

# Collective flavour transitions of supernova neutrinos

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Rencontres de Moriond EW 2009  
La Thuile, March 13, 2009



# Outline

- ★ Introduction
- ★ Self-interactions in 2-flavor scenario
- ★ Self-interactions in 3-flavour scenario
- ★ Conclusions

This talk is based on work in collaboration with G.L. Fogli, E. Lisi, A. Marrone and A. Mirizzi:

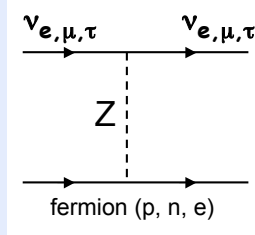
- arXiv: 0808.0807 [hep-ph]
- arXiv: 0812.3031 [hep-ph].

See references therein for credit to previous relevant papers.

# Introduction: neutrino interactions

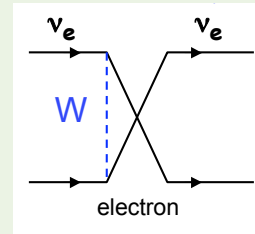
In a core-collapse supernova not only  $\nu$ -matter interactions are important ...

Neutrino-matter interactions:  
MSW flavour changes



all flavours

all neutrino flavors have neutral current (NC) interactions with ordinary matter background

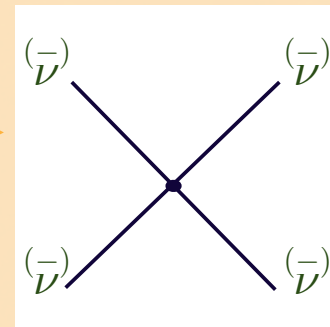


e-flavour only

only  $\nu_e$  has charged current (CC) interactions too

... but also  $\nu - \nu$  interactions

Neutrino-neutrino interactions:  
non-linear flavour conversions



all flavours

when an high density of  $\nu$  is present,  $\nu - \nu$  interactions become not negligible. Self-interactions induce large, non-MSW flavour change. Different neutrino flavours with different energies are coupled in their evolution history.



# Introduction: supernova relevance

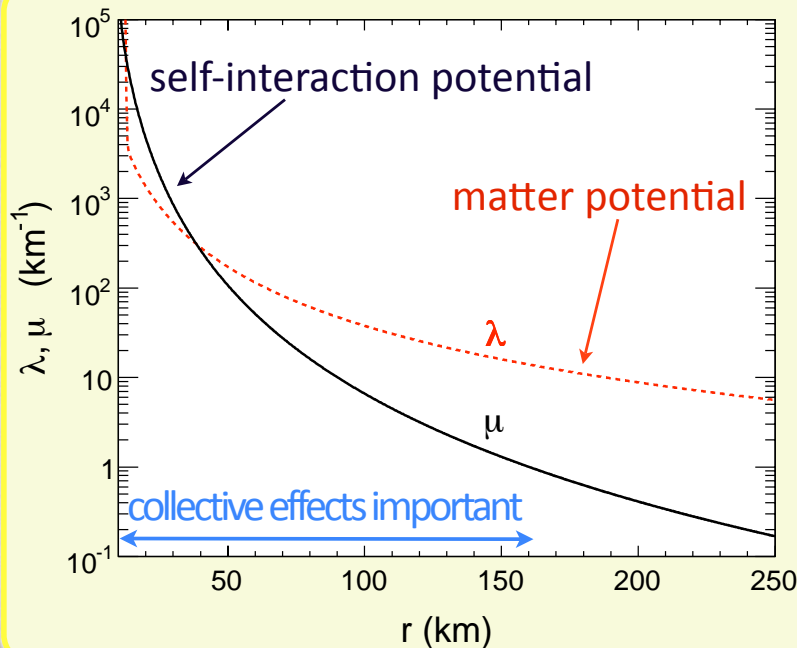
Core-collapse supernovae can help us improving our knowledge about neutrino physics and astrophysics. In fact, as a consequence of neutrino self-interactions, oscillated neutrino fluxes show great sensitivity to:

- ★ the non-zero mixing angle  $\theta_{13}$
- ★ the neutrino mass hierarchy

If the hierarchy is inverted and if  $\theta_{13} \neq 0$ , a full flavour conversion takes place. Otherwise oscillated fluxes are equal to non-oscillated ones.

In the following we use the inverted hierarchy scenario with  $\theta_{13} \neq 0$ .

effects of neutrino self-interactions



We use three main frequencies:

- ★ the self-interaction potential  $\mu = \sqrt{2}G_F(N + \bar{N})$
- ★ the matter potential  $\lambda = \sqrt{2}G_F N_e$
- ★ the vacuum frequency  $\omega = (\Delta m^2/2E)$ .

When

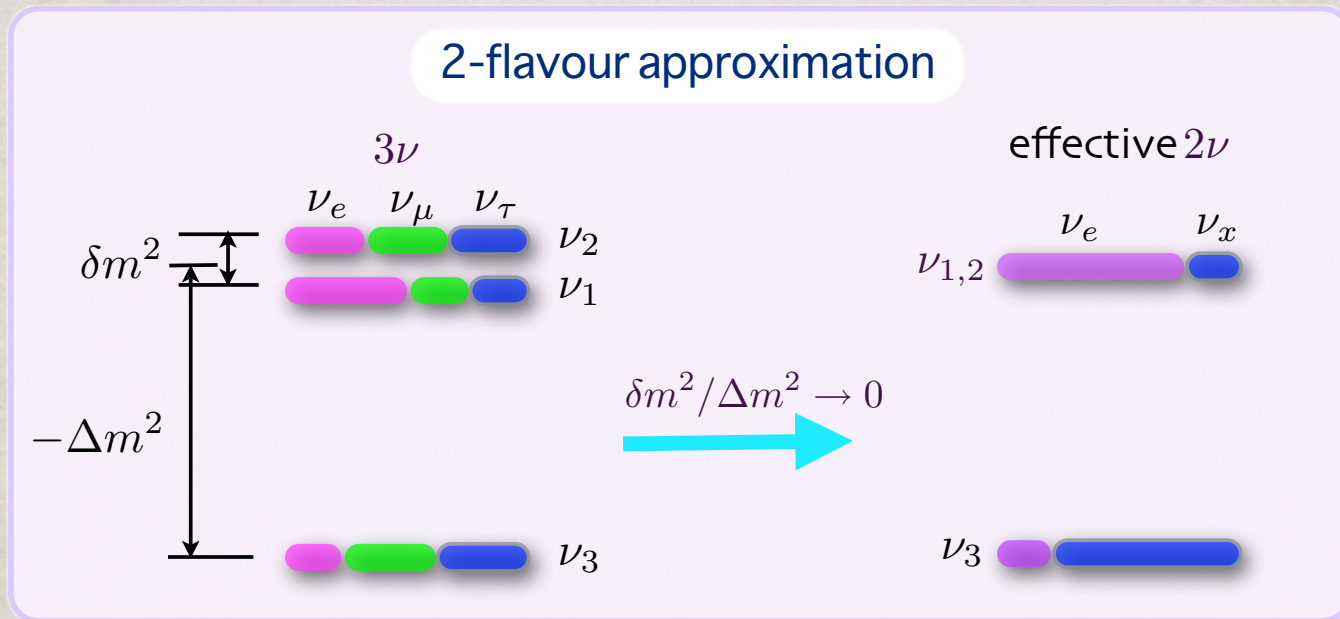
- $\mu > \omega$ : self-interactions dominant [ $r < 100$  km]
- $\lambda \sim \omega$ : MSW effects leading [ $r > 100$  km]
- $\lambda, \mu \ll \omega$ : vacuum oscillations

Self-interaction and matter potentials are chosen for  $t = 5$  s after the core-bounce.

