HIGGS-DEPENDENT LEPTOGENESIS [a]

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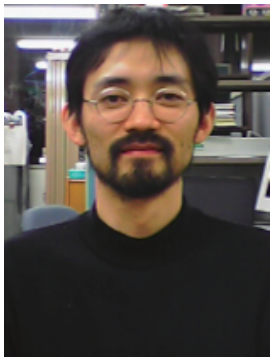

In this talk, we study electroweak-scale leptogenesis without resonance condition. If neutrino sector has higher dimensional operators with multi-Higgs bosons in addition to ordinary Yukawa interactions, interference of decay processes of TeV-scale heavy neutrinos induced by these interactions can give large CP asymmetry even if masses of heavy neutrinos are not degenerate. These effective Yukawa couplings contain phases coming from Higgs potential, which are the origin of CP violation. We find that this mechanism can generate enough baryon asymmetry of the Universe without unnatural fine tuning of any parameters.

1 Introduction

Leptogenesis [2] is one of the most promising mechanism to explain baryon asymmetry of the Universe (BAU). It requires the existence of CP violating (CPV) couplings which generate CP asymmetry in heavy neutrino decay processes, and neutrino Yukawa coupling is the one in usual cases. However in the case of TeV-scale heavy neutrinos with small Yukawa couplings, enough CP asymmetry cannot be generated by heavy neutrino decays because of the smallness of the Yukawa couplings unless masses of heavy neutrinos satisfy the resonance condition [3], which requires fine tuning between mass parameters.

In order to avoid this problem, we consider higher dimensional operators in neutrino Yukawa sector. It was pointed out in Ref.[4,5] that gauge invariant combination of Higgs fields can make mass hierarchy by higher dimensional operators in Yukawa sector similar to Froggatt-Nielsen model [6]. After the electroweak symmetry breaking (EWSB) by vacuum expectation values (VEVs) of Higgs bosons, effective Yukawa couplings written in mass eigenstates contain CP

[a] This talk is based on the work Ref. [1].

phases from Higgs sector as well as the ones in original Yukawa couplings. Since higher dimensional operators reduce the number of vertices, one-loop level diagram of heavy neutrino decay contains only one small neutrino Yukawa coupling unlike ordinary TeV-scale leptogenesis which contains three Yukawa couplings. Moreover, CP phases in original Yukawa couplings do not contribute to CP asymmetry because both tree and one-loop level diagrams have one Yukawa coupling with same generation indices, and these phases are cancelled out. Therefore interference between tree and one-loop level diagrams are not suppressed by additional small neutrino Yukawa couplings compared with the total decay rate and it can generate large CP asymmetry. The source of CPV is in the Higgs sector.

After introducing our model motivated by Ref.[4,5], we give a numerical example of leptogenesis to see that this mechanism can work.

2 The Model

We consider global $U(1)$ symmetric two Higgs doublet model with charge given by $Q(H_u) = h_u,\ Q(H_d) = h_d,\ Q(L_i) = -4h_u - 3h_d$ and $Q(N_{Ri}) = 0$. We assume that $h_u + h_d \neq 0$, then the combination $H_u H_d$ is not $U(1)$ invariant. In the neutrino sector, $U(1)$ invariant Yukawa interactions and Majorana mass term of the heavy neutrinos N_{Ri} are given by

$$\mathcal{L}_L = y^{\nu}_{ij} \bar{N}_{Ri} L_{Lj} H_u \left(\frac{H_u H_d}{M^2}\right)^{n^{\nu}_{ij}} - \frac{1}{2} N_{Ri} M_{Nij} N_{Rj} + h.c., \tag{1}$$

