. We show how coupled models free from non-adiabatic instabilities can be identified as a function of a generic coupling Q and of the dark energy equation of state w. In our analysis, we do not refer to any particular cosmic field. We also confront a viable class of model in which the interaction is directly proportional to the dark energy density to recent cosmological data. In that framework, we show the correlations between the dark coupling and several cosmological parameters allowing to *e.g.* larger neutrino mass than in uncoupled models.

1 Introduction

Interactions between dark matter and dark energy are still allowed by observational data today. At the level of the background evolution equations, one can introduce a coupling between these two sectors in the following way:

$$\dot{\rho}_{dm} + 3\mathcal{H}\rho_{dm} = Q\,, \tag{1}$$

$$\dot{\rho}_{de} + 3\mathcal{H}\rho_{de}(1+w) = -Q\,, \tag{2}$$

where $\rho_{dm}(\rho_{de})$ denotes the dark matter (dark energy) energy density, the dot indicates derivative with respect to conformal time $d\tau = dt/a$, $\mathcal{H} = \dot{a}/a$ and $w = P_{de}/\rho_{de}$ is the dark-energy equation of state (P denotes the pressure). We work with the Friedman-Robertson-Walker (FRW) metric, assuming a flat universe and pressureless dark matter $w_{dm} = P_{dm}/\rho_{dm} = 0$.

Q encodes the dark coupling and drives the energy exchange between dark matter and dark energy. For *e.g.* $Q < 0$ the energy flows from dark matter to dark energy. It also changes the way that dark matter and dark energy redshift acting as an extra contribution to their effective equation of state. In particular, for *e.g.* $Q < 0$, dark matter redshifts faster so that there is more dark matter in the past compared to uncoupled scenarios assuming that the dark matter density today is the same in the two models. Matter-radiation equality happens earlier and the growth of dark matter clustering is enhanced. This is one of the features which enable us to constraint the model with available cosmological data (see section 3).

In order to deduce the evolution of density and velocity perturbations in coupled models, we need an expression of the energy transfer in terms of the stress-energy tensor. We follow Ref.[1] in parameterizing the interaction as:

$$\nabla_\mu T^\mu_{(dm)\nu} = Q\, u_\nu^{(dm)}/a \;, \tag{3}$$

$$\nabla_\mu T^\mu_{(de)\nu} = -Q\, u_\nu^{(dm)}/a \;, \tag{4}$$

with $T^\mu_{(dm)\nu}$ and $T^\mu_{(de)\nu}$ the energy momentum tensors for the dark matter and dark energy components, respectively. The dark matter four velocity $u_\nu^{(dm)}$ is defined in the synchronous gauge, in terms of the fluid proper velocity $v^i_{(dm)}$, as $u_\nu^{(dm)} = a(-1, v^i_{(dm)})$, where $\mu = 0..3$ and $i = 1..3$. This choice of parameterization guaranties the conservation of the total stress energy tensor of the system while it avoids momentum transfer in the rest frame of dark matter in which case one can work in the synchronous gauge comoving with dark matter (*i.e* $v^i_{(dm)} = 0$).

We provide[2] a criteria associated to the dubbed *the doom factor* (see section 2.1) to identify the stability region of coupled models satisfying to Eq. (3) and (4). The doom factor is a function of the model parameters such as Q and w but it is defined independently of the explicit form of the coupling Q. Notice that the dark coupling terms which appears in the non-adiabatic dark energy pressure perturbations were first pointed out as a source for early time instabilities at large scales in Ref. [1].

Our results will then be illustrated within a successful class of models, in which Q is proportional to the dark energy density. Present data will be shown to allow for a sizeable interaction strength and to imply weaker cosmological limits on neutrino masses with respect to non-interacting scenarios.

2 Origin of non-adiabatic instabilities

Non-adiabatic instabilities arises at linear order in perturbations. Using the publicly available CAMB code[3], we preferred to work in the synchronous gauge comoving with dark matter.