Plot taken from G.L. Fogli, E. Lisi, A. Marrone, A. Mirizzi, I. Tamborra, arXiv: 0808.0807 [hep-ph]

# Self-interactions in 2-flavour scenario

Typical supernova neutrino energies [ $E \sim \mathcal{O}(10)$  MeV] are below threshold for  $\mu$  and  $\tau$  production via CC.  $\nu_\mu$  and  $\nu_\tau$  behave in a similar way, and are often denoted by  $\nu_x$ . As a consequence, in this context one may use the 2-flavour approximation, assuming  $\delta m^2 = 0$ .



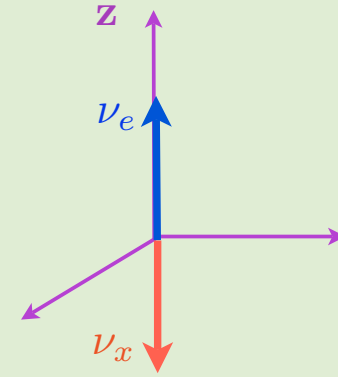
Some papers on collective effects in 2-flavour approximation:

- S. Pastor, G. Raffelt, arXiv: astro-ph/0207281
- H. Duan, G.M. Fuller, Y.Z. Qian, arXiv: astro-ph/0511275
- S. Hannestad, G. Raffelt, G. Sigl, Y.Y.Y. Wong, arXiv: astro-ph/0608695
- H. Duan, G.M. Fuller, J. Carlson, Y.Z. Quian, arXiv: astro-ph/0703776
- G. Raffelt, A.Y. Smirnov, arXiv: hep-ph/0705.1830
- G.L. Fogli, E. Lisi, A. Marrone, A. Mirizzi, arXiv: hep-ph/0707.1998



# Self-interactions in 2-flavour: evolution equations

For each neutrino energy mode  $i$ , decompose the  $2 \times 2$  (anti)neutrino density matrix over Pauli matrices to get the Bloch 3-vector. In the flavour basis, the Bloch vectors associated to  $\nu_e$  and  $\nu_x$  are respectively aligned and anti-aligned with the  $z$  axis.



Considering the Bloch vector's evolution equation for each energy mode, one has to solve a large system of non-linear differential equations:

$$\dot{\mathbf{P}}_i = \mathbf{H}_i \times \mathbf{P}_i = (+\omega_i \mathbf{B} + \lambda \mathbf{z} + \mu \mathbf{D}) \times \mathbf{P}_i$$

$$\dot{\bar{\mathbf{P}}}_i = \bar{\mathbf{H}}_i \times \bar{\mathbf{P}}_i = (-\omega_i \mathbf{B} + \lambda \mathbf{z} + \mu \mathbf{D}) \times \bar{\mathbf{P}}_i$$

vacuum term  
(with  $\mathbf{B}$  function of  
the mixing angle  $\theta_{13}$ )

matter term

self-interaction term with  
 $\mathbf{D} = \sum_i \mathbf{P}_i - \bar{\mathbf{P}}_i$

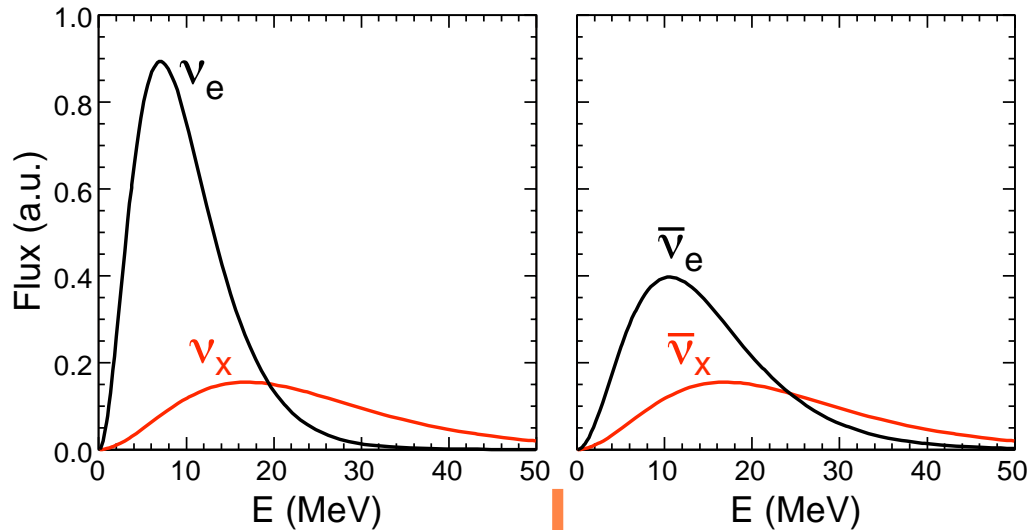
The third component of the Bloch vector ( $|\mathbf{P}_i| = 1$ ) is related to the survival probability:

$$P(\nu_e \rightarrow \nu_e) = \frac{1}{2} \left( 1 + \frac{P_z^f}{P_z^i} \right)$$