where we have neglected $\bar{N}_{Ri} L_{Lj} H_d^{\dagger} \left(H_u H_d / M^2\right)^{n^{\nu}_{ij}+1}$ term because these are more suppressed by ϵ defined below. Under the charge assignment presented above, n^{ν}_{ij} are universal and given by $(n^{\nu}_{ij}) = 3$. The electroweak symmetry and global $U(1)$ symmetry are broken by the VEVs of the Higgs bosons $\langle H_u \rangle = v_u = v \sin\beta$ and $\langle H_d \rangle = v_d = v \cos\beta$, where $v = 174$GeV. We define the suppression factor $\epsilon \equiv v_u v_d / M^2 = 10^{-2}$, and the cut-off scale of this model is $M = 1.23$ TeV when $\tan\beta = 1$. The scale of Majorana mass term of light neutrinos after the seesaw mechanism is proportional to $v^2 s_\beta^2 \epsilon^6 / M_N = 1.5 \times 10^{-2}$ eV for $M_N = 1$TeV. As for the charged lepton sector, similar discussion is possible and mass hierarchy are generated by powers of ϵ, such as $m_\tau \sim \epsilon v,\ m_\mu \sim 6\epsilon^2 v$ and $m_e \sim 3\epsilon^3 v$. Neutrino mass differences and mixing angles are obtained by appropriate choice of $\mathcal{O}(1)$ Yukawa couplings $y^{\nu,e}$. Expanding the neutral component of Higgs fields $(v + H_0)^{2n^{\nu}+1} / M^{2n^{\nu}}$ in Eq.1, we get mass term proportional to $\epsilon^{n^{\nu}} v$ and multi-Higgs terms proportional to $\epsilon^{n^{\nu}} H_0 (H_0/v)^{(0-2n^{\nu})}$, which have the common factor $\epsilon^{n^{\nu}}$. The existence of the $\epsilon^{n^{\nu}} H_0^2 / v$ term plays a crucial role in our scenario.

Higgs potential of this model upto quartic terms is given by

$$\begin{aligned} V &= m^2_{H_u} |H_u|^2 + m^2_{H_d} |H_d|^2 + \lambda_1 |H_u|^4 + \lambda_2 |H_d|^4 + \lambda_3 |H_u|^2 |H_d|^2 + \lambda_4 |H_u H_d|^2 && (2) \\ &+ \left[m^2 H_u H_d + \lambda_5 (H_u H_d)^2 + \lambda_6 |H_u|^2 H_u H_d + \lambda_7 |H_d|^2 H_u H_d + h.c. \right], && (3) \end{aligned}$$

where Eq.2 is $U(1)$ symmetric, and Eq.3 is explicit $U(1)$ breaking part, repspectively. While $m^2_{H_{u,d}}$ and $\lambda_{1,2,3,4}$ are real, m^2 and $\lambda_{5,6,7}$ are complex in general, which are the origin of CPV. The vacuum condition of the potential V requires that the parameter b defined as

$$b \equiv m^2 - v^2 s_\beta c_\beta \left(\lambda_4 + 2\lambda_5 - \lambda_6 \tan\beta - \lambda_7 \cot\beta\right), \tag{4}$$

to be real. Since the Higgs potential V has to be bounded from below to guarantee the EWSB, V should satisfy the vacuum stability conditions

$$\begin{aligned} &\lambda_1 > 0,\ \lambda_2 > 0,\ \lambda_3 + \sqrt{\lambda_1 \lambda_2} > 0, \\ \lambda_3 \ &+ \ \lambda_4 + 2\sqrt{\lambda_1 \lambda_2} - 2|\lambda_5| > 0 \ (\text{for } \lambda_6 = \lambda_7 = 0). \end{aligned} \tag{5}$$

Mass term of neutral Higgs bosons $H_0 = (\mathrm{Re}H_u^0, \mathrm{Re}H_d^0, \mathrm{Im}H_u^0, \mathrm{Im}H_d^0)^T$ is written as $V = (1/2)H_0^T M_0^2 H_0$, and the mass matrix M_0^2 has a zero eigenvalue which corresponds to the Goldstone boson eaten by Z gauge boson. Mass matrix M_0^2 is diagonalized by orthogonal transformation $O^T M_0^2 O = \mathrm{diag}(m_{h_1}^2, m_{h_2}^2, m_{h_3}^2, 0)$, and their mass eigenstates $h_a (a = 1 \cdots 4)$ are transformed from weak eigenstates H_0 as $h = O^T H_0$. Since CP is violated in the Higgs potential by complex parameters, real and imaginary elements of neutral Higgs bosons are mixed with each other. Off-diagonal blocks of M_0^2 and related elements of the matrix O vanish if all couplings are real, and in this case $h_{1,2}$ and h_3 correspond to the CP even and odd Higgs bosons, respectively (h_4 is the Goldstone mode).

