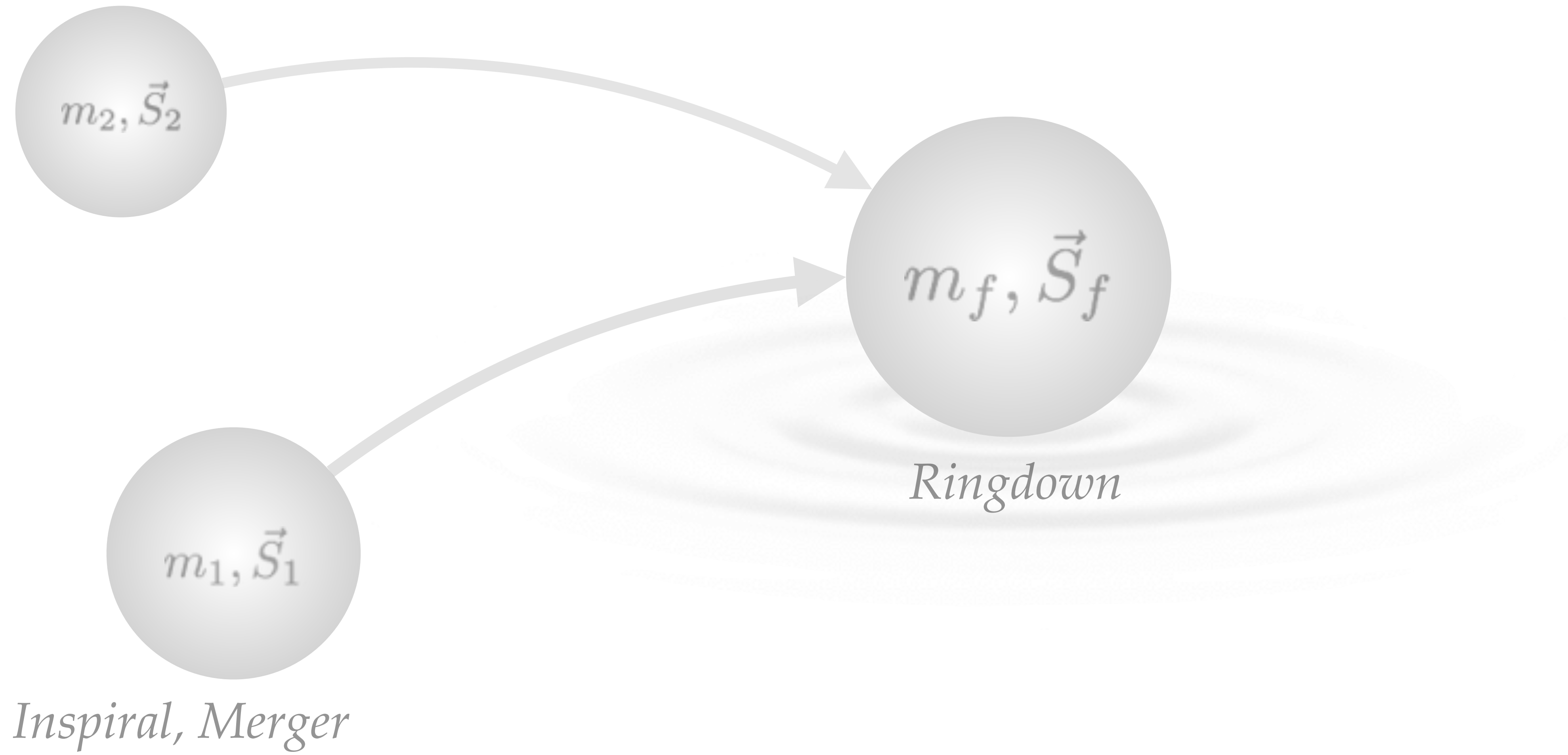


Context: Quasinormal models resulting from the merger of stellar mass BHs, and learning as much as we can from post-merger (ringdown) signals …

The Quasi-normal Modes of Black Holes

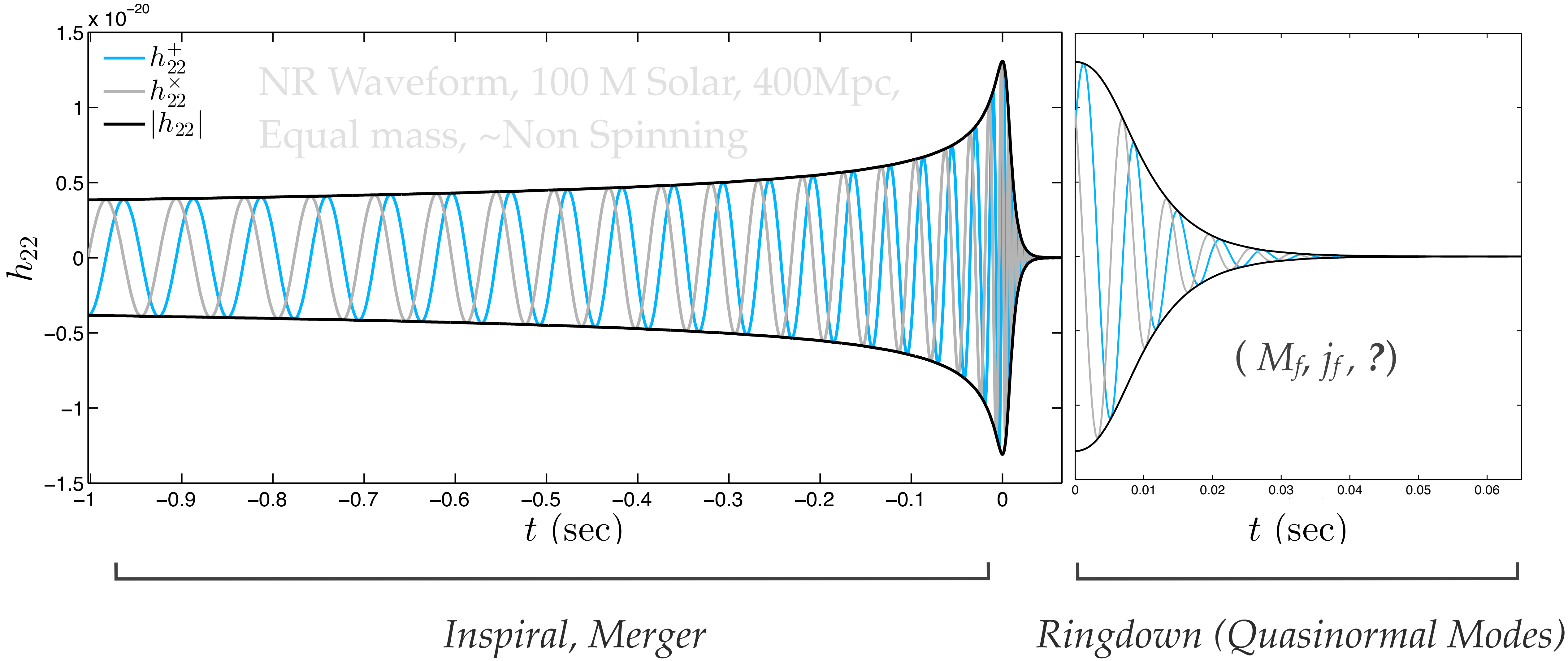
Review and Recent Updates

Lionel London

GWPAW, Annecy France

May 31st 2017

Context QNMs from Binary Black Hole Mergers



The **remnant black hole** (BH) is highly perturbed, with gravitational wave radiation that rings down. One goal of gravitational wave astronomy is to use our theoretical knowledge of BH QNMs to **learn more from detections**, and **determine the consistency (or inconsistency) of signals with GR**.

Overview Black Hole Quasinormal Modes, Review and Updates

A. The current context of black hole Quasinormal Modes (QNMs)
- QNMs as tools to help us learn about astrophysical BHs and Test GR

B. Review: QNMs and the **fundamental questions** about black holes
- Black hole stability (Perturbation Theory)
- Quasinormal mode structure (Kerr)

C. Review: Analytic and numerical relativity, **key aspects**
- Numerical Relativity *vs* Perturbation Theory
- QNM use in Binary BH (BBH) signal models