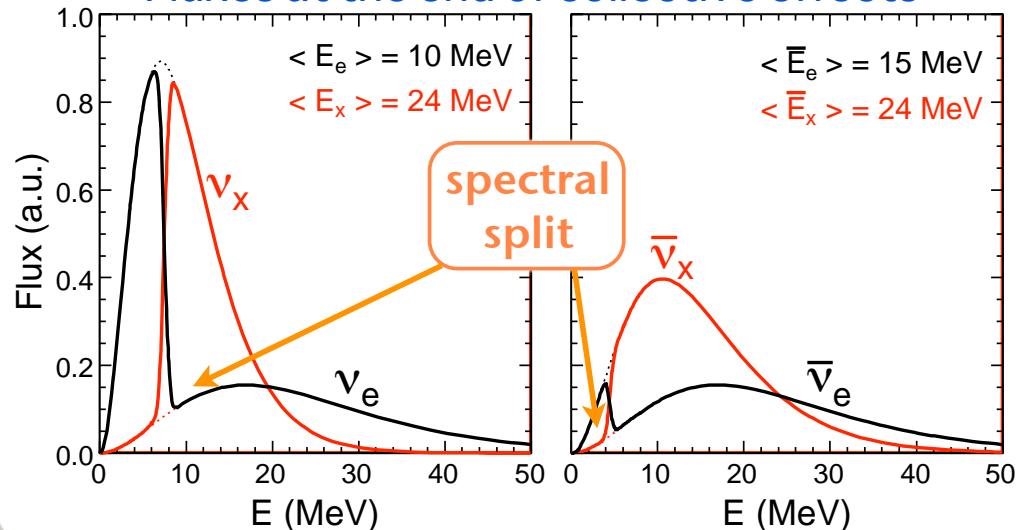
with  $P_z^i$  and  $P_z^f$  the initial and the final value of the third component of  $\mathbf{P}_i$ .

# Self-interactions in 2-flavour: numerical results

Initial fluxes



Fluxes at the end of collective effects



SPECTRAL SPLIT OF  $\nu$   
(AND MAYBE  $\bar{\nu}$ ) FLUXES  
APPEAR TO BE OBSERVABLE  
SIGNATURES OF  
SELF-INTERACTIONS IN  
INVERSE HIERARCHY FOR  
ANY  $\theta_{13} \neq 0$

As a result of collective effects for E greater than certain critical energies ( $E_c$  for  $\nu$  and  $\bar{E}_c$  for  $\bar{\nu}$ ) a full flavour change takes place.

low energy limit ( $E < E_c$ ):

$$F'_e = F_e^0$$

$$F'_x = F_x^0$$

high energy limit ( $E > E_c$ ):

$$F'_e = F_x^0$$

$$F'_x = F_e^0$$

# Self-interactions: 2-flavour vs 3-flavour

- ★ In a core-collapse supernova  $\nu_\mu$  and  $\nu_\tau$  have the same behavior because of typical energies. A two-flavour approximation give us a qualitative explanation of neutrino spectra features.
- ★ In inverted hierarchy and with  $\theta_{13} \neq 0$ , above certain critical energies a full flavour swap takes place. This is a robust signature of collective effects.
- ★ If the hierarchy is direct or if  $\theta_{13} = 0$ , the oscillated fluxes are equal to the non-oscillated ones.

But if we consider a three-flavour scenario, how do these features change?

We develop a three flavour scenario analysis including:

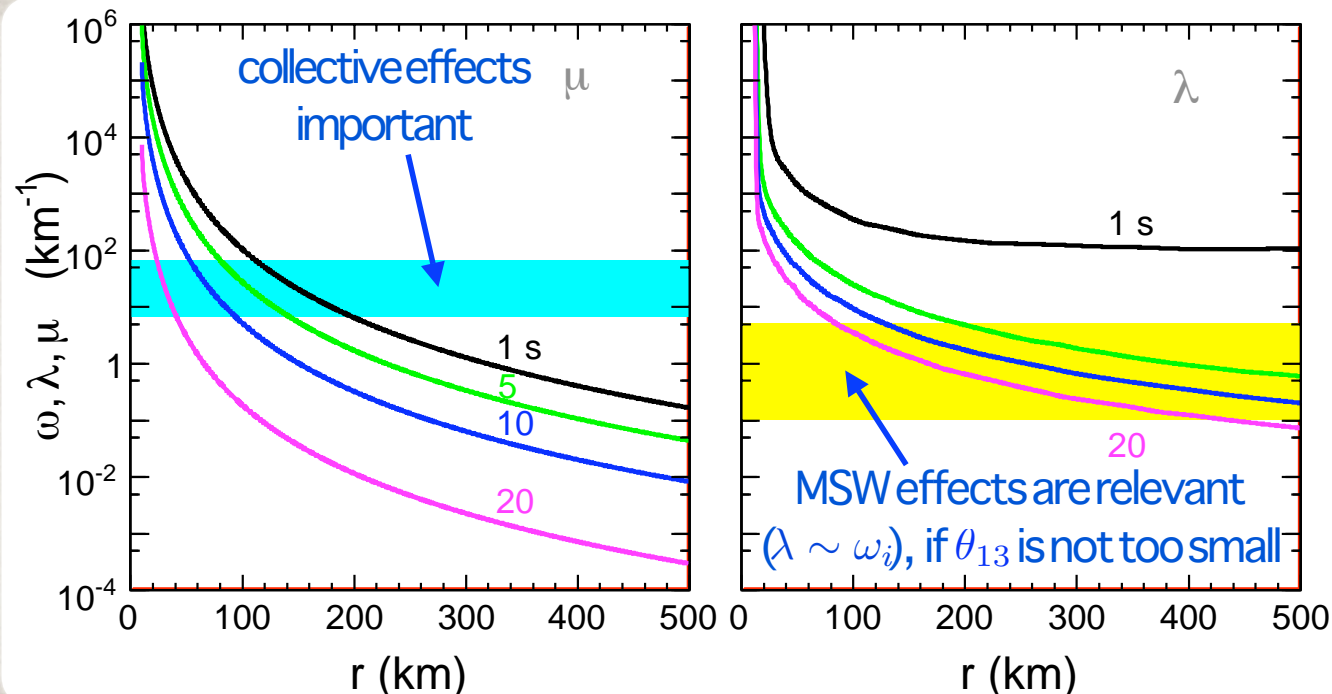
- ★ the mixing angles  $\theta_{12}$ ,  $\theta_{13}$  and  $\theta_{23}$
- ★ the mass differences  $\Delta m^2$  and  $\delta m^2$
- ★ 1-loop  $\nu_\mu - \nu_\tau$  matter interaction potential.

We consider the evolution for different times after the core-bounce, and check the evolution for the three flavour separately.



# Self-interactions in 3-flavour scenario

We consider four different times ( $t = 1, 5, 10, 20$  s) after the core-bounce.



We expect that:

- ★ collective effects take place at different radii for each  $t$  after the core-bounce
- ★ MSW effects take place after collective ones

We choose a small value for  $\theta_{13}$  ( $\sin^2 \theta_{13} = 10^{-6}$ ). In this way MSW effects are negligible for each  $t$  after the core-bounce.

