

# Phenomenology of A4 & S4 based models

Federica Bazzocchi, Vrij Universiteit Amsterdam

FBL. Merlo, S. Morisi 0902.2849

FBL. Merlo, S. Morisi 0901.2086  
Altarelli & Feruglio NP B 720 & 741, Altarelli, Feruglio & Lin NP B 775  
Hirsch, Morisi & Valle, PRD78

Rencontres de Moriond EW 2009, 7-14/03/09

**Neutrinos**

**Lepton mixing**

**Mass origin**

**Flavor symmetry**

**Model building**

**Neutrinos**

**Lepton mixing**

**Mass origin**

**Flavor symmetry**

**Model building**

**Phenomenology**

**Neutrinos**

**Lepton mixing**

**Mass origin**

**Flavor symmetry**

**Model building**

**Phenomenology**

**Comparison  
among models**

**Neutrinos**

**Lepton mixing**

**Mass origin**

**Flavor symmetry**

**Model building**

**Phenomenology**

**Comparison  
among models**

**Comparison  
with experiments**

2008

## Neutrinos & Lepton mixing

$$\begin{aligned}\Delta m_{sol}^2 &= 8.1(7.5 - 8.7) \cdot 10^{-5} \text{ eV}^2 \\ \Delta m_{atm}^2 &= 2.2(1.7 - 2.9) \cdot 10^{-3} \text{ eV}^2 \\ \sin^2 \theta_{12} &= 0.30(0.25 - 0.34) \\ \sin^2 \theta_{23} &= 0.50(0.38 - 0.64) \\ \sin^2 \theta_{13} &= 0(\leq 0.028) \leftarrow (\text{? } \sin \theta_{13} \neq 0 \text{ see Palazzo's talk on friday})\end{aligned}$$

M.Maltoni et al. New J.Phys.6:122,2004

2008

## Neutrinos & Lepton mixing

$$\begin{aligned}
 \Delta m_{sol}^2 &= 8.1(7.5 - 8.7) \cdot 10^{-5} \text{ eV}^2 \\
 \Delta m_{atm}^2 &= 2.2(1.7 - 2.9) \cdot 10^{-3} \text{ eV}^2 \\
 \sin^2 \theta_{12} &= 0.30(0.25 - 0.34) \\
 \sin^2 \theta_{23} &= 0.50(0.38 - 0.64) \\
 \sin^2 \theta_{13} &= 0(\leq 0.028) \leftarrow (\text{? } \sin \theta_{13} \neq 0 \text{ see Palazzo's talk on friday})
 \end{aligned}$$

M.Maltoni et al. New J.Phys.6:122,2004

Harrison, Perkins, Scott, PLB530 (2002)

$$U_{TB}^T M_\nu U_{TB} = M_\nu^{diag}$$

$$U_{TB} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \quad M_l^{diag}$$

2008

## Neutrinos & Lepton mixing

$$\begin{aligned}
 \Delta m_{sol}^2 &= 8.1(7.5 - 8.7) \cdot 10^{-5} \text{ eV}^2 \\
 \Delta m_{atm}^2 &= 2.2(1.7 - 2.9) \cdot 10^{-3} \text{ eV}^2 \\
 \sin^2 \theta_{12} &= 0.30(0.25 - 0.34) \\
 \sin^2 \theta_{23} &= 0.50(0.38 - 0.64) \\
 \sin^2 \theta_{13} &= 0(\leq 0.028) \leftarrow (\text{? } \sin \theta_{13} \neq 0 \text{ see Palazzo's talk on friday})
 \end{aligned}$$

M.Maltoni et al. New J.Phys.6:122,2004

Harrison, Perkins, Scott, PLB530 (2002)

$$U_{TB}^T M_\nu U_{TB} = M_\nu^{diag}$$

$$U_{TB} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

$M_l^{diag}$

TRI-maximal

Bi-maximal

$$\sin^2 \theta_{12} = 1/3, \sin^2 \theta_{23} = 1/2, \sin^2 \theta_{13} = 0$$

Suppose to have a model that predicts exact TBM...

$$U_{TB} \text{Diag}(m_1, m_2, m_3) U_{TB}^T = M_\nu$$

Suppose to have a model that predicts exact TBM...

$$U_{TB} \text{Diag}(m_1, m_2, m_3) U_{TB}^T = M_\nu$$

$$M_{\nu_{11}} + M_{\nu_{12}} = M_{\nu_{22}} + M_{\nu_{23}}$$

$$M_\nu = \begin{pmatrix} a+2c & b-c & b-c \\ b-c & 2c & a+b-c \\ b-c & a+b-c & 2c \end{pmatrix} \quad \begin{aligned} a &= \frac{1}{2}(m_1 - m_3) \\ b &= \frac{1}{4}(-m_1 + 2m_2 + m_3) \\ c &= \frac{1}{12}(m_1 + 2m_2 + 3m_3) \end{aligned}$$

Suppose to have a model that predicts exact TBM...

$$U_{TB} \text{Diag}(m_1, m_2, m_3) U_{TB}^T = M_\nu$$

$$M_{\nu_{11}} + M_{\nu_{12}} = M_{\nu_{22}} + M_{\nu_{23}}$$

$$M_\nu = \begin{pmatrix} a + 2c & b - c & b - c \\ b - c & 2c & a + b - c \\ b - c & a + b - c & 2c \end{pmatrix}$$

$$\begin{aligned} a &= \frac{1}{2}(m_1 - m_3) \\ b &= \frac{1}{4}(-m_1 + 2m_2 + m_3) \\ c &= \frac{1}{12}(m_1 + 2m_2 + 3m_3) \end{aligned}$$

**Input**

$\Delta m_{sol}^2$	$=$	$ m_2 ^2 -  m_1 ^2$
$\Delta m_{atm}^2$	$=$	$ m_3 ^2 -  m_2 ^2$

Suppose to have a model that predicts exact TBM...

$$U_{TB} \text{Diag}(m_1, m_2, m_3) U_{TB}^T = M_\nu$$

$$M_{\nu_{11}} + M_{\nu_{12}} = M_{\nu_{22}} + M_{\nu_{23}}$$

$$M_\nu = \begin{pmatrix} a + 2c & b - c & b - c \\ b - c & 2c & a + b - c \\ b - c & a + b - c & 2c \end{pmatrix}$$

$$\begin{aligned} a &= \frac{1}{2}(m_1 - m_3) \\ b &= \frac{1}{4}(-m_1 + 2m_2 + m_3) \\ c &= \frac{1}{12}(m_1 + 2m_2 + 3m_3) \end{aligned}$$

**Input**

$$\begin{aligned} \Delta m_{sol}^2 &= |m_2|^2 - |m_1|^2 \\ \Delta m_{atm}^2 &= |m_3|^2 - |m_2|^2 \end{aligned} \rightarrow$$

Suppose to have a model that predicts exact TBM...

$$U_{TB} \text{Diag}(m_1, m_2, m_3) U_{TB}^T = M_\nu$$

$$M_{\nu_{11}} + M_{\nu_{12}} = M_{\nu_{22}} + M_{\nu_{23}}$$

$$M_\nu = \begin{pmatrix} a + 2c & b - c & b - c \\ b - c & 2c & a + b - c \\ b - c & a + b - c & 2c \end{pmatrix}$$

**Input**

**Output**

$$\begin{aligned} a &= \frac{1}{2}(m_1 - m_3) \\ b &= \frac{1}{4}(-m_1 + 2m_2 + m_3) \\ c &= \frac{1}{12}(m_1 + 2m_2 + 3m_3) \end{aligned}$$

$$\begin{aligned} \Delta m_{sol}^2 &= |m_2|^2 - |m_1|^2 \\ \Delta m_{atm}^2 &= |m_3|^2 - |m_2|^2 \end{aligned}$$



$m_{ee}, m_\nu$

Suppose to have a model that predicts exact TBM...

$$U_{TB} \text{Diag}(m_1, m_2, m_3) U_{TB}^T = M_\nu$$

$$M_{\nu_{11}} + M_{\nu_{12}} = M_{\nu_{22}} + M_{\nu_{23}}$$

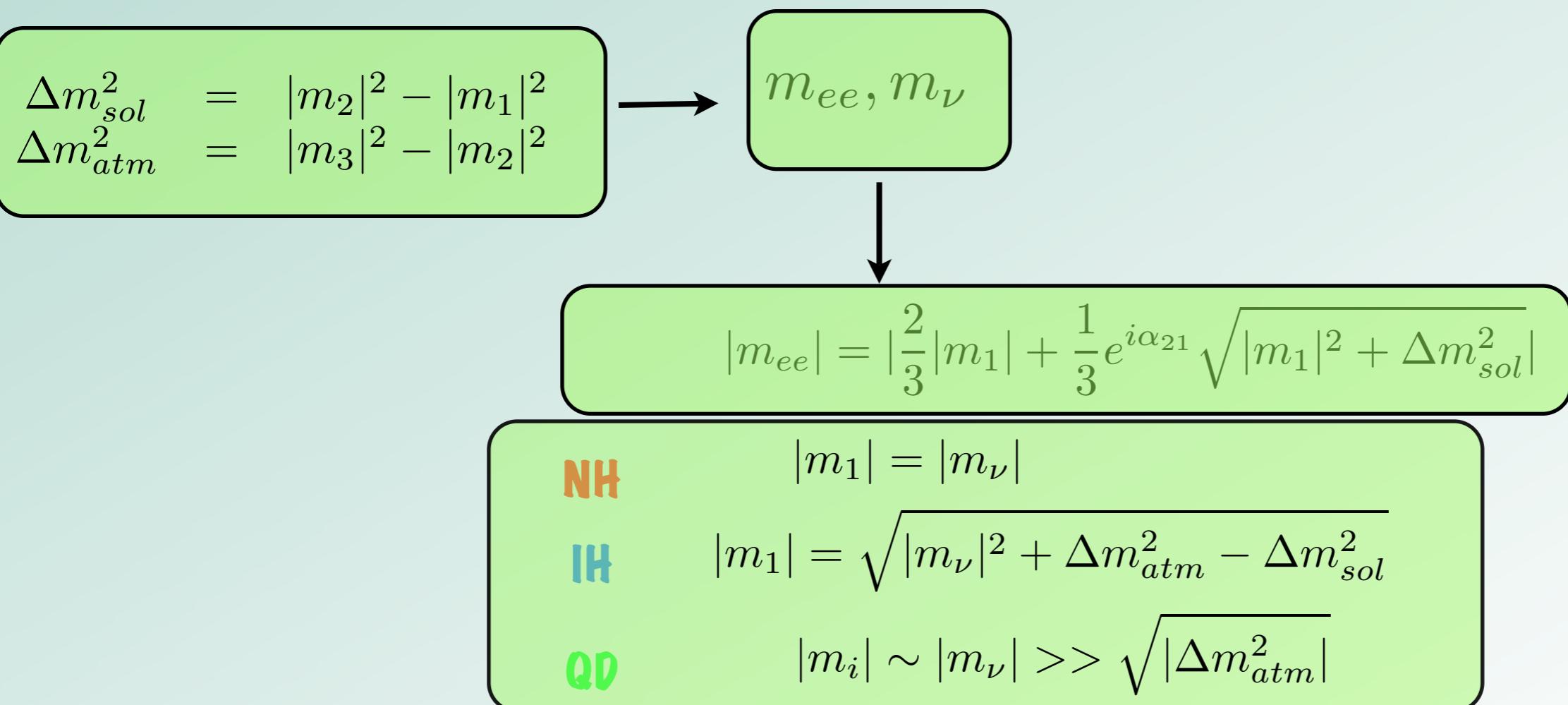
$$M_\nu = \begin{pmatrix} a+2c & b-c & b-c \\ b-c & 2c & a+b-c \\ b-c & a+b-c & 2c \end{pmatrix}$$