D. The current and **new questions** about BH QNMs
- What can we learn with QNMs?
- When can we learn it?

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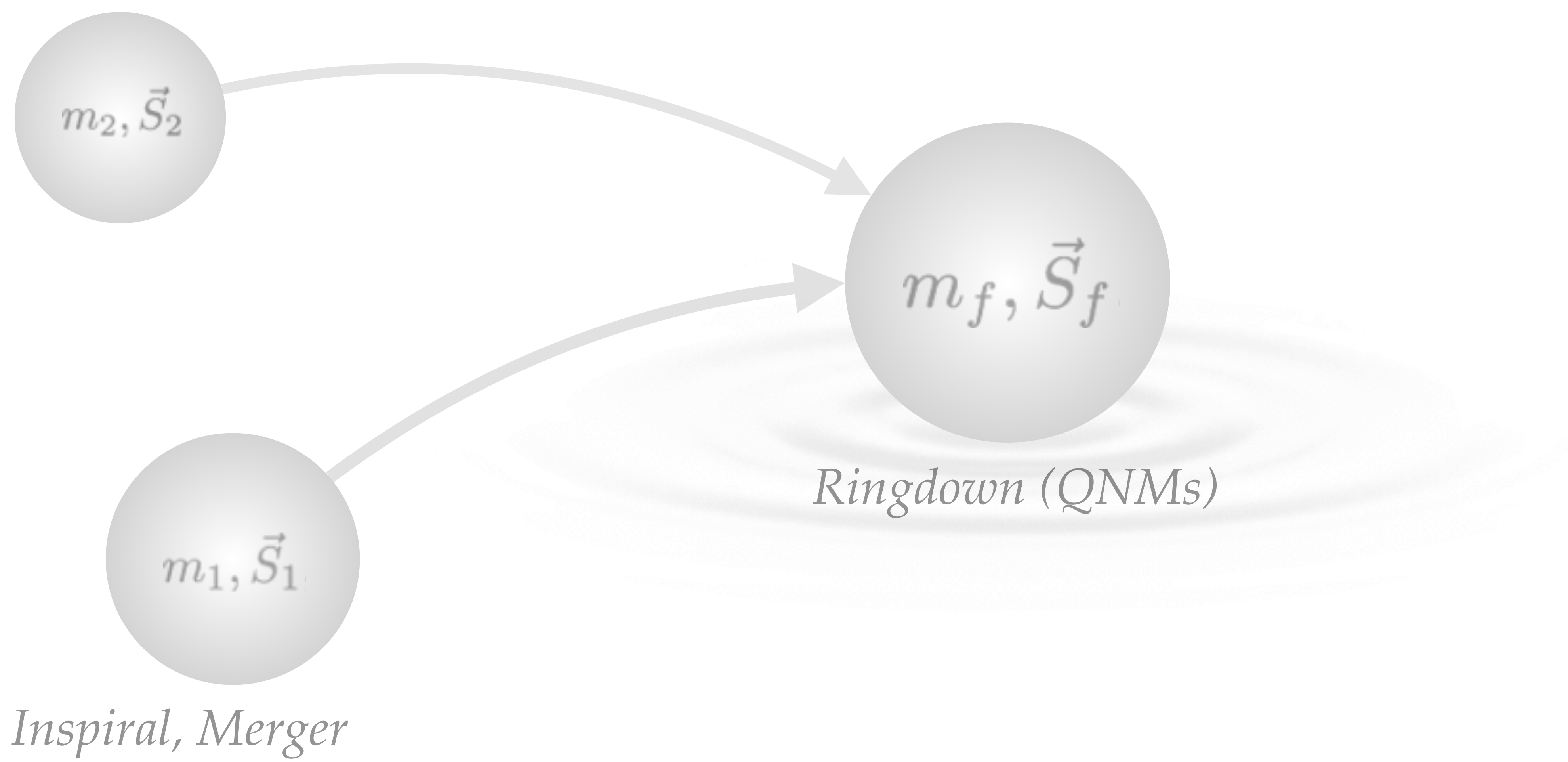
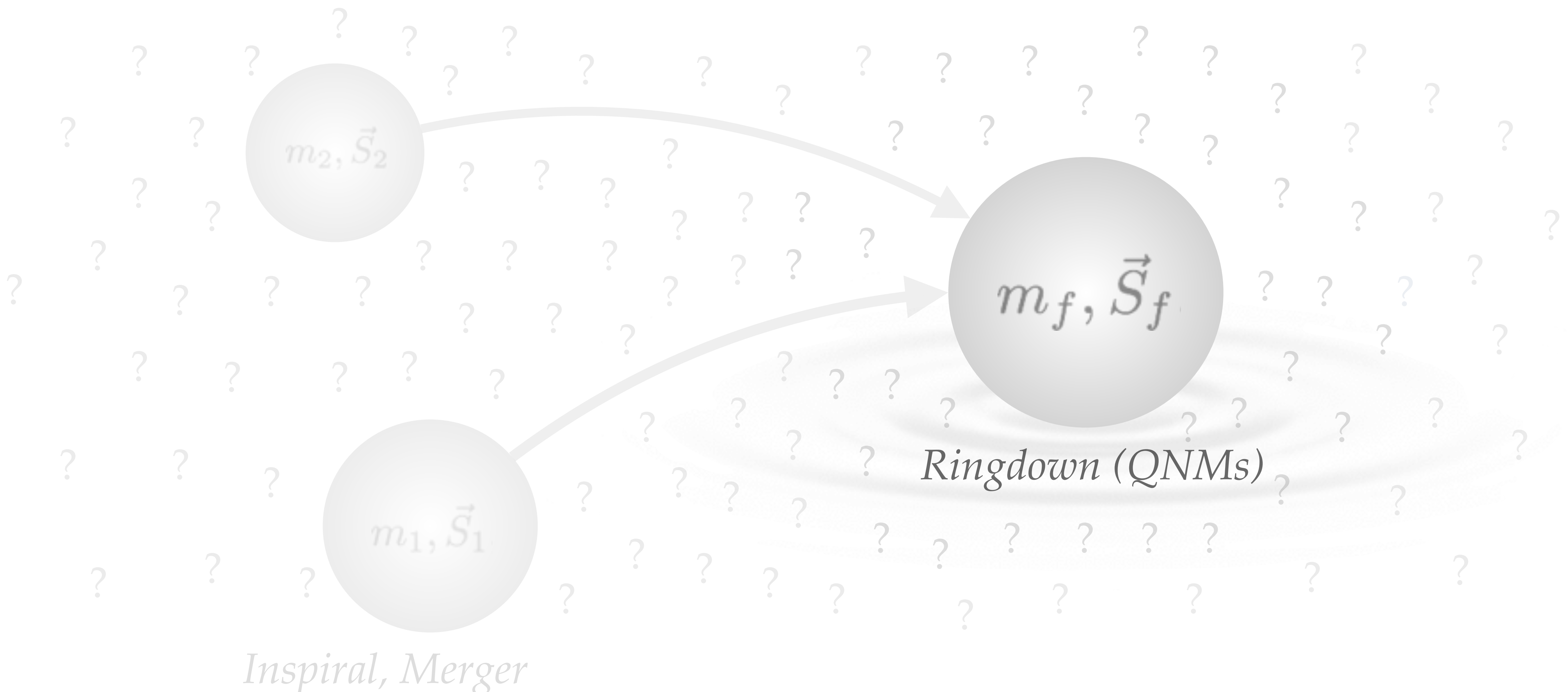
BH QNMs Learning about Astrophysical BHs and Testing GR



Current gravitational wave detections have been of black holes as described by GR — **but there are many notable limitations:** signal SNR, template accuracy, number of detections, etc ….

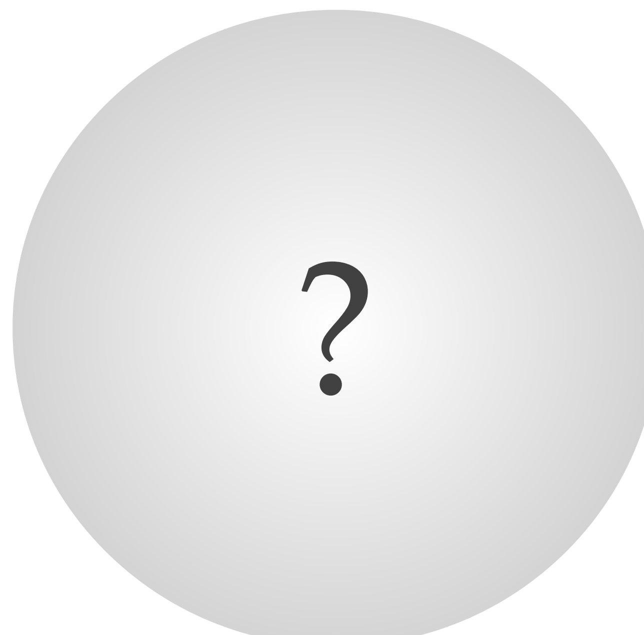
BH QNMs Learning about Astrophysical BHs and Testing GR



Many questions persist — Test the No Hair Theorem? BH Charge? Matter? Consistency with Numerical Relativity? Beyond GR?

… all stem from older and more fundamental questions …

The Fundamental Questions about Black Holes



Do black holes exist in nature?

The Fundamental Questions about Black Holes

Do black holes exist in nature?

$\longrightarrow$ *Are black holes stable?*

(*i.e.* BH Perturbation Theory)

Review: Black Hole Stability

- Regge + Wheeler, Edelstein (1957) (ten years prior to Wheeler's"black hole" terminology)
  - **Developed framework** for applying stability analysis to Schwarzschild black holes

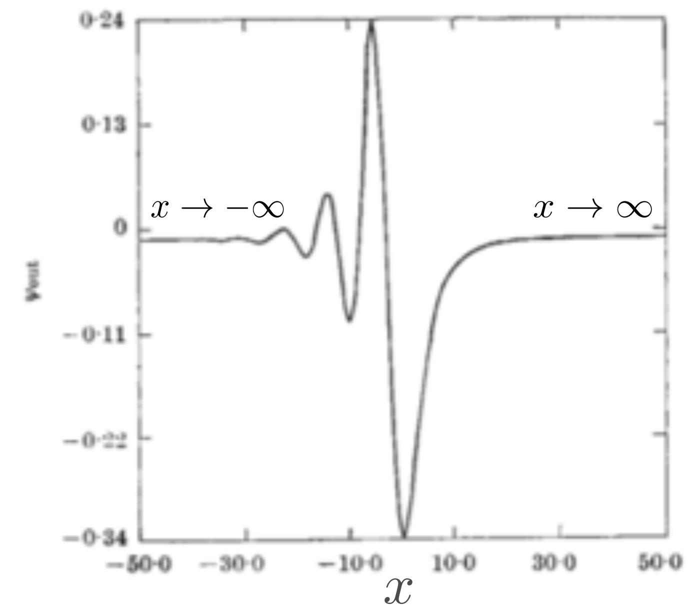
$$g'_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu} \to g'^{\mu\nu} = g^{\mu\nu} - g^{\mu\alpha} g^{\beta\nu} h^{\alpha\beta} + \mathcal{O}(h^2)$$

$$\delta R_{\mu\nu} = 0\,, \quad h_{\mu\nu} = \sum_{L=1}^{\infty} \sum_{M=-L}^{L} \sum_{n=1}^{10} C^n_{LM}(t,r)(Y_{LM})_{\mu\nu}(\theta,\varphi)$$

  - Claimed to have proven **stability (*i.e.* damped solutions)**, but did not consider the correct boundary conditions (coordinate singularity at r=2M)
- Kerr (1963)
  - Found solution to Einstein's field equations describing **spinning black holes**, parameterized by ***dimensionless*** spin, $j = S_z/M^2$, and mass, M:

$$ds^2 = (1 - 2Mr/\Sigma)dt^2 + (rj\,(4M^2 \sin^2\theta)/\Sigma)\,dtd\varphi - (\Sigma/\Delta)dr^2 - \Sigma d\theta^2$$
$$- (\sin^2\theta)(r^2 + M^2 j^2 + 2M^2 j^2 r (\sin^2\theta)/\Sigma)d\varphi^2$$

Review: Black Hole Stability



- Vishveshwara (1969, 1970) (Schwarzschild metric)
  - First to correctly apply open **boundary conditions** to the perturbative problem in Kruskal coordinates

$$x = r + 2M\, ln\left(\frac{r}{2M} - 1\right)$$

- Teukolsky (1972), Teukolsky & Press (1973) (Kerr metric)
  - Applied Newman Penrose formalism (1961) to the perturbation problem
  - Presented stability analysis of (spinning) Kerr BHs
  - Found **separable solution** with discrete spectra — the QNMs

$$h_+ - ih_\times = \int \mathrm{d}\omega \sum_{lm} R(r)_{-2}S_{lm}(\theta,\varphi)\, e^{i\omega t}$$

$\longrightarrow$ Teukolsky's Equations for $R(r)$ and ${}_{-2}S_{lm}(\theta,\varphi)$

$\longrightarrow$ Characteristic equations must be satisfied by:

$\tilde{\omega}_{lm}$ (QNM Frequency), and $\mathcal{E}_{lm}$ (Separation Constant)

$$\lim_{r\to\infty} R(r) = 1/r$$

$$\lim_{j\to 0}\, {}_{-2}Slm(\theta,\varphi) = {}_{-2}Ylm(\theta,\varphi)$$

Review: Black Hole Stability

- Leaver (1985) (Kerr metric)
  - **Analytic representation** (4D nonlinear) for QNMs of Schwarzschild and Kerr
  - Typically considered to be the **most accurate** method for QNM calculations
  - Uses continued fractions to solve characteristic equations for QNM frequency
- **1985 — 2014 (Abbrev.)**
  - Various methods for investigation of QNM frequencies: *e.g.* WKB, Laplace transform, Numerical Integration (Schutz, Will, Nollert, Schmidt, Krivan, Laguna, many many others)
  - Theoretical estimates for QNM measurability (Echeverria, Finn, Flanagan, Hughes, Cardoso, Berti, many others)
  - Black hole thermodynamics (Bardeen, Bekenstein, Hawking, others; **see Miriam's talk on the Area Theorem**)
  - **BBH QNM Excitations in Numerical Relativity** (Berti, Buonanno, Pan, Husa, Kamaretsos, Gossan, London, many others)

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Review: Structure of QNM Solution Space

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A schematic view of Leaver's method for calculating QNMs

Choose **BH mass**, **spin**, azimuthal eigenvalue, ***m***, and polar eigenvalue, ***l***

→

Iteratively **evaluate Leaver's equations**: (Separation Constant, Complex Frequency) —> "**work function**" value