2.1 Doom factor

Given our gauge choice, it is necessary to work out the expression for dark energy pressure perturbations δP_{de} in the rest frame for dark matter. It can be shown that in the presence of a dark coupling (see *e.g.* Ref. [4] and also Ref.[1]) it is given by:

$$\frac{\delta P_{de}}{\delta \rho_{de}} = \hat{c}^2_{s\,de} + 3(\hat{c}^2_{s\,de} - c^2_{a\,de})(1+w)\,(1+\mathbf{d})\,\frac{\mathcal{H}\theta_{de}}{k^2\delta_{de}}\,, \tag{5}$$

where $\delta\rho_{de}$ denotes the dark energy energy density perturbation and $\delta_{de} = \delta\rho_{de}/\rho_{de}$, $\theta_{de} \equiv \partial_i v^i_{(de)}$ is the divergence of the fluid proper velocity $v^i_{(de)}$, $\hat{c}^2_{s\,de}$ is the propagation speed of pressure fluctuations in the rest frame of dark energy and $c^2_{a\,de} = \dot{P}_{de}/\dot{\rho}_{de}$ is the so called "adiabatic sound speed". In the following, we work with constant equation of state w in which case $c^2_{a\,de} = w$ and we assume that our universe is in accelerating expansion today which implies that $w < -1/3$. Moreover we restrict our analysis to the case $\hat{c}^2_{s\,de} > 0$ and $\hat{c}^2_{s\,de} = 1$ will be assumed for numerical computation.

In Eq. (5) $\mathbf{d}$ refers to the doom factor which we have defined as:

$$\mathbf{d} \equiv \frac{Q}{3\mathcal{H}\rho_{de}(1+w)}\,. \tag{6}$$

We dub it so as it is precisely this extra factor, proportional to the dark coupling Q, which may induce non-adiabatic instabilities in the evolution of dark energy perturbations. Its sign will be determinant, as we show in the next section.

2.2 Growth equation

A cartoon equation of the growth equation governing the evolution of energy density linear perturbation for any species i, j is given by:

$$\delta_i'' = \underbrace{\boldsymbol{A}_i}_{\text{Exponential Growth or Oscillations}} \frac{\delta_i}{\boldsymbol{a}^2} + \underbrace{\boldsymbol{B}_i}_{\text{(Anti)Damping}} \frac{\delta_i'}{\boldsymbol{a}} + \overbrace{\boxed{\mathcal{F}(\rho_j, \delta_j, \delta_j'; \boldsymbol{j} \neq \boldsymbol{i})}}^{\text{leads when A,B negligible}}$$

where $\delta_i = \delta\rho_i/\rho_i$ and $' = \partial/\partial\, a$. The evolution of a perturbation depends on the relative weight of the three terms present in the equation *and* on their signs:

1. For positive A, the A and B terms taken by themselves would induce a rapid growth of the perturbation, which may be damped or antidamped (reinforced) depending on whether B is negative or positive, respectively[a]. In particular, for A and B both positive, the solution may enter in an exponentially growing, unstable, regime.

2. For negative A, in contrast, the A and B terms taken alone describe a harmonic oscillator, with oscillations damped (antidamped) if B is negative (positive). In the $A, B < 0$ regime, the third term may plays in fact the leading role.

In the standard uncoupled scenario dark matter perturbations behave as in case 1 above (with $A > 0$ and $B < 0$), while dark energy ones provide an example of behavior as in case 2. For coupled models, we concentrate on the case in which the dark-coupling terms dominate over the usual one in order to put forward the presence of non adiabatic instabilities (see Ref.[2] for more details). This *strong coupling regime* can be characterized by

$$|\mathbf{d}| \quad = \quad \left|\frac{Q}{3\mathcal{H}\rho_{de}(1+w)}\right| > 1\,, \tag{7}$$

which also guarantees that the interaction among the two dark sectors drives the non-adiabatic contribution to the dark energy pressure wave, see Eq. (5). At large scales and early times, it can be worked out that the main contributions of δ_{de} and δ'_{de} coefficients to the second order differential equation reduce to:

$$\delta''_{de} \quad \simeq \quad 3\,\mathbf{d}\,(\hat{c}^2_{s\,de}+1)\left(\frac{\delta'_{de}}{a} + 3\frac{\delta_{de}}{a^2}\frac{(\hat{c}^2_{s\,de}-w)}{\hat{c}^2_{s\,de}+1} + \frac{3(1+w)}{a^2}\delta[\,\mathbf{d}\,]\right) + \ldots \tag{8}$$

The sign of the coefficient B_e of δ'_{de} in this expression is crucial for the analysis of instabilities. Assuming $\hat{c}^2_{sde} > 0$, it reduces to the sign of the doom factor $\mathbf{d}$ defined in Eq. (6).