Now we can rewrite Yukawa interactions Eq.1 in the mass eigenstates. The $N_R - \nu_L$ interactions upto dimension-five operators are

$$\mathcal{L}_\nu = \bar{N}_i P_L \left(U_{MNS}\nu\right)_j \left(A_{ij}^a h_a + \frac{1}{v} B_{ij}^{ab} h_a h_b\right) + h.c., \tag{6}$$

where the effective Yukawa couplings A and B are given by

$$A_{ij}^a = \frac{(-1)^{n_{ij}^\nu}}{\sqrt{2}} y_{ij}^\nu \epsilon^{n_{ij}^\nu} \left[\left(1 - n_{ij}^\nu\right)(O_{1a} + iO_{3a}) - n_{ij}^\nu (O_{2a} + iO_{4a})\right], \tag{7}$$

$$\begin{aligned} B_{ij}^{ab} &= \frac{(-1)^{n_{ij}^\nu}}{8} y_{ij}^\nu \epsilon^{n_{ij}^\nu} \left[\frac{(O_{1a} + iO_{3a})}{s_\beta} + \frac{(O_{2a} + iO_{4a})}{c_\beta}\right] \\ &\times \left[n_{ij}^\nu \left(n_{ij}^\nu - 3\right)(O_{1b} + iO_{3b}) + n_{ij}^\nu \left(n_{ij}^\nu - 1\right)(O_{2b} + iO_{4b})\, c_\beta\right] + (a \leftrightarrow b). \end{aligned} \tag{8}$$

The effective Yukawa couplings A and B are complex because O is complex matrix as well as y^ν. However as we will see below, CP asymmetry in heavy neutrino decay processes is proportional to the combination $\mathrm{Im}[AB^*]$, and A and B have the same y^ν but different O structure. Therefore the phases in O contribute to CP asymmetry although those in y^ν are cancelled out.

The $N_R - e_L$ interactions with the charged Higgs boson ϕ^+ are

$$\mathcal{L}_e = (-1)^{n_{ij}^\nu} \epsilon^{n_{ij}^\nu} y_{ij}^\nu \bar{N}_i P_L e_j \phi^+ + h.c. \tag{9}$$

Three-point vertex of neutral Higgs bosons is

$$V_3 = v\, C_{abc} h_a h_b h_c, \tag{10}$$

and relevant elements of C_{abc} in the case of Eq.17 given below are

$$C_{211} = \frac{1}{\sqrt{2}} \lambda_1 s_\beta s_\theta c_\theta^2 + \frac{1}{\sqrt{2}} \lambda_2 c_\beta s_\theta^2 c_\theta + \mathcal{O}((\mathrm{Im}\lambda_5)^2), \tag{11}$$

$$\begin{aligned} C_{311} &= \mathrm{Im}\lambda_5 \Big[-\frac{2v^2}{m_{h1}^2 - m_{h3}^2} \frac{\cos(\theta+\beta)}{3\sqrt{2}} \left\{\lambda_1 s_\beta c_\theta (2c_\beta^2 - 3c_\theta^2) - \lambda_2 c_\beta s_\theta (2s_\beta^2 - 3s_\theta^2)\right\} \\ &+ \frac{2v^2}{m_{h2}^2 - m_{h3}^2} \sin(\theta+\beta) C_{211}\Big] + \mathcal{O}((\mathrm{Im}\lambda_5)^2), \end{aligned} \tag{12}$$

where θ is the mixing angle of CP even Higgs bosons h_1 and h_2, given by $\sin 2\theta = -\sin 2\beta\, m_{h3}^2/(m_{h2}^2 - m_{h1}^2)$. This vertex also contributes to CP asymmetry in decay process of heavy Majorana neutrinos. Next we study leptogenesis induced by these interactions.

3 Leptogenesis

CP asymmetry of decay mode $\varepsilon_i^j \equiv \varepsilon(N_{iR} \to \sum_{a=1}^3 \nu_{jL} h_a)$ is defined as

$$\varepsilon_i^j = \frac{\sum_a \left[\Gamma(N_{iR} \to \nu_{jL} h_a) - \Gamma(N_{iR} \to \bar{\nu}_{jL} \bar{h}_a)\right]}{\sum_{k,b} \left[\Gamma(N_{iR} \to \nu_{kL} h_b) + \Gamma(N_{iR} \to \bar{\nu}_{kL} \bar{h}_b)\right]} \equiv \frac{N_i^j}{\sum_k \Gamma_{Di}^{\nu k}}. \tag{13}$$

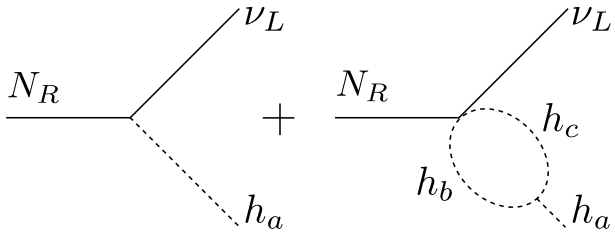

Figure 1: Diagrams of heavy Majorana neutrino decay into neutrino and heavy neutral Higgs boson. CP asymmetry is generated by interference of these diagrams because of complex couplings in the Higgs potential.