**Input**                           **Output**

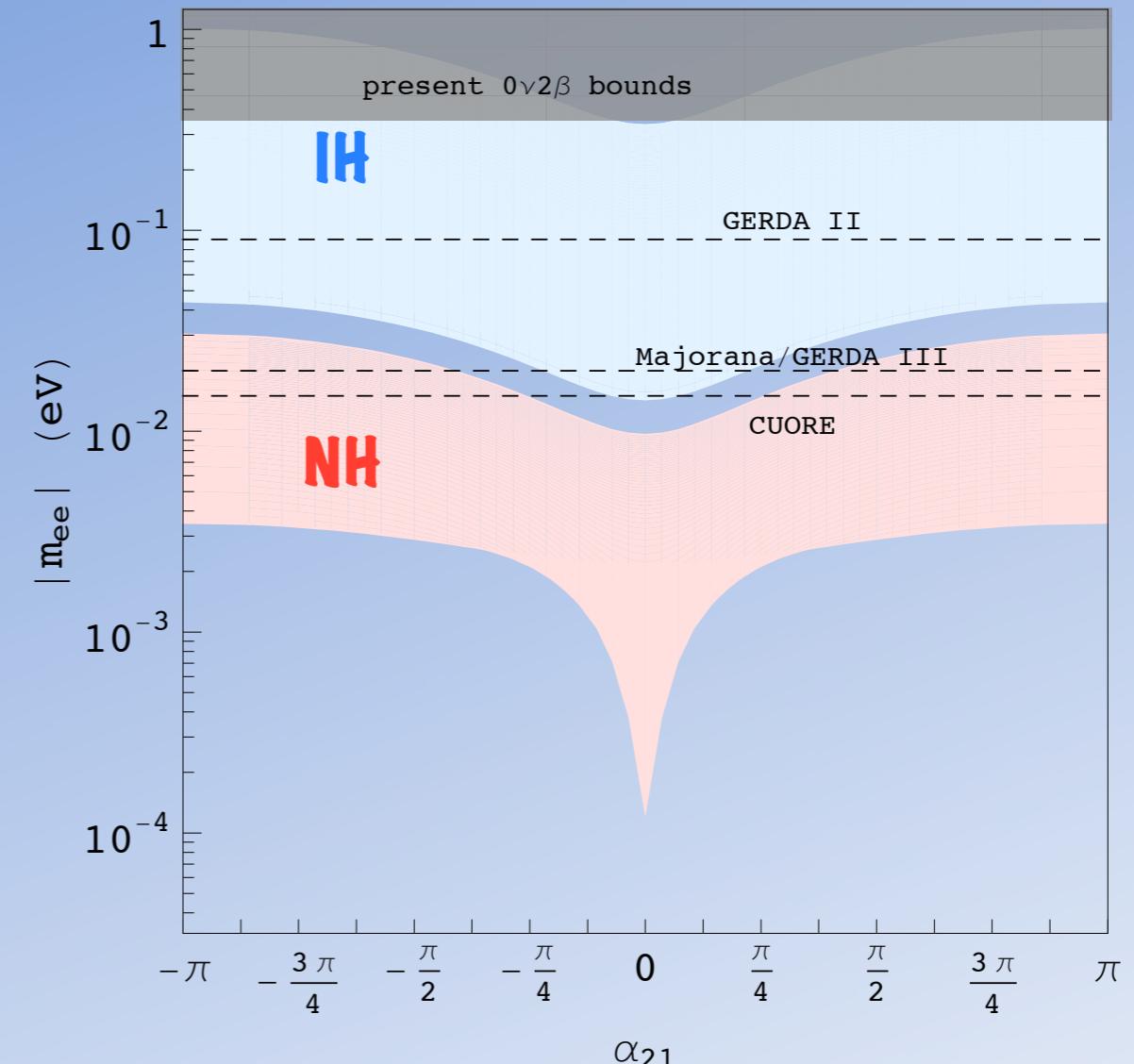
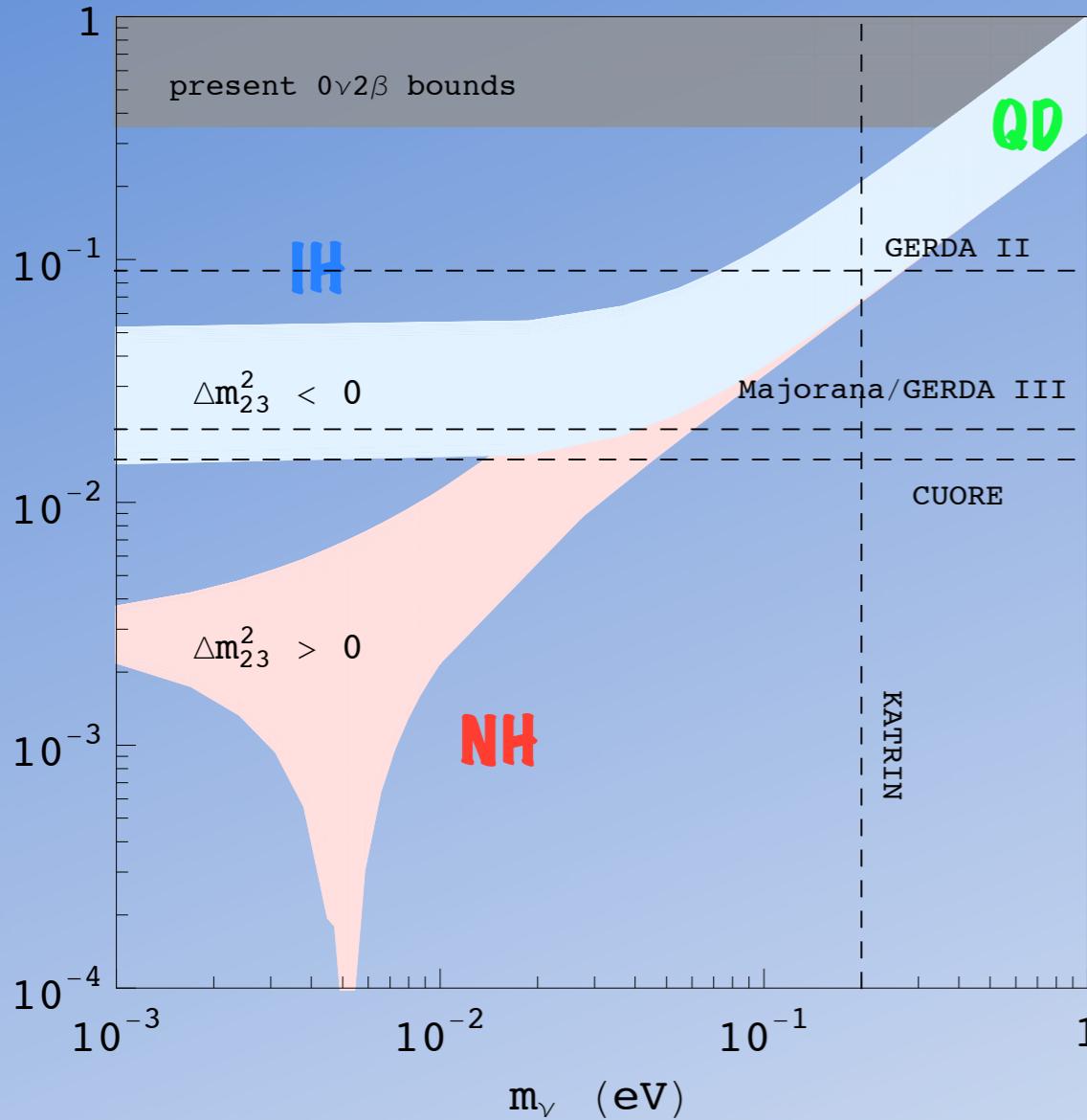
$$a = \frac{1}{2}(m_1 - m_3)$$

$$b = \frac{1}{4}(-m_1 + 2m_2 + m_3)$$

$$c = \frac{1}{12}(m_1 + 2m_2 + 3m_3)$$



# Model independent phenomenological predictions for exact TBM



# Discrete flavor symmetries

@ LO exact TBM!

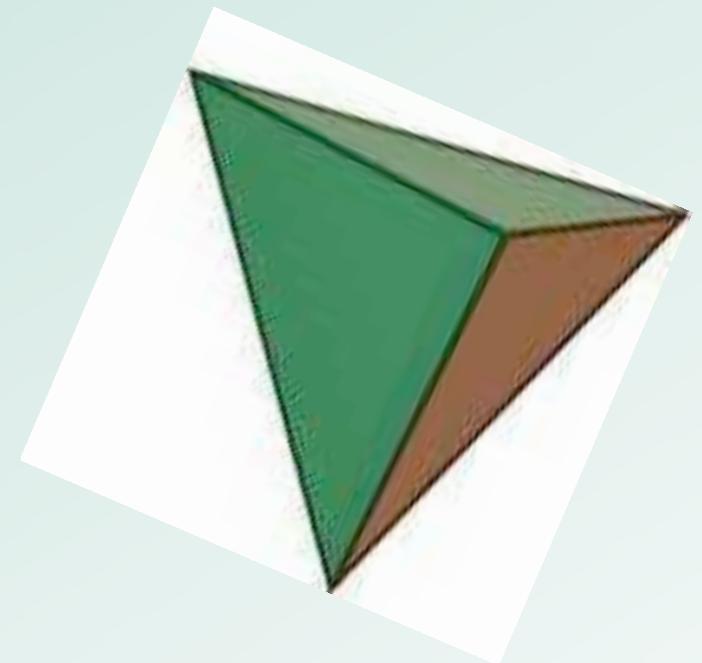
Ma & Rajasekaran PRD64,  
Babu et al. PLB552 (A4)  
Ma PLB632 (2006),  
Hagerdon et al. JHEP 06 042 (S4)

A4

- ★ even permutations of 4 objects (subgroup of  $S_4$ , tetrahedral symmetries)
- ★  $4!/2 = 12$  elements
- ★ generated by two basic permutations:  $S = (4\ 3\ 2\ 1)$  &  $T = (2\ 3\ 1\ 4)$
- ★  $S^2 = T^3 = (ST)^3 = 1 \rightarrow$  a representation of the group
- ★ 12 elements belong to 4 equivalence classes
- ★ 4 inequivalent representations  $1, 1', 1''$  &  $3$

