REVIEW OF V_{cb} AND V_{ub} MEASUREMENTS

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Flavour mixing is described within the Standard Model by the Cabibbo-Kobayashi-Maskawa matrix elements. With the high statistics collected by the experiments at the b-factories, the matrix elements $|V_{cb}|$ and $|V_{ub}|$ are measured with improved precision, allowing for more stringent tests of the Standard Model. In this paper, a review of the current status of their measurements is presented.

1 Introduction

The Standard Model (SM) accounts for flavor changing quark transition through the coupling of the V-A charged current operator to a W boson:

$$\mathcal{L}_W = -\sqrt{\frac{1}{2}} g \overline{u_{Li}} \gamma^\mu \overline{V_{ij}} d_{Lj} W^+_\mu + \text{h.c.} \tag{1}$$

where V_{ij} are the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix elements.

By convention, the mixing is expressed in terms of the V matrix operating on the charge -$e/3$ quark mass eigenstates (d, s and b):

$$\begin{pmatrix} d\,' \\ s\,' \\ b\,' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{2}$$

Generation changing transitions between quarks are possible because the off-diagonal elements are not zero. The values of the CKM matrix elements are fundamental parameters of the SM and cannot be predicted. In the following, a review of the current values of $|V_{cb}|$ and $|V_{ub}|$ measured by the BaBar, Belle and Cleo experiments is presented. The averages of the Heavy Flavour Averaging Group (HFAG) [1] will also be quoted.

2 $|V_{cb}|$ Measurements

The CKM matrix element $|V_{cb}|$ is measured from the semileptonic inclusive and exclusive $b \to cl\nu$ decays. At the parton level, this decay rate can be calculated accurately; it is proportional to $|V_{cb}|^2$ and depends on the quark masses, m_b and m_c. To relate measurements of the semileptonic B-meson decay rate to $|V_{cb}|$, the parton-level calculations have to be corrected for effects of strong interactions.

2.1 Inclusive Measurements

In the kinetic-mass scheme the Heavy Quark Expansion (HQE) to $\mathcal{O}(1/m_b^3)$ for the rate Γ_{SL} of semileptonic decays $B \to X_c l^- \nu$ can be expressed as [2]:

$$\begin{aligned}
\Gamma_{SL} &= \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 (1 + A_{ew}) A_{pert}(r,\mu) \\
&\times \left[z_0(r) \left(1 - \frac{\mu_\pi^2 - \mu_G^2 + \frac{\rho_D^4 + \rho_{LS}^3}{c^2 m_b}}{2c^4 m_b^2} \right) \right. \\
&- 2(1-r)^4 \frac{\mu_G^2 + \frac{\rho_D^3 + \rho_{LS}^3}{c^2 m_b}}{c^4 m_b^2} + d(r) \frac{\rho_D^3}{c^6 m_b^3} \\
&+ \left. \mathcal{O}(1/m_b^4) \right].
\end{aligned} \qquad (3)$$

This expansion contains six parameters: the running kinetic masses of the b- and c-quarks, $m_b(\mu)$ and $m_c(\mu)$, and four non-perturbative parameters μ_π, μ_G, ρ_D, and ρ_{LS}: . The parameter μ denotes the Wilson normalization scale that separates effects from long- and short-distance dynamics. The ratio $r = m_c^2/m_b^2$ enters in the tree level phase space factor $z_0(r) = 1 - 8r + 8r^3 - r^4 - 12r^2 \ln r$ and in the function $d(r) = 8 \ln r + 34/3 - 32r/3 - 8r^2 + 32r^3/3 - 10r^4/3$. The factor $1 + A_{ew}$ accounts for electroweak corrections. It is estimated to be $1 + A_{ew} \cong (1 + \alpha/\pi \ln M_Z/m_b)^2 = 1.014$, where α is the electromagnetic coupling constant. The quantity A_{pert} accounts for perturbative contributions and is estimated to be $A_{pert}(r,\mu) \approx 0.908$. The moments of the hadronic-mass and electron-energy distributions in semileptonic B decays $B \to X_c l^- \nu$ and the moments of the photon-energy spectrum in $B \to X_s \gamma$ decays depend on the same set of parameters.

