UNIFICATION IN EXTRA-DIMENSIONAL SCENARIO

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In this work, we briefly describe how the characteristics of gauge, Yukawa and quartic coupling evolution change in the presence of universal extra dimensions. The gauge coupling unification scale depends on compactification radius R, and is much lower in comparison to the four-dimensional case. Later, we mention that the supersymmetric extension of this scenario requires a much larger value of R^{-1}, in order that the gauge couplings remain perturbative up to the unification scale.

1 Introduction

In the standard model (SM), the gauge, Yukawa and quartic scalar couplings run logarithmically with the energy scale. Although the gauge couplings do not all meet at a point, they tend to unify near 10^{15} GeV. Such a high scale is beyond the reach of any present or future experiments. Extra dimensions accessible to SM fields have the virtue, thanks to the couplings' power law running, of bringing the unification scale down to an explorable range. Higher dimensional theories, with radii of compactification around an inverse TeV, have been investigated from the perspective of high energy experiments, phenomenology, string theory, cosmology, and astrophysics. Our concern here is a specific framework, called the Universal Extra Dimension (UED) scenario, where there is a single flat extra dimension, which is accessed by all the SM particles [1]. The extra dimension is compactified on an S_1/Z_2 orbifold $i.e$ a circle of radius R with a Z_2 orbifolding identifying $y \to -y$, where y denotes the fifth compactified coordinate. The orbifolding is crucial in generating *chiral* zero modes for fermions. After integrating out the compactified dimension, the 4-dimensional Lagrangian can be written involving the zero mode and the KK modes.

Constraints on the UED scenario from $g-2$ of the muon [2], flavour changing neutral currents [3], $Z \to b\bar{b}$ decay [4], the ρ parameter [1,5], several other electroweak precision tests and implications from hadron collider studies, all conclude that $R^{-1} \gtrsim 300$ GeV.

2 Renormalisation Group Equations

We now come to the technical meaning of RG running in a higher dimensional context. Like all other extra-dimensional models, from a 4-dimensional point of view, the UED scenario too is non-renormalisable due to the infinite multiplicity of the KK states. So 'running' of couplings as a function of the energy scale μ ceases to make sense. What we should say is that the couplings receive finite quantum corrections whose size depend on some explicit cutoff[b] Λ. The corrections originate from the ΛR number of KK states which lie between the scale R^{-1} where the first KK states are excited and the cutoff scale Λ. The couplings will have a power law dependence on Λ as a result of the KK summation. This cutoff is interpreted as the scale where a paradigm shift occurs when some new renormalisable physics underlying our effective non-renormalisable framework surfaces.

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[a] Talk presented by Swarup Kumar Majee.

[b] The beta functions are coefficients of the divergence $1/\epsilon$ in a 4-dimensional theory. Here, a second kind of divergence appears when the finite beta functions get corrections from each layer of KK states which are summed over. This summation is truncated at a scale Λ.

We now lay out the strategy followed to compute the RG correction to the gauge couplings from the KK modes. The first step is obviously the calculation of the contribution from a given KK level which has both Z_2-even and -odd states. The first step KK excitation occurs at the scale R^{-1} (modulo the zero mode mass). Up to this scale the RG evolution is logarithmic, controlled by the SM beta functions with coefficients $41/10$, $-19/6$, -7 for $U(1)$, $SU(2)$ and $SU(3)$ gauge groups respectively. Between R^{-1} and $2R^{-1}$, the running is still logarithmic but with beta functions modified due to the first KK level excitations, and so on. Every time a KK threshold is crossed, new resonances are sparked into life, and new sets of beta functions rule till the next threshold arrives. This is what, for the gauge couplings, depicted in the left panel of Fig. 1 for $R^{-1} = 1$ TeV. The beta function contributions are the same, for each of the ΛR KK levels, which, in effect, can be summed. After this, the scale dependence is not logarithmic any more, it shows power law behaviour. This illustration shows that if $\Lambda R \gg 1$, then to a very good accuracy the calculation basically boils down to computing the number of KK states up to the cutoff scale. For one extra dimension up to the energy scale E this number is $S = ER$, and $E^{\max} = \Lambda$. Then if $\beta^{\rm SM}$ is a generic SM beta function valid during the logarithmic running up to R^{-1}, beyond that scale one should replace it as

$$\beta^{\rm SM} \to \beta^{\rm SM} + (S-1)\tilde{\beta}, \tag{1}$$

where $\tilde{\beta}$ is a generic contribution from a single KK level while different co-efficients corresponding to the $U(1)$, $SU(2)$ and $SU(3)$ gauge group are $\tilde{b}_1 = \frac{81}{10}$, $\tilde{b}_2 = \frac{7}{6}$, $\tilde{b}_3 = -\frac{5}{2}$.