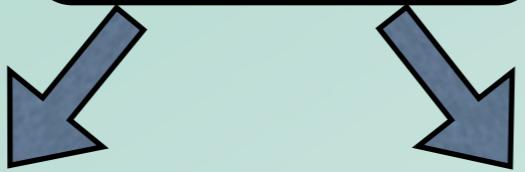
S4

- ★ permutations of 4 objects (tetrahedral symmetries)
- ★  $4! = 24$  elements
- ★  $S^4 = T^3 = 1, ST^2S = T \rightarrow$  a representation of the group
- ★ 24 elements belong to 5 equivalence classes
- ★ 5 inequivalent representations  $1_1, 1_2, 2, 3_1, 3_2$



$A_4, S_4$

$A_4, S_4$



$A_4, S_4$

$Z_3$   
charged leptons

$Z_2, Z_2 \times Z_2$   
neutrinos

$A_4, S_4$

$Z_3$   
charged leptons

$Z_2, Z_2 \times Z_2$   
neutrinos

$$G_{Z_3}^T M_l M_l^\dagger G_{Z_3} = M_l M_l^\dagger$$

(or  $G_{Z_3}^T M_l G_{Z_3} = M_l$  )

$$G_{Z_2}^T M_\nu G_{Z_2} = M_\nu$$

$A_4, S_4$

$Z_3$   
charged leptons

$Z_2, Z_2 \times Z_2$   
neutrinos

$$G_{Z_3}^T M_l M_l^\dagger G_{Z_3} = M_l M_l^\dagger$$

(or  $G_{Z_3}^T M_l G_{Z_3} = M_l$  )

$U_{Z_3}$

$$G_{Z_2}^T M_\nu G_{Z_2} = M_\nu$$

$U_{Z_2}$

$$U_{lep} = U_{Z_3}^\dagger U_{Z_2} = U_{TB}$$

$A_4, S_4$

$Z_3$   
charged leptons

$Z_2, Z_2 \times Z_2$   
neutrinos

$$G_{Z_3}^T M_l M_l^\dagger G_{Z_3} = M_l M_l^\dagger$$

(or  $G_{Z_3}^T M_l G_{Z_3} = M_l$  )

$$G_{Z_2}^T M_\nu G_{Z_2} = M_\nu$$

$U_{Z_3}$

$U_{Z_2}$

$$U_{lep} = U_{Z_3}^\dagger U_{Z_2} = U_{TB}$$

the group splits differently in charged lepton and neutrino sector not to get trivial mixing !

choose a basis for the  $A_4, S_4$  generators in which the charged lepton are diagonal

neutrinos can get a mass in different ways

effective operator (EF)

type I see-saw (SSI)

type II see-saw (SSII)

type III see-saw (SSIII)

choose a basis for the  $A_4, S_4$  generators in which the charged lepton are diagonal

neutrinos can get a mass in different ways

flavour symmetry

effective operator (EF)

$$\frac{1}{\Lambda} LL h_u h_u$$

type I see-saw (SSI)

$$M_\nu \sim -m_D M_R^{-1} m_D^T$$

type II see-saw (SSII)

$$LL\Phi$$

type III see-saw (SSIII)

$$M_\nu \sim -m_{l\Sigma} M_\Sigma^{-1} m_{l\Sigma}^T$$

choose a basis for the  $A_4, S_4$  generators in which the charged lepton are diagonal

neutrinos can get a mass in different ways

flavour symmetry

effective operator (EF)

$$\frac{1}{\Lambda} LL h_u h_u$$

$$\frac{1}{\Lambda} LL h_u h_u \frac{\phi_i}{\Lambda_F}$$

type I see-saw (SSI)

$$M_\nu \sim -m_D M_R^{-1} m_D^T$$

$$\begin{aligned} m_D &\sim L h_u \nu^c \frac{\phi_i}{\Lambda_F} \\ M_R &\sim \nu^c \nu^c \phi_i \end{aligned}$$

type II see-saw (SSII)

$$LL\Phi$$

$$LL\Phi \frac{\phi_i}{\Lambda_F}$$

type III see-saw (SSIII)

$$M_\nu \sim -m_{l\Sigma} M_\Sigma^{-1} m_{l\Sigma}^T$$

$$\begin{aligned} m_{l\Sigma} &\sim L h_u \Sigma \frac{\phi_i}{\Lambda_F} \\ M_\Sigma &\sim \Sigma \Sigma \phi_i \end{aligned}$$

# Comparing models: A4

AF-EF

$$M_\nu = v \begin{pmatrix} a+2c & -c & -c \\ -c & 2c & a-c \\ -c & a-c & 2c \end{pmatrix}$$

AF-SSI

$$M_\nu = v \begin{pmatrix} \frac{1}{3a} + \frac{2}{3}\frac{1}{(a+3c)} & \frac{1}{3a} - \frac{1}{3}\frac{1}{(a+3c)} & \frac{1}{3a} - \frac{1}{3}\frac{1}{(a+3c)} \\ \frac{1}{3a} - \frac{1}{3}\frac{1}{(a+3c)} & \frac{1}{3a} + \frac{2}{3}\frac{1}{(a+3c)} & \frac{1}{3a} - \frac{1}{3}\frac{1}{(a+3c)} \\ \frac{1}{3a} - \frac{1}{3}\frac{1}{(a+3c)} & \frac{1}{3a} - \frac{1}{3}\frac{1}{(a+3c)} & \frac{1}{3a} + \frac{2}{3}\frac{1}{(a+3c)} \end{pmatrix}$$

HMV

$$M_\nu = v \begin{pmatrix} \frac{2}{3}a^2 + \frac{1}{3}b^2 & -\frac{1}{3}a^2 + \frac{1}{3}b^2 & -\frac{1}{3}a^2 + \frac{1}{3}b^2 \\ -\frac{1}{3}a^2 + \frac{1}{3}b^2 & \frac{1}{6}a^2 + \frac{1}{3}b^2 + \frac{1}{2}c^2 & \frac{1}{6}a^2 + \frac{1}{3}b^2 - \frac{1}{2}c^2 \\ -\frac{1}{3}a^2 + \frac{1}{3}b^2 & \frac{1}{6}a^2 + \frac{1}{3}b^2 - \frac{1}{2}c^2 & \frac{1}{6}a^2 + \frac{1}{3}b^2 + \frac{1}{2}c^2 \end{pmatrix}$$

# Comparing models: A4

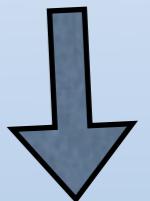
**AF-EF**

$$M_\nu = v \begin{pmatrix} a+2c & -c & -c \\ -c & 2c & a-c \\ -c & a-c & 2c \end{pmatrix}$$

**AF-SSI**

$$M_\nu = v \begin{pmatrix} \frac{1}{3a} + \frac{2}{3} \frac{1}{(a+3c)} & \frac{1}{3a} - \frac{1}{3} \frac{1}{(a+3c)} & \frac{1}{3a} - \frac{1}{3} \frac{1}{(a+3c)} \\ \frac{1}{3a} - \frac{1}{3} \frac{1}{(a+3c)} & \frac{1}{3a} + \frac{2}{3} \frac{1}{(a+3c)} & \frac{1}{3a} - \frac{1}{3} \frac{1}{(a+3c)} \\ \frac{1}{3a} - \frac{1}{3} \frac{1}{(a+3c)} & \frac{1}{3a} - \frac{1}{3} \frac{1}{(a+3c)} & \frac{1}{3a} + \frac{2}{3} \frac{1}{(a+3c)} \end{pmatrix}$$

Fl-structure one in  
MR the other in mD



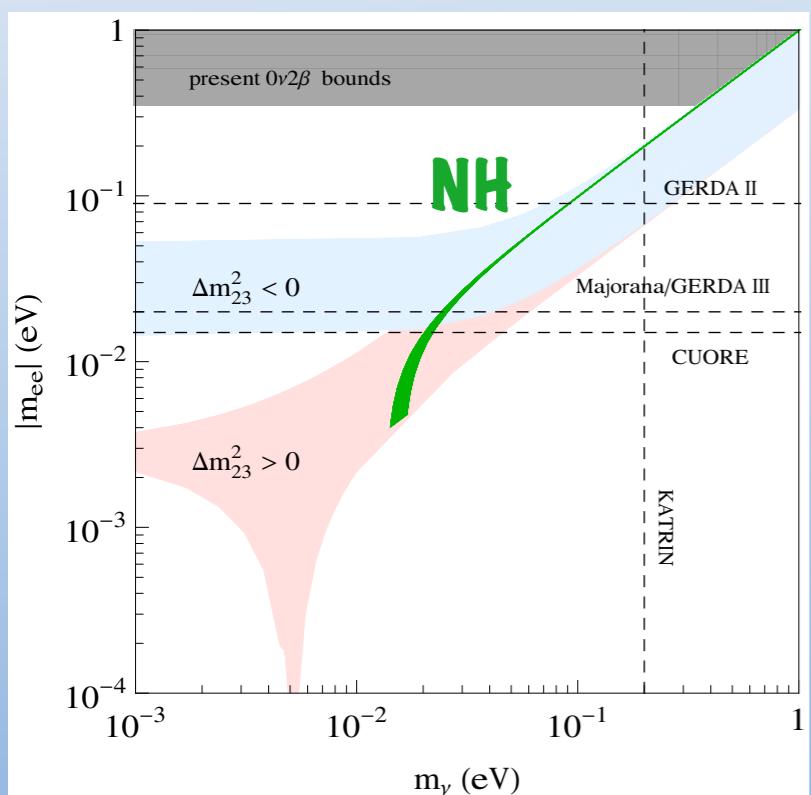
**HMV**

$$M_\nu = v \begin{pmatrix} \frac{2}{3}a^2 + \frac{1}{3}b^2 & -\frac{1}{3}a^2 + \frac{1}{3}b^2 & -\frac{1}{3}a^2 + \frac{1}{3}b^2 \\ -\frac{1}{3}a^2 + \frac{1}{3}b^2 & \frac{1}{6}a^2 + \frac{1}{3}b^2 + \frac{1}{2}c^2 & \frac{1}{6}a^2 + \frac{1}{3}b^2 - \frac{1}{2}c^2 \\ -\frac{1}{3}a^2 + \frac{1}{3}b^2 & \frac{1}{6}a^2 + \frac{1}{3}b^2 - \frac{1}{2}c^2 & \frac{1}{6}a^2 + \frac{1}{3}b^2 + \frac{1}{2}c^2 \end{pmatrix}$$

# Comparing models: A4

**AF-EF**

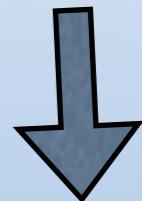
$$M_\nu = v \begin{pmatrix} a+2c & -c & -c \\ -c & 2c & a-c \\ -c & a-c & 2c \end{pmatrix}$$