BaBar [3] and Belle [4] have performed a combined fit to all these moments to extract values for $|V_{cb}|$, the quark masses m_b and m_c, the total semileptonic branching fraction $\mathcal{B}(B \to X_c l^- \nu)$, and the non-perturbative HQE parameters. The fitted value of $|V_{cb}|$, using expressions in the kinetic scheme, is $|V_{cb}| = (41.67 \pm 0.43 \pm 0.08 \pm 0.58) \times 10^{-3}$, where the errors are due to the global fit, the B lifetime and theory, respectively. However it should be noted that that a fit just to the $B \to X_c l^- \nu$ moments tends to give a value of m_b about 1σ higher than the one from $B \to X_c l^- \nu$ and $B \to X_s \gamma$ moments combined as shown in Fig. 1. This incertitude impacts the $|V_{ub}|$ extraction where m_b is used as input in the fitting procedure. In addition the χ^2/NDF of the fit is 29.7/57, a quite small value that can possibly come from an improper treatment of correlations between the different moments. The most recent result from Belle [4] does not shown the dependence of the value of m_b on the set of moment used.

2.2 Exclusive Measurements

The determination of $|V_{cb}|$ from exclusive $b \to cl\nu$ decays is based on the $B \to D^{(*)} l\nu$ decays, for which, in the assumption of infinite b and c quark masses, the form factors describing the $B \to D^{(*)}$ transitions depend only on the product, w, of the initial, v, and final, v', state hadron

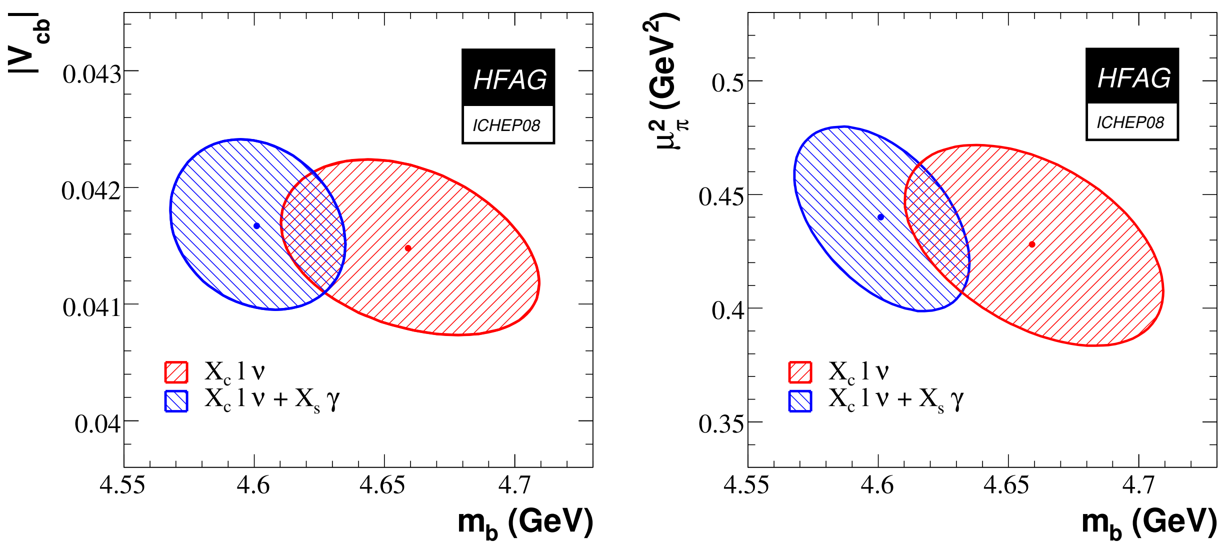

Figure 1: $\Delta_{\chi^2} = 1$ contours for the fit to all moments and the fit to the $B \to X_c l^- \nu$ data only. $|V_{cb}|$ vs m_b (left) and μ_π^2 vs m_b (right).