Some papers on collective effects in 3-flavour approximation:

- A. Esteban-Pretel, S. Pastor, R. Tomas, G. Raffelt, G. Sigl, arXiv: astro-ph/0712.1137
- B. Dasgupta, A. Dighe, arXiv: hep-ph/0712.3798
- H. Duan, G.M. Fuller and Y.Z. Qian, arXiv: 0801.1363 [hep-ph]
- B. Dasgupta, A. Dighe, A. Mirizzi and G.G. Raffelt, arXiv: 0801.1660 [hep-ph]

Plot taken from G.L. Fogli, E. Lisi, A. Marrone, I. Tamborra, arXiv: 0812.3031 [hep-ph]

# Self-interactions in 3-flavour: evolution equations

For each neutrino mode  $i$ , decompose the  $3 \times 3$  (anti)neutrino density matrix over Gell-Mann matrices to get the Bloch 8-vector.

Self-interactions may be explained in terms of evolution equations for the Bloch vectors of  $\nu$  ( $\mathbf{P}_i$ ) and  $\bar{\nu}$  ( $\bar{\mathbf{P}}_i$ ):

$$\begin{aligned}\dot{\mathbf{P}}_i &= [+(\omega_{L,i}\mathbf{B}_L + \omega_{H,i}\mathbf{B}_H) + \mathbf{v} + \mu\mathbf{D}] \times \mathbf{P}_i \\ \dot{\bar{\mathbf{P}}}_i &= [-(\omega_{L,i}\mathbf{B}_L + \omega_{H,i}\mathbf{B}_H) + \mathbf{v} + \mu\mathbf{D}] \times \bar{\mathbf{P}}_i\end{aligned}$$

two vacuum terms  
[with  $\omega_{H,i} = (\Delta m^2/2E)$  and  $\omega_{L,i} = (\delta m^2/2E)$ ]

matter term

self-interaction term with  
 $\mathbf{D} = \sum_i \mathbf{P}_i - \bar{\mathbf{P}}_i$

We numerically solve a large system of non-linear differential equations<sup>\*</sup>.

A linear combination of  $P_3$  and  $P_8$  ( $|\mathbf{P}_i| = 2/\sqrt{3}$ ) is related to  $P(\nu_e \rightarrow \nu_e)$  and is the analogous of the third component in the two-flavour limit:

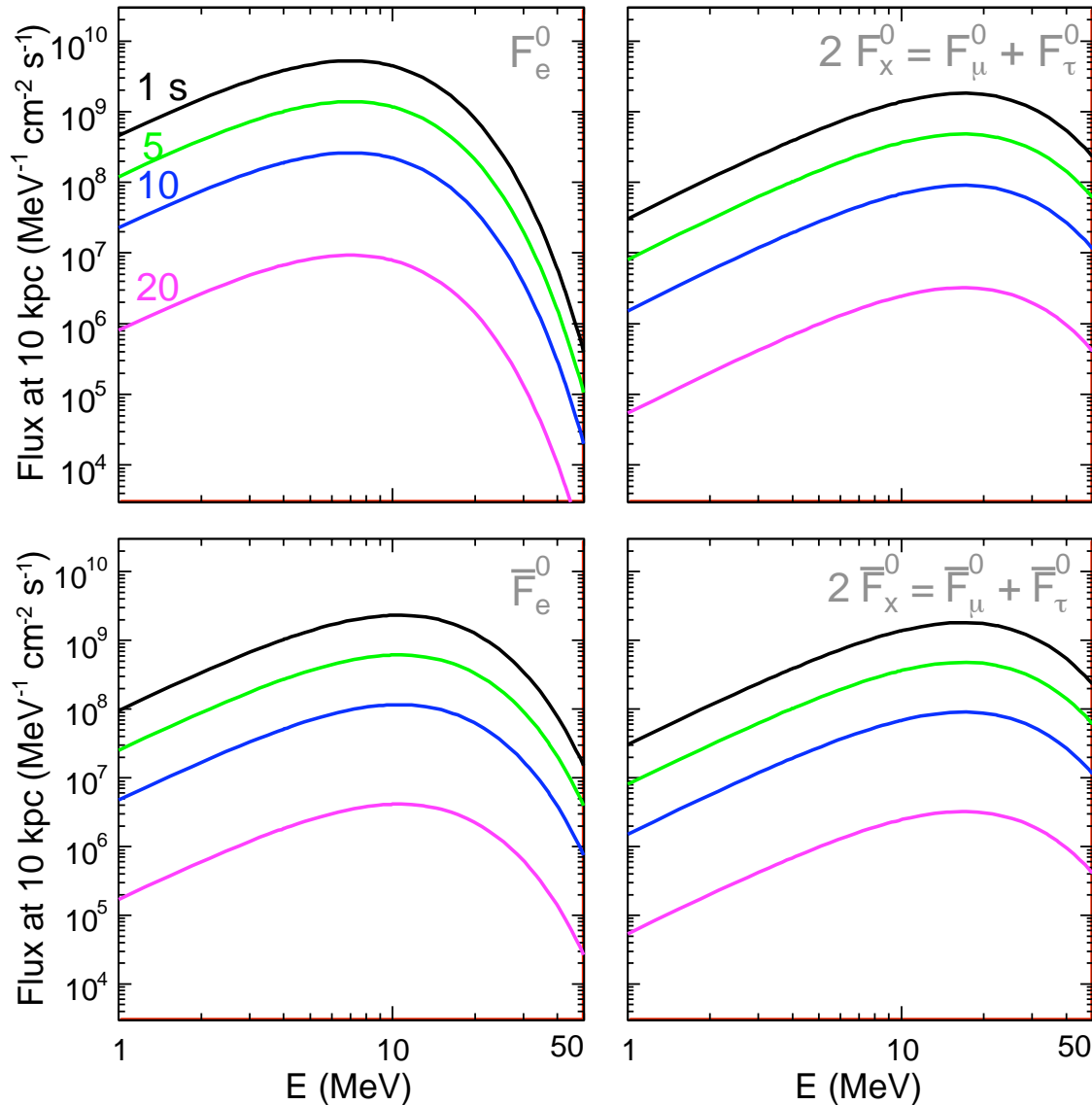
$$P_z \rightarrow P_3 + \frac{P_8}{\sqrt{3}}$$

<sup>\*</sup> G.L. Fogli, E. Lisi, A. Marrone, I. Tamborra, arXiv: 0812.3031 [hep-ph]



