
Unification in Extra-Dimensional Scenarios

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&

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In collaboration with

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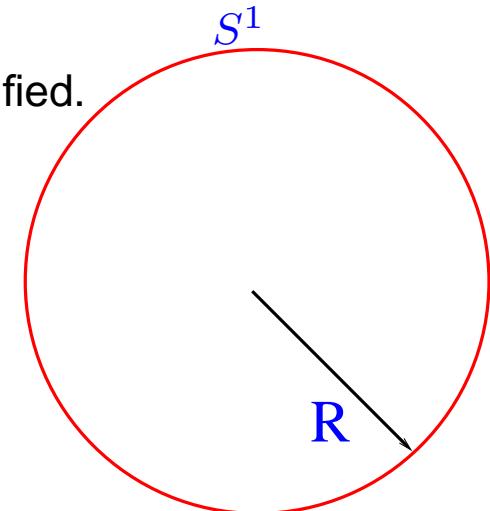
UED

- In UED model each particle can access all dimensions.

Appelquist, Cheng, Dobrescu

- We consider only *one space-type extra dimension (y)*

So our co-ordinate system : $\{\mathbf{x}(t, \vec{x}), y\}$ where, y is compactified.



UED

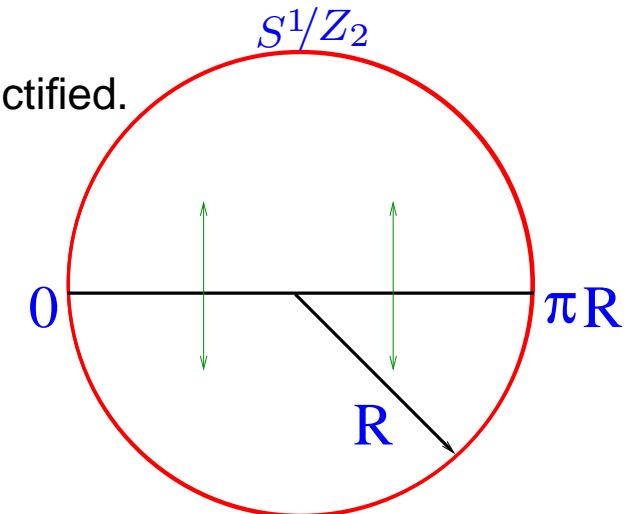
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So our co-ordinate system : $\{\mathbf{x}(t, \vec{x}), y\}$ where, y is compactified.

- Translational symmetry **breaks** $\Rightarrow p_5$ i.e. KK number (n) is **not conserved**. But **KK parity $\equiv (-1)^n$** is **conserved**.
- All the SM particles now have **KK-modes**.



$$A_\mu(x, y) = \frac{\sqrt{2}}{\sqrt{2\pi}R} A_\mu^{(0)}(x) + \frac{2}{\sqrt{2\pi}R} \sum_{n=1}^{\infty} A_\mu^{(n)}(x) \cos \frac{ny}{R},$$

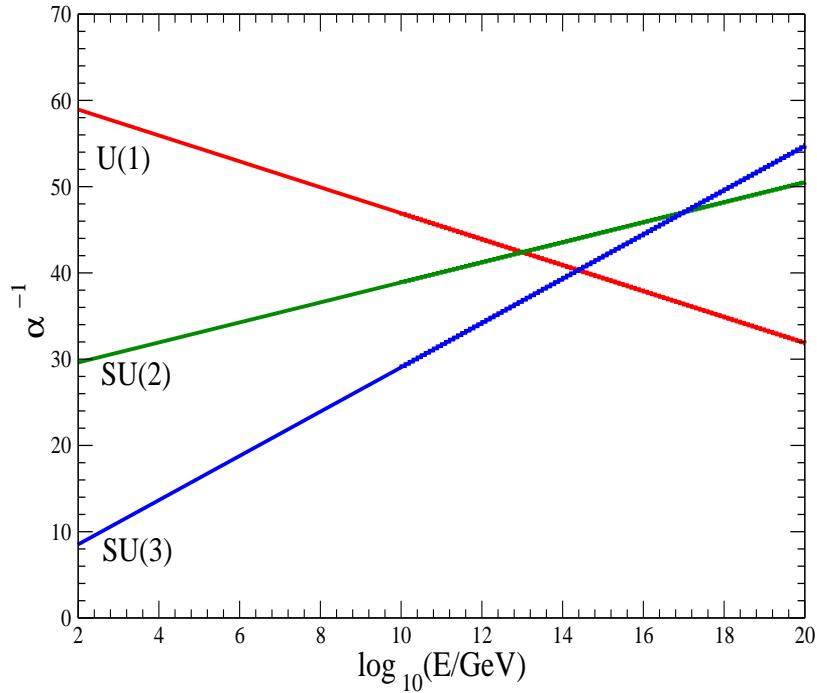
$$A_5(x, y) = \frac{2}{\sqrt{2\pi}R} \sum_{n=1}^{\infty} A_5^{(n)}(x) \sin \frac{ny}{R}.$$

$$\mathcal{Q}_i(x, y) = \frac{\sqrt{2}}{\sqrt{2\pi}R} \left[\binom{u_i}{d_i}_L(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[\mathcal{Q}_{iL}^{(n)}(x) \cos \frac{ny}{R} + \mathcal{Q}_{iR}^{(n)}(x) \sin \frac{ny}{R} \right] \right].$$