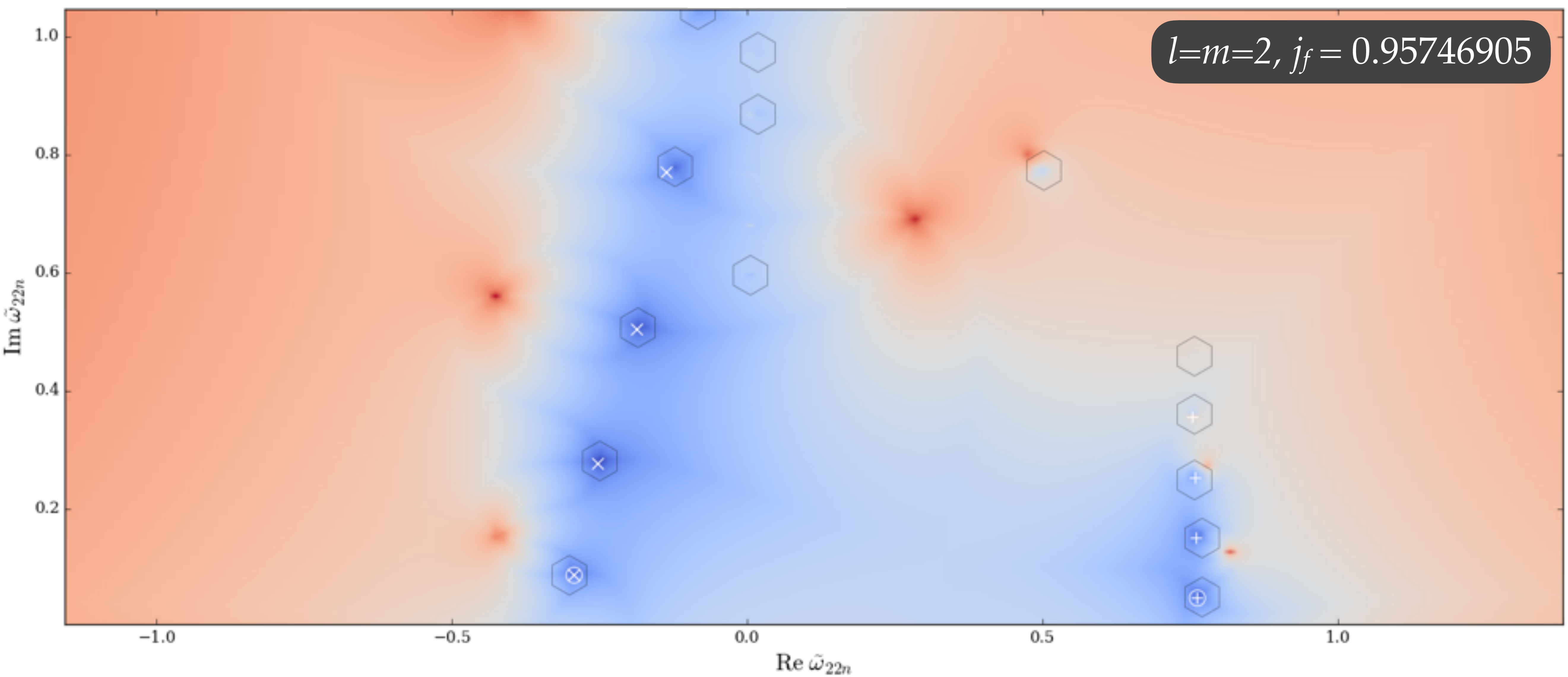
→

QNM Solutions correspond to roots of the work function

4D Optimization problem:

- Complex valued QNM frequency: $\tilde{\omega}_{lmn} = \omega_{lmn} + i/\tau_{lmn}$ ($M = G = c = 1$)
- Complex valued separation constant: $\mathcal{E}_{lmn}$ (*approximated analytically by Berti et al*)

Review: Structure of QNM Solution Space



Example: Blue (Low work function values), **Red** (High). Using Berti's approximation for the separation constants allows 2D visualization of QNM solution space

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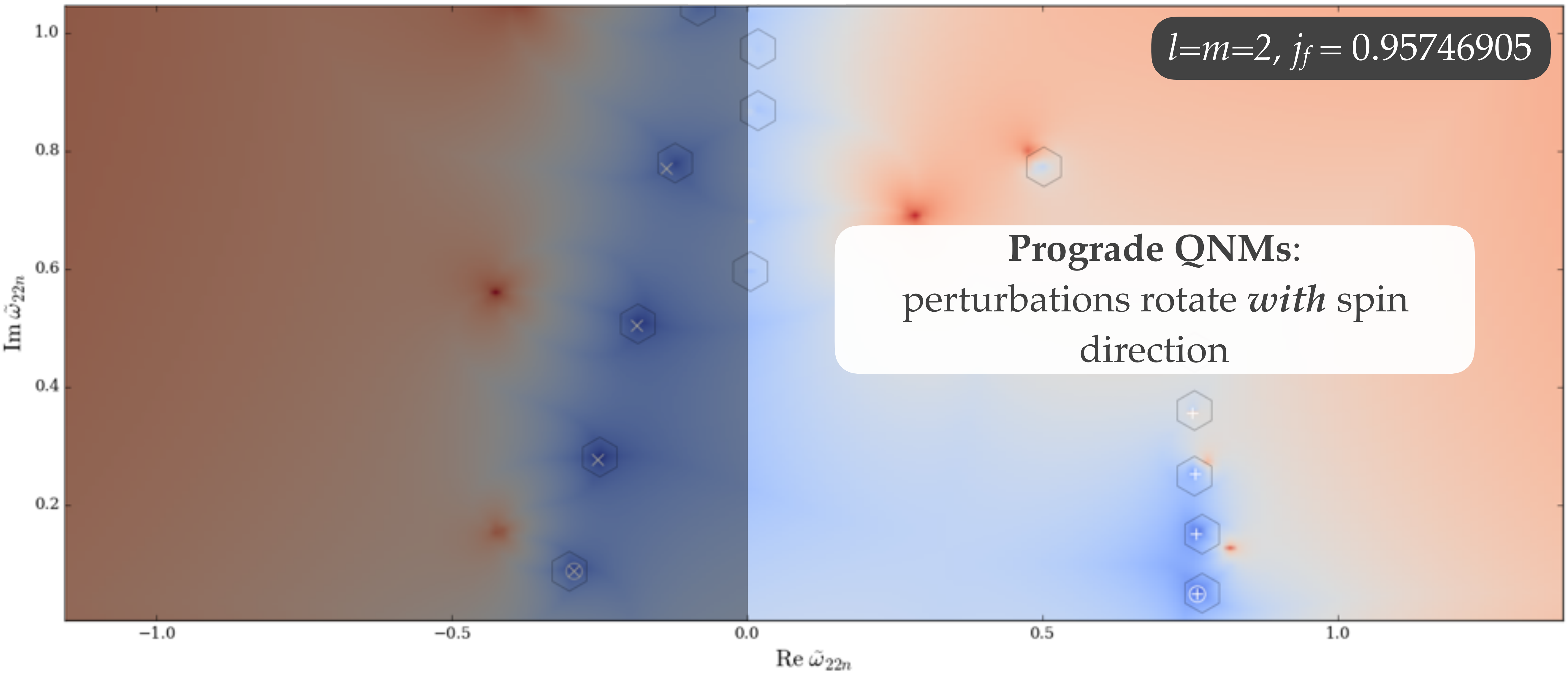

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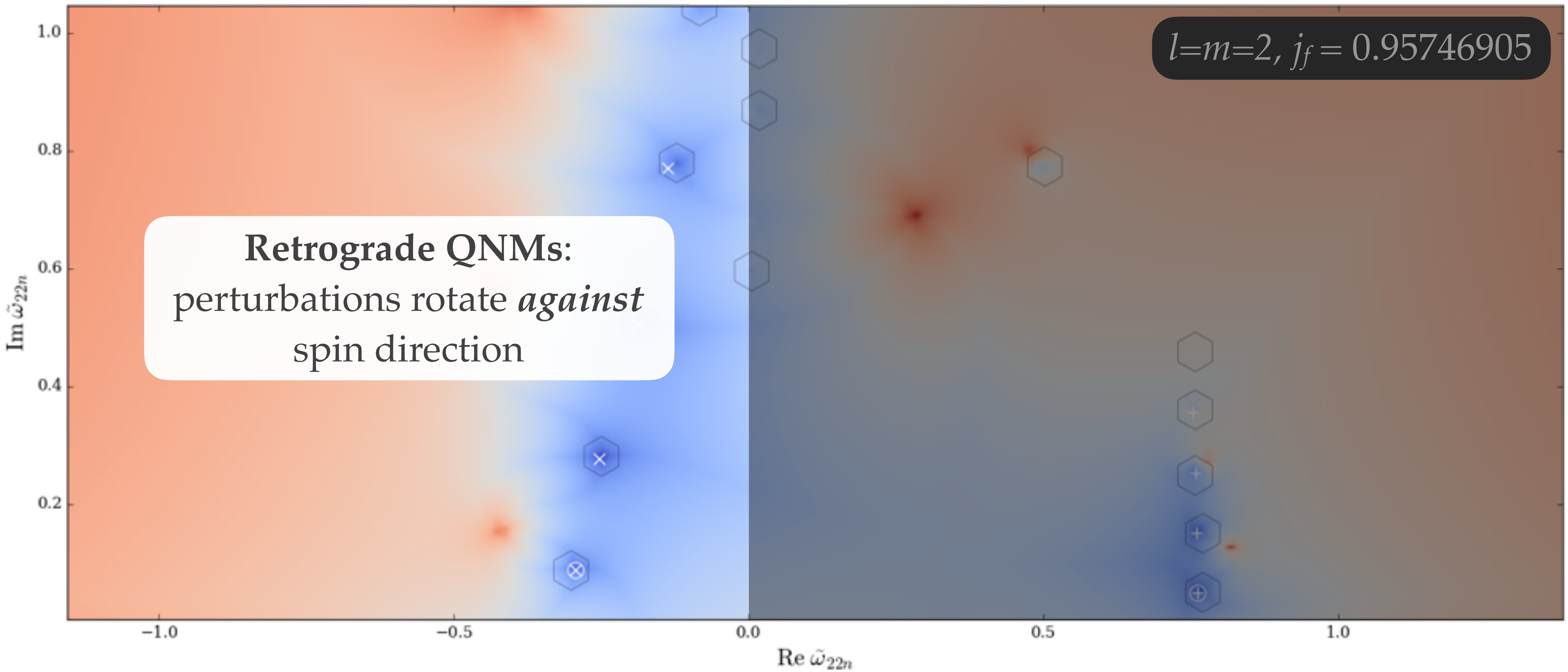