**AF-SSI**

$$M_\nu = v \begin{pmatrix} \frac{1}{3a} + \frac{2}{3} \frac{1}{(a+3c)} & \frac{1}{3a} - \frac{1}{3} \frac{1}{(a+3c)} & \frac{1}{3a} - \frac{1}{3} \frac{1}{(a+3c)} \\ \frac{1}{3a} - \frac{1}{3} \frac{1}{(a+3c)} & \frac{1}{3a} + \frac{2}{3} \frac{1}{(a+3c)} & \frac{1}{3a} - \frac{1}{3} \frac{1}{(a+3c)} \\ \frac{1}{3a} - \frac{1}{3} \frac{1}{(a+3c)} & \frac{1}{3a} - \frac{1}{3} \frac{1}{(a+3c)} & \frac{1}{3a} + \frac{2}{3} \frac{1}{(a+3c)} \end{pmatrix}$$

Fl-structure one in  
MR the other in mD



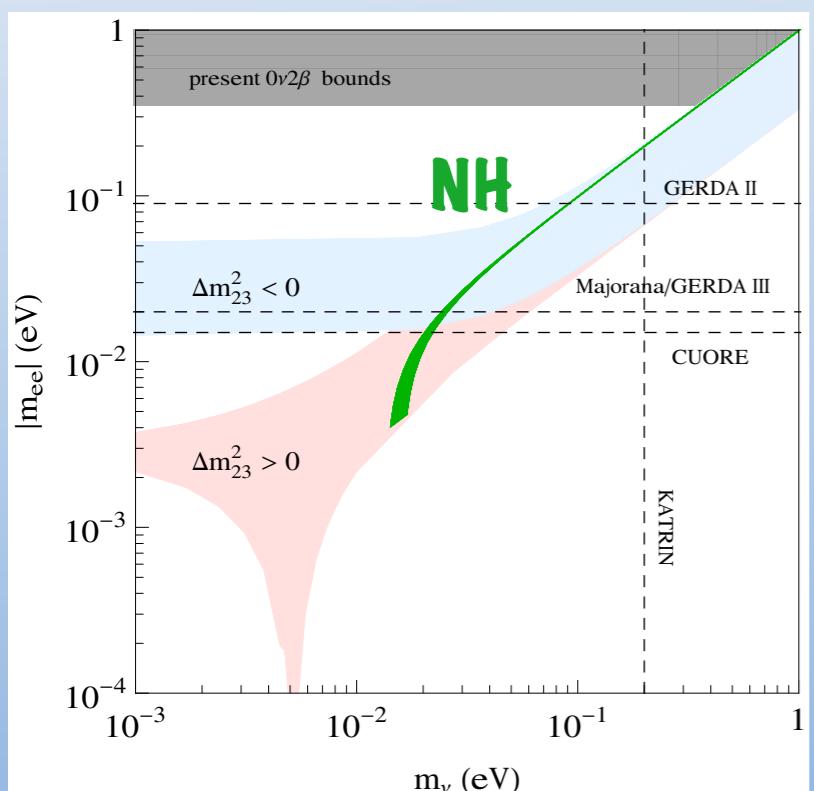
**HMV**

$$M_\nu = v \begin{pmatrix} \frac{2}{3}a^2 + \frac{1}{3}b^2 & -\frac{1}{3}a^2 + \frac{1}{3}b^2 & -\frac{1}{3}a^2 + \frac{1}{3}b^2 \\ -\frac{1}{3}a^2 + \frac{1}{3}b^2 & \frac{1}{6}a^2 + \frac{1}{3}b^2 + \frac{1}{2}c^2 & \frac{1}{6}a^2 + \frac{1}{3}b^2 - \frac{1}{2}c^2 \\ -\frac{1}{3}a^2 + \frac{1}{3}b^2 & \frac{1}{6}a^2 + \frac{1}{3}b^2 - \frac{1}{2}c^2 & \frac{1}{6}a^2 + \frac{1}{3}b^2 + \frac{1}{2}c^2 \end{pmatrix}$$

# Comparing models: A4

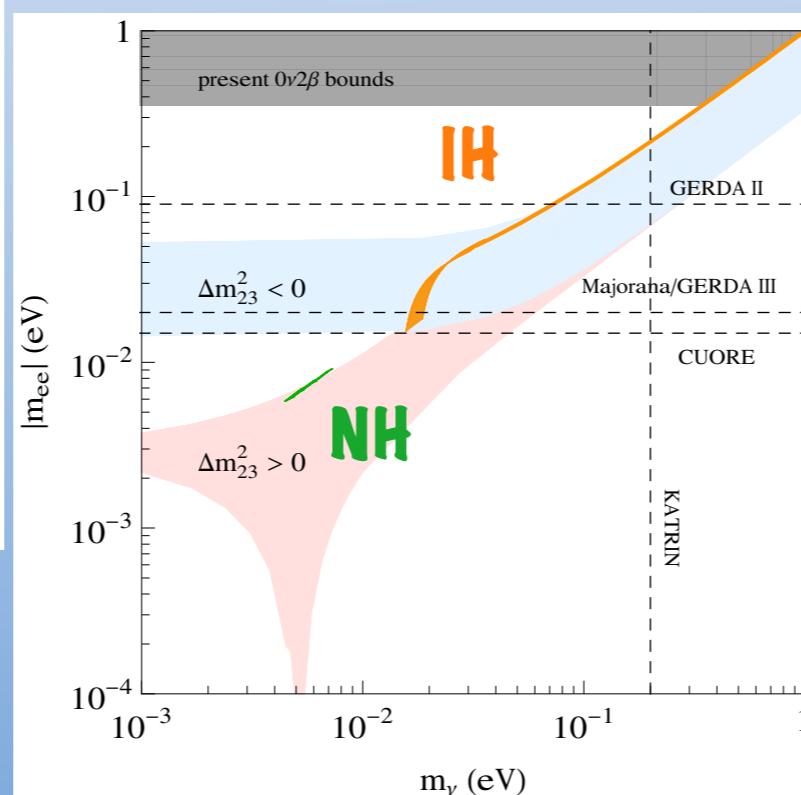
**AF-EF**

$$M_\nu = v \begin{pmatrix} a+2c & -c & -c \\ -c & 2c & a-c \\ -c & a-c & 2c \end{pmatrix}$$

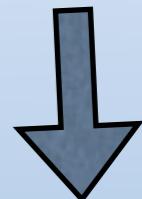


**AF-SSI**

$$M_\nu = v \begin{pmatrix} \frac{1}{3a} + \frac{2}{3} \frac{1}{(a+3c)} & \frac{1}{3a} - \frac{1}{3} \frac{1}{(a+3c)} & \frac{1}{3a} - \frac{1}{3} \frac{1}{(a+3c)} \\ \frac{1}{3a} - \frac{1}{3} \frac{1}{(a+3c)} & \frac{1}{3a} + \frac{2}{3} \frac{1}{(a+3c)} & \frac{1}{3a} - \frac{1}{3} \frac{1}{(a+3c)} \\ \frac{1}{3a} - \frac{1}{3} \frac{1}{(a+3c)} & \frac{1}{3a} - \frac{1}{3} \frac{1}{(a+3c)} & \frac{1}{3a} + \frac{2}{3} \frac{1}{(a+3c)} \end{pmatrix}$$



Fl-structure one in  
MR the other in mD



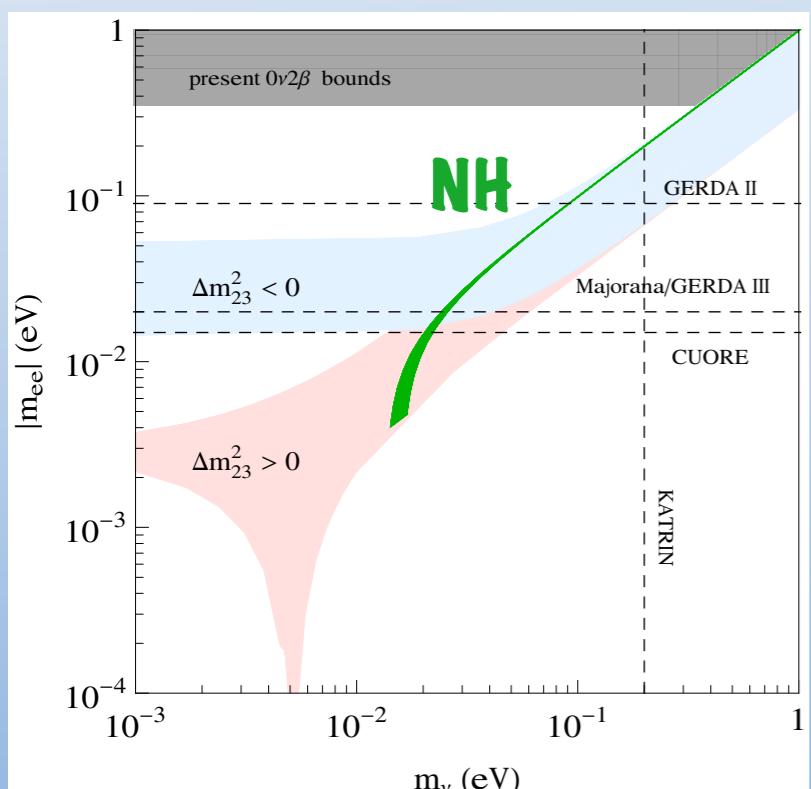
**HMV**

$$M_\nu = v \begin{pmatrix} \frac{2}{3}a^2 + \frac{1}{3}b^2 & -\frac{1}{3}a^2 + \frac{1}{3}b^2 & -\frac{1}{3}a^2 + \frac{1}{3}b^2 \\ -\frac{1}{3}a^2 + \frac{1}{3}b^2 & \frac{1}{6}a^2 + \frac{1}{3}b^2 + \frac{1}{2}c^2 & \frac{1}{6}a^2 + \frac{1}{3}b^2 - \frac{1}{2}c^2 \\ -\frac{1}{3}a^2 + \frac{1}{3}b^2 & \frac{1}{6}a^2 + \frac{1}{3}b^2 - \frac{1}{2}c^2 & \frac{1}{6}a^2 + \frac{1}{3}b^2 + \frac{1}{2}c^2 \end{pmatrix}$$

