CKM ELEMENTS FROM SQUARK-GLUINO LOOPS

ANDREAS CRIVELLIN
Institut für Theoretische Teilchenphysik, Karlsruhe Institute of Technology – Universität Karlsruhe
D-76128 Karlsruhe, Germany

I present results for the finite renormalization of the Cabibbo-Kobayashi-Maskawa (CKM) matrix induced by gluino–squark diagrams in the MSSM with non-minimal sources of flavour violation. Subsequently I derive bounds on the flavour–off–diagonal elements of the squark mass matrices by requiring that the radiative corrections to the CKM elements do not exceed the experimental values. The constraints on the associated dimensionless quantities $\delta_{ij}^{d\,LR}$, $j > i$, are stronger than the bounds from flavour-changing neutral current (FCNC) processes if gluino and squarks are heavier than 500 GeV. The results imply that it is still possible to generate all observed flavour violation from the soft supersymmetry-breaking terms without conflicting with present-day data on FCNC processes. Therefore we reappraise the idea that only Y_{33}^{q} is different from zero at tree-level and all other Yukawa couplings are generated radiatively. This model solves the SUSY flavour as well as the SUSY CP problem. The presented results are published in [1].

1 Introduction

The generic Minimal Supersymmetric Standard Model (MSSM) contains a plethora of new sources of flavour violation, which reside in the supersymmetry–breaking sector. Especially the additional flavour violation in the squark sector can be dangerously large because the squark-quark-gluino vertex, which involves the strong coupling constant, is in general not flavour diagonal. This potential failure of the MSSM to describe the small flavour violation observed in experiment is known as the "SUSY flavour problem".

We have only partial information from experiment about the quark mass matrices and therefore also about the Yukawa matrices. Not the whole matrices but only their singular values (physical masses) and the misalignment between the rotations of left-handed fields, needed to obtain the mass eigen-basis, (the CKM matrix) are known. This is the reason why it is useful to work in the so called super-CKM basis. We arrive at the super-CKM basis by applying the same rotations which are needed to diagonalize the quark mass matrices to the squark fields:

$$\tilde{q}^{int} = \begin{pmatrix} \tilde{q}_L^{\rm int} \\ \tilde{q}_R^{\rm int} \end{pmatrix} \to \tilde{q}^{SCKM} = \begin{pmatrix} U_L^q & 0 \\ 0 & U_R^q \end{pmatrix} \cdot \begin{pmatrix} \tilde{q}_L^{\rm int} \\ \tilde{q}_R^{\rm int} \end{pmatrix} = \begin{pmatrix} \tilde{q}_L^{SCKM} \\ \tilde{q}_R^{SCKM} \end{pmatrix} \tag{1}$$

Here the superscript "int" means interaction eigenstates and the matrices $U^q_{L,R}$ are determined by the requirement that they diagonalize the tree-level quark mass matrices:

$$U_L^{u\dagger} \mathbf{m}_u^{(0)} U_R^u = \mathbf{m}_u^{(D)}, \qquad\qquad U_L^{d\dagger} \mathbf{m}_d^{(0)} U_R^d = \mathbf{m}_d^{(D)} \tag{2}$$

In the super-CKM basis the squark mass matrices in the down and in the up sector contain bilinear terms $\mathrm{M}^2_{\tilde{q}}$, $\mathrm{M}^2_{\tilde{u}}$and $\mathrm{M}^2_{\tilde{d}}$ as well as the trilinear terms $A^{u,d}$ which originate from the soft SUSY breaking and are flavour non-diagonal, in general. All other terms are flavour diagonal in the super-CKM basis and originate from the spontaneous breakdown of $SU(2)_L$. Since the squark mass matrices are hermitian they can be diagonalised by a unitary transformations of the squark fields:

$$\tilde{q}^{SCKM} \to \tilde{q}^{mass} = W^{\tilde{q}} \cdot \tilde{q}^{SCKM}, \qquad\qquad M_{\tilde{q}}^{2\,(D)} = W^{\tilde{q}\dagger} M_{\tilde{q}}^2 W^{\tilde{q}} \tag{3}$$