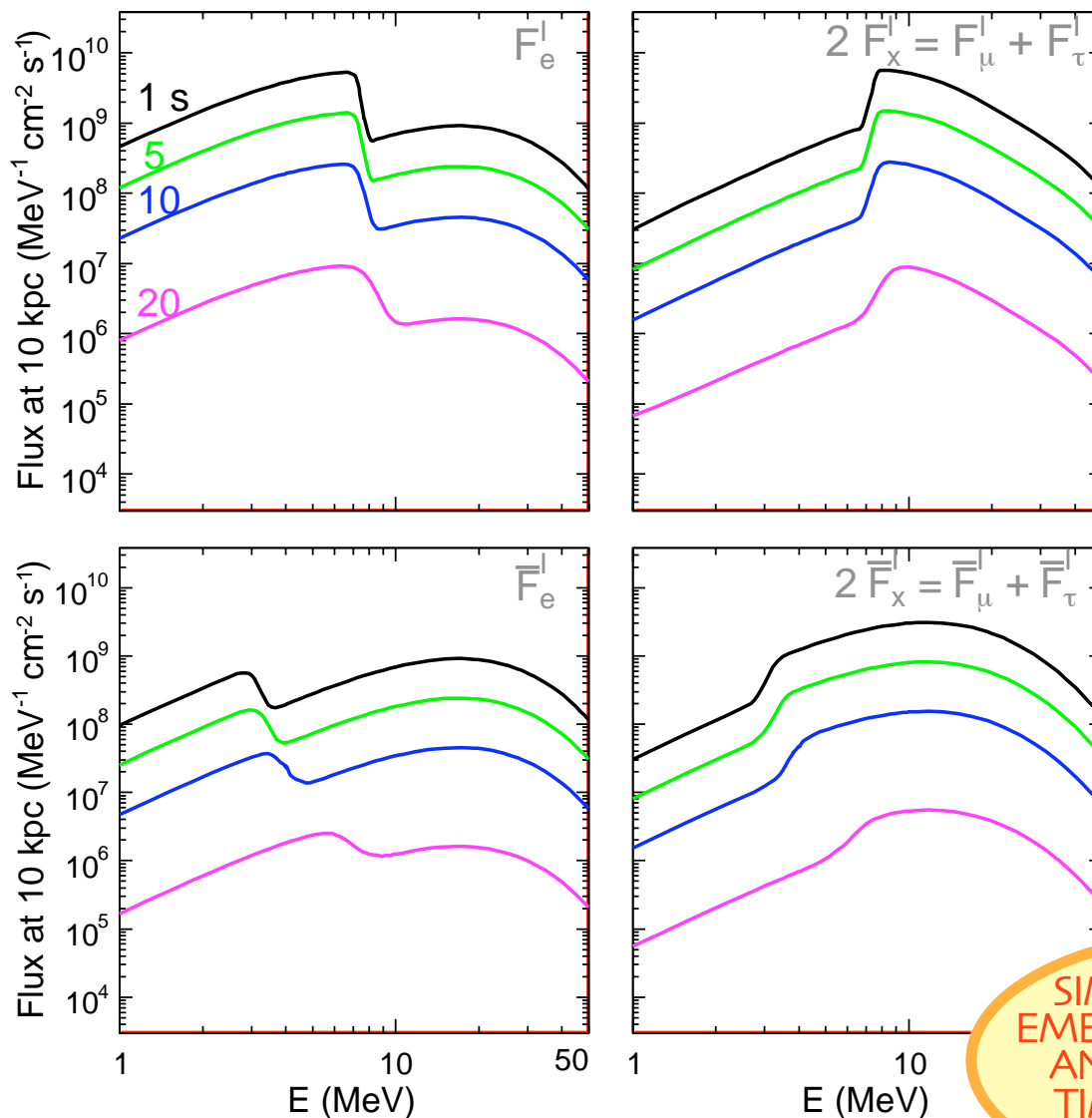
# Self-interactions in 3-flavour: numerical results

Initial fluxes



# Self-interactions in 3-flavour: numerical results

## Fluxes at the end of collective effects

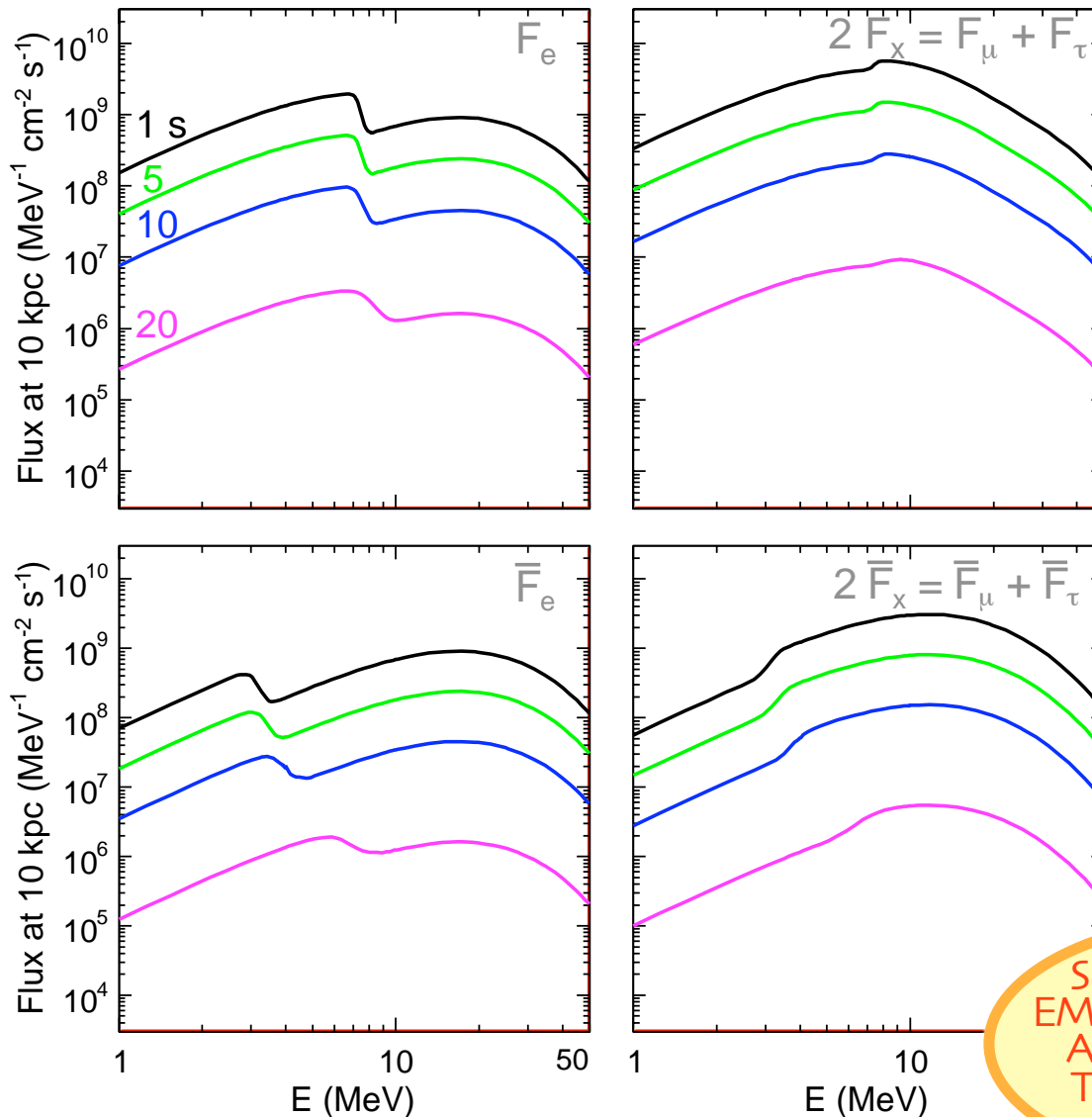


SIMILAR FEATURES  
EMERGE AT ALL TIMES,  
AND THUS ALSO IN  
TIME INTEGRATED  
SPECTRA.