Effects of KK-modes on RGE

$$16\pi^2 E \frac{dg_i}{dE} = b_i g^3 = \beta_{SM}(g) \Rightarrow \alpha_i^{-1}(E) = \alpha_i^{-1}(M_Z) - \frac{b_i}{2\pi} \ln \frac{E}{M_Z}$$

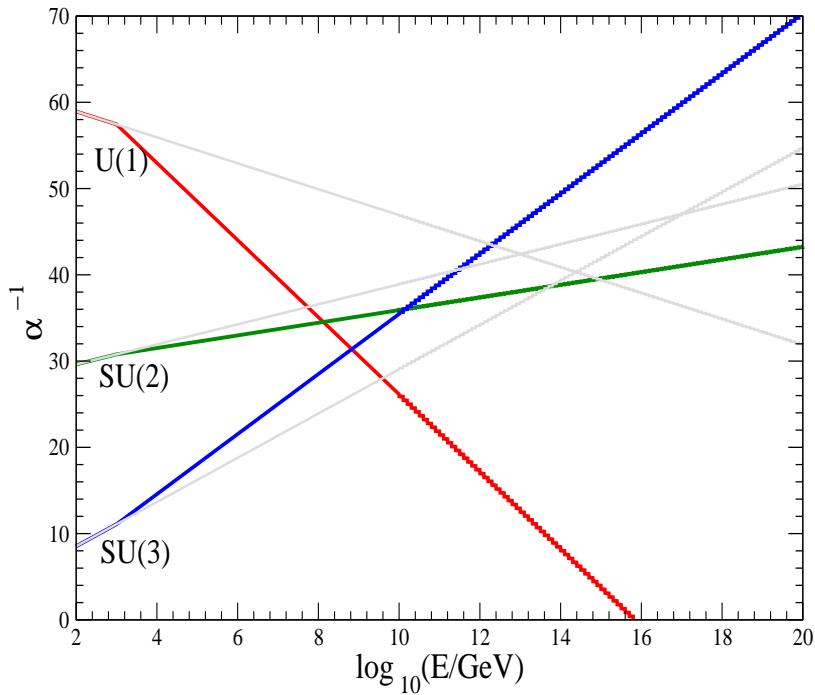
$$b_1 = \frac{41}{10}, \quad b_2 = -\frac{19}{6}, \quad b_3 = -7$$