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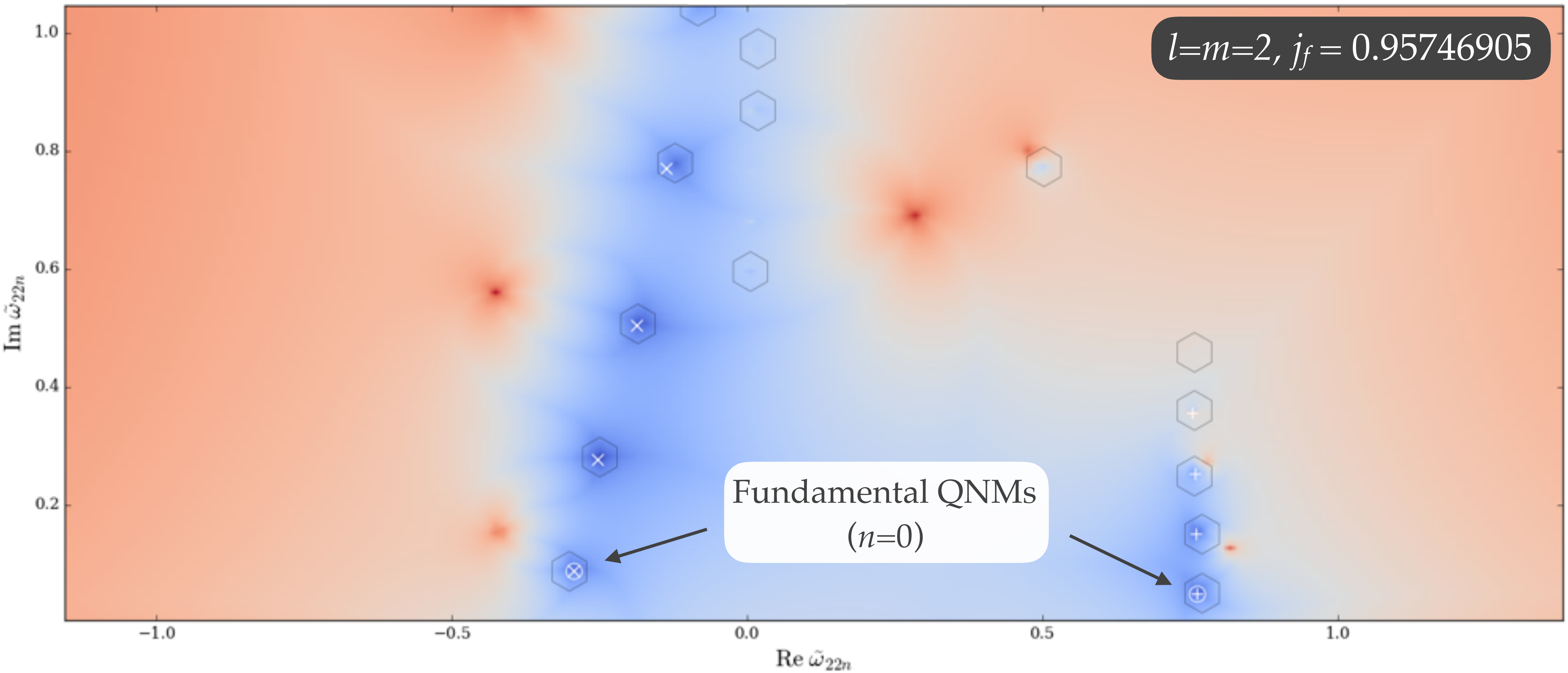

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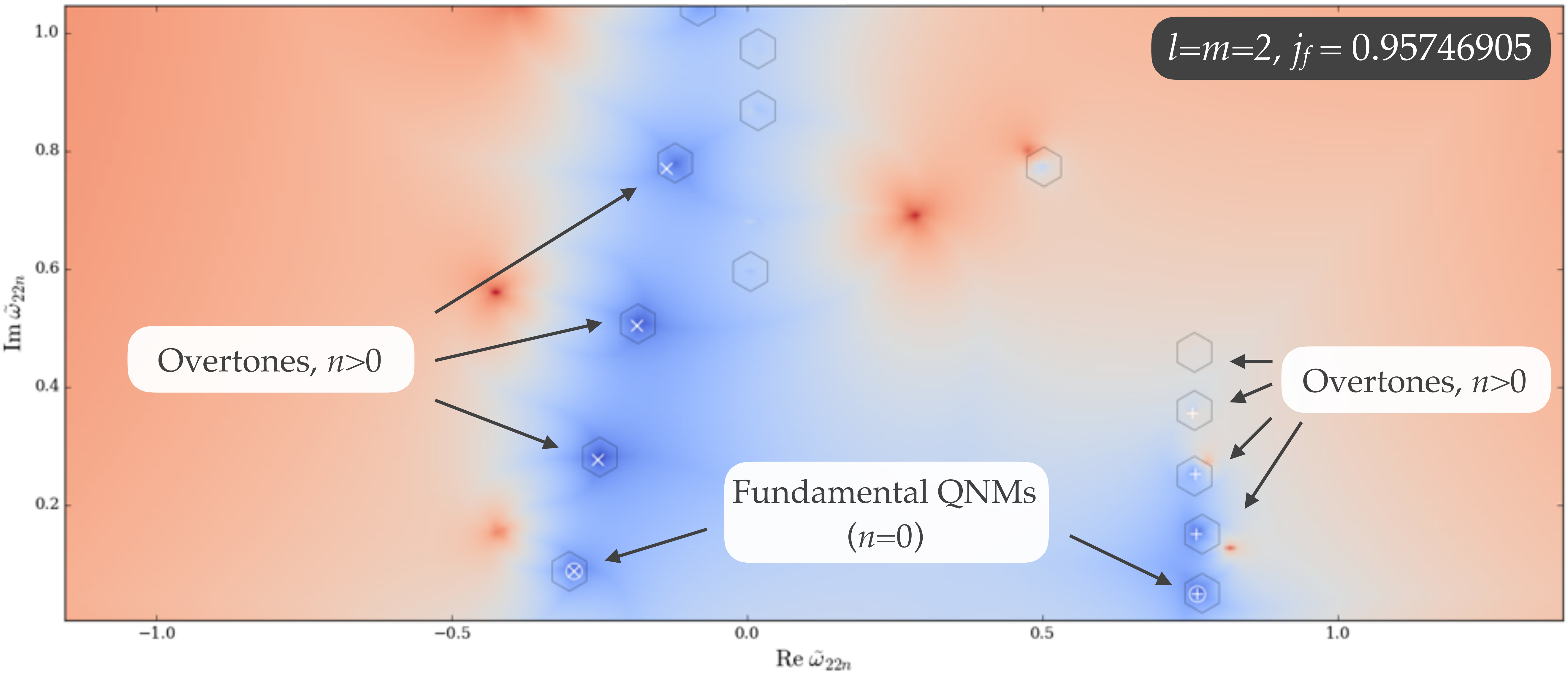

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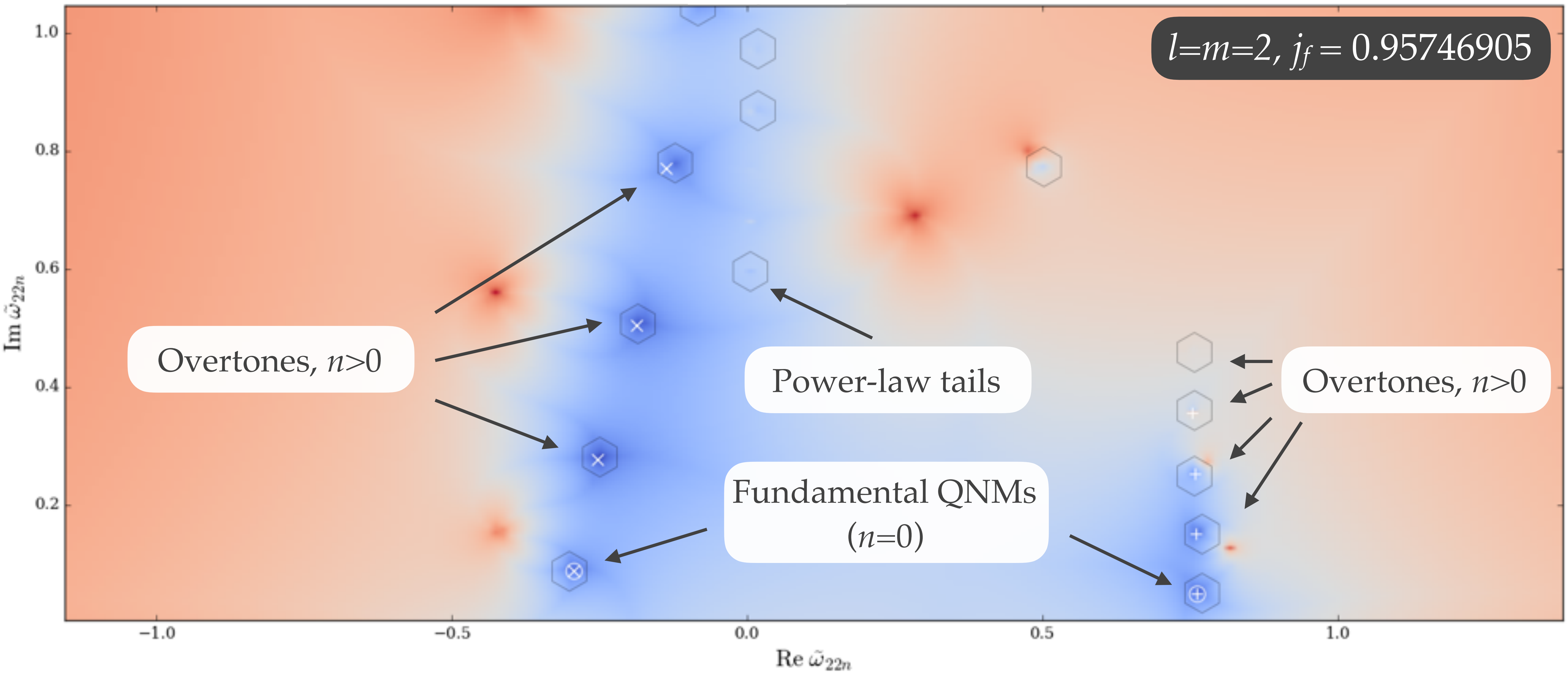

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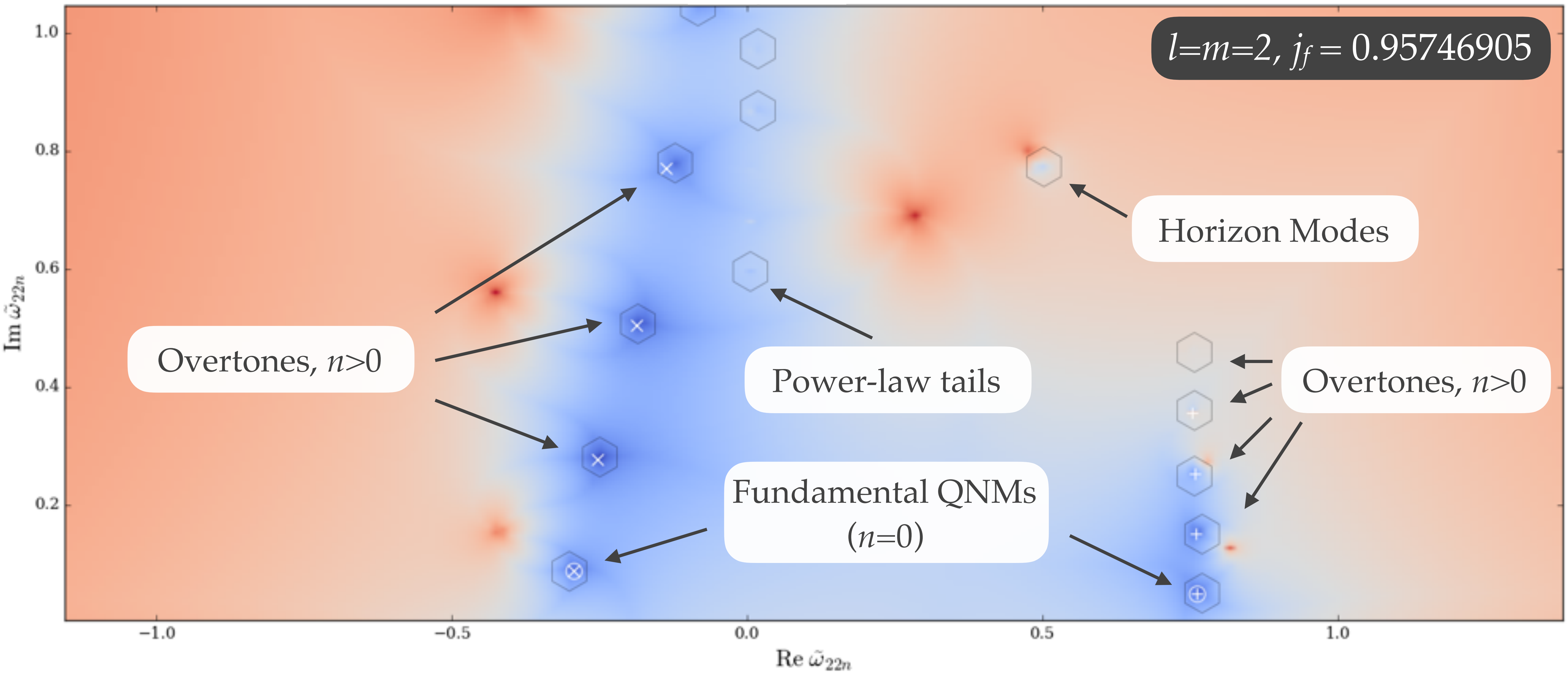

