A new method for incorporating precession and higher-order modes in searches for compact binaries

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Modeled searches for compact binaries

- Current deployed modeled searches for compact binaries are restricted to *aligned spin* systems, that do not precess, and include only the dominant $l = 2, m = \pm 2$ harmonics
- However the ability to detect precessing systems can be an important discriminant for formation channels (e.g. common evolution *vs* dynamical capture); likewise, higher-order modes can be important for higher mass, high mass ratio systems (see talk by I. Harry)
- In this talk I describe a new technique under development for including arbitrary modes, and the unanswered questions in its implementation that we are still investigating

Preliminaries: Response at an interferometer

- The gravitational waveform received at an interferometer with perpendicular arms may be written as:

$$h(t) = \frac{r_0}{r}\,\mathrm{Re}\left(\left[(F_+(\alpha,\delta,\psi) + iF_\times(\alpha,\delta,\psi) \right] \left[\sum_{lm} h_{lm}(\vec{\mu}; t_0)\, {}_{-2}Y_{lm}(\iota,\phi) \right] \right)$$

- Here we have separated out *intrinsic* parameters $\vec{\mu}$ from the *extrinsic*: $r, \alpha, \delta, \psi, \iota, \phi$ and t_0.
- The antenna pattern functions F_+ and $F_\times$ are normally written as trigonometric functions of the three angles, and the ${}_{-2}Y_{lm}$ are spin-weighted spherical harmonics

Preliminaries: Statistics

- In colored Gaussian noise, the probability that a particular stream of data $s(t)$ is observed given that a signal $h(t)$ is present is proportional to $e^{-\frac{1}{2}(s-h|s-h)}$, where the inner-product is defined as:

$$(h|g) = 4\,\mathrm{Re}\int_0^\infty \frac{\tilde{h}(f)\tilde{g}^*(f)}{S_n(f)}\,df$$

- For a statistic, we may either *maximize* the probability over the extrinsic parameters (*F*-statistic; Jaranowski *et al* Phys. Rev. **D58** 063001) or *marginalize* (*B*-statistic; Prix & Krishnan Class. Quant. Grav. **26** 204013)

Comparing aligned spin to precessing & HOM

- If we consider non-precessing systems where all modes with $l > 2$ are negligible, then the maximization over r, ϕ, ψ and ι may be effectively performed analytically, leading to the usual *F*-statistic. Maximization over t_0 can be efficiently accomplished using the Fast Fourier Transform
- In coincident (as opposed to coherent) searches, we first analyze the data in each interferometer independently, and then combine triggers above a threshold using a coincident statistic. When analyzing data at a single IFO for aligned spin, we may also analytically maximize over sky location (α, δ) as well, leaving only intrinsic parameters to be searched over
- <u>*None*</u> of this analytic maximization works so straightforwardly when modes other than $l = 2, |m| = 2$ are significant

Overview of previous work

- Several authors considered template families for precessing signals (Apostolatos; Grandclemént & Kalogera; Buonanno, Chen & Vallisneri [BCV2]).
- More recently, more sophisticated precessing models (SEOBNRv3: Pan *et al* Phys. Rev. **D89**, 084006; IMRPhenomP: Hannam *et al* PRL **113**, 151101) have been proposed and used in parameter estimation.
- Building on BCV2, Pan *et al* developed the *Physical Template Family* search. (Phys. Rev. **D69**, 104017) This search considered single-spin systems with all five $l = 2$ modes, restricted by polynomial constraints. But those constraints were expensive to solve and never fully implemented.
- Harry *et al* (Phys. Rev. **D94** 024012) proposed the *Sky Max SNR* search. It analytically maximizes over sky location, and uses a grid search over the usual intrinsic parameters as well as the inclination angle ι.

Matrix elements and the rotation group

- It was previously observed (Dhurandhar & Tinto MNRAS **234** 663–676) that the antenna pattern functions can be expressed as linear combinations of the matrix elements of the rotation group, SO(3)
- It is also true that the spin-weighted spherical harmonics can be expressed in terms of these matrix elements:

$$D^l_{-ms}(\phi, \theta, \psi) = (-1)^m \sqrt{\frac{4\pi}{2l+1}}\, {}_sY_{lm}(\theta, \phi)\, e^{-is\psi}$$

- So we can either set ψ to zero, or introduce a second (redundant) polarization angle (compare to Harry & Fairhurst, Phys. Rev. **D83** 084002)

New coordinates

- This observation means that we can re-express our first equation for $h(t)$ entirely in terms of modes depending on intrinsic parameters (in the Fourier domain), a single amplitude, and matrix elements of two elements of the rotation group: one describing the transformation from the source to radiation frame, and another describing the transformation from radiation to detector frame
- The familiar expressions correspond to coordinatizing SO(3) using three *Euler angles* to describe a rotation. But we can maximize (*F*-statistic) or marginalize (*B*-statistic) using whichever coordinates on SO(3) are most convenient.