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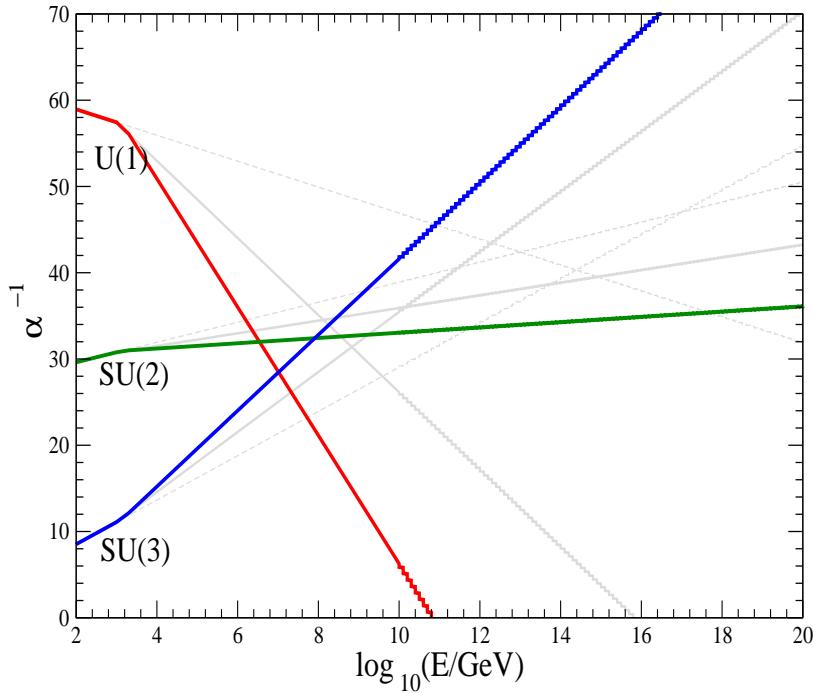
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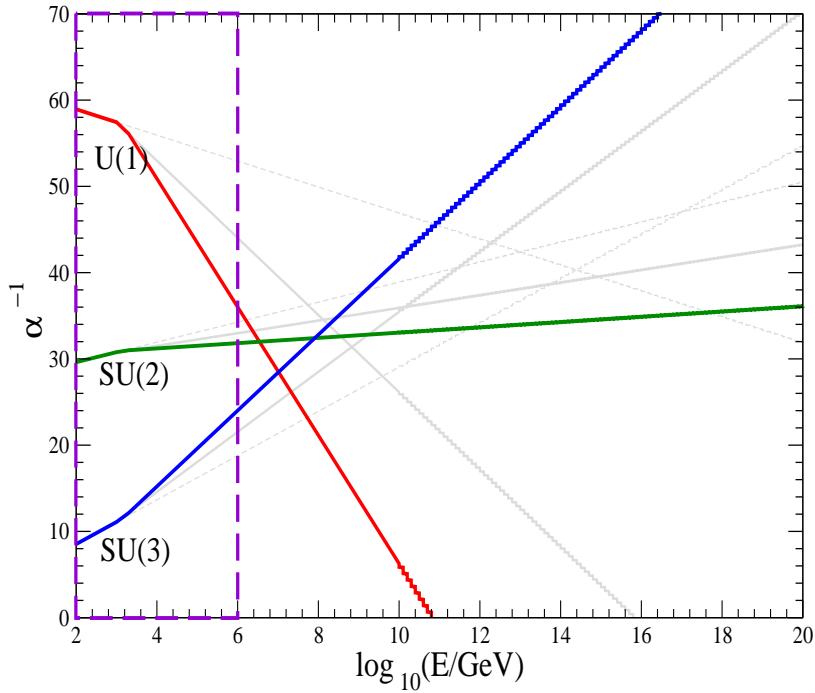
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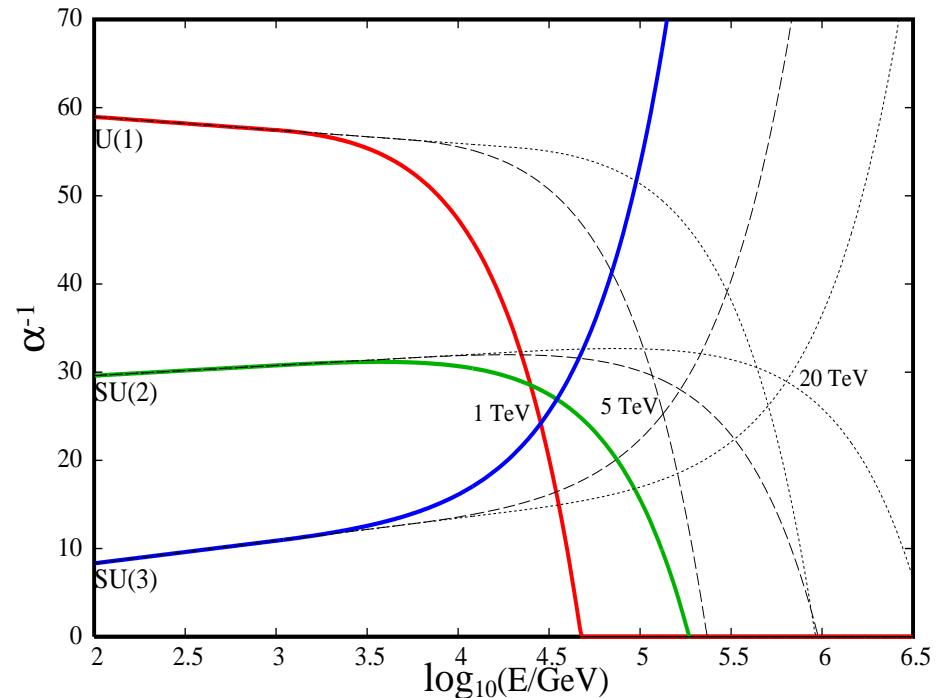
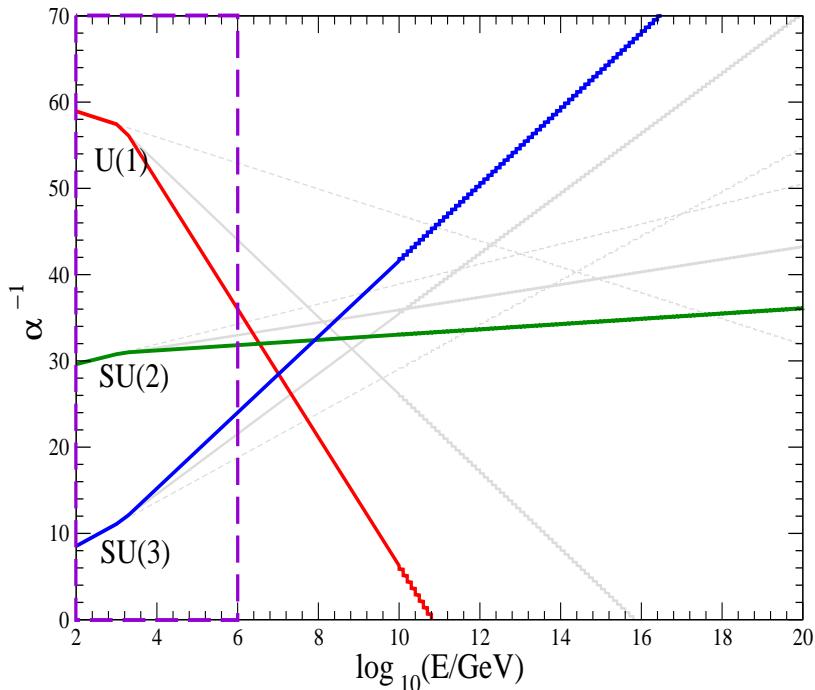