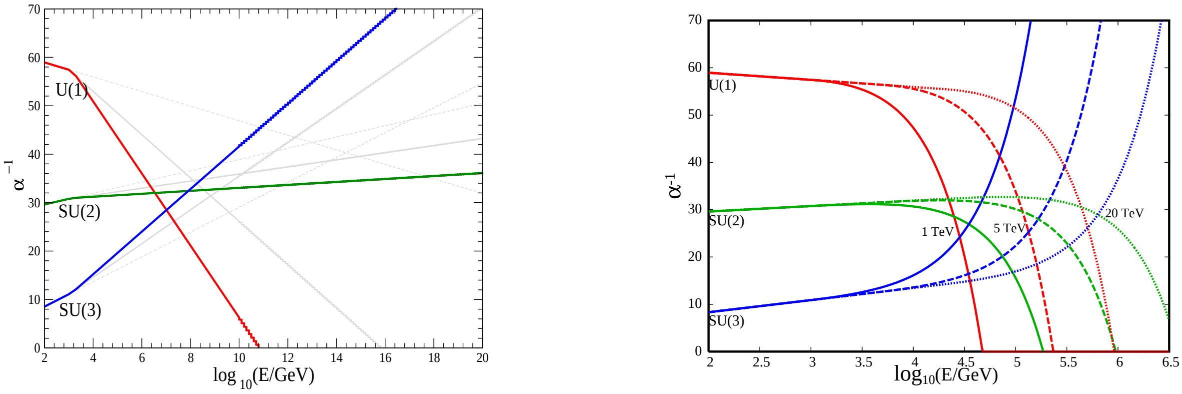
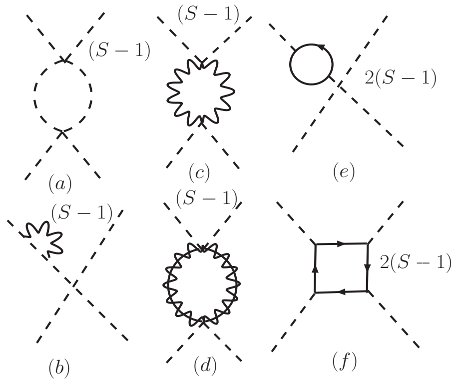


Figure 1: Left panel: Standard model gauge coupling evolution and it's modification due to first and second KK-mode contribution. Right panel: power law evolution of the gauge couplings in UED are shown.

Irrespective of whether we deal with the 'running' of gauge, Yukawa, or quartic scalar couplings, the structure of Eq. (1) would continue to hold. Clearly, the S dependence reflects power law running. Evolution of gauge couplings in UED for $R^{-1} = 1$, 5, and 20 TeV are shown in the right panel of Fig. 1. The running is fast, as expected, and the couplings nearly meet around 30, 138 and 525 TeV, respectively. It is not hard to provide an intuitive argument for such low unification scales and how they vary with R: roughly speaking, ΛR is order $\ln(M_{\rm GUT}/M_W) \sim \ln(10^{15})$, where $M_{\rm GUT}$ is the 4-dimensional GUT scale, i.e. the effect of a slow logarithmic running over a large scale is roughly reproduced by a fast power law sprint over a short track.

3 Quartic couplings and bounds on Higgs mass

The Feynman diagrams that contribute to the power law evolution of quartic coupling (in Landau gauge) are shown in the left panel of Fig 2. More explicitly, consider the Figs. 2d from the left panel. This graph proceeds through the exchange of adjoint A_5 scalars and yield non-vanishing contributions. This is a new diagram, and there is no analogous diagram in the standard model. As we examine contributions from individual KK states, we see that due to the argument of fermion chirality, not in all diagrams do the cosine and sine mode states both *simultaneously* contribute. This accounts for a relative factor of 2 between the two types of diagrams. For example, Fig. 2a has a multiplicating factor $(S-1)$, while for Fig. 2e the factor is $2(S-1)$.



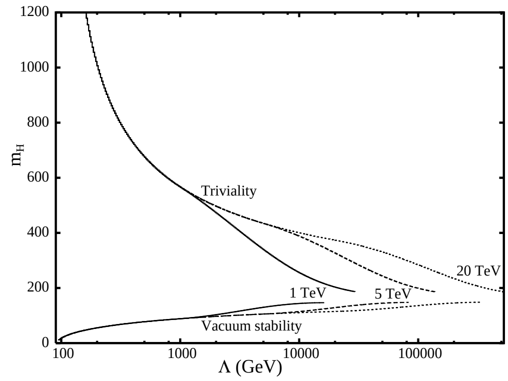


Figure 2: Left-panel: Diagrams contributing to quartic coupling evolution in the Landau gauge. Solid (broken) lines correspond to fermions (SM scalars), while wavy lines (wavy+solid lines) represent ordinary gauge bosons (fifth components of gauge bosons). Right-panel: Variation of Higgs mass with the cut-off.