These decay processes are generated by interactions Eqs.6 and 10 through the diagrams of Fig.1, and the most general form of Eq.13 is

$$N_i^j = -\frac{1}{16\pi^2}\frac{1}{16\pi}M_{Ni}\sum_{a,b,c=1}^{3} C_{abc}\mathrm{Im}\left[J_{a,bc}\right]\mathrm{Im}\left[\left(A^a U_{MNS}\right)_{ij}\left(B^{bc}U_{MNS}\right)^*_{ij}\right]\left(1-\frac{m^2_{h_a}}{M^2_{Ni}}\right)^2 \quad (14)$$

$$\Gamma^{\nu k}_{Di} = \frac{1}{16\pi}M_{Ni}\sum_{a=1}^{3}\left|\left(A^a U_{MNS}\right)_{ik}\right|^2\left(1-\frac{m^2_{h_a}}{M^2_{Ni}}\right)^2, \quad (15)$$

where $J_{a,bc}$ is the loop function defined as $J_{a,bc} = \int_0^1 dx \ln\left[x(x-1)+(1-x)r_b + xr_c\right]$ and $r_{b,c} = m^2_{h_{b,c}}/m^2_{h_a}$. The loop function $J_{a,bc}$ has imaginary part when mass of the final state (h_a) is larger than the sum of intermediate Higgs masses, that is, $m_{h_b} + m_{h_c} < m_{h_a}$. The heavy neutrinos N_{iR} decay into charged leptons and charged Higgs bosons as well by the coupling Eq.9 with the decay rate $\Gamma^{ek}_{Di} = \Gamma(N_{iR} \to e^{\mp}_{kL}\phi^{\pm})$:

$$\Gamma^{ek}_{Di} = \frac{1}{16\pi}M_{Ni}\epsilon^{2n^\nu_{ik}}\left|y^\nu_{ik}\right|^2\left(1-\frac{m^2_\phi}{M^2_{Ni}}\right)^2. \quad (16)$$

The total decay rate of N_{iR} is defined as $\Gamma_{Di} = \Gamma^\nu_{Di} + \Gamma^e_{Di}$, where $\Gamma^{\nu(e)}_{Di} \equiv \sum_{k=1}^3 \Gamma^{\nu(e)k}_{Di}$.

Now we give a numerical example. We assume non-degenerate heavy neutrino masses $(M_N)_{ij} = M_N\mathrm{diag}(0.5, 1.25, 1.5)$, the following Higgs parameters

$$\lambda_1 = 0.2,\ \lambda_2 = 0.5,\ \mathrm{Im}\lambda_5 \neq 0,\ m^2 = (300\mathrm{GeV})^2 + 2iv^2 s_\beta c_\beta \mathrm{Im}\lambda_5,\ \text{others} = 0, \quad (17)$$

and $\tan\beta = 1$. The imaginary part of m^2 is constrained by the vacuum condition Eq.4. In these parameters, CPV coupling $\mathrm{Im}\lambda_5$ is the only free parameter. In these choice of parameters, masses of neutral Higgs bosons are approximately $h_1 \sim 140\mathrm{GeV},\ h_2 \sim 480\mathrm{GeV},\ h_3 \sim 440\mathrm{GeV}$. In this example, decay modes of $h_{2,3}$ final states with two h_1 internal states in the loop give non-vanishing CP asymmetry. Three components of nine CP asymmetries ε^j_i are plotted in Fig.2 as a function of $\mathrm{Im}\lambda_5$, where the shaded region is prohibited by the vacuum stabilization condition Eq.5. The dependence of ε^j_i on $\mathrm{Im}\lambda_5$ is almost linear. In our choice of Yukawa couplings, decay process $N_{3R} \to \sum_a \nu_{2L}h_a$ gives the dominant contribution and we define $\varepsilon^2_3 \equiv \varepsilon_{\max}$. One can see that large CP asymmetry ($< 10^{-3}$) is generated.