In the conventions of Ref. [2] the full 6×6 mass matrix is parametrized by

$$M^2_{\tilde{q}} = \begin{pmatrix}
\left(M^{\tilde{d}}_{1L}\right)^2 & \Delta^{\tilde{d}LL}_{12} & \Delta^{\tilde{d}LL}_{13} & \Delta^{\tilde{d}LR}_{11} & \Delta^{\tilde{d}LR}_{12} & \Delta^{\tilde{d}LR}_{13} \\
\Delta^{\tilde{d}LL^*}_{12} & \left(M^{\tilde{d}}_{2L}\right)^2 & \Delta^{\tilde{d}LL}_{23} & \Delta^{\tilde{d}RL^*}_{12} & \Delta^{\tilde{d}LR}_{22} & \Delta^{\tilde{d}LR}_{23} \\
\Delta^{\tilde{d}LL^*}_{13} & \Delta^{\tilde{d}LL^*}_{23} & \left(M^{\tilde{d}}_{3L}\right)^2 & \Delta^{\tilde{d}RL^*}_{13} & \Delta^{RL*}_{23} & \Delta^{\tilde{d}LR}_{33} \\
\Delta^{\tilde{d}LR^*}_{11} & \Delta^{\tilde{d}RL}_{12} & \Delta^{\tilde{d}RL}_{13} & \left(M^{\tilde{d}}_{1R}\right)^2 & \Delta^{\tilde{d}RR}_{12} & \Delta^{\tilde{d}RR}_{13} \\
\Delta^{\tilde{d}LR^*}_{12} & \Delta^{\tilde{d}LR*}_{22} & \Delta^{\tilde{d}RL}_{23} & \Delta^{\tilde{d}RR^*}_{12} & \left(M^{\tilde{d}}_{2R}\right)^2 & \Delta^{\tilde{d}RR}_{23} \\
\Delta^{\tilde{d}LR^*}_{13} & \Delta^{\tilde{d}LR^*}_{23} & \Delta^{\tilde{d}LR^*}_{33} & \Delta^{\tilde{d}RR^*}_{13} & \Delta^{\tilde{d}RR^*}_{23} & \left(M^{\tilde{d}}_{3R}\right)^2
\end{pmatrix} \tag{4}$$

Anticipating the smallness of the off-diagonal elements $\Delta^{\tilde{q}\,XY}_{ij}$ (with $X, Y = L$ or R) it is possible to treat them pertubatively as squark mass terms [2,3,4,5]. It is customary to define the dimensionless quantities

$$\delta^{q\,XY}_{ij} = \frac{\Delta^{\tilde{q}\,XY}_{ij}}{\frac{1}{6}\sum\limits_s \left[M^2_{\tilde{q}}\right]_{ss}}. \tag{5}$$

Note that the chirality-flipping entries $\delta^{q\,XY}_{ij}$ with $X \neq Y$, even though they are dimensionless, do not stay constant if all SUSY parameters are scaled by a common factor of a, but rather decrease like $1/a$.

In the current era of precision flavour physics stringent bounds on the parameters $\delta^{q\,XY}_{ij}$ have been derived from FCNC processes, by requiring that the gluino–squark loops do not exceed the measured values of the considered observables [2,6,7,8,9,10,11]

We will show in the next section that even more stringent bounds on these quantities can be obtained if we apply a fine-tuning argument which assumes the absence of large accidental cancellations between different contributions to the CKM matrix.

