Neutrino Yukawa couplings and signatures at future colliders

Mario Kadastik, Martti Raidal, Liis Rebane

NICPB

Moriond EW, 2009

The model and new particles

To account for the neutrino masses we extend SM with a simple scalar triplet: $(\Phi^0, \Phi^{\pm}, \Phi^{\pm\pm})$ with quantum numbers (3,2).
The piece of the lagrangian of interest:

$$\mathcal{L}_{\text{lepton}} = (Y_\Phi)_{ij}[\Phi^0 \nu_i \nu_j + \Phi^{\pm} \frac{\nu_i \ell_j^{\mp} + \ell_i^{\mp} \nu_j}{\sqrt{2}} + \Phi^{\pm\pm} \ell_i^{\mp} \ell_j^{\mp}] + h.c.,$$

The Yukawa couplings in turn are related to neutrinos:

$$(\mathcal{M}_\nu)_{ij} = (Y_\Phi)_{ij} \frac{\mu v^2}{m_\Phi^2}$$

where μ is the triplet coupling to SM Higgs and v the SM Higgs vev.

Relation to neutrinos

The branching ratios are defined as relative decay widths among all possible decay channels:

$$BR_{ij} = \frac{\Gamma_{ij}}{\sum_{kl} \Gamma_{kl} + \Gamma_{WW}}$$

The decay width in turn gives the relation to the neutrino sector through the Yukawa coupling:

$$\Gamma_{ij} \equiv \Gamma(\Phi^{\pm\pm} \to \ell_i^{\pm} \ell_j^{\pm}) = \begin{cases} \frac{1}{8\pi} |(Y_\Phi)_{ii}|^2 m_{\Phi^{\pm\pm}} & i = j, \\ \frac{1}{4\pi} |(Y_\Phi)_{ij}|^2 m_{\Phi^{\pm\pm}} & i \neq j, \end{cases}$$

Through the above relation the $\Phi^{\pm\pm}$ branching ratios are heavily dependent on the neutrino parameters e.g.:

$$(BR_\Phi)_{ij} = f_{ij}\,(\theta_{13}, \theta_{23}, \theta_{12}, \delta, \alpha_1, \alpha_2, \text{sign}(\Delta m_{23}), m_0)$$

We can't use BR-s directly, but we can use their relations:

$$\frac{\mathrm{BR}_{ij}}{\mathrm{BR}_{kl}} = \frac{\Gamma_{ij}}{\Gamma_{kl}} = \frac{N_{kl} |(Y_\Phi)_{ij}|^2}{N_{ij} |(Y_\Phi)_{kl}|^2} \qquad N_{ij} = \begin{cases} 1, & i \neq j \\ 2, & i = j \end{cases}$$

That limits us to 5 possible equations with 7 unknown variables, so we use tri-bi-maximal mixing as a guideline:

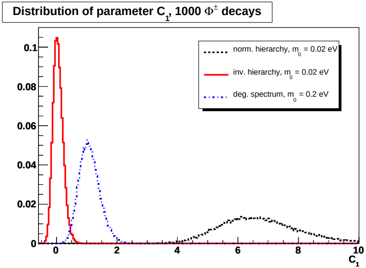
$$\sin^2\theta_{12} = 1/3, \quad \sin^2\theta_{23} = 1/2, \quad \sin^2\theta_{13} = 0$$

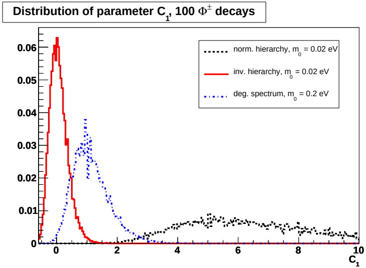
Mass hierarchy determination

- Through combining the $\mu\mu$, $\mu\tau$, ee and $e\mu$ channel branching ratios we can define a characteristic dimensionless constant:

$$C_1 \equiv \frac{2\mathrm{BR}_{\mu\mu} + \mathrm{BR}_{\mu\tau} - \mathrm{BR}_{ee}}{\mathrm{BR}_{ee} + \mathrm{BR}_{e\mu}} = \frac{-m_1^2 + m_2^2 + 3m_3^2}{2m_1^2 + m_2^2}.$$

- This dimensionless variable is independent of the phases and uniquely determines the neutrino hierarchy:
  - $C_1 > 1$ – normal mass hierarchy,
  - $C_1 < 1$ – inverted mass hierarchy,
  - $C_1 \approx 1$ – degenerate masses.