Similar second order differential equations for δ_{de} were obtained in Ref. [5] for particular expressions of the dark coupling Q and an analytical form of their solutions where derived in

[a] Obviously, for $|B| >> |A|$ a negative B would prevent the onset of growth for any sign of A.

order to determine when δ_{de} blows up. In particular, the results of Ref. [5] confirm those of Ref. [1] for positive $Q \propto \rho_{dm}$ and $1+w > 0$. In comparison, our approach gives rise to general conditions to avoid instabilities as a function of the sign of the doom factor independently of the exact form of the dark coupling Q. Indeed, as previously argued, a positive $\mathbf{d}$ acts as an antidamping source in the growth Eq. (8). Whenever $\mathbf{d} > 1$, the overall sign of the A_e coefficient of δ_{de}, resulting from the last two terms in Eq. (8), is also positive and it triggers an exponential runaway growth of the dark energy perturbations. Large scale instabilities arise then and the universe appears to be non viable.

3 A viable model: $Q \propto \rho_{de}$

We have developed a method to determine if a coupled model satisfying Eq. (3) and (4) suffer or not from non-adiabatic instabilities. We can now easily verify that for

$$Q = \xi \mathcal{H} \rho_{de}, \tag{9}$$

we have a rather simple and viable model for specific combination of $1 + w$ and of the dimensionless constant coupling ξ. In this model, the doom factor of Eq. (6) is given by:

$$\mathbf{d} = \frac{\xi}{3(1+w)} \,. \tag{10}$$

Its sign defines the (un)stable regimes. When $\mathbf{d} < 0$, that is, for $\xi < 0$ and $1 + w > 0$ (or $\xi > 0$ and $1 + w < 0$), no instabilities are expected. On the contrary, when ξ and $1 + w$ have the same sign, instabilities will develop at early times whenever $\mathbf{d} > 1$.

In Ref. [2] we confirmed these results numerically. Also notice that they are in agreement with those of Ref. [5], which first pointed out that coupled models with $Q = \xi \mathcal{H} \rho_{de}$ can be stable for $1 + w < 0$. They restricted though their stability analysis to the $\xi > 0$ case.

In the following, we confront the model satisfying to Eq. (9) to cosmological data restricting ourselves to negative couplings and $w > -1$. This guarantees that instability problems in the dark energy perturbation equations are avoided for all values[b] of ξ.

4 Cosmological Constraints from data for $Q = \xi \mathcal{H} \rho_{de}$

We explored the current constraints on the dark energy-dark matter coupling ξ using the publicly available package `cosmomc` [6]. The latter was modified in order to include the coupling among the dark matter and dark energy components. More details on the cosmological model and on the priors adopted can be found in Ref. [2]. The datasets which were taken into account in the analysis are:

1. WMAP 5-year data [7,8]
2. prior on the Hubble parameter of 72 ± 8 km/s/Mpc from the Hubble key project (HST) [9]
3. Super Novae (SN) data [10]
4. $H(z)$ data at $0 < z < 1.8$ from galaxy ages [11]
5. large scale structure data (LSS data) from the Sloan Digital Sky Survey [12]

[b]For $Q = \xi \mathcal{H} \rho_{de}$, the dark energy density is always positive, all along the cosmic evolution and since its initial moment. To ensure that the same happens with the dark matter density, all values of $w < 0$ are acceptable for $\xi < 0$, while for positive ξ it is required that $\xi \lesssim -w$.