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$$\tilde{b}_1 = \frac{81}{10}, \quad \tilde{b}_2 = -\frac{7}{6}, \quad \tilde{b}_3 = -\frac{5}{2}$$



$$16\pi^2 E \frac{dg_i}{dE} = \beta_{SM}(g) + (S - 1)\beta_{UED}(g)$$

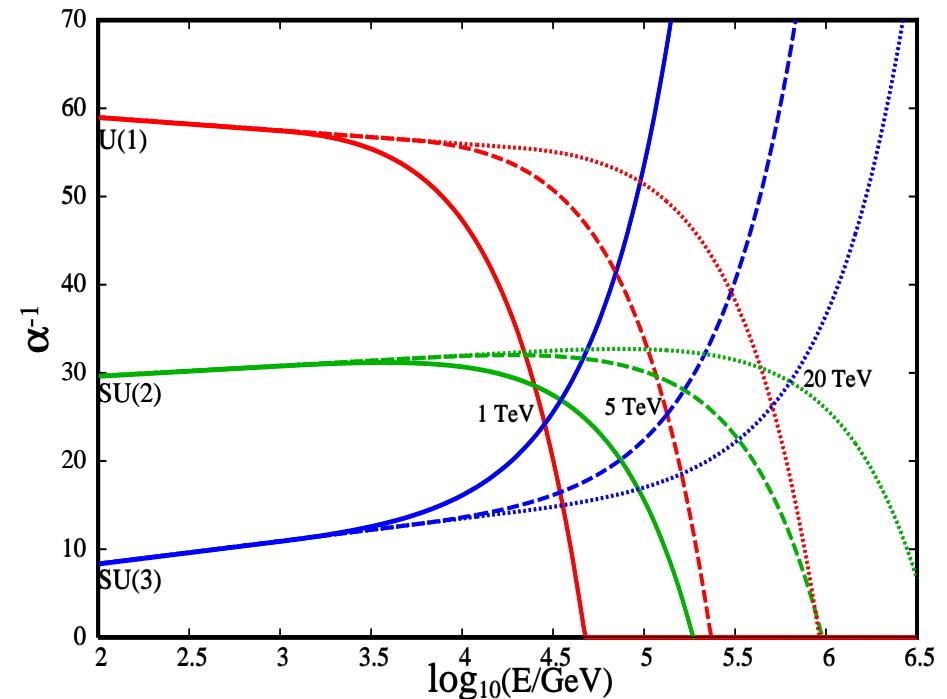
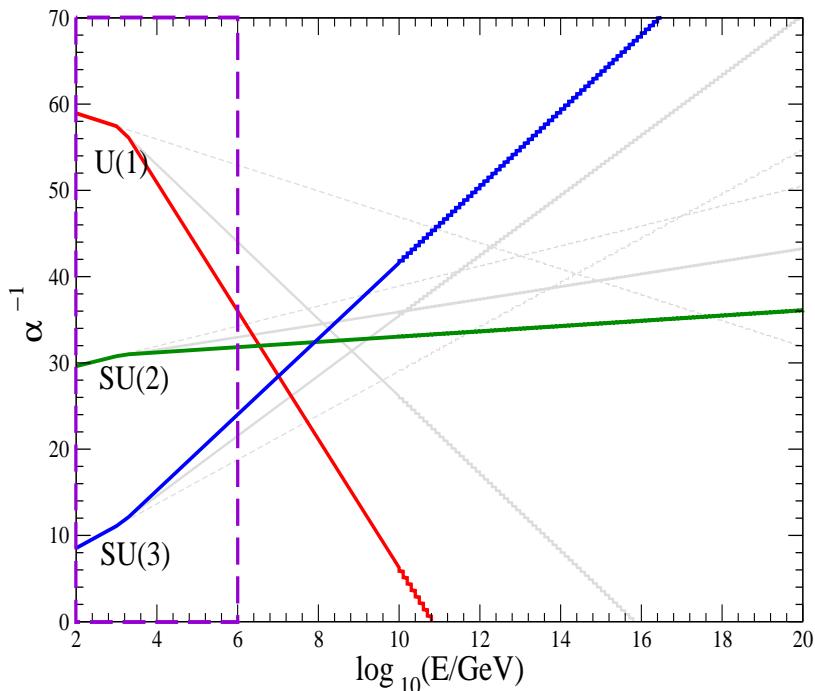
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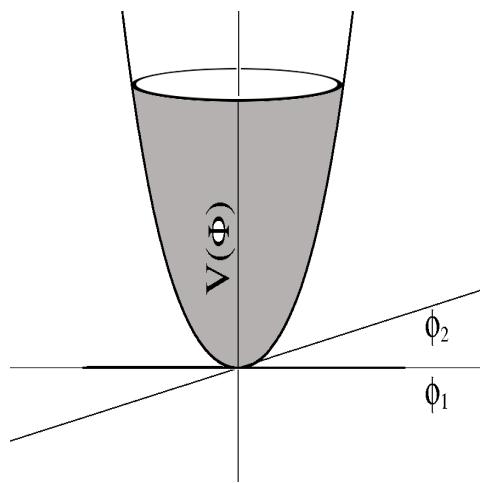
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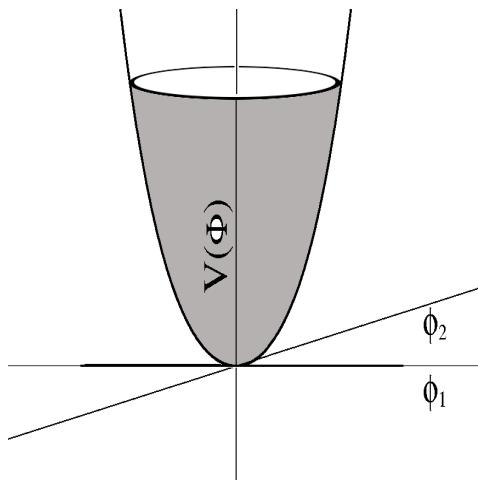
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Standard Model Higgs