# Self-interactions in 3-flavour: numerical results

## Fluxes at the Earth



SIMILAR FEATURES  
EMERGE AT ALL TIMES,  
AND THUS ALSO IN  
TIME INTEGRATED  
SPECTRA.

# Self-interactions in 3-flavour: $\nu_e - \nu_x$ oscillated fluxes

In the three flavour case the oscillated fluxes can be expressed as simple combinations of the non oscillated ones:

low energy limit ( $E < E_c$ ):

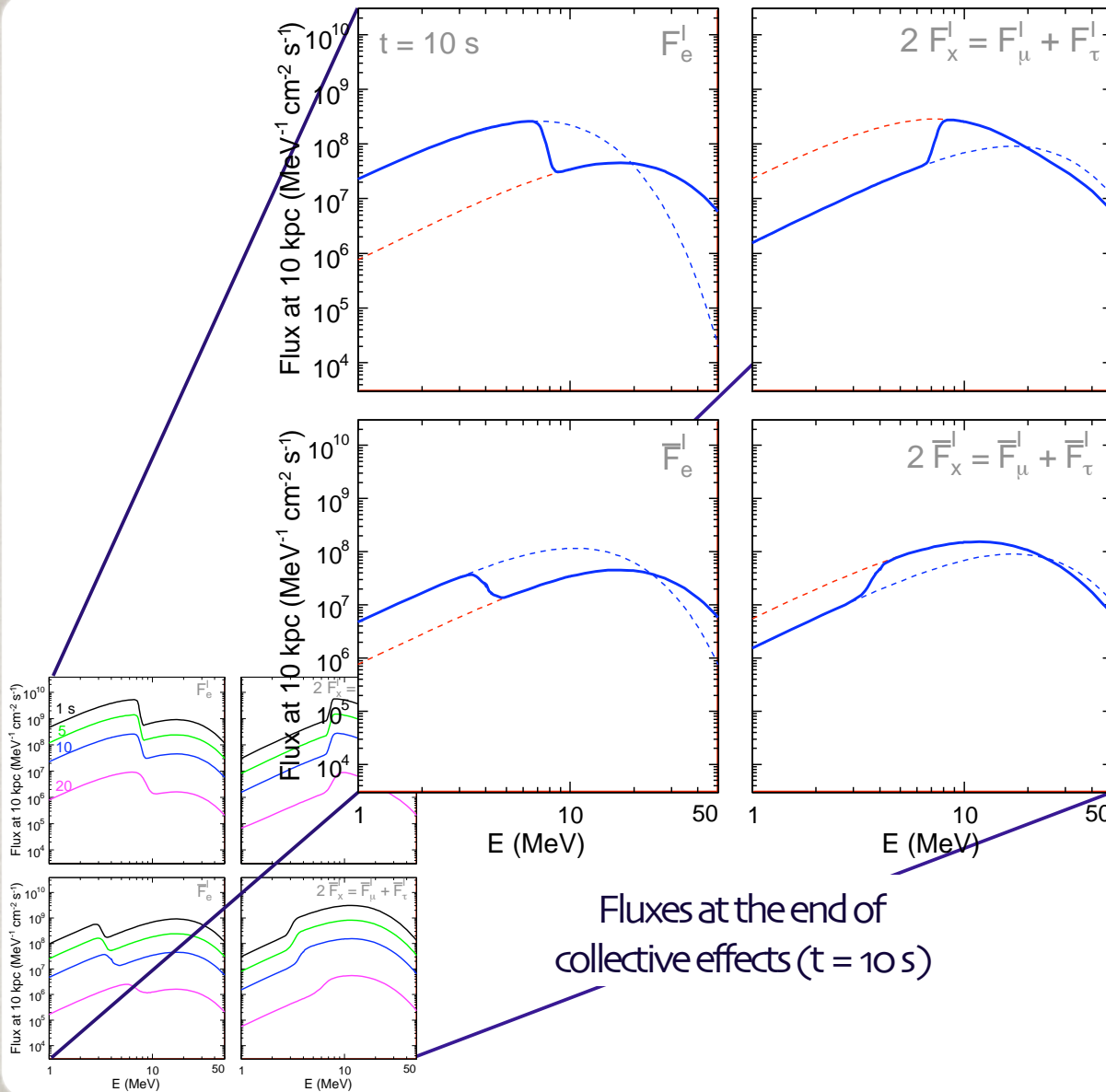
$$F'_e = F_e^0$$

$$2F'_x = 2F_x^0$$

high energy limit ( $E > E_c$ ):