# Comparing models: A4

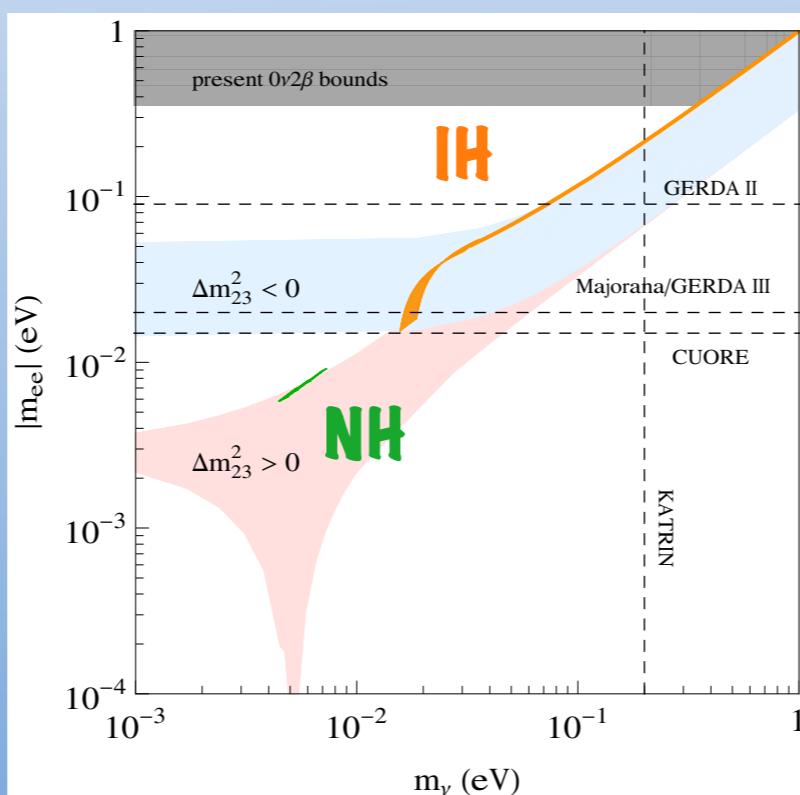
**AF-EF**

$$M_\nu = v \begin{pmatrix} a+2c & -c & -c \\ -c & 2c & a-c \\ -c & a-c & 2c \end{pmatrix}$$

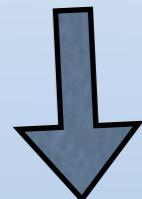


**AF-SSI**

$$M_\nu = v \begin{pmatrix} \frac{1}{3a} + \frac{2}{3} \frac{1}{(a+3c)} & \frac{1}{3a} - \frac{1}{3} \frac{1}{(a+3c)} & \frac{1}{3a} - \frac{1}{3} \frac{1}{(a+3c)} \\ \frac{1}{3a} - \frac{1}{3} \frac{1}{(a+3c)} & \frac{1}{3a} + \frac{2}{3} \frac{1}{(a+3c)} & \frac{1}{3a} - \frac{1}{3} \frac{1}{(a+3c)} \\ \frac{1}{3a} - \frac{1}{3} \frac{1}{(a+3c)} & \frac{1}{3a} - \frac{1}{3} \frac{1}{(a+3c)} & \frac{1}{3a} + \frac{2}{3} \frac{1}{(a+3c)} \end{pmatrix}$$

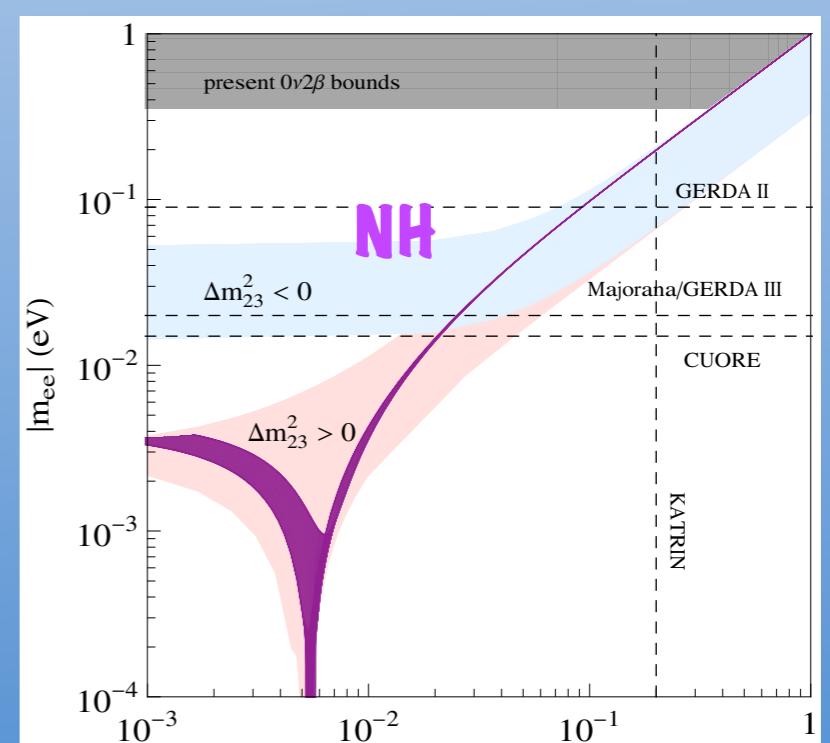


Fl-structure one in  
MR the other in mD



**HMV**

$$M_\nu = v \begin{pmatrix} \frac{2}{3}a^2 + \frac{1}{3}b^2 & -\frac{1}{3}a^2 + \frac{1}{3}b^2 & -\frac{1}{3}a^2 + \frac{1}{3}b^2 \\ -\frac{1}{3}a^2 + \frac{1}{3}b^2 & \frac{1}{6}a^2 + \frac{1}{3}b^2 + \frac{1}{2}c^2 & \frac{1}{6}a^2 + \frac{1}{3}b^2 - \frac{1}{2}c^2 \\ -\frac{1}{3}a^2 + \frac{1}{3}b^2 & \frac{1}{6}a^2 + \frac{1}{3}b^2 - \frac{1}{2}c^2 & \frac{1}{6}a^2 + \frac{1}{3}b^2 + \frac{1}{2}c^2 \end{pmatrix}$$



# Comparing models: S4

BMM-EF, SS II

$$M_\nu = v \begin{pmatrix} 2c & b-c & b-c \\ b-c & 2c & b-c \\ b-c & b-c & 2c \end{pmatrix}$$

BMM-SSI, SS III

$$M_\nu = v \begin{pmatrix} \frac{1}{6b} + \frac{2}{3} \frac{1}{(3c-b)} & \frac{1}{6b} - \frac{1}{3} \frac{1}{(3c-b)} & \frac{1}{6b} - \frac{1}{3} \frac{1}{(3c-b)} \\ \frac{1}{6b} - \frac{1}{3} \frac{1}{(3c-b)} & \frac{1}{6b} + \frac{2}{3} \frac{1}{(3c-b)} & \frac{1}{6b} - \frac{1}{3} \frac{1}{(3c-b)} \\ \frac{1}{6b} - \frac{1}{3} \frac{1}{(3c-b)} & \frac{1}{6b} - \frac{1}{3} \frac{1}{(3c-b)} & \frac{1}{6b} + \frac{2}{3} \frac{1}{(3c-b)} \end{pmatrix}$$

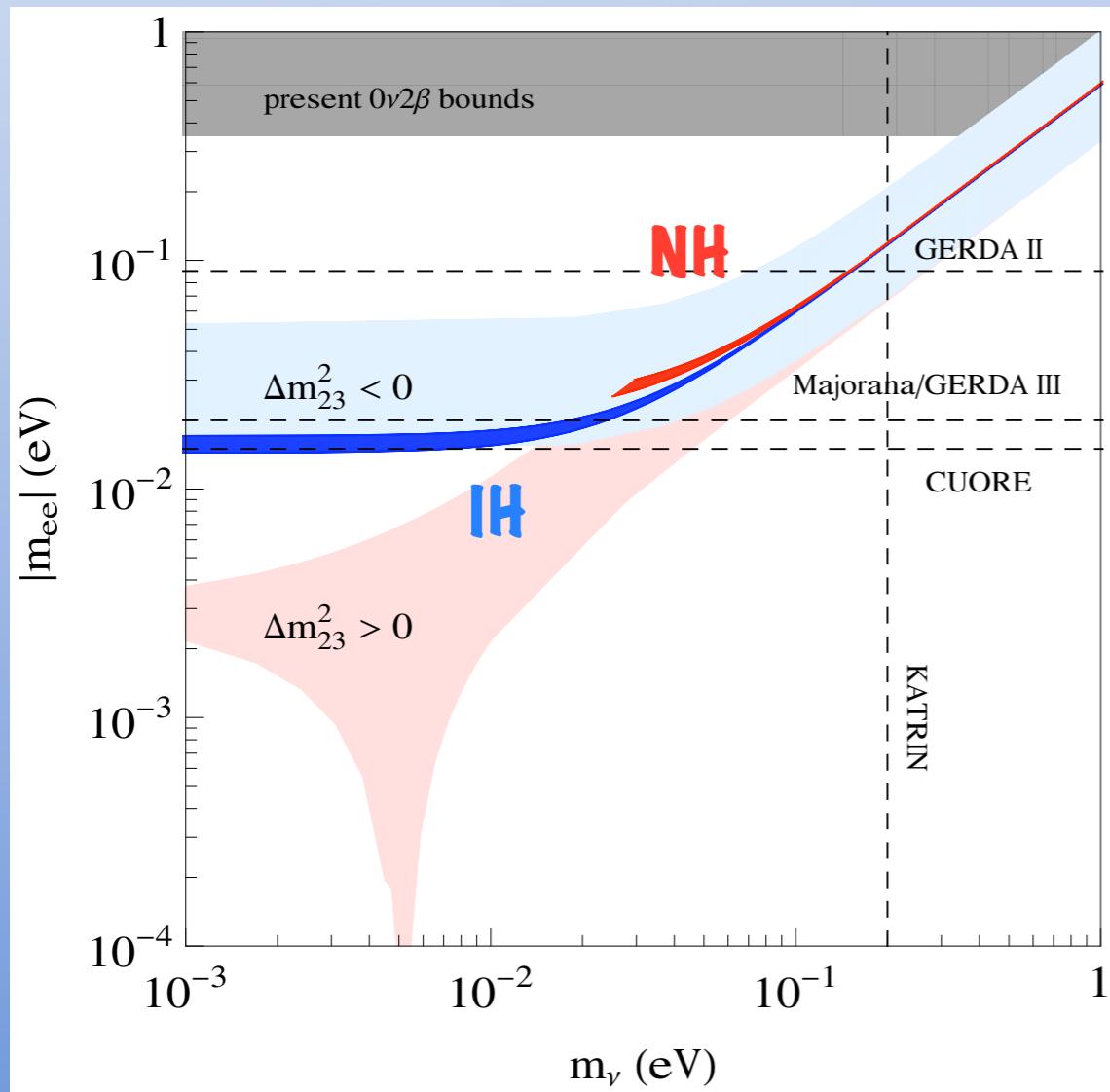
# Comparing models: S4

**BMM-EF, SS II**

$$M_\nu = v \begin{pmatrix} 2c & b-c & b-c \\ b-c & 2c & b-c \\ b-c & b-c & 2c \end{pmatrix}$$