Next we discuss Boltzmann equations of the heavy neutrinos η_{Ni} and η_{Δ_j}, where $\Delta_j = B/3 - L_j$. The parameter η_X is defined as $\eta_X = n_X/n_\gamma$, where n_X is number density of X and photon number density is given by $n_\gamma = 2T^3\zeta(3)/\pi^2$. We solve the Boltzmann equations:

$$\frac{d\eta_{Ni}}{dz} = -\frac{z}{n_\gamma H(z=1)}\left(\frac{\eta_{Ni}}{\eta^{eq}_{Ni}} - 1\right)\gamma_{Di}, \quad (18)$$

$$\frac{d\eta_{\Delta j}}{dz} = -\frac{z}{n_\gamma H(z=1)}\sum_{i=1}^{3}\left[\left(\frac{\eta_{Ni}}{\eta^{eq}_{Ni}} - 1\right)\epsilon^j_i\gamma^\nu_{Di} - \frac{\eta_{\Delta\nu j}}{2\eta^{eq}_{\nu j}}\gamma^{\nu j}_{Di} - \frac{\eta_{\Delta ej}}{2\eta^{eq}_{ej}}\gamma^{ej}_{Di}\right], \quad (19)$$

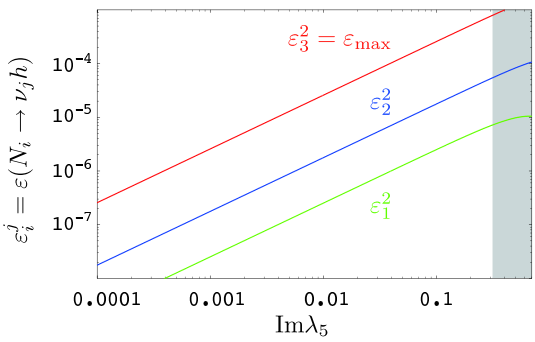

Figure 2: CP asymmetry ε_i^j as a function of $\mathrm{Im}\lambda_5$. Decay process $N_{3R} \to \sum_a \nu_{2L} h_a$ gives the largest asymmetry in our choice of Yukawa couplings. Shaded region is prohibited by the vacuum stability condition.

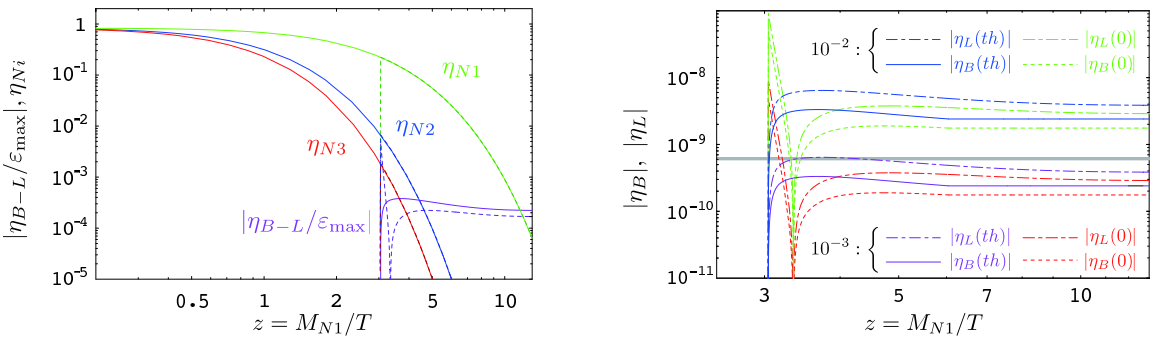

Figure 3: Left panel: Behavior of $|\eta_{B-L}/\varepsilon_{\max}|$ and η_{Ni} for $\mathrm{Im}\lambda_5 = 10^{-2}$ in the case of thermal (solid curves) and zero (dashed curves) N_i abundances. Right panel: Lepton and baryon asymmetry for $\mathrm{Im}\lambda_5 = 10^{-2}$ (blue and green curves) and 10^{-3} (purple and red curves). Parentheses (th) and (0) denote thermal and zero initial abundances of N_i at $z = z_c$, respectively. The horizontal band is the experimental value $\eta_B^{\rm exp} = (6.05 - 6.37) \times 10^{-10}$.

where $\eta_{\Delta j}$ and asymmetry of left-handed neutrino (charged lepton) $\eta_{\Delta\nu(e)j}$ are related to each other by the "A-matrix" [7] defined as $\eta_{\Delta\nu(e)j} = A^{\nu(e)}_{jk}\eta_{\Delta k}$ with

$$A^\nu = \frac{1}{279}\begin{pmatrix} -89 & 4 & 4 \\ 4 & -89 & 4 \\ 4 & 4 & -89 \end{pmatrix}, \quad A^e = \frac{1}{279}\begin{pmatrix} -80 & 13 & 13 \\ 13 & -80 & 13 \\ 13 & 13 & -80 \end{pmatrix}. \tag{20}$$