The quartic coupling [c] evolution equation can, thus, be written as

$$16\pi^2 \frac{d\lambda}{dt} = \beta_\lambda^{\rm SM} + \beta_\lambda^{\rm UED} \tag{2}$$

The expressions for $\beta_\lambda^{\rm SM}$ can be found e.g. in Ref [7]. The UED beta functions are given by

$$\begin{aligned} \beta_\lambda^{\rm UED} &= (S-1)\left[3g_2^4 + \frac{6}{5}g_2^2 g_1^2 + \frac{9}{25}g_1^4 - 3\lambda(3g_2^2 + \frac{3}{5}g_1^2) + 12\lambda^2\right] \\ &+ 2(S-1)\left[4\left(Y_l + 3Y_u + 3Y_d\right)\lambda - 4\sum_{l,u,d}\left(y_l^4 + 3y_u^4 + 3y_d^4\right)\right]. \end{aligned} \tag{3}$$

The evolution of λ has interesting bearings on the Higgs mass and put some bounds on the Higgs mass on the grounds of 'triviality' and 'vacuum stability' [8]. The 'triviality' argument requires that λ stays away from the Landau pole, i.e. remains finite, all the way to the cutoff scale Λ. The condition that $1/\lambda(\Lambda) > 0$ can be translated to an upper bound on the Higgs mass (m_H) at the electroweak scale when the cutoff of the theory is Λ. This has been plotted in the right panel of Fig. 2 (the upper curves) for three different values of R. A given point on that curve (for a given R) corresponds to a maximum allowed m_H at the weak scale; for a larger m_H the coupling λ becomes infinite at some scale less than Λ and the theory ceases to be perturbative. Clearly, this $m_H^{\rm max}$ varies as we vary the cutoff Λ . The argument of 'vacuum stability' relies on the requirement that the scalar potential be always bounded from below, i.e. $\lambda(\Lambda) > 0$. This can be translated to a lower bound $m_H^{\rm min}$ at the weak scale. The lower set of curves in right panel of Fig. 2 (for three values of R^{-1}) represent the 'vacuum stability' limits, the region below the curve for a given R being ruled out. Recalling that the cutoff is where the gauge couplings tend to unify, we observe that the Higgs mass is limited in the narrow zone

$$148 \lesssim m_H \lesssim 186 \text{ GeV} \tag{4}$$

[c] The Yukawa RG equations can be found in Ref [6].

in all the three cases, for a zero mode top quark mass of 174.2 GeV. Admittedly, our limits are based on one-loop corrections only. That the upper and lower limits are insensitive to the choice of R is not difficult to understand, as what really counts is the number of KK states, given by the product ΛR, which, as mentioned before, is nearly constant, order $\ln(10^{15})$.

4 Supersymmetric UED

What happens if we take the supersymmetric (SUSY) version of UED? A 5-dimensional $N = 1$ supersymmetry when perceived from a 4-dimensional context contains two different $N = 1$ multiplets forming one $N = 2$ supermultiplet. In the RG evolution, in this case, two energy scales will come into play. The first of these is the supersymmetry scale, called M_S, which we take to be 1 TeV. Beyond M_S, supersymmetric particles get excited and their contributions must be included in the RG evolution. The second scale is that of the compactified extra dimension $1/R$, which we take to be larger than M_S.

The gauge coupling evolution must now be specified for three different regions and can be written as

$$b_i^{\rm tot} = b_{io} + \Theta(E - M_S)\ (b_{is} - b_{i0}) + \Theta(E - \frac{1}{R})\ (S-1)\ \tilde{b}_i, \tag{5}$$

The first of these is when $E < M_S$ where the SM with the additional scalar doublet [d] beta functions are in control. In this region $b_{1o} = \frac{21}{5},\ \ b_{2o} = -\frac{10}{3},\ \ b_{3o} = -7$. Once M_S is crossed and up until $1/R$, we also have the superpartners of the SM particles pitching in with their effects. The contributions of the SM particles and their superpartners together are given by $b_{1s} = \frac{33}{5},\ \ b_{2s} = 1,\ \ b_{3s} = -3$. Finally, when the KK-modes are excited ($E > 1/R$) one has further contributions from the individual modes $\tilde{b}_1 = \frac{66}{5},\ \ \tilde{b}_2 = 10,\ \ \tilde{b}_3 = 6$.

Not unexpectedly, for the SUSY UED case, gauge unification is possible. We observe that the introduction of this plethora of KK excitations of the SM particles and their superpartners radically changes the beta functions; so much so, that the gauge couplings tend to become non-perturbative before unification is achieved. In order that all of them remain perturbative during the entire RG evolution, the onset of the KK dynamics has to be sufficiently delayed. This requirement imposes $R^{-1} \gtrsim 5.0 \times 10^{10}$ GeV. In effect, this implies that the twin requirements of a SUSY-UED framework as well as perturbative gauge coupling unification pushes the detectability of the KK excitations well beyond the realm of the LHC.

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[d]SUSY requires two complex scalar doublets.