$$F'_e = F_x^0$$

$$2F'_x = F_e^0 + F_x^0$$





# Self-interactions in 3-flavour: $\nu_\mu - \nu_\tau$ oscillated fluxes

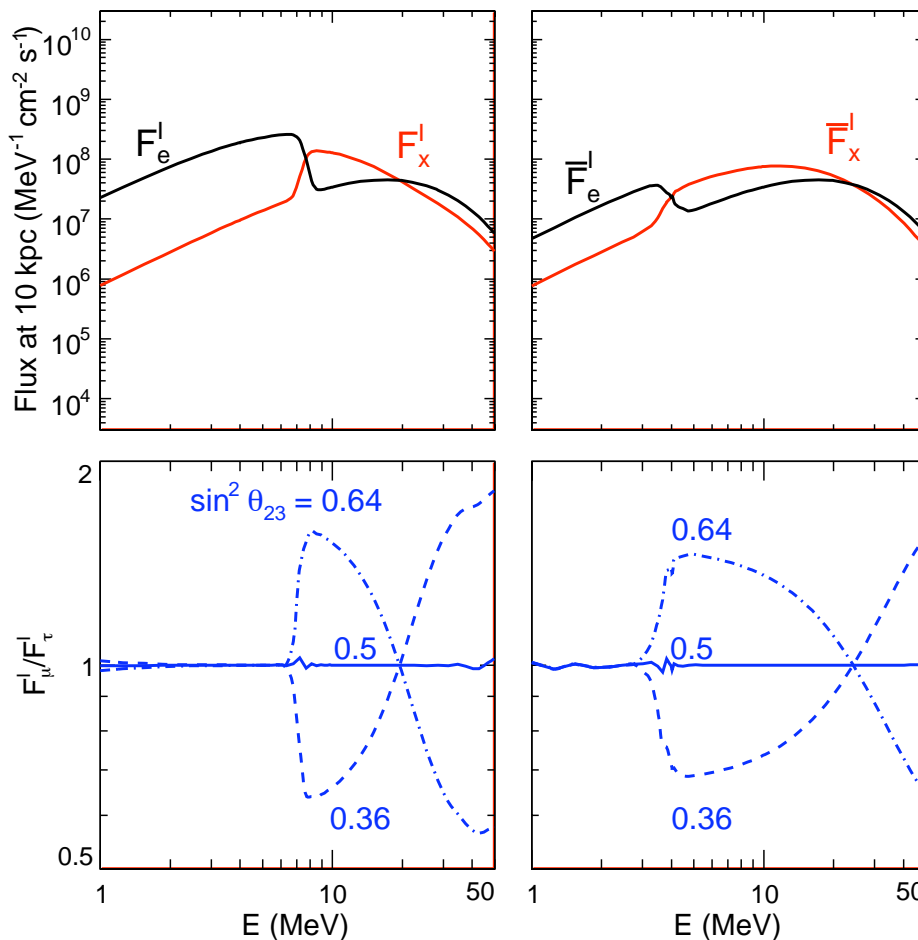
We explore the  $\nu_\mu - \nu_\tau$  mixing for different values of  $\theta_{23}$ .  $\nu_x$  oscillated flux is not sensitive to  $\theta_{23}$ .

When  $\theta_{23}$

- ★ belongs to the first octant, we have a leading  $\nu_e \rightarrow \nu_\tau$  conversion
- ★ belongs to the second one, we have a leading  $\nu_e \rightarrow \nu_\mu$  conversion
- ★ is maximal ( $\theta_{23} = \pi/4$ ),  $\nu_\mu$  and  $\nu_\tau$  fluxes are exactly equal.

This different behavior between  $\nu_\mu$  and  $\nu_\tau$  fluxes, although is not observable nowadays, could be detected in a future supernova explosion.

Fluxes at the end of collective effects ( $t = 10$  s)



# Self-interactions: 2-flavour vs 3-flavour's remarks

- ★ The three flavour case is similar, but does not reduce to the two flavour one.
- ★ In particular the coupled evolution equations are sensitive to the absolute luminosity for each flavour. It makes a difference if the total neutrino-luminosity is distributed on two or three flavours.

	$2\nu$	$3\nu$
low energy limit ( $E < E_c$ ):	$\begin{aligned} F'_e &= F_e^0 \\ F'_x &= F_x^0 \end{aligned}$	$\begin{aligned} F'_e &= F_e^0 \\ 2F'_x &= 2F_x^0 \end{aligned}$
high energy limit ( $E > E_c$ ):	$\begin{aligned} F'_e &= F_x^0 \\ F'_x &= F_e^0 \end{aligned}$	$\begin{aligned} F'_e &= F_x^0 \\ 2F'_x &= F_e^0 + F_x^0 \end{aligned}$

- ★ The split features are the same at different times. This may be more “visible” collective effects in the next supernova explosion.
- ★ Minor three-flavour features are basically unobservable.



# Conclusions

- ★ Neutrino-neutrino interactions are not negligible when neutrino density is high, like in core-collapse supernovae.
- ★ Collective effects are important to improve our knowledge about the neutrino mass hierarchy and the mixing angle  $\theta_{13}$ .
- ★ In direct hierarchy or if  $\theta_{13} = 0$ , the oscillated spectra are equal to the non-oscillated ones.
- ★ In inverted hierarchy and for any  $\theta_{13} \neq 0$ :
  - spectral split takes place with the same features at different times after the core-bounce. This feature may make collective effects more evident in the next supernova explosion.
  - in a two-flavour scenario a full flavour swap takes place
  - in a three-flavour scenario oscillated fluxes can be expressed as simple combinations of non-oscillated ones. An “effective” two flavour reduction is not possible because all three flavour evolve.
  - $\nu_x$  is not sensitive to different values of  $\theta_{23}$ .
  - the effect of  $\delta m^2 \neq 0$  and 1-loop  $\nu_\mu - \nu_\tau$  corrections are negligible.

Waiting for the  
next supernova  
explosion...thanks for  
your attention!