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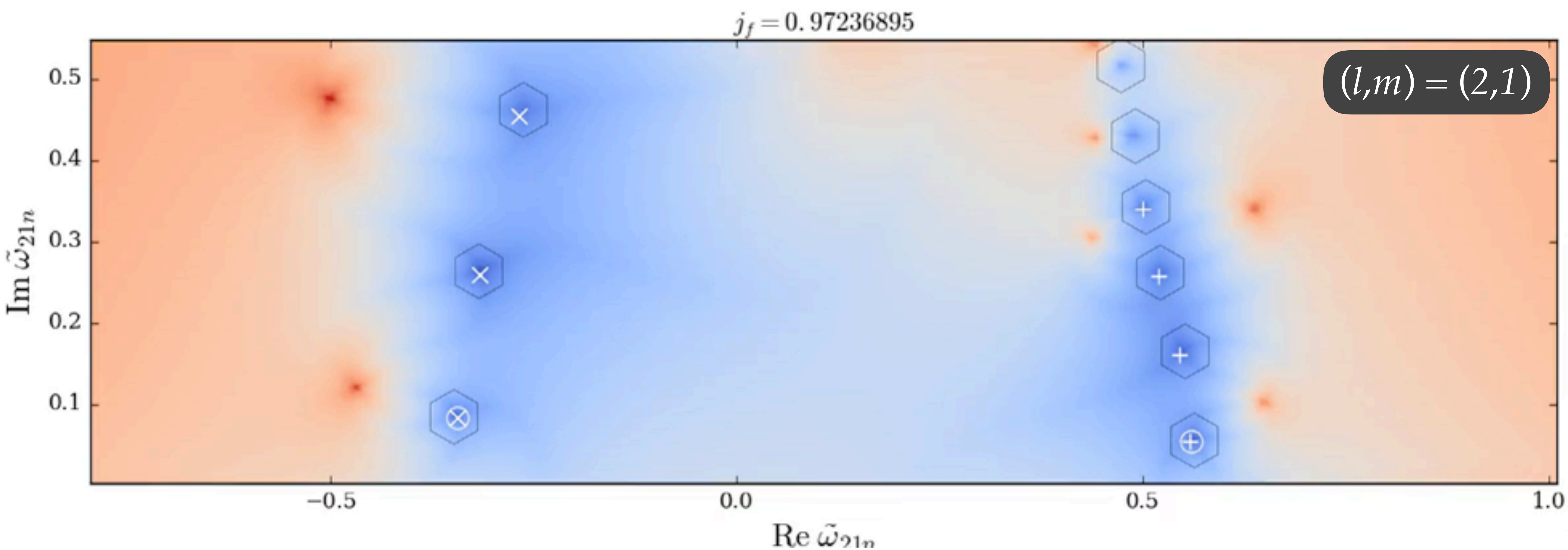
Review: Structure of QNM Solution Space

Nontrivial behavior in the limit extremal BH spin ($j_f \sim 1$):
solution branching, and nonzero / **zero damping**



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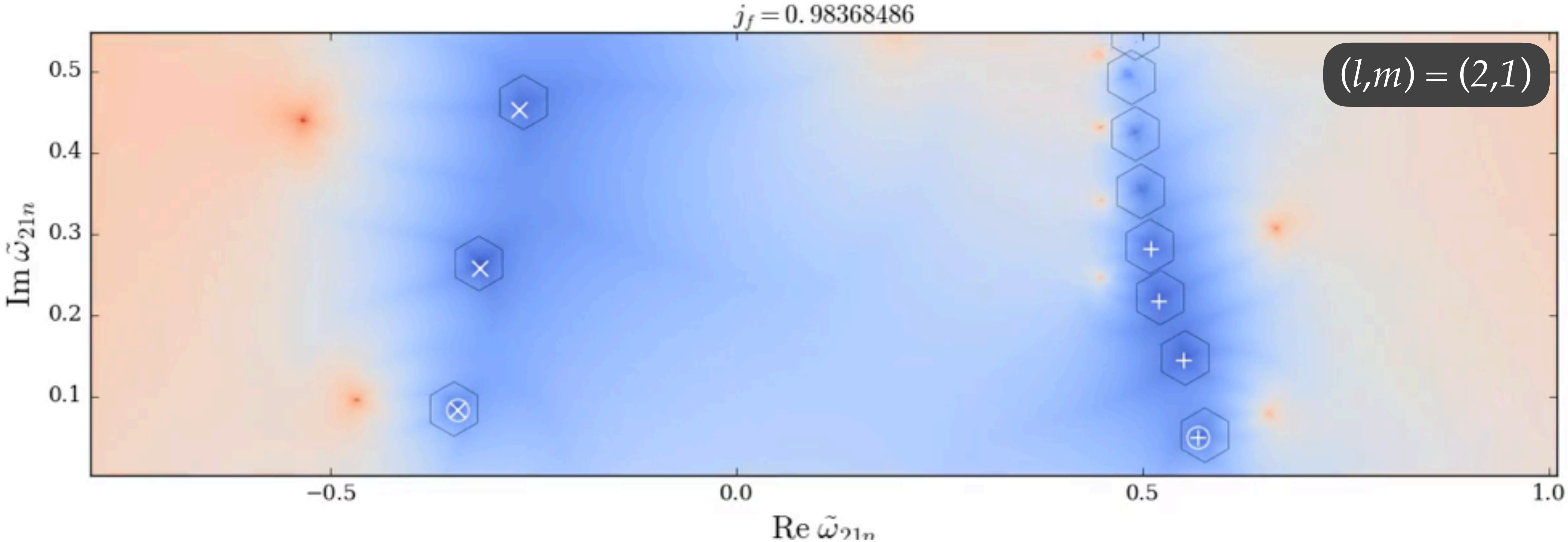

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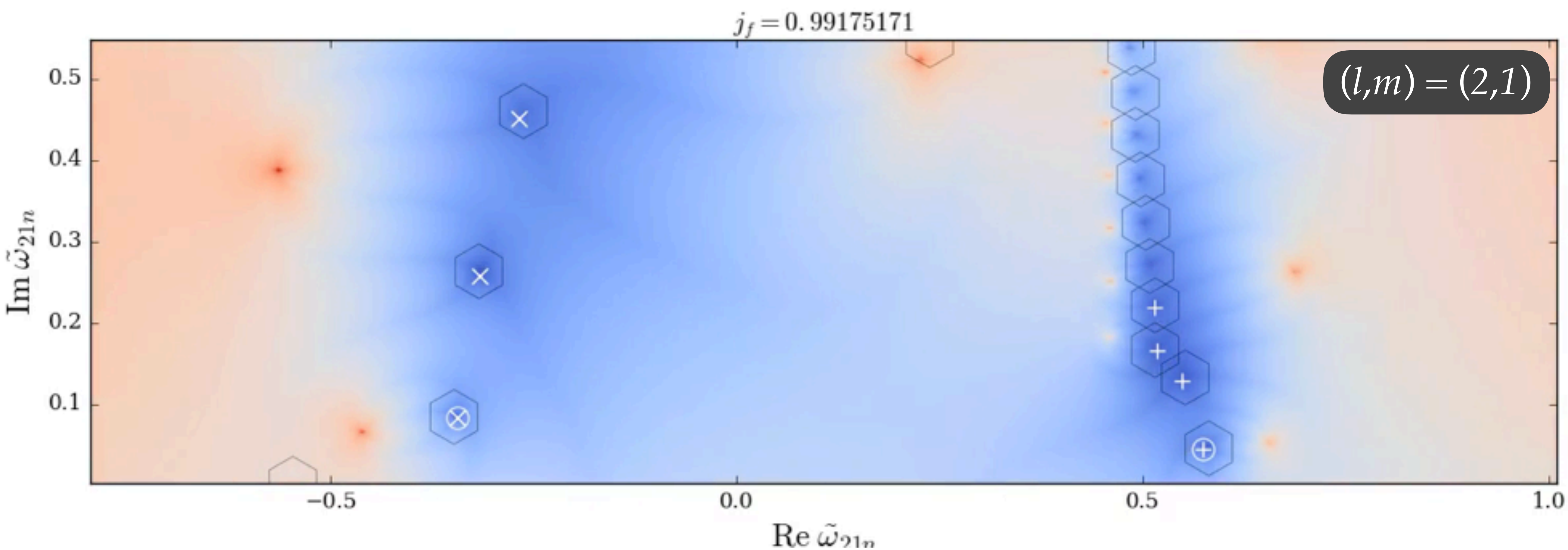

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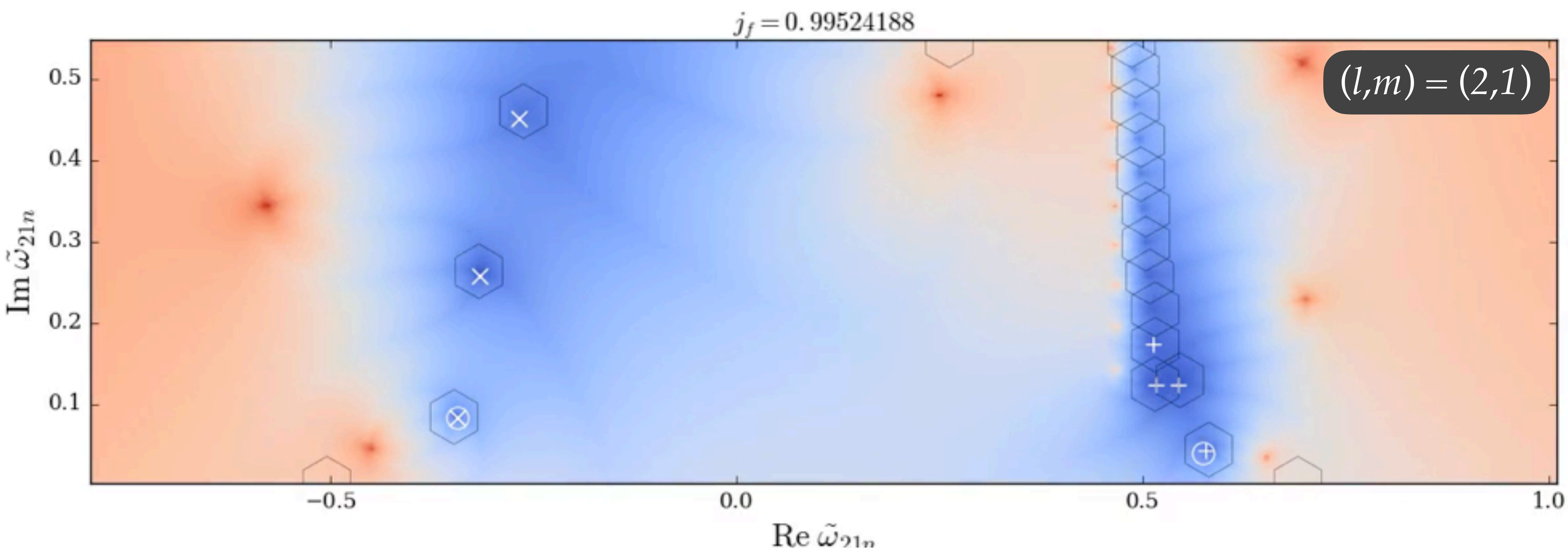