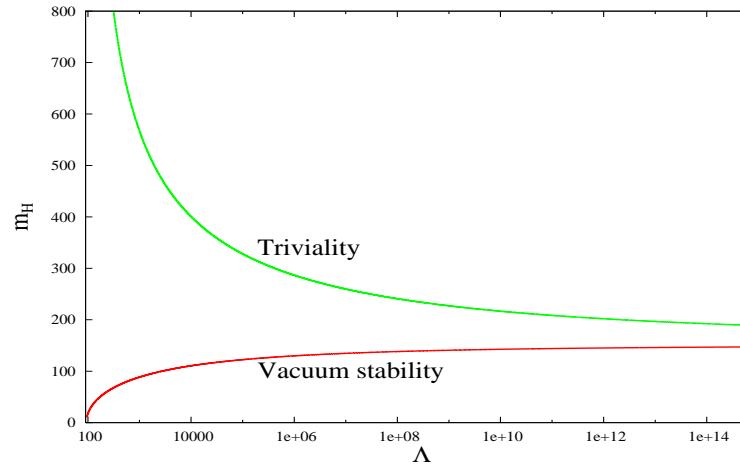
$$V = m^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

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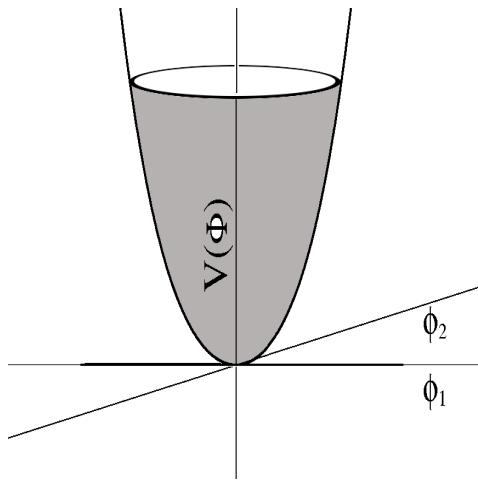
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$$\frac{d\lambda}{d \ln E} = f(g_i, y_l, y_q, \lambda)$$