2 Renormalization of the CKM matrix

The simplest diagram (and at least for our discussion the most important one) in which this new flavour and chirality violations induced by the squark mass matrices enters is a potentially flavour-changing self-energy with a squark and a gluino as virtual particles. Since the SUSY

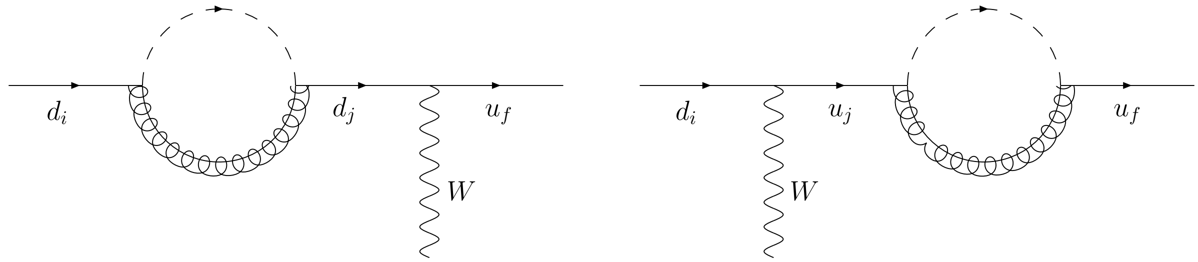

Figure 1: One-loop corrections to the CKM matrix from the down and up sectors contributing to ΔU_L^d and ΔU_L^u in Eq. (9), respectively.

particles are much heavier than the five lightest quarks, it is possible in the calculation of these diagrams to expand in the external momentum, unless one external quark is the top. In the following we consider the self-energies with only light external quarks. The case with a top quark as an external quarks is discussed in [1]. Direct computation of the diagram gives:

$$\Sigma^{q\,LR}_{fi}(p^2=0) = \frac{2m_{\tilde{g}}}{3\pi}\alpha_s(M_{\mathrm{SUSY}})\sum_{s=1}^{6} W^{\tilde{q}}_{f+3,s} W^{\tilde{q}*}_{is} B_0\left(m_{\tilde{g}}, m_{\tilde{q}_s}\right) \tag{6}$$

$$\Sigma^{q\,RL}_{fi}(p^2=0) = \frac{2m_{\tilde{g}}}{3\pi}\alpha_s(M_{\mathrm{SUSY}})\sum_{s=1}^{6} W^{\tilde{q}}_{f,s} W^{\tilde{q}*}_{i+3,s} B_0\left(m_{\tilde{g}}, m_{\tilde{q}_s}\right) \tag{7}$$

For our definition of the loop-function B_0 see appendix of [1]. This self-energy has several important properties:

- It is finite and independent of the renormalization scale.
- It is always chirality-flipping.
- It does not decouple but rather converges to a constant if all SUSY parameter go to infinity.
- It satisfies $\Sigma^{q\,LR}_{fi} = \Sigma^{q\,RL\,*}_{if}$.
- It is chirally enhanced by an approximate factor of $\frac{\left|A^q_{fi}\right|}{M_{SUSY}\left|Y^q_{fi}\right|}$ or $\frac{v\tan\beta}{M_{SUSY}}$ compared to the tree-level quark coupling. These factors may compensate for the loop suppression factor of $1/(16\pi^2)$.

In the case when this self-energy is flavour conserving, it renormalizes the corresponding quark mass in a rather trivial way:

$$m^{(0)}_{q_i} \to m_{q_i} = m^{(0)}_{q_i} + \Sigma^{q\,LR}_{ii} \tag{8}$$

Since the self-energy is finite the introduction of a counter-term is optional. In minimal renormalization schemes the counter-term is absent and in the one-shell scheme it just equals $-\Sigma^{q\,LR}_{ii}$. Since we will later consider the possibility that the light quark masses are generated exclusively via these loops, meaning $m_{q_i} = \Sigma^{q\,LR}_{ii}$, it is most natural and intuitive to choose a minimal renormalization scheme like $\overline{\mathrm{MS}}$.