New coordinates (II)

- For our purposes, it is much more convenient to use *Cayley-Klein* or quaternionic coordinates; they are also closely related to *Euler-Rodrigues* coordinates.
- For ER coordinates, we specify a unit vector $\hat{n}$ and an angle θ. The quaternionic and Cayley-Klein coordinates are then:

$$\alpha_0 = \cos\theta$$
$$\alpha_i = \sin\theta\,\hat{n}_i \qquad \text{for } i \in \{1,2,3\}$$

$$U \equiv (\alpha_0 - i\alpha_3)$$
$$V \equiv (\alpha_2 - i\alpha_1)$$
$$U\overline{U} + V\overline{V} = 1$$

First result: polynomial expression

- It is now possible to appeal to the well-studied representation theory of SO(3), and observe that in terms of the Cayley-Klein coordinates, *all matrix elements are polynomials*. Moreover, we have seen that the only constraint among these parameters is the single constraint $U\overline{U} + V\overline{V} = 1$, which is also polynomial
- Thus, for any number of additional modes, maximizing over the extrinsic angular variables can be transformed into maximizing a polynomial, subject to a polynomial constraint
- When marginalizing over these variables, the measure is also comparatively simple, if using uniform-in-volume priors

Example: single detector, precessing

- Consider the signal observed at a single IFO, for a precessing source where all modes with $l > 2$ are negligible. If we define:

$$A^4 = \frac{r_0}{r}\sqrt{F_+^2 + F_\times^2} \qquad e^{2i\psi} = \frac{F_+ + iF_\times}{\sqrt{F_+^2 + F_\times^2}}$$

then:

$$h(t) = A^4 \,\mathrm{Re}\left[D^2_{-22} h_{22}(t) + D^2_{-21} h_{21}(t) \right.$$
$$\left. + D^2_{-20} h_{20}(t) + D^2_{-2-1} h_{2-1}(t) + D^2_{-2-2} h_{2-2}(t)\right]$$

Example (cont'd)

- In terms of the U, V variables, can show:

$$D^2_{-22} \propto U^4 \qquad D^2_{-2-1} \propto UV^3$$
$$D^2_{-21} \propto U^3 V \qquad D^2_{-2-2} \propto V^4$$
$$D^2_{-20} \propto U^2 V^2$$

- If we then define $X = AU, Y = AV$, we have two unconstrained complex coordinates, and:

$$h(t) = \mathrm{Re}\left[h_{22}(t)\, X^4 + h_{21}(t)\, X^3 Y + h_{20}(t)\, X^2 Y^2 + h_{2-1}(t)\, XY^3 + h_{2-2}(t)\, Y^4\right]$$

Maximizing over extrinsic parameters

- Even in this simple case where we only consider a single detector, when we minimize $(s - h|s - h)$ over our X, Y variables, we will get an eighth-order polynomial in two complex (equivalently, four real) variables. This is highly non-trivial to solve!
- Currently, investigating best way to do this. Considering two techniques from computational algebraic geometry, each of which have been used to solve parametric systems. There is an expensive, off-line part of the computation that only needs to be done once, and then a faster part that is done for each instance of the problem (i.e., data realization)
- May require hierarchical approach: find points of interest with something cheap to compute (e.g. quadrature sum of matched-filter with all modes) and then deploy the maximization over a subset of candidates.

Extending to multi-detector

- Because the antenna functions can also be expressed in terms of matrix elements, we can also consider data from multiple interferometers and consider a statistic that either maximizes or marginalizes $\sum(s-h|s-h)$ over all detectors
- Key new complication is that there is a time delay depending on the (unknown) sky position. A few possibilities:
  - Search over all sky positions: a coherent search (expensive)
  - Treat timing of single IFO triggers as exact, to determine or constrain sky position
  - Model time-dependence of SNR series near the peak (trigger time) and so express it analytically in terms of polynomial variables, and apply the same techniques

Summary

- Including the effects of precession or higher-order modes could be important for detecting interesting classes of signals.
- For both computational efficiency and sensitivity, we would like our search to not just matched-filter against additional modes, but also quasi-analytically maximize or marginalize over extrinsic parameters
- Naively, this looks daunting, as it involves complicated trigonometric functions of the extrinsic variables
- A better choice of coordinates, however, can reduce this to a polynomial optimization problem, which is well-studied in applied mathematics
- But still more work needed to know which solution technique is most efficient, and how efficient it is

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