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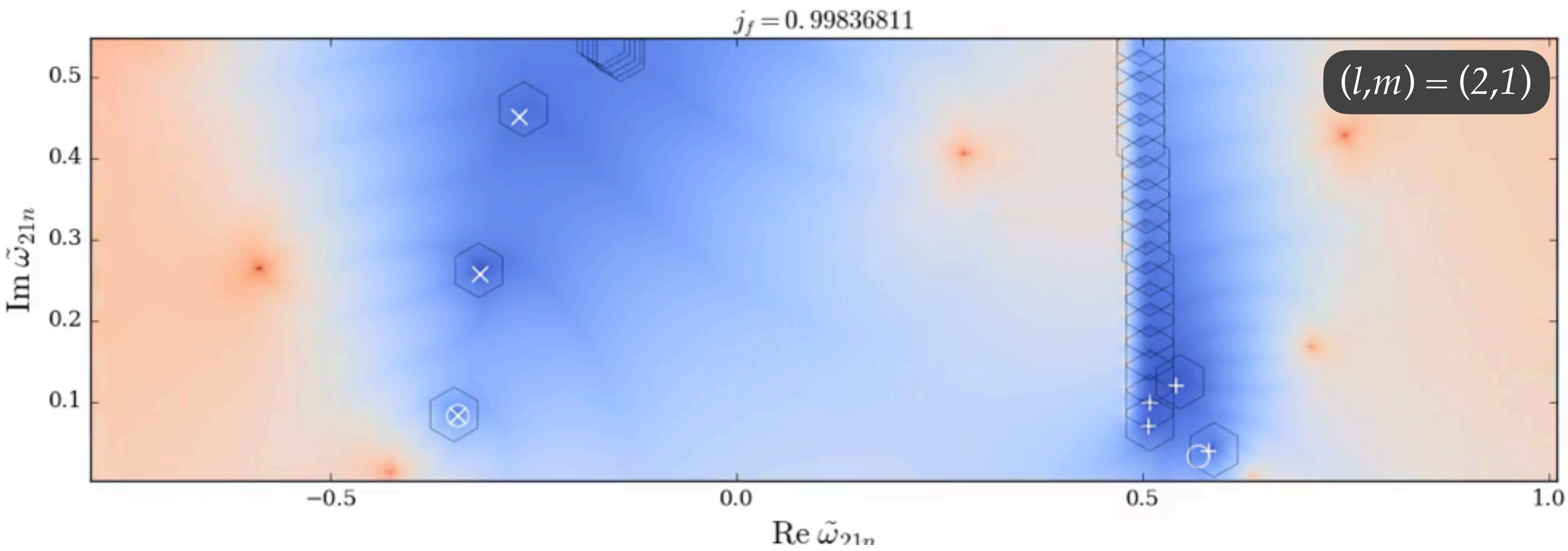

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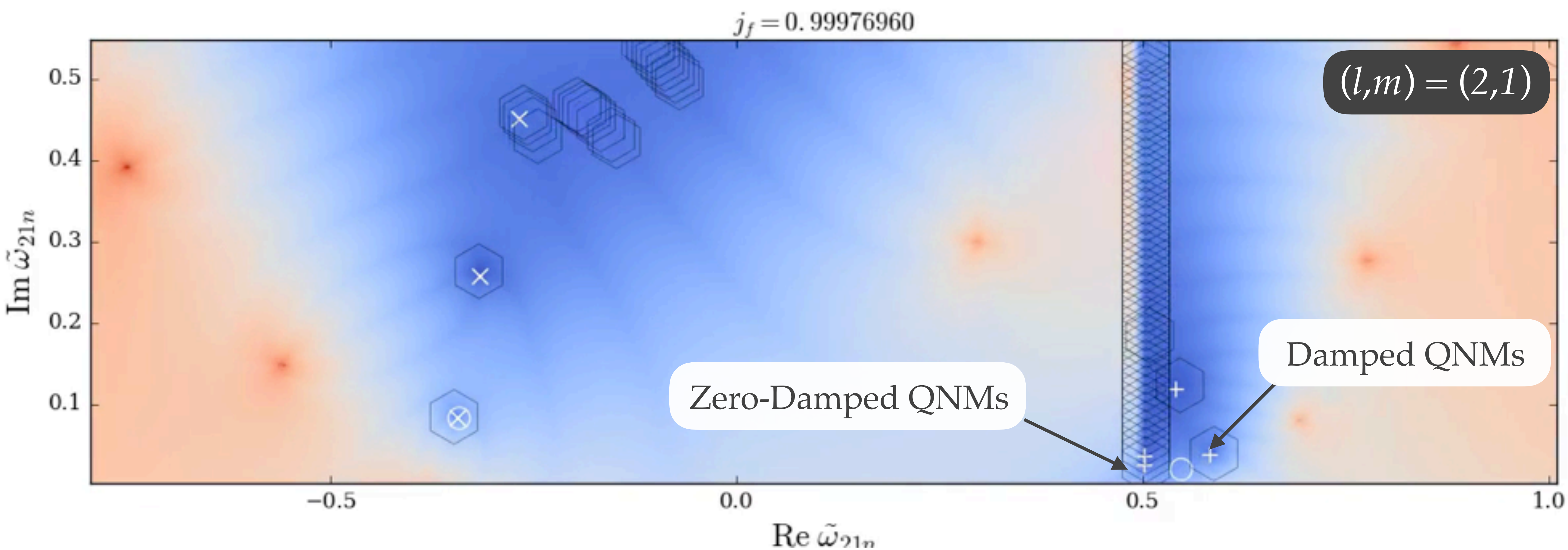

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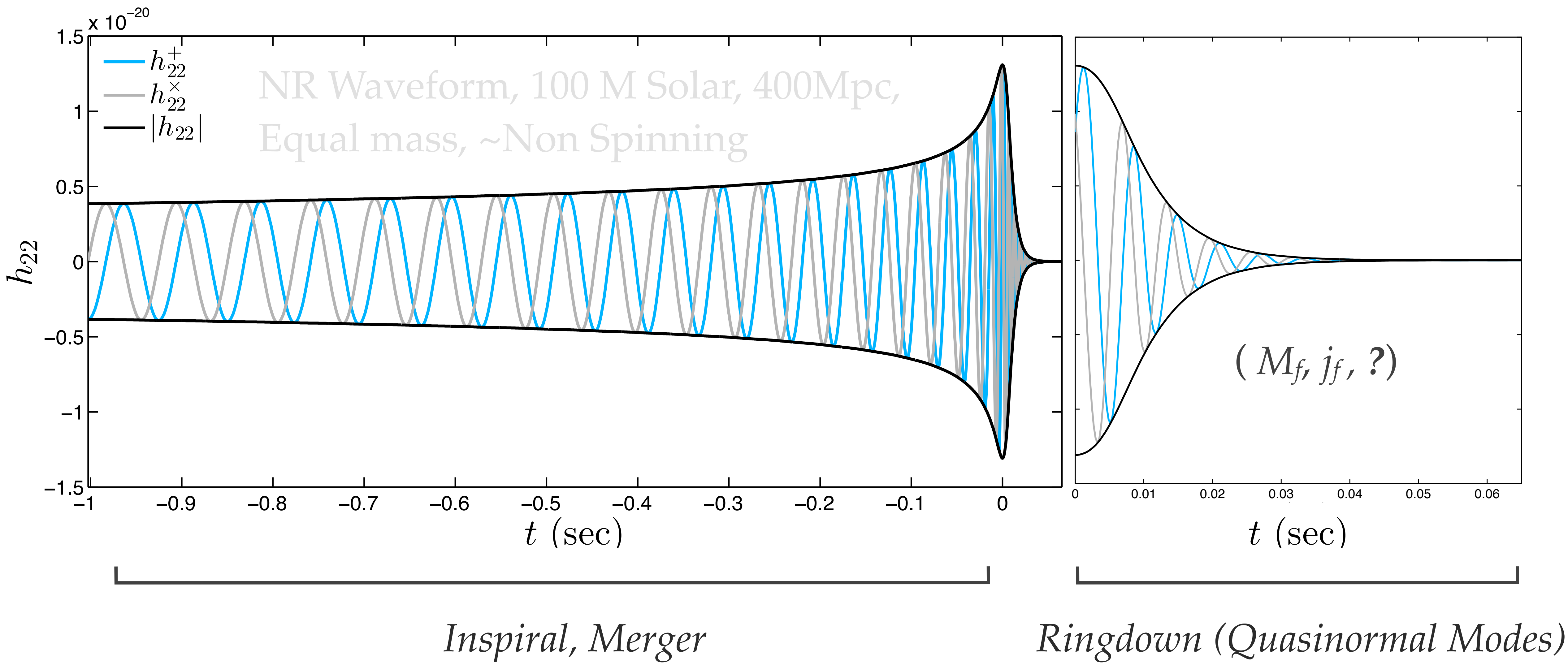
Recap: Black Hole Stability

- **Non-extremal Kerr BHs are stable** under generic perturbations.
- There is a **significant amount structure in the QNM solution space**. This enhances the prospects for testing GR with astrophysical QNMs.
- Of all QNM solutions, **the fundamental modes are the least-damped**, and thus are the **top priority** for analysis of data from experiment.

- **Select Topical Reviews**
  - "On quasinormal modes of asymptotically anti-de Sitter black holes" (Warnick 2013)
  - **"**Quasinormal modes of black holes and black branes**"** (Berti *et al* 2009)
  - "Quasinormal modes of stars and black holes" (Kokkotas & Schmidt 1999)
  - "Quasinormal modes: the characteristic "sound" of black holes and neutron stars" (Nollert 1999)

However, for rigorous QNMs to be useful in LIGO data analysis, **there is an ongoing need to synthesize analytic QNM theory with Numerical Relativity results**.

Review: QNM use in BBH signal models



GW detection and parameter estimation are significantly assisted by model (template) signals. The 2005 advances in Numerical Relativity (NR) (*eg* Baker *et al*) have enabled GW models that encompass **Inspiral, Merger and Ringdown** (IMR). Despite the success of these models, **there have been (and remain) difficulties relating to ringdown** …

Review: QNM use in BBH signal models

Overview

- Starting with **Ajith's 2007 work**, Kerr QNMs have been used to parameterize **Phenomenological waveform models**.
- **IMRPhenomD** (Khan *et al* and Husa *et al* 2015) and the related **IMRPhenomP** (Schmidt, Hannam et al 2013) have been heavily used in the analysis of GW150914 and subsequent events.
- In parallel, **Effective One Body** (EOB) approaches to BBH inspiral have been extended using Kerr QNMs (Damour, Buonanno, Pan, Taracchini, others)
- The related **SEOBNRv2** (Taracchini *et al* 2013) has also been heavily used in post-detection LIGO data analysis.