four-velocities, $w = v \times v'$, and relies on a parametrization of the form factors using the Heavy Quark Symmetry (HQS) [5] and a non-perturbative calculation of the form factor normalization at w = 1, which corresponds to the maximum momentum transfer to the leptons. The form factors for $B \to Dl\nu$ and for $B \to D^* l\nu$ decays are G(w) and F(w), respectively. BaBar and Belle adopt the form factor parametrization from Caprini et al. [6], and lattice QCD to correct the normalization of the form factor at w = 1, due to the finite quark masses. Experimentally, the w spectrum is measured and $F(1)|V_{cb}|$ and $G(1)|V_{cb}|$ are obtained from an extrapolation of the measured w spectrum to 1. Several analyses from BaBar [7] and Belle [8], which adopt different experimental techniques, were recently presented. The bi-dimensional plots of the form factor at w = 1 times $|V_{cb}|$ versus the slope parameter for the form factors ρ^2 is shown in Fig. 2. The fitted values are $G(1)|V_{cb}| = (42.4 \pm 1.6) \times 10^{-3}$ and $F(1)|V_{cb}| = (35.4 \pm 0.5) \times 10^{-3}$ rispectively. Assuming: $G(1) = 1.074 \pm 0.018 \pm 0.016$ [9] and $F(1) = 0.924 \pm 0.012 \pm 0.019$ [10], where the errors are statistical and systematical, respectively, and appling a 1.07 QCD correction factor, the values $|V_{cb}| = (39.7 \pm 1.4 \pm 0.9) \times 10^{-3}$ and $|V_{cb}| = (38.1 \pm 0.6 \pm 0.9) \times 10^{-3}$ are obtained, for $B \to Dl\nu$ and for $B \to D^* l\nu$ decays, respectively. The two results are completely consistent.

The 2σ discrepancy between the value of $|V_{cb}|$ extracted from the moment analysis and the one coming from $B \to D^{(*)} l\nu$ decays using the Lattice QCD form factor calculations, is still an open question.

3 $|V_{ub}|$ Measurements

Semileptonic inclusive and exclusive $b \to ul\nu$ decays are used to measure the CKM matrix element $|V_{ub}|$. Different experimental and theoretical approaches are involved, thus providing complementary ways to extract $|V_{ub}|$.