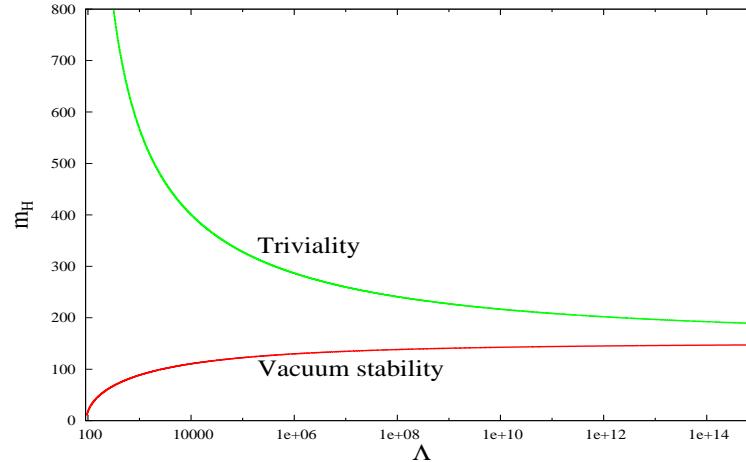


Hambye, Riesselmann; Sirlin, Zucchini; Lindner

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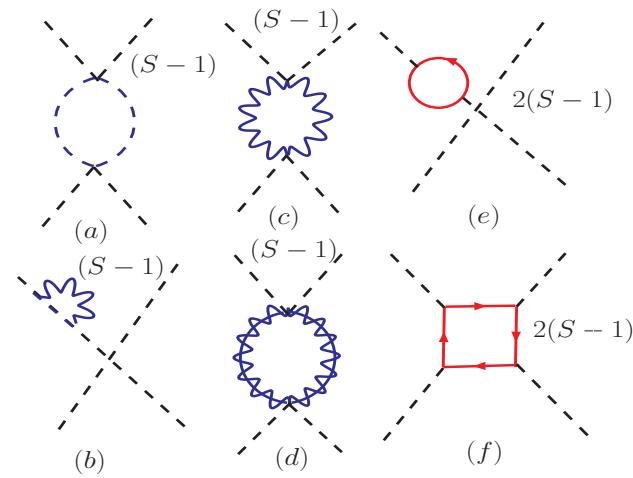
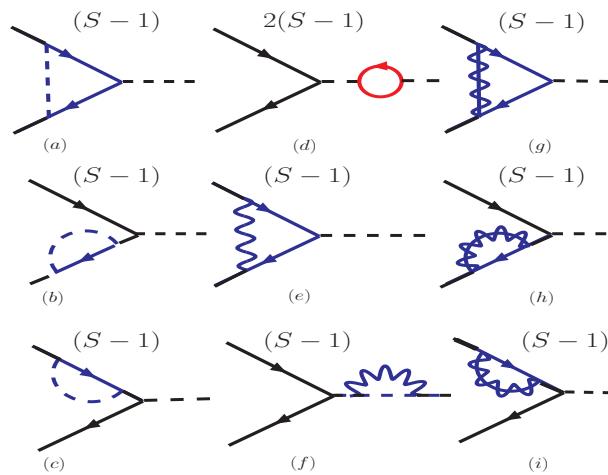


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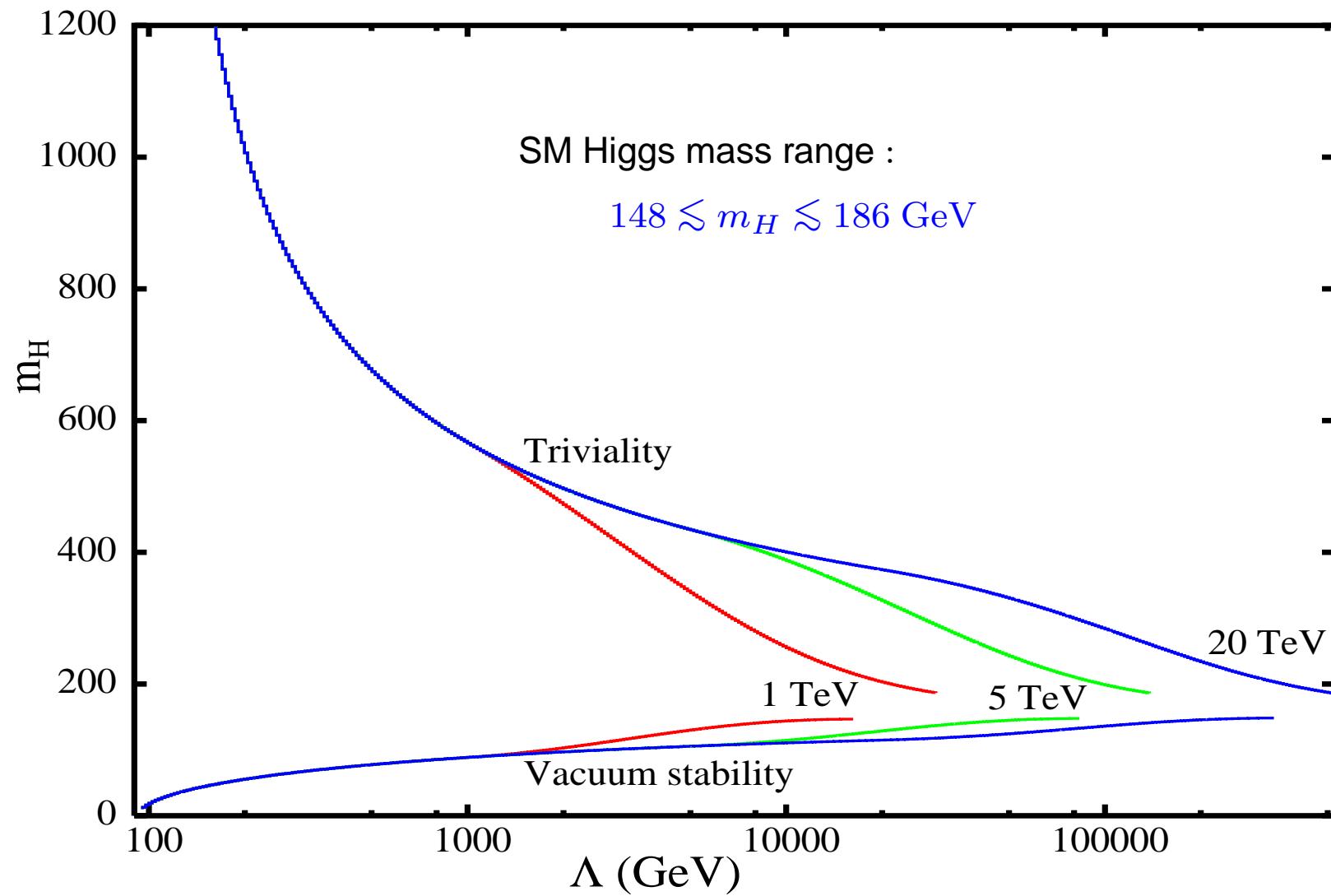