Status

All of these models require the tuning to NR simulations to incorporate how much each QNM is excited. However**, until recently, there was no general and robust way to model fundamental QNM excitations from NR simulations ….**

Review: QNM use in BBH signal models

The problem lies in a difference of perspective

- **Numerical Relativity**: Spherical Harmonic Multipoles, ${}_{-2}Y_{lm}$ **(Orthogonal in *l*)**
  - $rh = r(h_+ - ih_\times) = \sum_{l,m} h^{NR}_{lm} \, {}_{-2}Y_{lm}(\theta,\phi)$
  - $h^{NR}_{lm} = \int_\Omega rh \, {}_{-2}\bar{Y}_{lm} d\Omega$
- **Perturbation Theory**: Spheroidal Harmonic Multipoles, ${}_{-2}S_{lm}$ **(<u>Not</u> orthogonal in *l*)**
  - $rh = \sum_{lmn} h^{PT}_{lmn} \, {}_{-2}S_{lm}(\theta,\phi;\tilde{\omega}_{lmn} j_f)$
  - $h^{PT}_{lmn} = A_{lmn} \left[e^{i\tilde{\omega}_{lmn} t} \right]$

$$\rightarrow h^{NR}_{lm} = \sum_{l'n} A_{l'mn} e^{i\tilde{\omega}_{l'mn} t} \int_\Omega {}_{-2}\bar{Y}_{lm} \, {}_{-2}S_{l'mn} \mathrm{d}\Omega$$

The Spherical multipoles of NR are sums of QNMs

— i.e. "mode mixing"

Aside: Mixing of QNMs in NR Waveforms

$$\rightarrow h^{NR}_{lm} = \sum_{l'n} A_{l'mn} e^{i\tilde{\omega}_{l'mn}t} \int_{\Omega} {}_{-2}\bar{Y}_{lm}\, {}_{-2}S_{l'mn} \mathrm{d}\Omega$$

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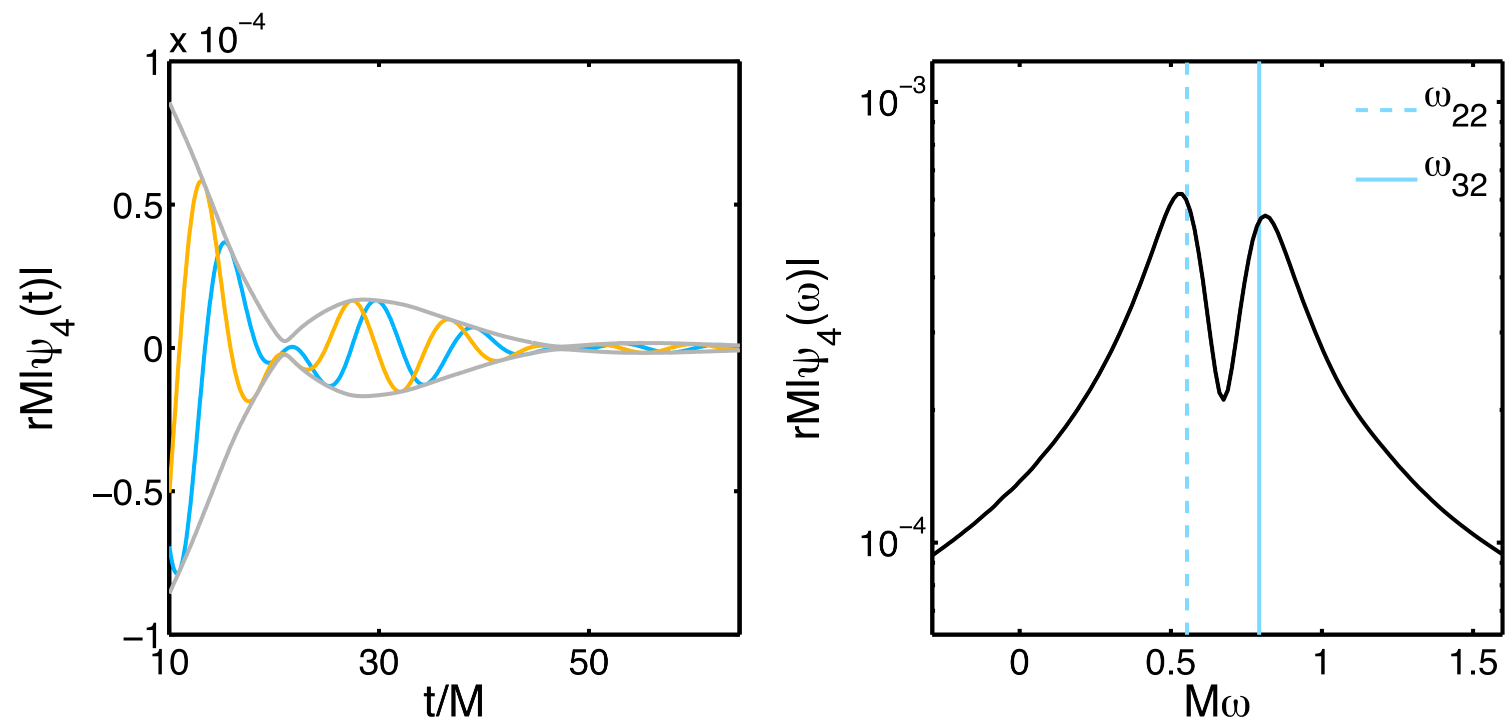
Effect on time domain ringdown. Figure: NR ringdown waveform for (l,m)=(3,2), equal mass, non-spinning.



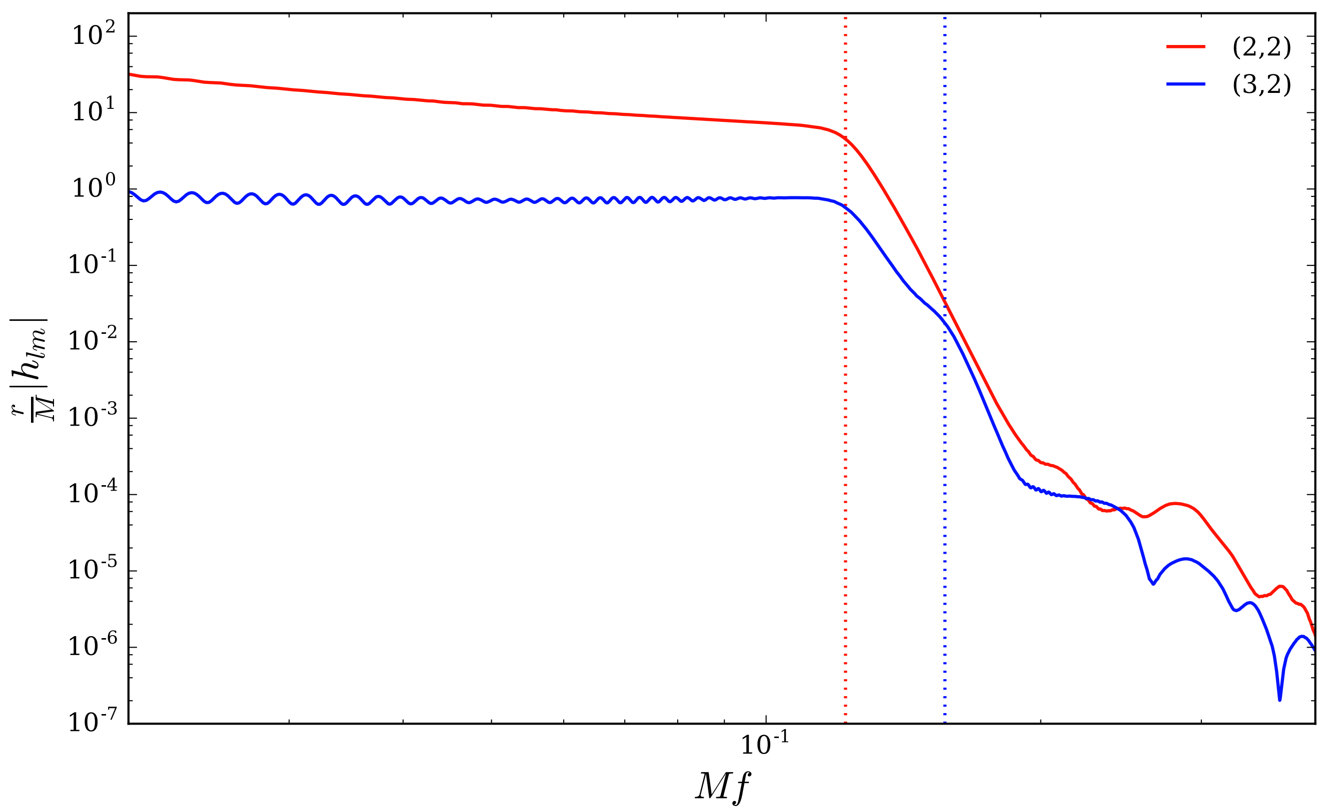
In the time domain, the **mixing of QNMs due to use of a spherical basis causes "beating"**, while a simple decaying sinusoid might naively be expected. **This effect complicates the time domain modeling of post-merger higher multipoles**.

Aside: Mixing of QNMs in NR Waveforms

$$\rightarrow h_{lm}^{NR} = \sum_{l'n} A_{l'mn} e^{i\tilde{\omega}_{l'mn} t} \int_{\Omega} {}_{-2}\bar{Y}_{lm} \, {}_{-2}S_{l'mn} \mathrm{d}\Omega$$

The Spherical multipoles of NR are sums of QNMs — i.e. "mode mixing"

Effect on frequency domain IMR waveform. Figure: NR IMR waveform for **(l,m)=(2,2) and (3,2)**, equal mass, dimensionless aligned spins of 0.85 (z-hat).



There are **similar complications for the frequency domain IMR waveform**.

Review: QNM use in BBH signal models

Overview

- Early studies focused primarily on the **(l,m)=(2,2), where mode mixing is minimal**. (2005-2013)
- Kamaretsos *et al* (2011) modeled the **QNM amplitudes** for non-spinning BBH systems in the Spherical basis (effectively treating the remnant as non-spinning)
- L. London *et al* (2014) developed a method for estimating Spheroidal content from NR waveforms, and modeled **QNM amplitudes and relative phases** for non spinning systems using complex polynomial regression.
- L. London *et al* (paper in preparation) have **used QNMs to develop the first IMR waveform model for non-precessing BBH sources**. **(See Sebastian Khan's talk)**

Update — A QNM Waveform Model for Non-Precessing BBH Systems

There is a clear need to extend result to precessing BBH systems. Towards this goal, I have recently developed a **effective-spin model for QNM excitation**, which is to be used as the **GR prediction** when perform more general data analysis on astrophysical signals.