**BMM-SSI, SS III**

$$M_\nu = v \begin{pmatrix} \frac{1}{6b} + \frac{2}{3} \frac{1}{(3c-b)} & \frac{1}{6b} - \frac{1}{3} \frac{1}{(3c-b)} & \frac{1}{6b} - \frac{1}{3} \frac{1}{(3c-b)} \\ \frac{1}{6b} - \frac{1}{3} \frac{1}{(3c-b)} & \frac{1}{6b} + \frac{2}{3} \frac{1}{(3c-b)} & \frac{1}{6b} - \frac{1}{3} \frac{1}{(3c-b)} \\ \frac{1}{6b} - \frac{1}{3} \frac{1}{(3c-b)} & \frac{1}{6b} - \frac{1}{3} \frac{1}{(3c-b)} & \frac{1}{6b} + \frac{2}{3} \frac{1}{(3c-b)} \end{pmatrix}$$



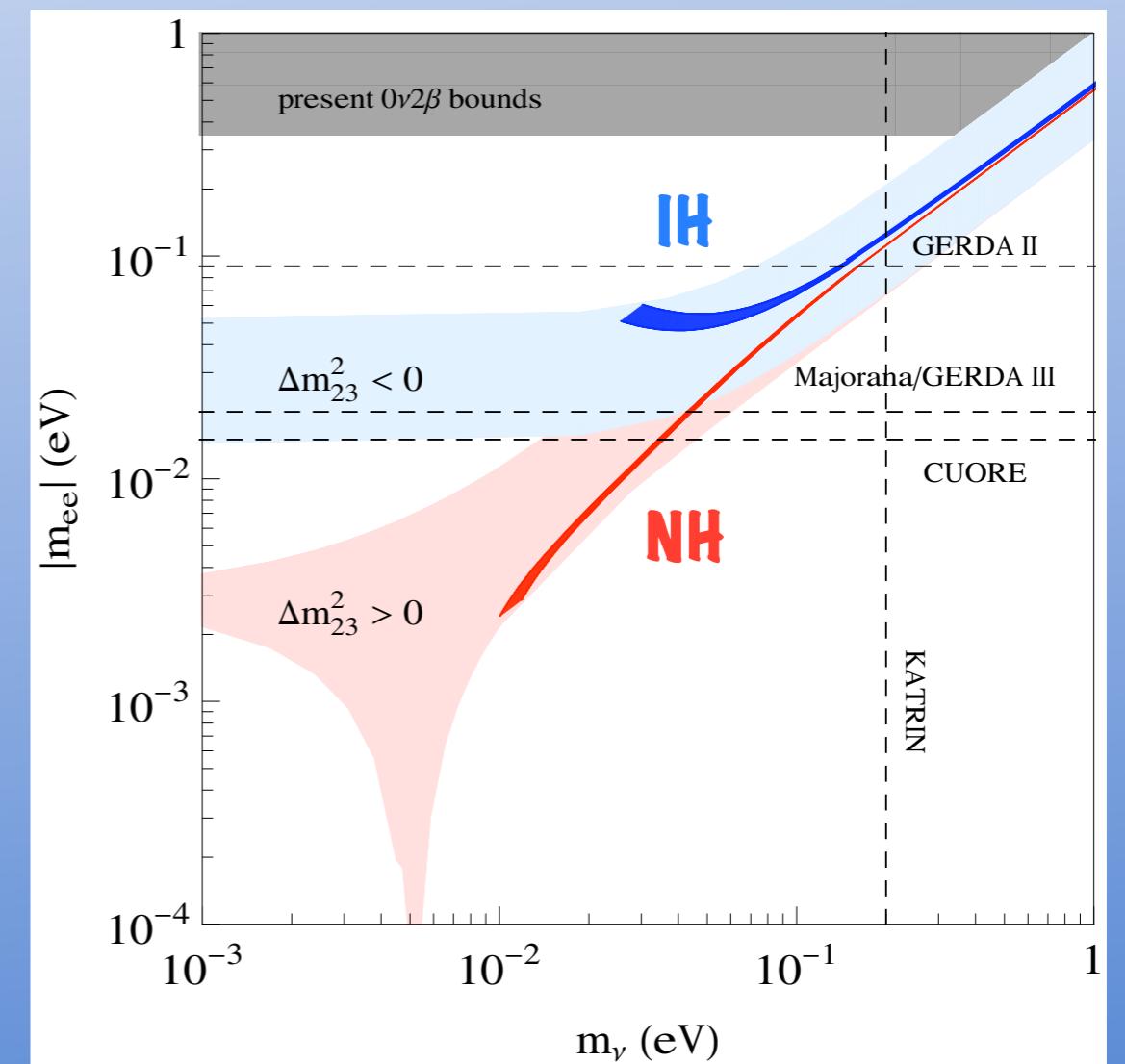
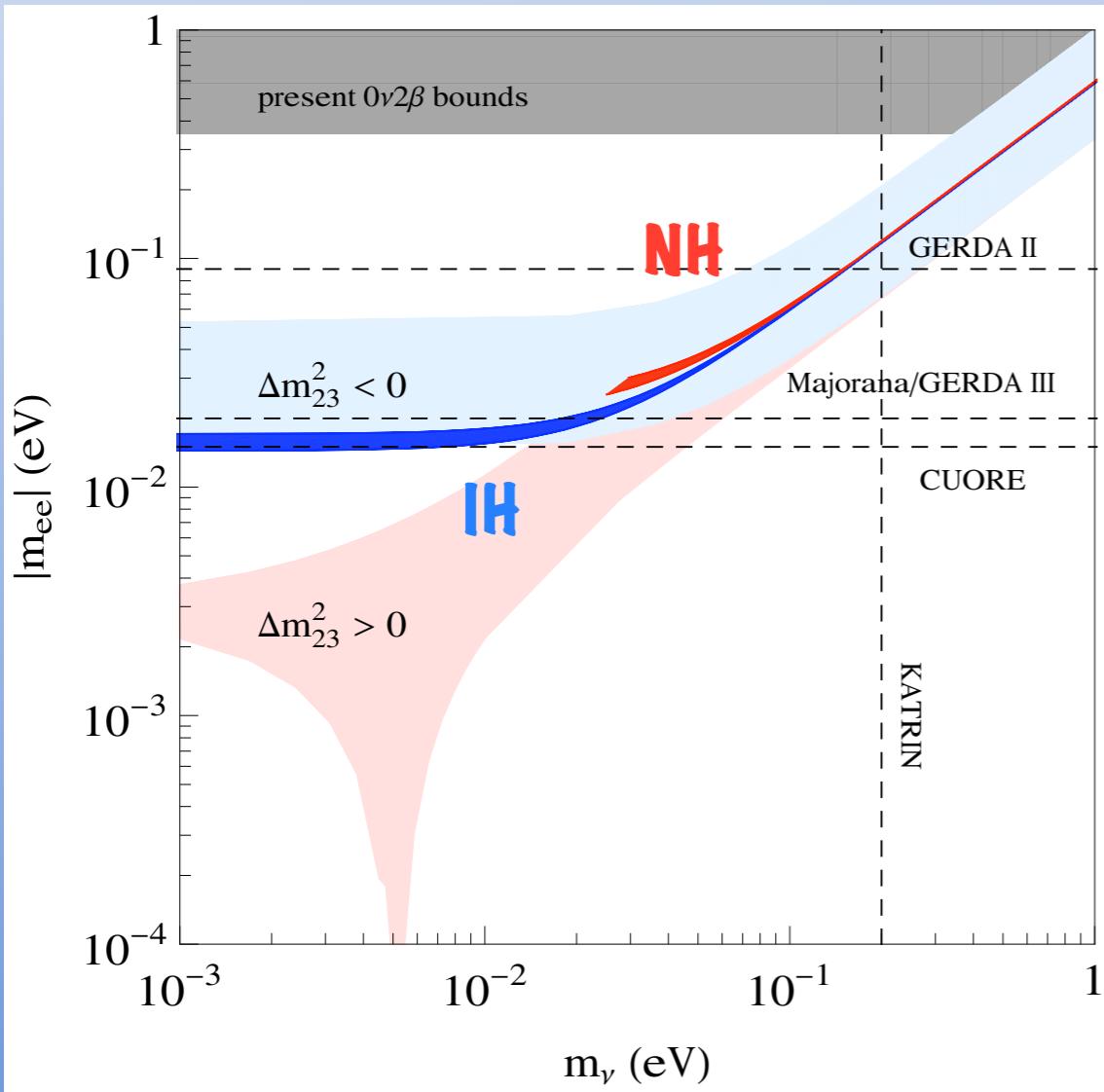
# Comparing models: S4

**BMM-EF, SS II**

$$M_\nu = v \begin{pmatrix} 2c & b-c & b-c \\ b-c & 2c & b-c \\ b-c & b-c & 2c \end{pmatrix}$$

**BMM-SSI, SSIII**

$$M_\nu = v \begin{pmatrix} \frac{1}{6b} + \frac{2}{3} \frac{1}{(3c-b)} & \frac{1}{6b} - \frac{1}{3} \frac{1}{(3c-b)} & \frac{1}{6b} - \frac{1}{3} \frac{1}{(3c-b)} \\ \frac{1}{6b} - \frac{1}{3} \frac{1}{(3c-b)} & \frac{1}{6b} + \frac{2}{3} \frac{1}{(3c-b)} & \frac{1}{6b} - \frac{1}{3} \frac{1}{(3c-b)} \\ \frac{1}{6b} - \frac{1}{3} \frac{1}{(3c-b)} & \frac{1}{6b} - \frac{1}{3} \frac{1}{(3c-b)} & \frac{1}{6b} + \frac{2}{3} \frac{1}{(3c-b)} \end{pmatrix}$$