$$V = m^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

Hambye, Riesselmann; Sirlin, Zucchini; Lindner



Higgs Mass



Conclusion

1. Low gauge coupling unification scales can be achieved without introducing non-perturbative gauge couplings. The **unification scale** depends on R , and is approximately given by $\Lambda \sim (25 - 30)/R$.
2. The ‘**triviality**’ and ‘**vacuum stability**’ bounds on the Higgs mass have been studied in the context of power law evolution. This limits the Higgs mass in the range $148 \lesssim m_H \lesssim 186 \text{ GeV}$ at the one-loop level.
3. It is also interesting to discuss the UED extension of the supersymmetric model. We find that if low energy SUSY is realised in Nature, then **the requirement of perturbative gauge coupling unification pushes the inverse radius of compactification all the way up to $\sim 10^{10} \text{ GeV}$** .

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Thank You !

BACK UP

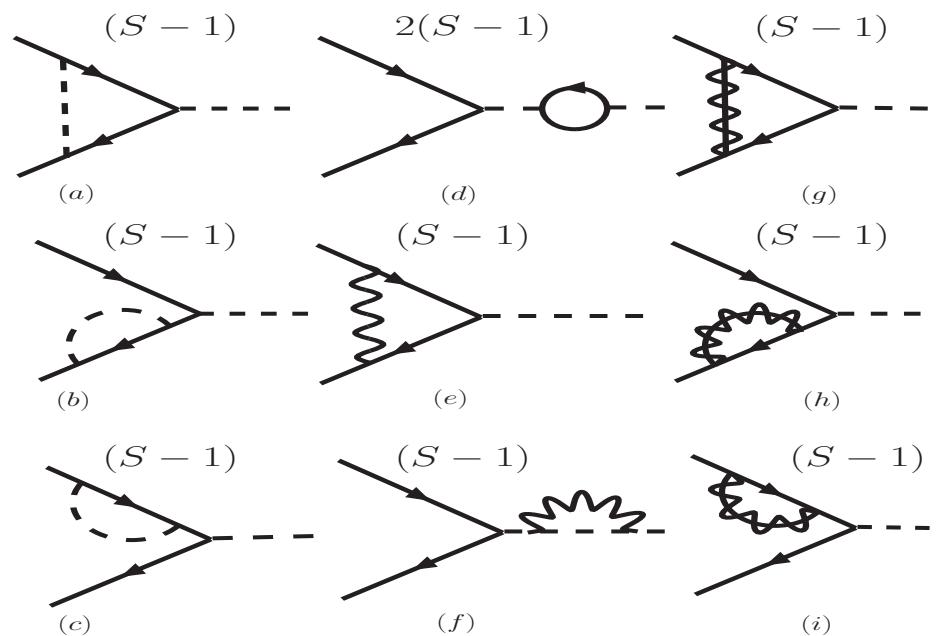
Supersymmetric UED

- gauge coupling evolution for three different regions.
 - $E < M_S \equiv$ SM with the additional complex scalar doublets:
$$b_{1o} = \frac{21}{5}, \quad b_{2o} = -\frac{10}{3}, \quad b_{3o} = -7$$
 - $M_S < E < 1/R$, superpartners of the SM particles will come into play, contribution of the superpartners with the SM ones are :
$$b_{1s} = \frac{33}{5}, \quad b_{2s} = 1, \quad b_{3s} = -3$$
 - $E > 1/R$ KK-modes of each particle will now contribute and contributions from the individual modes are :