The thermally averaged decay rates $\gamma_{Di}, \gamma_{Di}^{\nu(e)}$ and $\gamma_{Di}^{\nu(e)j}$ are obtained from $\Gamma_{Di}, \Gamma_{Di}^{\nu(e)}$ and $\Gamma_{Di}^{\nu(e)j}$ as $\gamma_{Di}^{\nu(e)j} = n_{Ni}^{\rm eq}(K_1(z_i)/K_2(z_i))\Gamma_{Di}^{\nu(e)j}$, $\gamma_{Di}^{\nu(e)} = \sum_{j=1}^3 \gamma_{Di}^{\nu(e)j}$ and $\gamma_{Di} = \gamma_{Di}^\nu + \gamma_{Di}^e$. K_1 and K_2 are the modified Bessel functions and $z_i = M_{Ni}/T$, hereafter we define $z \equiv z_1$ as usual. The left panel of FIG.3 shows $|\eta_{B-L}/\varepsilon_{\max}|$ and η_{Ni} as a function of z for the case of thermal (solid curves) and zero (dashed curves) N_i abundances at the critical temperature T_c, $z_c \equiv M_{N1}/T_c = 3.04$. In both cases, $B-L$ asymmetry $\eta_{B-L} = \sum_j \eta_{\Delta j}$ is generated at z_c. While $|\eta_{B-L}/\varepsilon_{\max}|$ is estimated in the case of $\mathrm{Im}\lambda_5 = 10^{-2}$ in Fig.3, other values of $\mathrm{Im}\lambda_5$ give similar results.

Fast sphaleron processes convert lepton asymmetry to baryon asymmetry [8]. However, sphaleron processes are not always active for $T < T_c$. The sphaleron rate $\Gamma_{\Delta(B+L)}$ for the temperature $M_W(T) \ll T \ll M_W(T)/\alpha_W$ is given by $\Gamma_{\Delta(B+L)} \sim M_W(M_W/(\alpha_W T))^3 (M_W/T)^3 \exp[-E_{sp}/T]$ [9,10] where α_W is $SU(2)_L$ fine structure constant, M_W is W-boson mass and sphaleron energy $E_{sp} \sim M_W/\alpha_W$. Just below T_c, sphaleron is faster than the expansion of the Universe, that is, $\Gamma_{\Delta(B+L)}/H(z=1) \gg 1$. In this region, η_{B-L} is related to baryon and lepton asymmetry $\eta_{B,L}$

by sphaleron effect with the temperature-depedent rate given by [11,12]

$$\eta_B = \frac{16T^2 + 10v(T)^2}{46T^2 + 31v(T)^2}\eta_{B-L}, \quad \eta_L = -\frac{30T^2 + 21v(T)^2}{46T^2 + 31v(T)^2}\eta_{B-L}. \tag{21}$$

Sphaleron rate $\Gamma_{\Delta(B+L)}$ decreases below T_c by the Boltzmann factor $\exp[-E_{sp}/T]$, and it reaches $\Gamma_{\Delta(B+L)}(z_d)/H(z = 1) = 1$ at $z_d = 6.04$. Since sphaleron processes become slow and are switched off for $z > z_d$, we can make approximation that baryon asymmetry is almost constant in this region. The final results for baryon and lepton asymmetry in our model are shown in the right panel of Fig.3 for $\mathrm{Im}\lambda_5 = 10^{-2}$ (blue and green curves) and 10^{-3} (red and purple curves). In both cases of thermal and zero N_i abundances denoted by (th) and (0), we can obtain enough baryon asymmetry [13].

4 Conclusions

We have discussed electroweak-scale leptogenesis without resonance condition in two Higgs doublet model with global $U(1)$ symmetry. Since Higgs bosons make mass hierarchy of matter fermions, there are higher dimensional operators with multi-Higgs bosons in neutrino Yukawa sector, and these couplings induce new diagrams of heavy neutrino decay which contain one small neutrino Yukawa coupling same as the tree level diagram. Although CP violating phases in original Yukawa couplings do not contribute to CP asymmetry because these are cancelled out, those coming from the Higgs potential do contribute. Therefore one can obtain large CP asymmetry by decay process of non-degenerate TeV-scale heavy neutrinos. There is small window of temperature where sphalerons are active and baryon asymmetry is created even below the critical temperature. We have shown that we do not need unnatural fine tuning of any parameters to obtain baryon asymmetry of the Universe in our mechanism.

Acknowledgments

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