The renormalization of the CKM matrix is a bit more involved. There are two possible contributions, the self-energy diagrams and the proper vertex correction. The vertex diagrams involving a W coupling to squarks are not chirally enhanced and moreover suffer from gauge

Table 1: Comparisons of our constraints on $\delta_{ij}^{q\,XY}$ with the constraints obtained from FCNC processes and vacuum stability bounds.

quantity	our bound	bound from FCNC's		bound from VS [15]
$\lvert\delta_{12}^{d\,LR}\rvert$	≤ 0.0011	≤ 0.006	K mixing [7]	$\leq 1.5\times 10^{-4}$
$\lvert\delta_{13}^{d\,LR}\rvert$	≤ 0.0010	≤ 0.15	B_d mixing [9]	≤ 0.05
$\lvert\delta_{23}^{d\,LR}\rvert$	≤ 0.010	≤ 0.06	$B\to X_s\gamma; X_s l^+l^-$ [10]	≤ 0.05
$\lvert\delta_{13}^{d\,LL}\rvert$	≤ 0.032	≤ 0.5	B_d mixing [9]	–
$\lvert\delta_{12}^{u\,LR}\rvert$	≤ 0.011	≤ 0.016	D mixing [11]	$\leq 1.2\times 10^{-3}$
$\lvert\delta_{13}^{u\,LR}\rvert$	≤ 0.062	—		≤ 0.22
$\lvert\delta_{23}^{u\,LR}\rvert$	≤ 0.59	—		≤ 0.22

cancellations with non-enhanced pieces from the self-energies. Therefore we only need to consider self-energies, just as in the case of the electroweak renormalization of V in the SM [12]. The two diagrams shown in Fig 1 contribute at the one loop level. According to [13] they can be treated in the same way as one-particle-irreducible vertex corrections. Computing theses diagrams we receive the following corrections to the CKM matrix:

$$V^{(0)} \to V = \left(1+\Delta U_L^{u\dagger}\right) V^{(0)} \left(1+\Delta U_L^{d}\right) \tag{9}$$

with

$$\Delta U_L^q = \begin{pmatrix} 0 & \frac{1}{m_{q_2}}\Sigma_{12}^{q\,LR} & \frac{1}{m_{q_3}}\Sigma_{13}^{q\,LR} \\ \frac{-1}{m_{q_2}}\Sigma_{21}^{q\,RL} & 0 & \frac{1}{m_{q_3}}\Sigma_{23}^{q\,LR} \\ \frac{-1}{m_{q_3}}\Sigma_{31}^{q\,RL} & \frac{-1}{m_{q_3}}\Sigma_{32}^{q\,RL} & 0 \end{pmatrix} \tag{10}$$

In Eq. (10) we have discarded small quark-mass ratios. Just as in the case of the mass-renormalization we choose a minimal renormalization scheme which complies with the use of the super-CKM basis (see [1] for details). It is easily seen from Eq. (10) that the corrections are antihermitian which is in agreement with the demanded unitarity of the CKM matrix at one-loop. Our corrections are independent of the renormalization scale μ. The choice $\mu = M_{SUSY}$ avoids large logarithms in $\Sigma_{ij}^{q\,LR}$, so that we have to evaluate it at this scale. This means we must also evaluate the quark masses appearing in Eq. (10) at the scale M_{SUSY}.

We can now receive constraints on the off-diagonal elements of the squark mass matrices from Eq. (9) by applying a fine-tuning argument. Large accidental cancellations between the SM and supersymmetric contributions are, as already mentioned in the introduction, unlikely and from the theoretical point of view undesirable. Requiring the absence of such cancellations is a commonly used fine-tuning argument, which is also employed in standard FCNC analyses of the $\delta_{ij}^{q\,XY}$'s [2,6,7,8,9,10,11]. Analogously, we assume that the corrections due to flavour-changing SQCD self-energies do not exceed the experimentally measured values for the CKM matrix elements quoted in the Particle Data Table (PDT) [14]. To this end we set the tree-level CKM matrix $V^{(0)}$ equal to the unit matrix and generate the measured values radiatively. For $m_{\tilde{q}} = m_{\tilde{g}} = 1000\text{GeV}$ we receive the constraints quoted in table 1.