# Comparing models: A4 vs S4

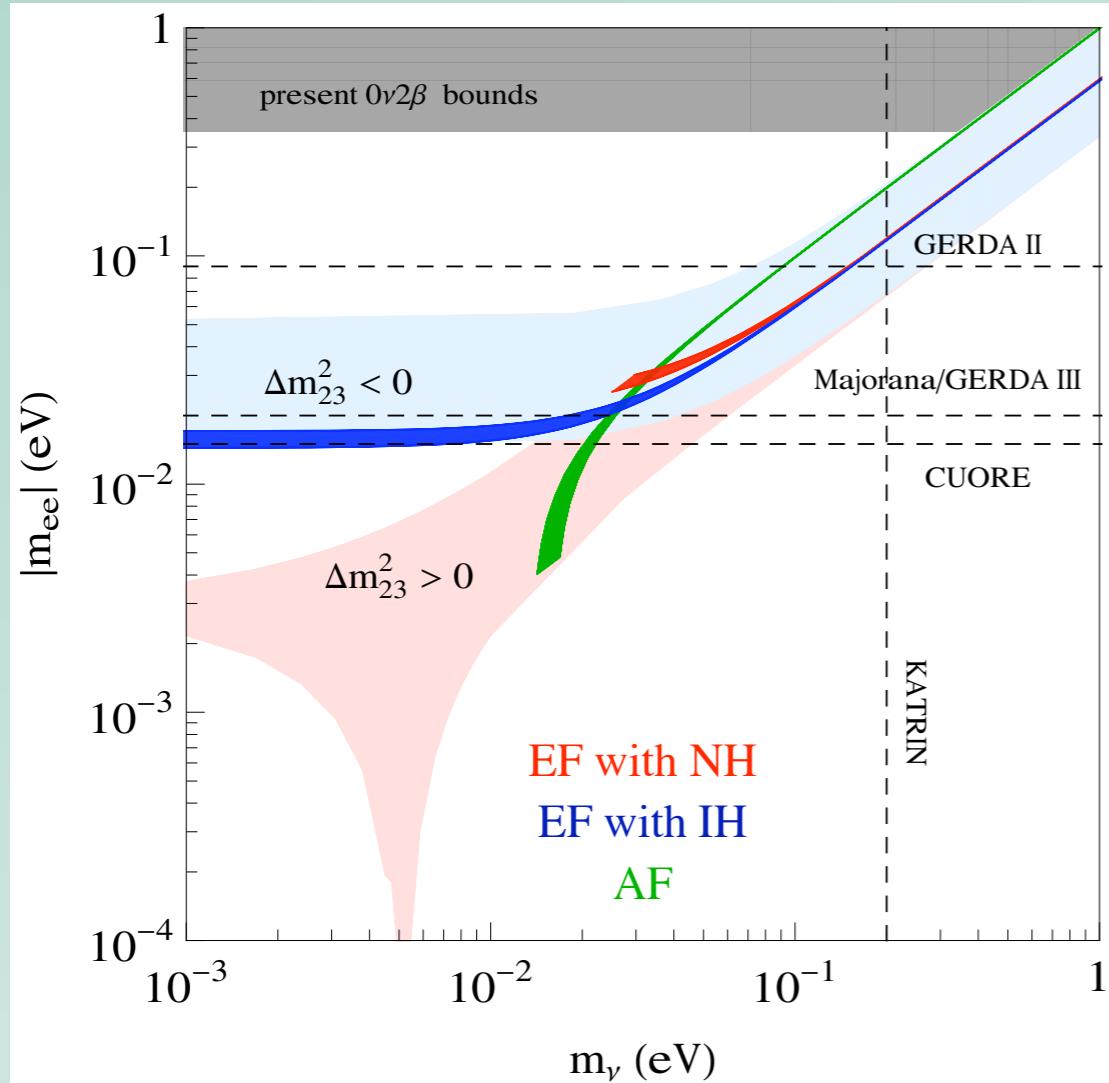
EFSII

SSI,SSIII

# Comparing models: A4 vs S4

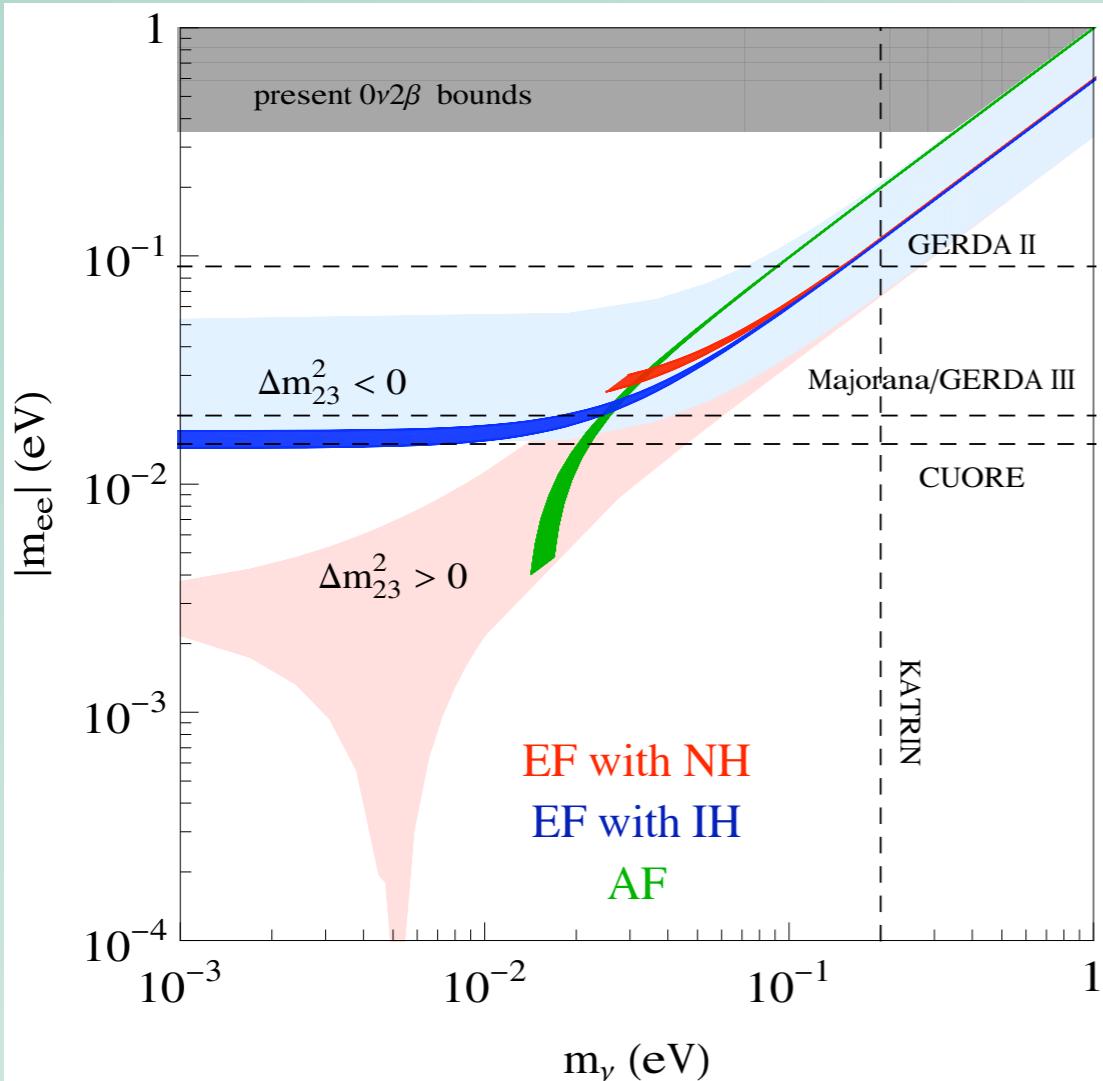
EESSII

SSI,SSIII

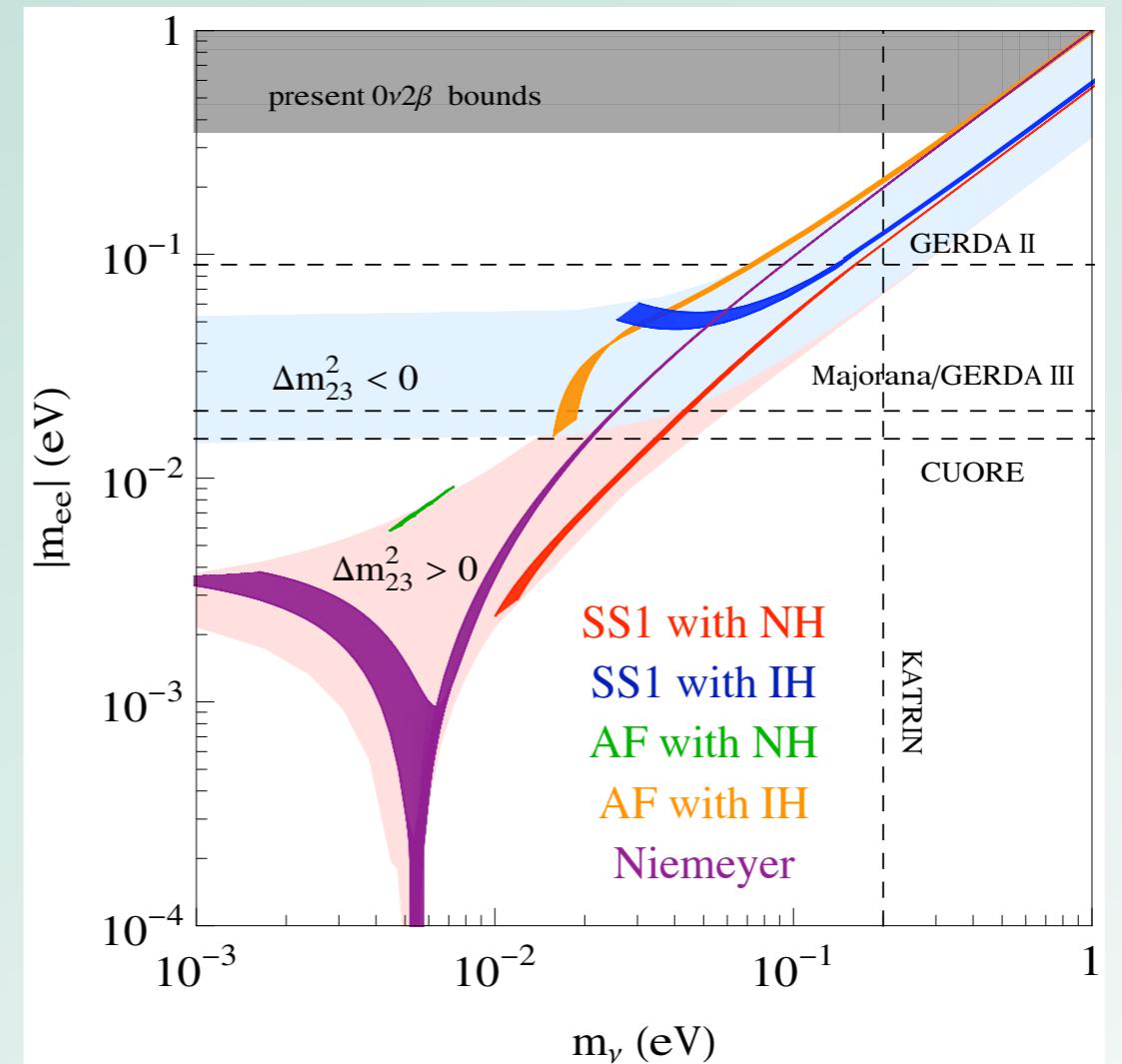


# Comparing models: A4 vs S4

**EFSII**



**SSI,SSIII**



# Comparing models: A4 vs S4

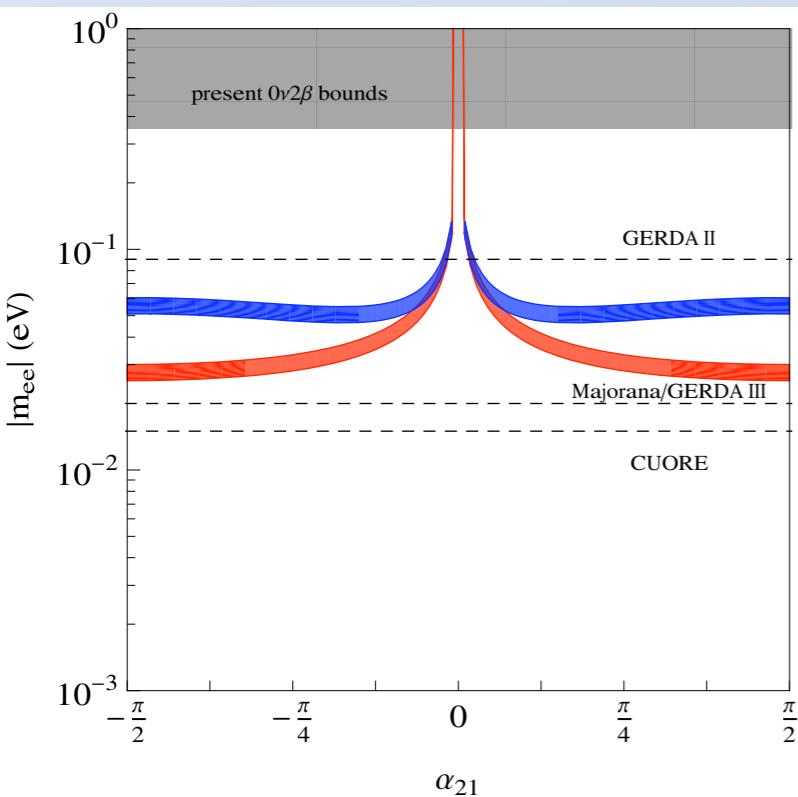
EFSII (S4)