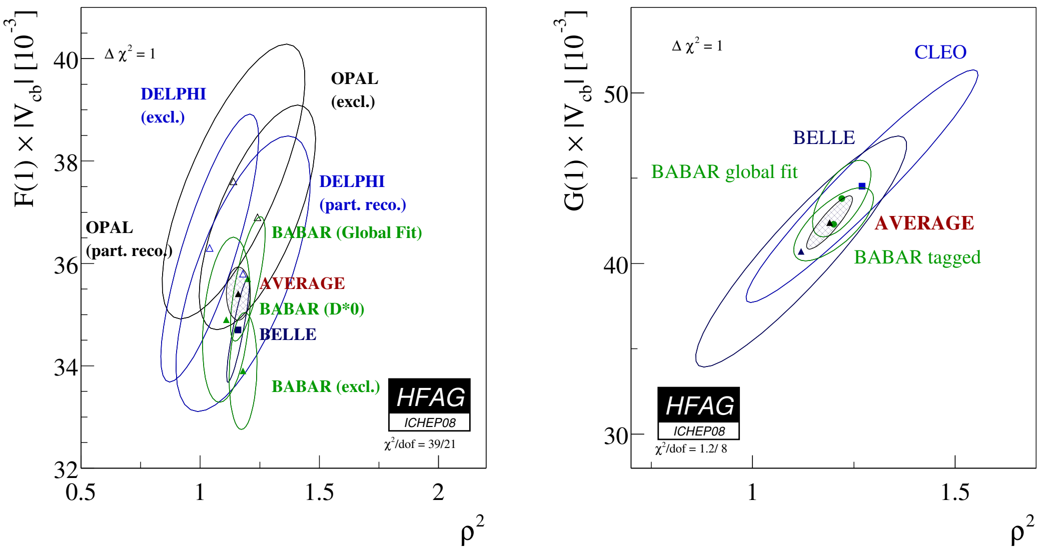

Figure 2: $G(1)|V_{cb}|$ (left) and $F(1)|V_{cb}|$ (right) versus the form factor slope parameter ρ^2.

3.1 Inclusive Measurements

The measurement of the inclusive decays rate for $B \to X_u l^- \nu$ decays is affected by a large background of the order $|V_{ub}/V_{cb}|^2 = 1/50$, due to $B \to X_c l^- \nu$ decays. Stringent kinematic cuts are applied to select regions of the phase space in which the $B \to X_c l^- \nu$ background can be kept under control. Thus, only a partial branching fraction, limited to the particular kinematic region selected, is measured and needs to be estrapolated to the full phase space.

Whilst the total branching fraction can be computed using HQE and QCD perturbation theory, the partial rate needs further theoretical tools, which have been the subject of intense theoretical effort, expecially in the last years. Different approches have been used: BLNP [11] (a shape function approach, where the shape function represents the momentum distribution function of the b quark in the B meson), DGE [12] (a resummation based approach), GGOU [13] (an HQE based structure function parametrization approach) and ADFR [14] (a soft gluon resummation and analytic time-like QCD coupling approach). Concerning BLNP, recent NNLO corrections [15] were presented. The models depend strongly on the b quark mass, except for ADFR, so it is very important to use a precise determination of this quantity. BaBar [16] and Belle [17] have applied kinematic cuts using the following variables: the lepton energy (E_l), the invariant mass of the hadron final state (M_X), the light-cone distribution ($P^+ = E_X - |p_X|$, E_X and $|p_X|$ being the energy and the magnitude of the 3-momentum of the hadronic system) and a two dimensional distribution in the electron energy and s^{max}, the maximal M_X^2 at fixed q^2 and E_l. The results obtained by these methods and the corresponding averages are shown in Fig. 3.

The values of $|V_{ub}|$ obtained using diffent kinematical cuts and exctracted using the same theoretical approch are consistent. On the contrary, different theoretical approches give $|V_{ub}|$ values that are somehow different.

Very recently, a preliminary result from Belle using an innovative multivariate analysis [18], in which 90% of the total rate is measured, has been presented. This experimental measurement