$$\tilde{b}_1 = \frac{66}{5}, \quad \tilde{b}_2 = 10, \quad \tilde{b}_3 = 6$$
- Thus, beyond $1/R$, the total contribution is given by
$$b_i^{\text{tot}} = b_{i0} + \Theta(E - M_S) (b_{is} - b_{i0}) + \Theta(E - \frac{1}{R}) (S - 1) \tilde{b}_i$$
- *Perturbative requirement* imposes $R^{-1} \gtrsim 5.0 \times 10^{10}$ GeV.

Yukawa Couplings

$$16\pi^2 \frac{dy_f}{dt} = \beta_{y_f}^{\text{SM}} + \beta_{y_f}^{\text{UED}},$$

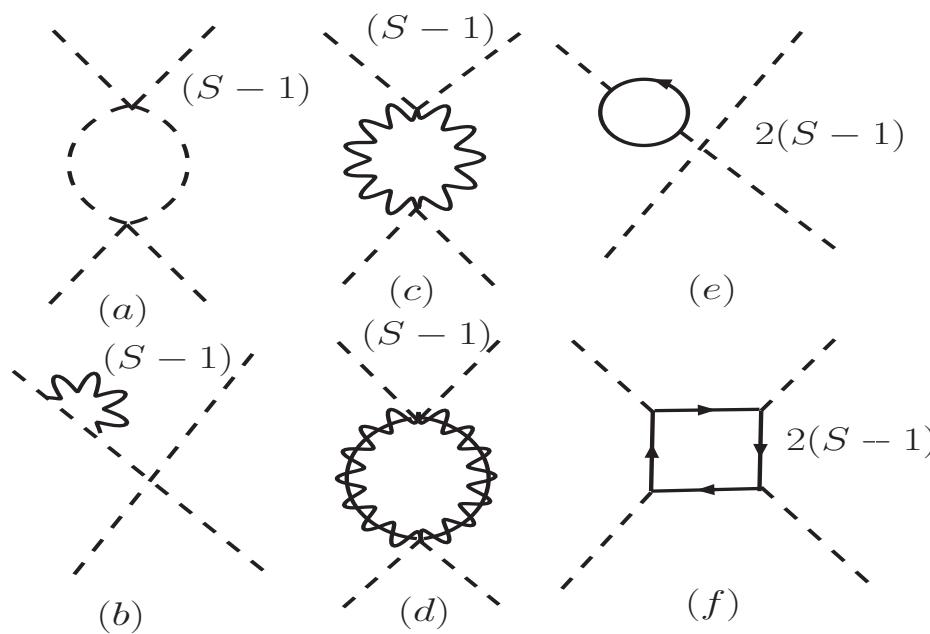
where $t = \ln E$



$$\begin{aligned}\beta_{y_l}^{\text{UED}} &= (S-1) \left[-\left(\frac{21}{8}g_2^2 + \frac{129}{40}g_1^2\right) + \frac{3}{2}y_l^2 \right] y_l + 2(S-1) [Y_l + 3Y_u + 3Y_d] y_l, \\ \beta_{y_u}^{\text{UED}} &= (S-1) \left[-(12g_3^2 + \frac{21}{8}g_2^2 + \frac{9}{8}g_1^2) + \frac{3}{2}(y_u^2 - y_d^2) \right] y_u + 2(S-1) [Y_l + 3Y_u + 3Y_d] y_u, \\ \beta_{y_d}^{\text{UED}} &= (S-1) \left[-(12g_3^2 + \frac{21}{8}g_2^2 + \frac{9}{40}g_1^2) + \frac{3}{2}(y_d^2 - y_u^2) \right] y_d + 2(S-1) [Y_l + 3Y_u + 3Y_d] y_d\end{aligned}$$

with $Y_l = \sum_l y_l^2$, $Y_d = \sum_d y_d^2$, and $Y_u = \sum_u y_u^2$.

Quartic Couplings



$$16\pi^2 \frac{d\lambda}{dt} = \beta_\lambda^{\text{SM}} + \beta_\lambda^{\text{UED}}$$

$$\begin{aligned}\beta_\lambda^{\text{UED}} &= (S - 1) \left[3g_2^4 + \frac{6}{5}g_2^2g_1^2 + \frac{9}{25}g_1^4 - 3\lambda(3g_2^2 + \frac{3}{5}g_1^2) + 12\lambda^2 \right] \\ &+ 2(S - 1) \left[4(Y_l + 3Y_u + 3Y_d)\lambda - 4 \sum_{l,u,d} (y_l^4 + 3y_u^4 + 3y_d^4) \right].\end{aligned}$$