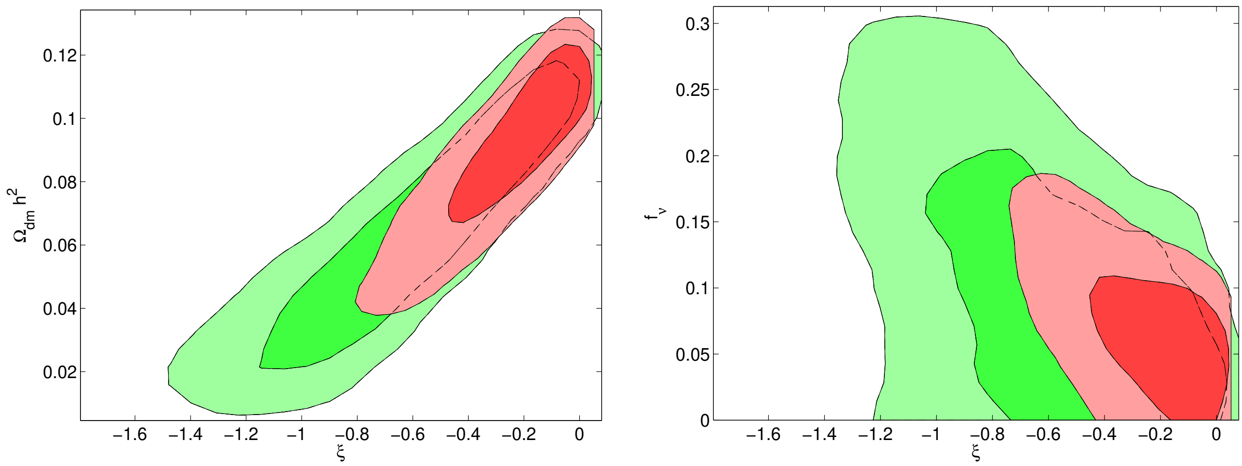

Figure 1: *Scenario with $Q \propto \rho_{de}$. Left (right) panel: 1σ and 2σ marginalized contours in the ξ–$\Omega_{dm}h^2$ (ξ–f_ν) plane. The largest, green contours show the current constraints from WMAP (5 year data), HST, SN and $H(z)$ data. The smallest, red contours show the current constraints from WMAP (5 year data), HST, SN, $H(z)$ and LSS data.*

The data analysis was carried out into two runs, the first run includes the datasets from 1 to 4 while in the second run the fifth dataset is added.

Figure 1 (left panel) illustrates the 1 and 2σ marginalized contours in the ξ–$\Omega_{dm}h^2$ plane where Ω_{dm} is today's ratio between dark matter energy density and critical energy density. The results from the two runs described above are shown. Notice that a huge degeneracy is present, being ξ and $\Omega_{dm}h^2$ positively correlated. The shape of the contours can be easily understood following our discussion in section 1. In a universe with a negative dark coupling ξ, dark matter redshift faster. As a consequence, the matter content in the past is higher than in the standard uncoupled scenario for a fixed dark matter density today. The amount of *intrinsic* dark matter (which is directly proportional to $\Omega_{dm}h^2$) needed to reproduce the LSS data should decrease as the dark coupling becomes more and more negative. We also see that LSS data in the second run (red contours in Fig. 1) give rise to the most stringent constraint on the coupling ξ.

The right panel of Fig. 1 shows the correlation among the fraction of matter energy-density in the form of massive neutrinos f_ν and the dark coupling ξ. The relation between the neutrino fraction used here f_ν and the neutrino mass for N_ν degenerate neutrinos reads

$$f_\nu = \frac{\Omega_\nu h^2}{\Omega_{dm}h^2} = \frac{\sum m_\nu}{93.2\mathrm{eV}} \cdot \frac{1}{\Omega_{dm}h^2} = \frac{N_\nu m_\nu}{93.2\mathrm{eV}} \cdot \frac{1}{\Omega_{dm}h^2} \ . \qquad (11)$$

Neutrinos can indeed play a relevant role in large scale structure formation and leave key signatures in several cosmological data sets, Specially, non-relativistic neutrinos in the recent Universe suppress the growth of matter density fluctuations and galaxy clustering. This effect can be compensated by the existence of a coupling between the coupled sectors, given that in this model negative couplings enhance the growth of matter density perturbations.

5 Conclusion

In this talk, we discuss the origin of non-adiabatic instabilities in coupled models satisfying to Eqs. (3) and (4). We show that the sign of the doom factor **d**

$$\mathbf{d} \equiv \frac{Q}{3\mathcal{H}\rho_{de}(1+w)} \qquad (12)$$

which is a function of the dark coupling Q characterizes the (un)stable regime. In particular, when $\mathbf{d}$ is positive and sizeable, $\mathbf{d} > 1$, the dark-coupling dependent terms may dominate the evolution of dark energy perturbations, which will then enter a runaway, unstable, exponential growth regime.

In a class of viable model in which $Q = \xi \mathcal{H} \rho_{de}$ we have then studied the constraints from cosmological data on the dimensionless coupling ξ. This analysis was carried out in the $\xi < 0$ and positive $(1+w)$ region of the parameter space which offers the best agreement with data on large scale structure formation. Both w and ξ are not very constrained from data, and it can be shown [2] that substantial values for both parameters, near -0.5, are easily allowed. Furthermore, ξ turns out to be positively correlated with $\Omega_{dm}h^2$ and larger neutrino fraction f_ν is allowed for negative ξ.

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