Note that our constraints are all much stronger than the FCNC bounds. The FCNC bounds in addition decouple. This means they vanish like $1/a^2$ if all SUSY masses are scaled with a.

The vacuum stability bounds are stronger than our ones for the $\delta_{12}^{q\,LR}$ elements and they are non-decoupling like our bounds. However, the analysis of [15] only takes tree-level Yukawa coupling into account and the small Yukawa couplings are modified by the very same loop effects which enter $\Delta U^q_{L,R}$ in Eq. (10).

3 The Model

The smallness of the Yukawa couplings of the first two generations suggests that these couplings are generated through radiative corrections [16]. In the context of supersymmetric theories these loop-induced couplings arise from diagrams involving squarks and gluinos. Although the B factories have confirmed the CKM mechanism of flavour violation with very high precision, leaving little room for new sources of FCNCs, the possibility of radiative generation of quark masses and of the CKM matrix still remains valid as proven in section 2, even for SUSY masses well below 1 TeV, if the sources of flavour violation are the trilinear terms [1]. Of course, the heaviness of the top quark requires a special treatment of Y^t and the successful bottom-tau Yukawa unification suggests to keep tree-level Yukawa couplings for the third generation. At large $\tan\beta$, this idea gets even more support from the successful unification of the top and bottom Yukawa coupling, as suggested by some GUT models. Radiative Yukawa interactions from SUSY-breaking terms have been considered earlier in Refs [17,18,19].

In the modern language of Refs. [20,21] the global $[U(3)]^3$ flavour symmetry of the gauge sector (here we do not consider neutrinos) is broken to $[U(2)]^3 \times [U(1)]$ by the Yukawa couplings of the third generation. Here the three $U(2)$ factors correspond to rotations of the left-handed doublets and the right-handed singlets of the first two generation quarks in flavour space, respectively.

This means we have

$$Y^q = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y^q \end{pmatrix}, \quad V^{(0)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{11}$$

in the tree-level Lagrangian. We next assume that the soft breaking terms $\Delta_{ij}^{\tilde{q}\,LL}$ and $\Delta_{ij}^{\tilde{q}\,RR}$ possess the same flavour symmetry as the Yukawa sector, which implies that $\mathbf{M}_{\tilde{q}}$, $\mathbf{M}_{\tilde{d}}$ and $\mathbf{M}_{\tilde{u}}$ are diagonal matrices with the first two entries being equal. For transitions involving the third generation the situation is different because flavour violation can occur not only because of a misalignment between A^u and A^d but also due to a misalignment with the Yukawa matrix. So the elements $A^{u,d}_{j3}$ do not only generate the CKM matrix at one-loop, they also act as a source of non-minimal flavour violation and thus can be constrained by FCNC processes.

This model has several advantages compared with the generic MSSM:

- Flavour universality holds for the first two generations. Thus our Model is minimally flavour violating according to the definition of [20] with respect to the first two generations since the quark and the squark mass matrices are diagonal in the same basis. This provides a explanation of the precise agreement between theory and experiment in K and D physics.
- The SUSY flavour problem is reduced to the quantities $\delta^{q\,RL}_{13,23}$. However, these flavour-changing elements are less constrained from FCNCs and might even explain a possible new CP phase indicated by recent data on B_s mixing.
- The flavour symmetry of the Yukawa sector protects the quarks of the first two generations from a tree-level mass term.
- The model is economical: Flavour violation and SUSY breaking have the same origin. Small quark masses and small off-diagonal CKM elements are explained by a loop suppression.

- The SUSY CP problem is substantially alleviated by an automatic phase alignment [19]. In addition, the phase of μ does not enter the EDMs at the one-loop level, because the Yukawa couplings of the first two generations are zero.