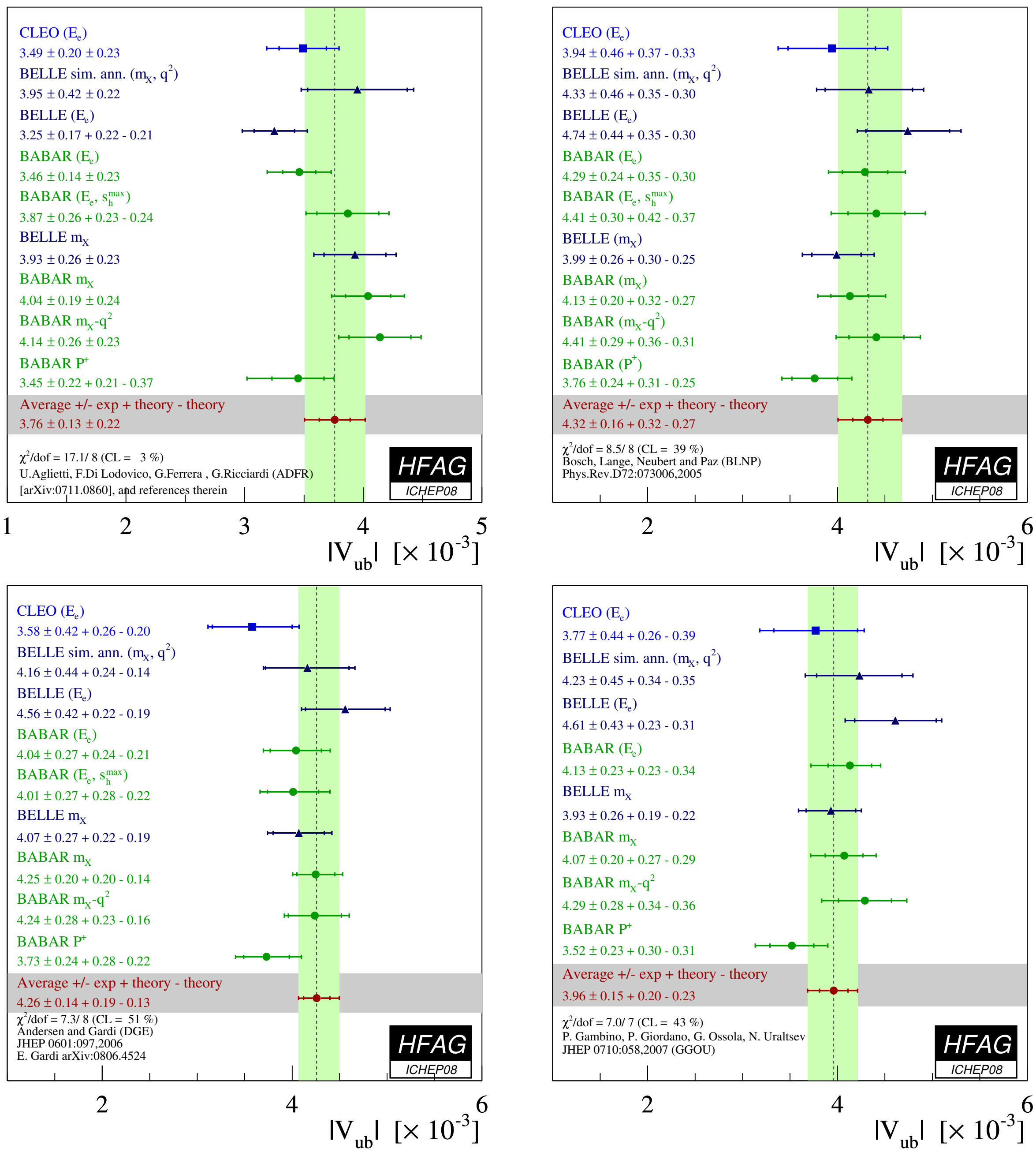

Figure 3: Inclusive $|V_{ub}|$ measurements.