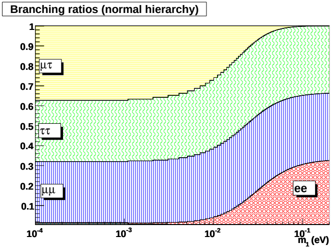
Lowest neutrino mass determination

- For the normal mass hierarchy:

$$m_2^2 = m_1^2 + \Delta m_{sol}^2$$

$$m_3^2 = m_1^2 + \Delta m_{sol}^2 + \Delta m_{atm}^2$$

$$m_1^2 = \frac{\Delta m_{sol}^2(4 - C_1) + 3\Delta m_{atm}^2}{3(C_1 - 1)}$$

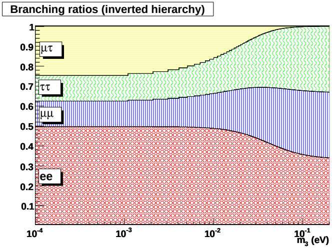


- For the inverted mass hierarchy:

$$m_2^2 = m_3^2 + \Delta m_{atm}^2$$

$$m_1^2 = m_3^2 + \Delta m_{atm}^2 - \Delta m_{sol}^2$$

$$m_3^2 = \frac{\Delta m_{sol}^2(1 + 2C_1) - 3C_1\Delta m_{atm}^2}{3(C_1 - 1)}$$



Determination of phase difference

We define again a dimensionless variable:

$$C_2 \equiv \frac{\mathrm{BR}_{e\mu}}{\mathrm{BR}_{ee}} = \frac{2(m_1^2 + m_2^2 - 2m_1 m_2 \cos\Delta\alpha)}{4m_1^2 + m_2^2 + 4m_1 m_2 \cos\Delta\alpha}$$

For the normal hierarchy $\Delta\alpha$ becomes:

$$\cos\Delta\alpha = \frac{(4 - 5C_2)m_1^2 + (2 - C_2)\Delta m_{sol}^2}{4(1 + C_2)m_1\sqrt{m_1^2 + \Delta m_{sol}^2}}$$

For the inverted however:

$$\cos\Delta\alpha = \frac{2(2C_2 - 1)\Delta m_{sol}^2 + (4 - 5C_2)(\Delta m_{atm}^2 + m_3^2)}{4(1 + C_2)\sqrt{(\Delta m_{atm}^2 + m_3^2)(m_3^2 + \Delta m_{atm}^2 - \Delta m_{sol}^2)}}$$

which can be approximated as

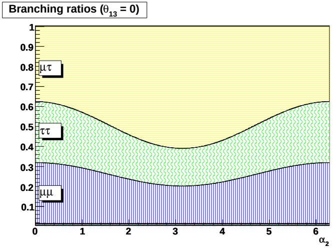
$$\cos\Delta\alpha = \frac{4 - 5C_2}{4(1 + C_2)} + \mathcal{O}\left(\frac{\Delta m_{sol}^2}{\Delta m_{atm}^2}\right)$$

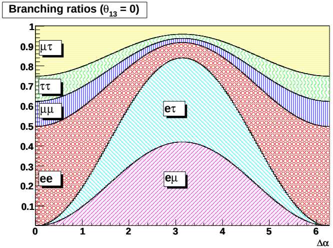
To get the Majorana phases we can then use the following equation:

$$C_3 \equiv \frac{2\mathrm{BR}_{\mu\mu} - \mathrm{BR}_{\mu\tau}}{\mathrm{BR}_{ee} + \mathrm{BR}_{e\mu}} = \frac{2m_3(\cos\alpha_1 m_1 + 2\cos\alpha_2 m_2)}{2m_1^2 + m_2^2}$$

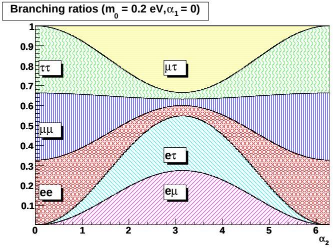
Plots for $\Delta\alpha$ dependance

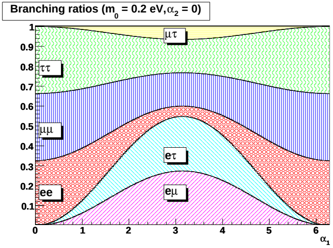
Normal and inverse hierarchy:





Degenerate masses:





Conclusions

- If the Higgs triplet is responsible for neutrino masses and found in HEP experiments like the LHC then better determination of the neutrino sector parameters is possible
- Determination of the mass hierarchy and also the lowest mass is straightforward already soon after the discovery
- Determination of the Majorana phases requires a big amount of statistics and could even require a precision measurement machine like the ILC
- Our results are valid even when our assumption of the tri-bi-maximal mixing is wrong, they just need slight adjustments in the final formulas to account for the small corrections
- For further details, please refer to:
  Kadastik, M., Raidal, M., Rebane, L., "Direct determination of neutrino mass parameters at future colliders", Phys.Rev.D77:115023,2008, arXiv:0712.3912

What if $\theta_{13} \neq 0$?

- For tri-bi-maximal mixing the following has to hold:

$$\mathrm{BR}_{e\mu} = \mathrm{BR}_{e\tau} \quad \& \quad \mathrm{BR}_{\tau\tau} = \mathrm{BR}_{\mu\mu}$$

- If these relations don't hold then tri-bi-maximal mixing does not hold and we usually have to use more branching ratios to determine the same quantities
- Changes in θ_{12} and θ_{23} only change the coefficients in the equations, however θ_{13} creates qualitative differences
- As an example if we assume θ_{13} to still be small the C_1 parameter takes the following form:

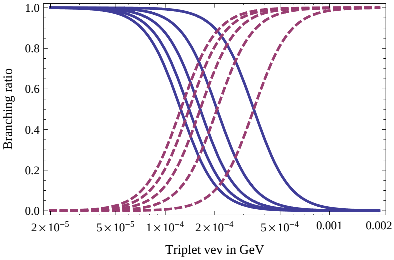
$$C_1' \equiv \frac{2\mathrm{BR}_{\mu\mu} + 2\mathrm{BR}_{\tau\tau} + 2\mathrm{BR}_{\mu\tau} - 2\mathrm{BR}_{ee}}{2\mathrm{BR}_{ee} + \mathrm{BR}_{e\mu} + \mathrm{BR}_{e\tau}} = \frac{-m_1^2 + m_2^2 + 3m_3^2}{2m_1^2 + m_2^2} + \mathcal{O}(\sin^2\theta_{13})$$

WW channel and vev

The relationship between the WW channel and the leptons is depending on the decay widths. The WW decay width is:

$$\Gamma_{WW} \equiv \Gamma(\Phi^{\pm\pm} \to W^{\pm}W^{\pm}) = kv_{\Phi}^2.$$

And the relative contribution to leptons or WW channel depends on the value of the triplet vev as:



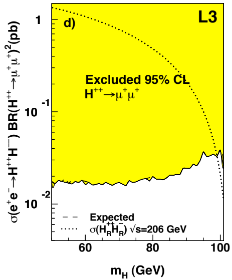
Blue lines - leptonic decays,
red dashed lines - decays to WW.

Different curves represent different $\Phi^{\pm\pm}$ masses with higher masses moving the transition point to the right.

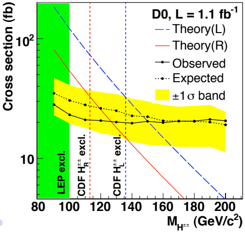
Previous collider searches

It can easily be: BR($\Phi^{\pm\pm} \rightarrow \ell_i \ell_j$) = 1/6 $\forall$ i,j

LEP searches:
The searches at the various LEP experiments varied a lot, but in general the result was a limit of 100 GeV. Taking into account possible branching ratios this can be scaled down to 75-80GeV. Pic from hep-ex/0309076v1.



Tevatron searches:
D0 has searched for only muons and set a limit at 150 GeV, CDF has included other channels, however only looking one at a time. Using realistic branching ratios sets the limit below LEP limit. Pic from 0803.1534v1.



Mixing of the mass states

As has become evident from experiments, like the quarks the neutrino flavor and mass eigenstates differ. Their relation is as follows:

$$(\mathcal{M}_\nu)_{ij} = U^* m_\nu^D U^\dagger \quad \text{with} \quad m_\nu^D = \text{diag}\,(m_1, m_2, m_3)$$

with U parameterized as follows:

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \times$$

$$\times \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$c_{ij} = \cos\theta_{ij}$$
$$s_{ij} = \sin\theta_{ij}$$