SSI,SSIII (S4)

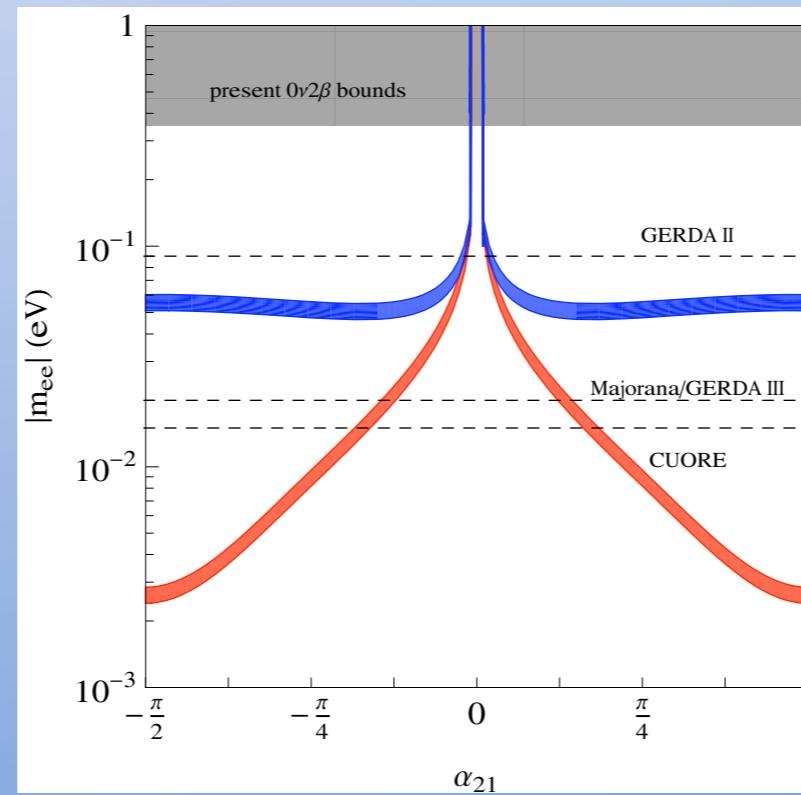
HMV (A4)

# Comparing models: A4 vs S4

EESII (S4)



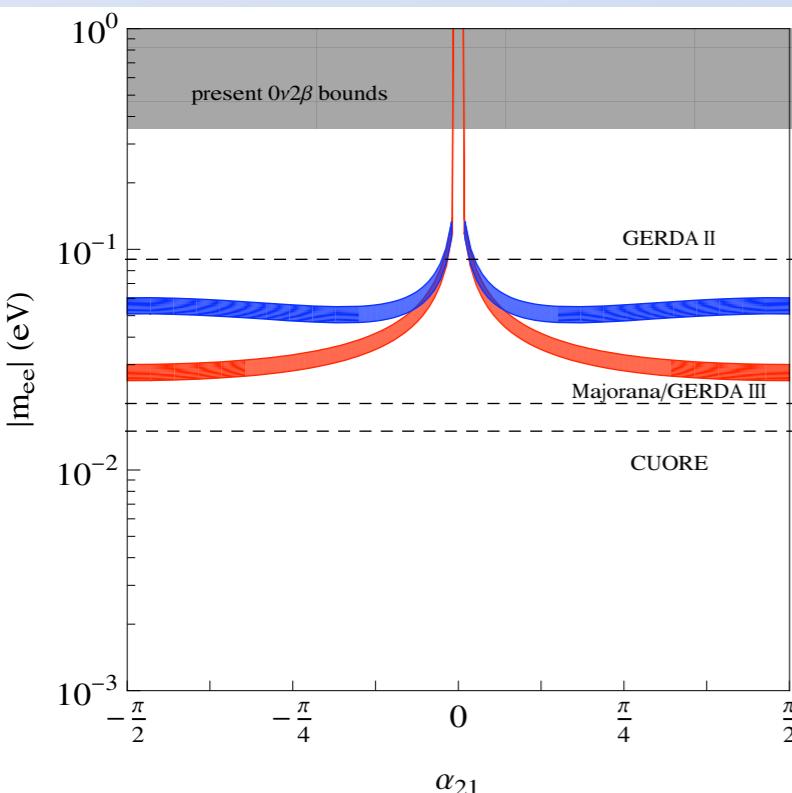
SSI,SSIII (S4)



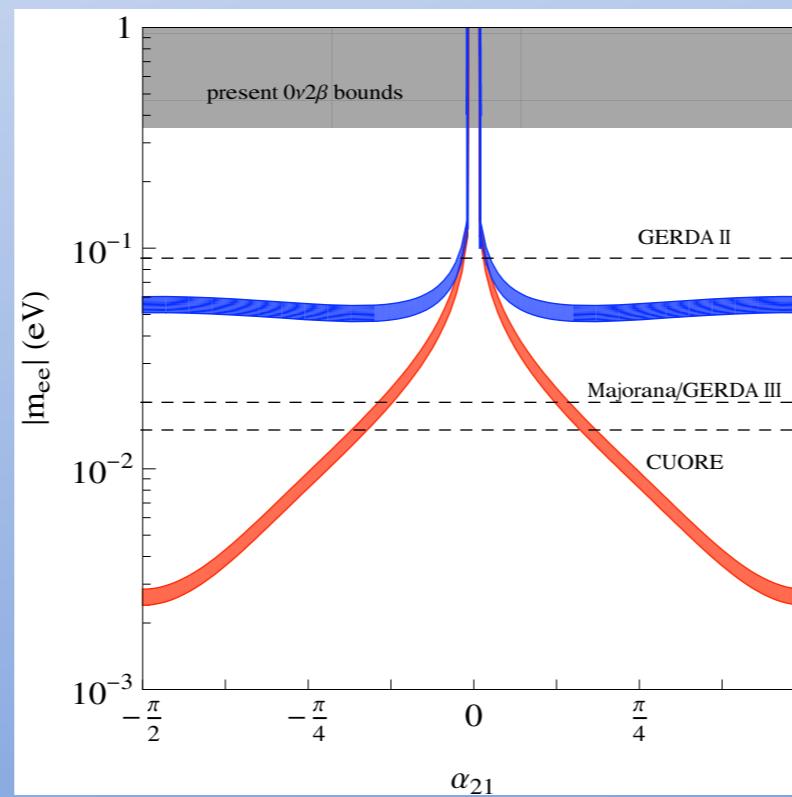
HMV (A4)

# Comparing models: A4 vs S4

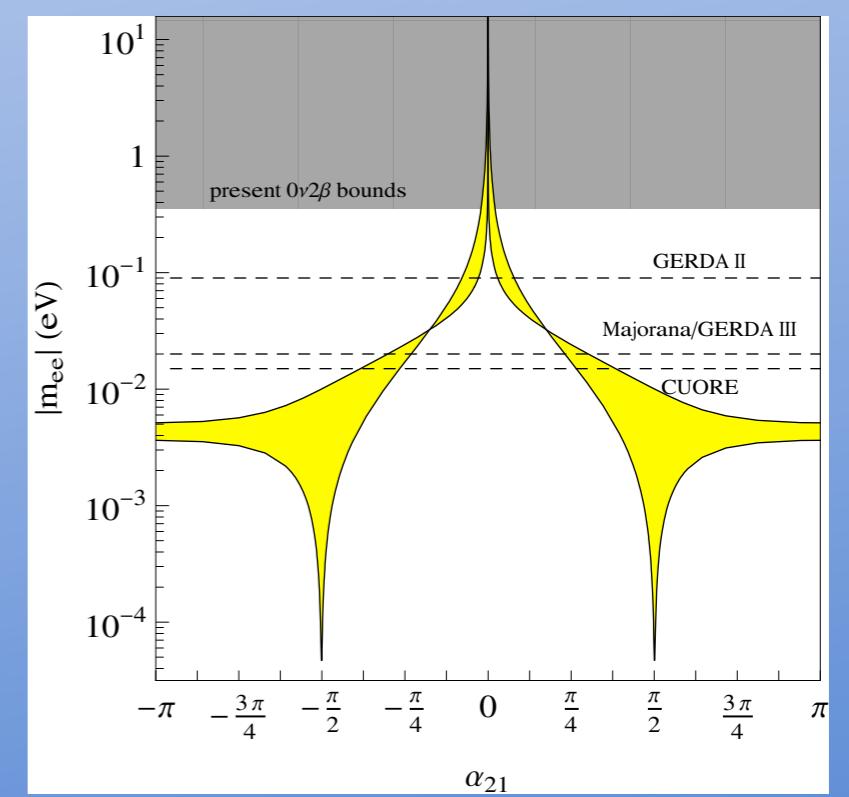
EFSII (S4)



SSI,SSIII (S4)



HMV (A4)



# Conclusions

- ✓ Model building investigations to explain neutrino masses and lepton mixing (also quarks) is a very active field
- ✓ Discrete flavor symmetry (even if they present some technical problems) works impressively well for lepton mixing
- ✓ Many flavor symmetry & many models: phenomenology may allow us to distinguish them
- ✓ Future experiments could rule out some model realizations
- ✓ Possible further discriminations in the study of leptogenesis (work in progress)

# Conclusions

- ✓ Model building investigations to explain neutrino masses and lepton mixing (also quarks) is a very active field
- ✓ Discrete flavor symmetry (even if they present some technical problems) works impressively well for lepton mixing
- ✓ Many flavor symmetry & many models: phenomenology may allow us to distinguish them
- ✓ Future experiments could rule out some model realizations
- ✓ Possible further discriminations in the study of leptogenesis (work in progress)

Thanks !



# Comparing models: A4

HMV