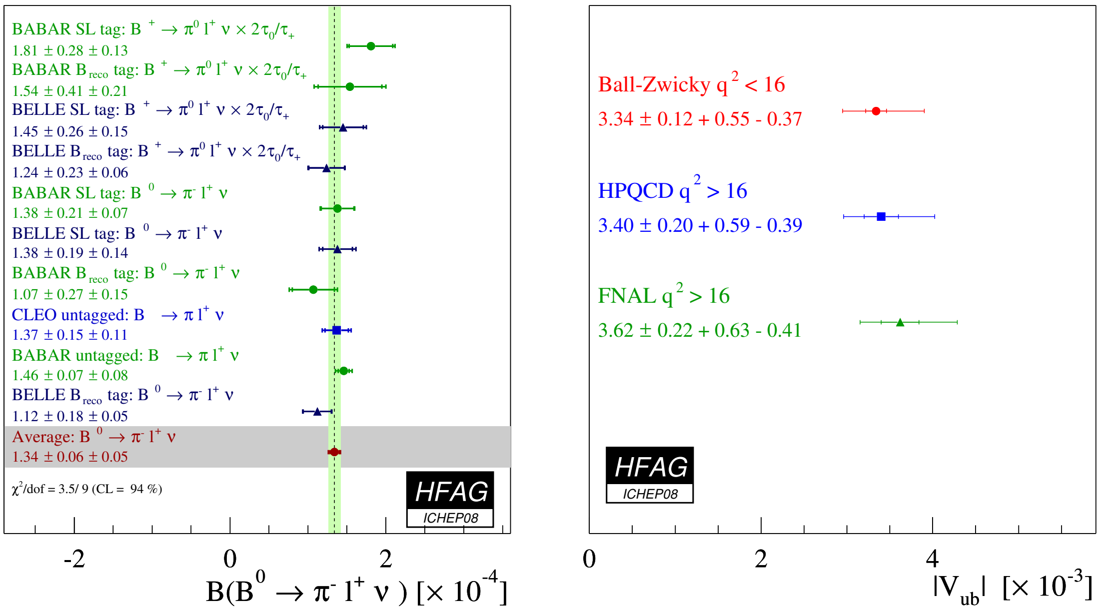

Figure 4: $B \to \pi l \nu$ branching fractio measurements (left) and $|V_{ub}|$ values extracted using different theoretical calculations (right).

is extremely interesting as it will help in a further understanding of $|V_{ub}|$ from inclusive decays.

3.2 Exclusive Measurements

$|V_{ub}|$ can be extracted from exclusive charmless semileptonic decays, $B \to \pi, \rho, \eta, \eta', \omega l\nu$, where the corresponding rate is related to $|V_{ub}|$ by the form factor(s) $f(q^2)$, where q^2 is the momentum transfer squared to the lepton pair. Non perturbative methods for the calculation of the form factors include unquenched lattice QCD, like the HPQCD [19] and Fermilab/MILC [20] calculations, and QCD light cone sum rules [21].

BaBar [22], Belle [23], and Cleo [24] have perfomed measurements of $B \to \pi l \nu$ decays exploiting different analysis techniques that fall into two broad classes: untagged and tagged, depending on whether the B in the event that does not decay into the $\pi l \nu$ final state is tagged or not. The untagged method has higher statistic and higher background, while the B tagging reduces significantly the background at a price of a reduced statistics. The results are presented for the full q^2, $q^2 > 16\ GeV^2$ and $q^2 < 16\ GeV^2$ ranges. The last two phase space regions correspond to regions where the lattice and QCD light cone sum rule calculations of the form factors are restricted to, respectively. The corresponding measurements of the total branching ratio for all the collaborations and their average is shown in Fig. 4(left plot). From the average, and using both lattice QCD and QCD light cone sum rules, the value of $|V_{ub}|$ are extracted (Fig. 4, right plot). The $|V_{ub}|$ results coming from different theoretical calculations are consistent among themselves. However the uncertainties from form factors calculation are the dominant systematic in the $|V_{ub}|$ extraction.

In a recent paper by Bailey at al.[25], the $B \to \pi l \nu$ 12 bin q^2 spectrum measured by BaBar [26] has been used to extract $|V_{ub}| = (3.38 \pm 0.36) \times 10^{-3}$.

Moreover, experimental measurements of the $B \to \pi, \rho, \eta, \eta', \omega l\nu$ branching ratio have been performed by BaBar[27], Belle[23] and Cleo[28] and will provide a test of the $|V_{ub}|$ extraction from $B \to \pi l\nu$ decays, once the corresponding form factors will be computed.

4 Summary

A significant progress has been made in the past years thanks to the b-factory measurements of $|V_{cb}|$ and $|V_{ub}|$ and to a remarkable theoretical effort. However the dominant systematics are the one coming from the theoretical calculation used to extract the CKM matrix elements from the experimental observables. More data and theoretical progress will improve our knowledge of $|V_{cb}|$ and $|V_{ub}|$.

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