4 Conclusions

We have computed the renormalization of the CKM matrix by chirally-enhanced flavour-changing SQCD effects in the MSSM with generic flavour structure [1]. We have derived upper bounds on the flavour-changing off-diagonal elements $\Delta_{ij}^{\tilde{q}\,XY}$ of the squark mass matrices by requiring that the supersymmetric corrections do not exceed the measured values of the CKM elements. For $M_{\text{SUSY}} \geq 500\,\text{GeV}$ our constraints on *all* elements $\Delta_{ij}^{\tilde{d}\,LR}$, $i < j$, are stronger than the constraints from FCNC processes. As an important consequence, we conclude that it is possible to generate the observed CKM elements completely through finite supersymmetric loop diagrams [17,18] without violating present-day data on FCNC processes. In this scenario the Yukawa sector possesses a higher flavour symmetry than the trilinear SUSY breaking terms. Additional applications to charged Higgs and chargino couplings are considered in [1].

Acknowledgments

This work is supported by BMBF grant 05 HT6VKB and by the EU Contract No. MRTN-CT-2006-035482, "FLAVIAnet". I am grateful to the organizers for inviting me to this conference. I like to thank Lars Hofer for reading the manuscript and many useful discussions. I am grateful to Ulrich Nierste for the collaboration on the presented work [1].

References

1. A. Crivellin and U. Nierste, Phys. Rev. D **79** (2009) 035018.
2. F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, Nucl. Phys. B **477** (1996) 321.
3. L. J. Hall, V. A. Kostelecky and S. Raby, Nucl. Phys. B **267** (1986) 415.
4. M. Misiak, S. Pokorski and J. Rosiek, Adv. Ser. Direct. High Energy Phys. **15**, 795 (1998).
5. A. J. Buras, A. Romanino and L. Silvestrini, Nucl. Phys. B **520** (1998) 3.
6. J. S. Hagelin, S. Kelley and T. Tanaka, Nucl. Phys. B **415** (1994) 293.
7. M. Ciuchini *et al.*, JHEP **9810** (1998) 008.
8. F. Borzumati, C. Greub, T. Hurth and D. Wyler, Phys. Rev. D **62** (2000) 075005.
9. D. Becirevic *et al.*, Nucl. Phys. B **634**, 105 (2002).
10. L. Silvestrini, Ann. Rev. Nucl. Part. Sci. **57** (2007) 405.
11. M. Ciuchini, E. Franco, D. Guadagnoli, V. Lubicz, M. Pierini, V. Porretti and L. Silvestrini, Phys. Lett. B **655** (2007) 162. A. Denner and T. Sack, Nucl. Phys. B **347** (1990) 203.
12. A. Denner and T. Sack, Nucl. Phys. B **347** (1990) 203.
13. H. E. Logan and U. Nierste, Nucl. Phys. B **586** (2000) 39.
14. C. Amsler *et al.* [Particle Data Group], Phys. Lett. B **667** (2008) 1.
15. J. A. Casas, A. Lleyda and C. Munoz, Nucl. Phys. B **471** (1996) 3. J. A. Casas and S. Dimopoulos, Phys. Lett. B **387** (1996) 107.
16. S. Weinberg, Phys. Rev. Lett. **29** (1972) 388. S. Weinberg, Phys. Rev. D **5** (1972) 1962.
17. W. Buchmuller and D. Wyler, Phys. Lett. B **121** (1983) 321.
18. J. Ferrandis and N. Haba, Phys. Rev. D **70**, 055003 (2004).
19. F. Borzumati, G. R. Farrar, N. Polonsky and S. D. Thomas, Nucl. Phys. B **555** (1999) 53.
20. G. D'Ambrosio, G. F. Giudice, G. Isidori and A. Strumia, Nucl. Phys. B **645** (2002) 155.
21. R. S. Chivukula and H. Georgi, Phys. Lett. B **188** (1987) 99.