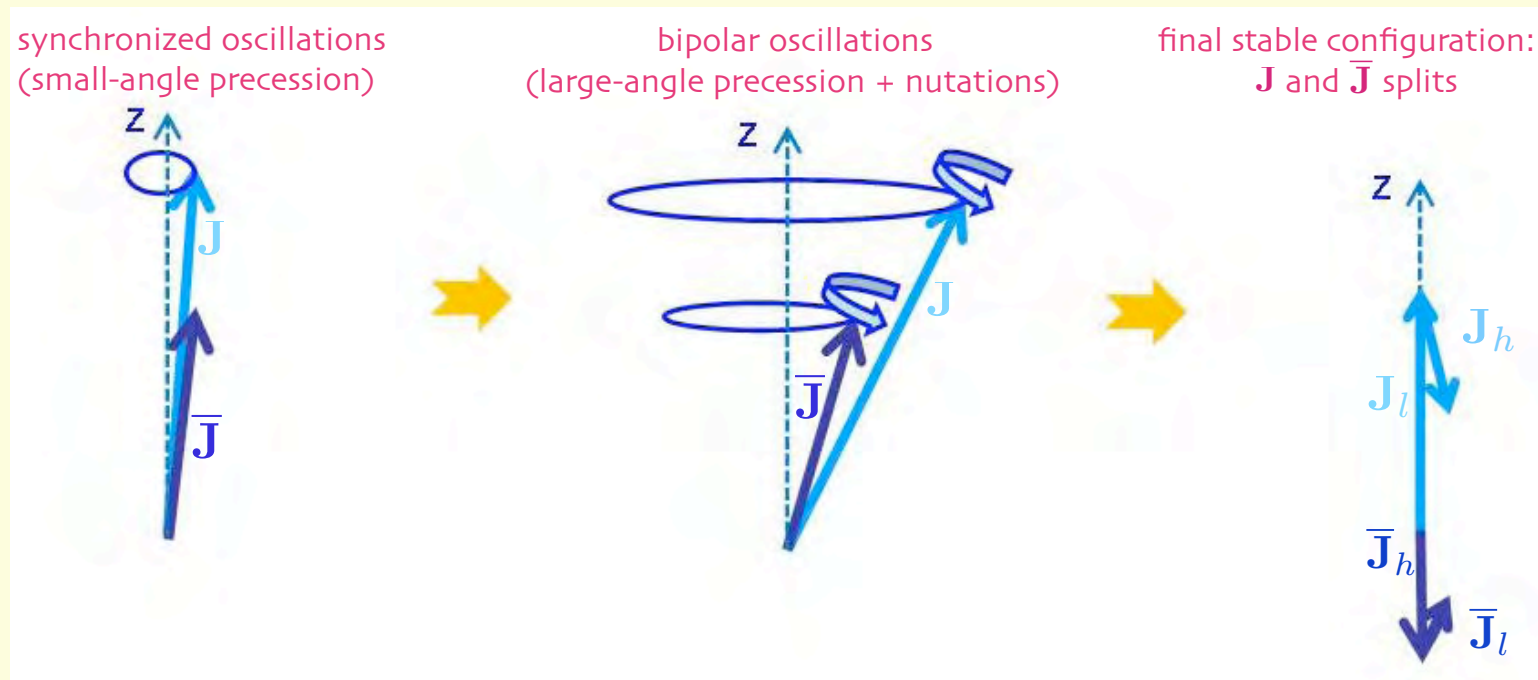


Backup slides

# Self-interactions in 2-flavour: global evolution

When the collective effects are dominant, the alignment approximation is valid: the Bloch vectors are closely pinned to their sum ( $\mathbf{P}_i \parallel \mathbf{J}$  and  $\bar{\mathbf{P}}_i \parallel \bar{\mathbf{J}}$ ). The flavour evolution can be understood in terms of the global vectors  $\mathbf{J} = \sum_i \mathbf{P}_i$  and  $\bar{\mathbf{J}} = \sum_i \bar{\mathbf{P}}_i$ .

At the end, alignment approximation is no longer valid (split).

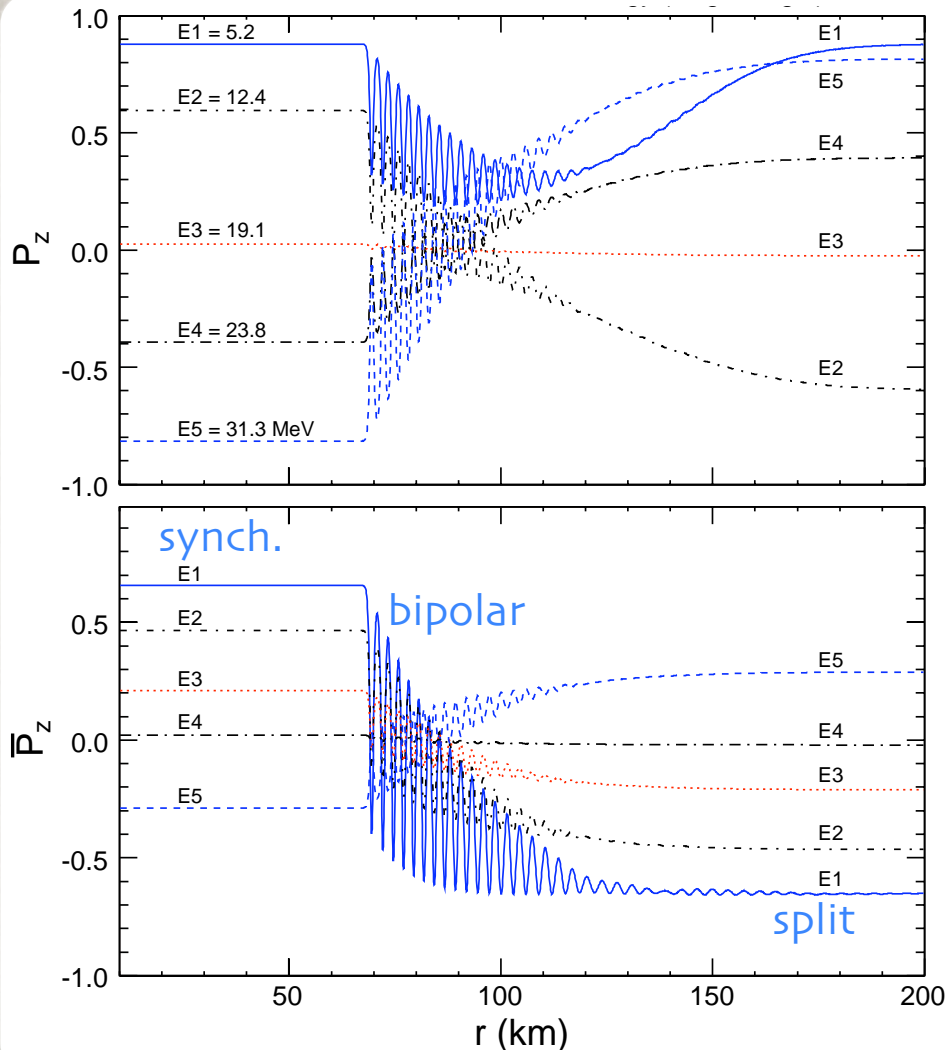


$\theta_{13}$  sets initial misalignment of  $\mathbf{J}$  and  $\bar{\mathbf{J}}$  with vertical. Its specific value is not much relevant (provided that  $\theta_{13} > 0$ ). Only for  $\theta_{13} = 0$  initial conditions are frozen.



# Self-interactions in 2-flavour: spectral split

In terms of the third components of the Bloch vectors at different energies, we have:



neutrinos

For  $E > E_c (\simeq 7 \text{ MeV}) : P_z^f = -P_z^i$

full flavour conversion

For  $E < E_c (\simeq 7 \text{ MeV}) : P_z^f = P_z^i$

absence of flavour conversion

antineutrinos

For  $E > \bar{E}_c (\simeq 4 \text{ MeV}) : \bar{P}_z^f = -\bar{P}_z^i$

full flavour conversion

# Self-interactions in 2-flavour: spectral split

## Neutrino spectral split

- ★ The alignment approximation is not able to explain the neutrino spectral split.
- ★ The critical energy  $E_c$  is predicted using the fact that  $\#\nu_e - \#\bar{\nu}_e = \text{const.}$
- ★ Alternatively, the neutrino spectral split can be explained using the **adiabatic hypothesis**.

According to this hypothesis, when  $\mu$  slowly decreases:

$$\mathbf{P}_i || \mathbf{H} \quad \text{and} \quad \overline{\mathbf{P}}_i || \overline{\mathbf{H}}$$

It is applicable to the end of the collective effects.

## Antineutrino spectral split (G.L. Fogli, E. Lisi, A. Marrone, A. Mirizzi, I. Tamborra, arXiv: 0808.0807 [hep-ph])

- ★ The antineutrino split is related to the breaking of the global alignment
- ★ The antineutrino split depends on the adiabatic hypothesis violation [it happens at low energies when  $\mu \sim \omega$ ]
- ★ The critical energy  $\overline{E}_c$  is a function of the matter potential and the mixing angle.