Update: A QNM Waveform Model for NonPrecessing Systems

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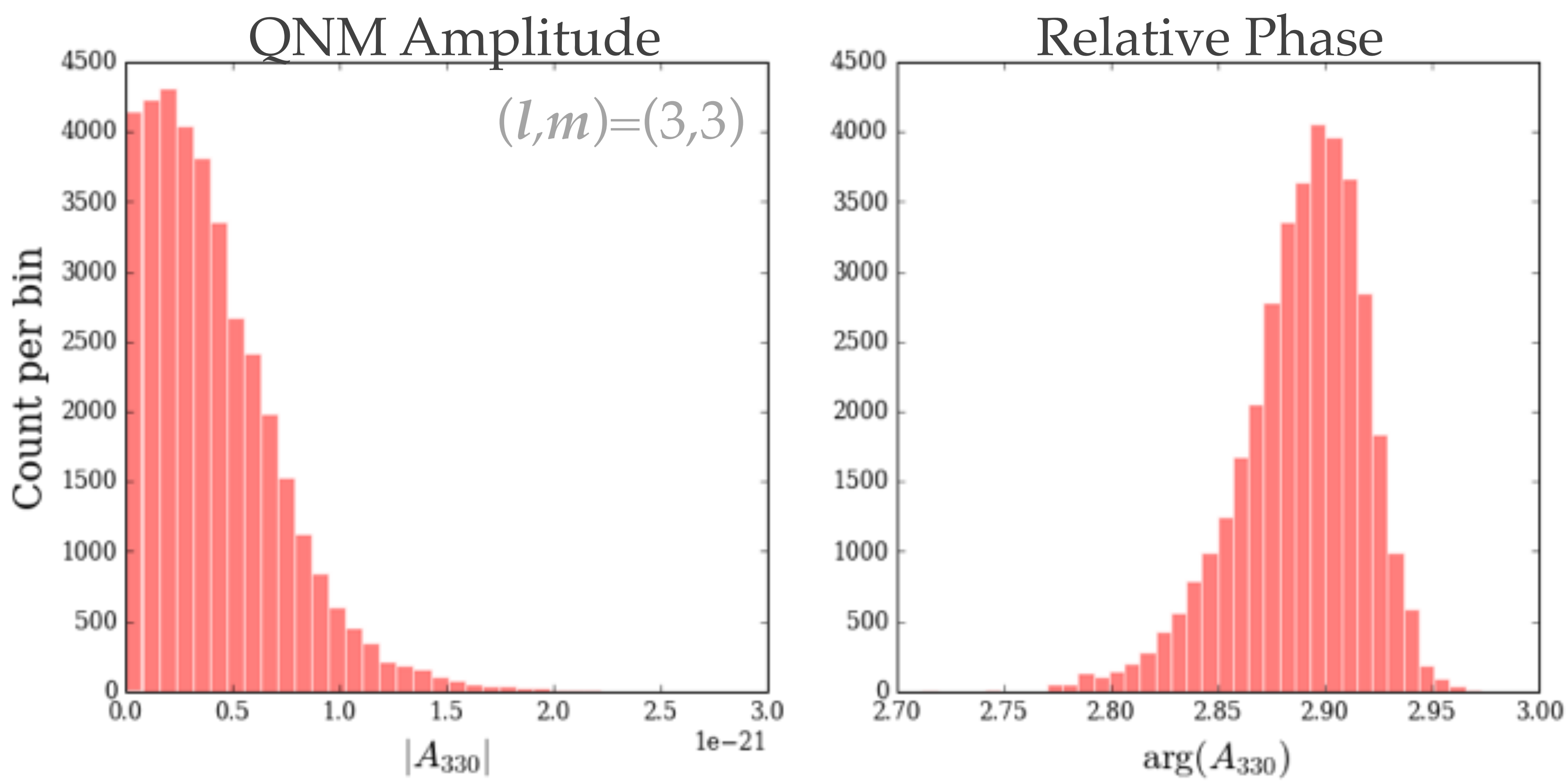
Example usage: **Estimated QNM amplitude (GR prediction) for GW150914**



Current and New Questions about BH QNMs

Current Questions

- Given GR predictions for QNM excitations (*e.g.* London et al 2014)
  - Can we perform a test of the No Hair Theorem? **(See Chandra's talk today, and Archisman's talk on Thursday)**

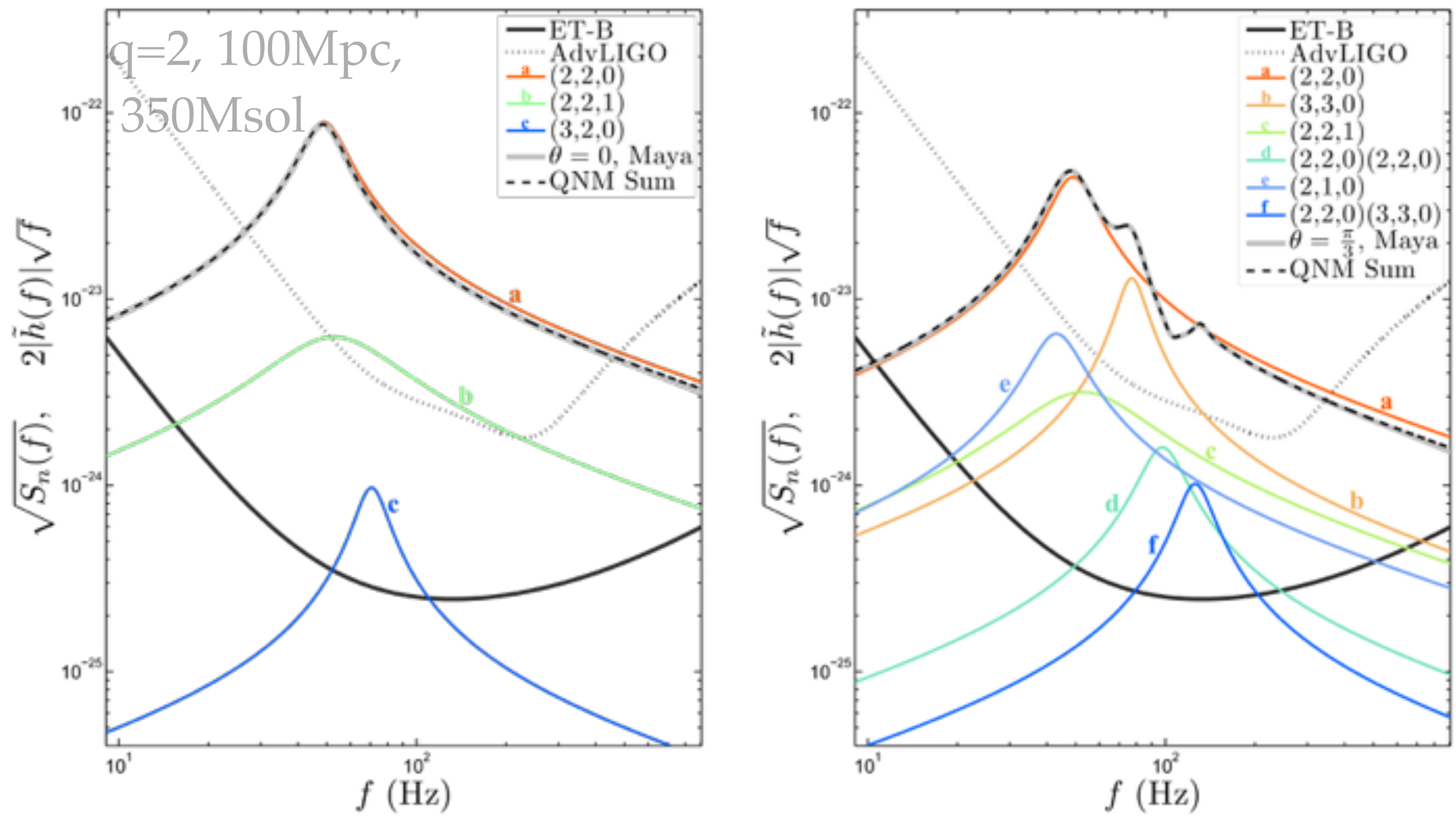
Can we "coherently stack ringdown signals" to improve statistics? This has been preliminarily investigated by Yang, Yagi et al (2017) **(also see Felipe's talk)**

Can we use QNM information to develop IMR models with higher multipoles **(See Sebastian's talk)**

New (more recent) Questions

- Can we distinguish BH QNMs from BH mimickers? (*e.g.* Cardoso *et al* 2016)
- Beyond GR effects? **(See Archisman's talk on Thursday)**
- What will be needed to model QNM excitations for precessing systems?
- Will the 2nd order (nonlinear) QNMs be relevant for GW astronomy? (London et al 2014)

When can we expect to confidently extract multiple QNMs?

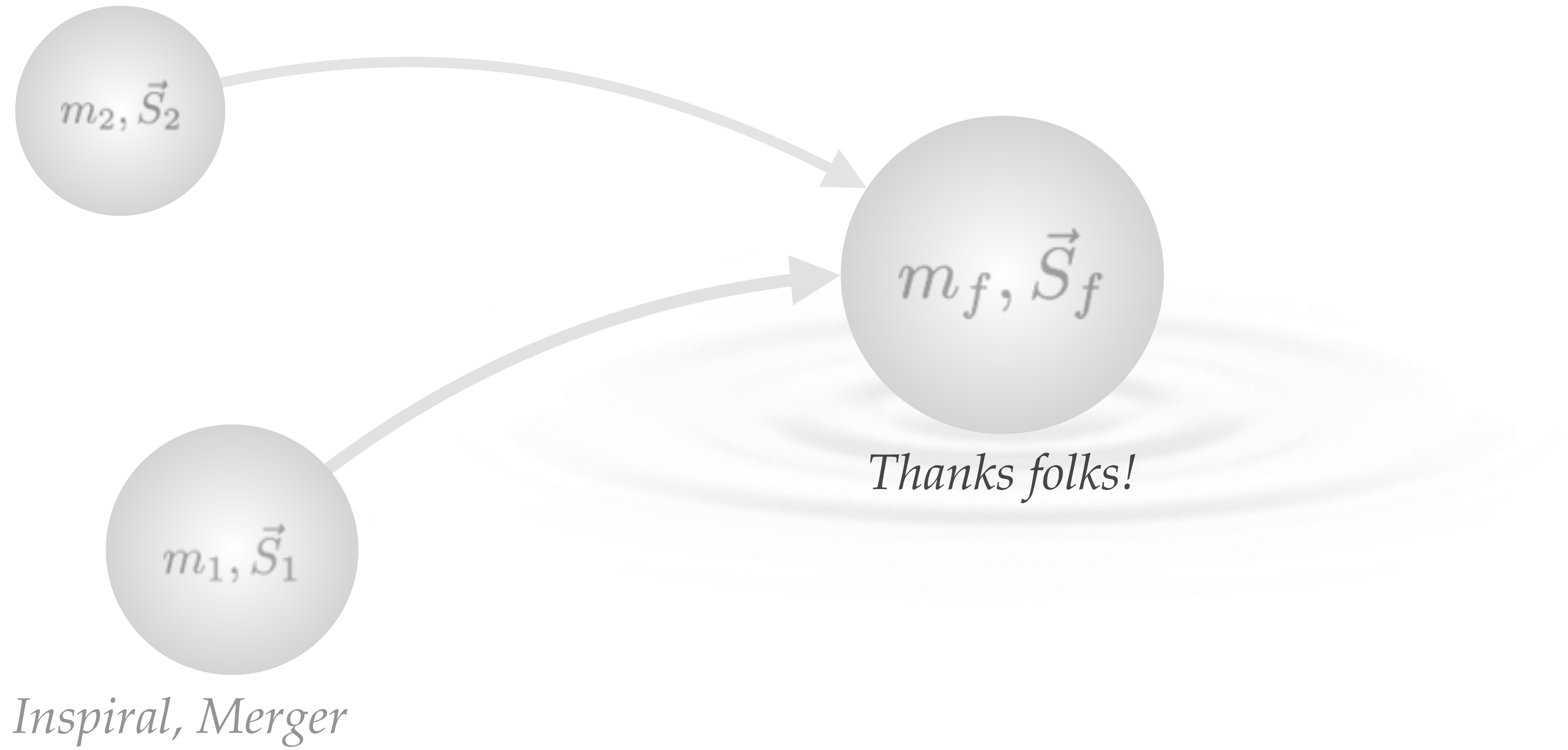


(Figure from London *et al* 2014)

- Most evidence suggests extraction QNM signals will be routine with the **Einstein telescope** (*e.g.* Maselli *et al* 2017, Gossan et al 2011, Berti, Cardoso, London, others)
- **However, there are always new possibilities on the horizon …**

Concluding Remarks

- We've seen that the topic of **BH perturbation theory grew out of pondering the very existence of BHs in nature**.
- QNM solution space is generally rich with features. **The fundamental QNMs are the least damped, and therefore the most likely to be observed first**.
- QNM solutions represent eigenfunctions of perturbed Einstein equations, but **NR simulations are currently needed to inform how much each QNM is excited**.
- Models of QNM excitation can **assist with tests of GR, and inform IMR signal models (some updates in later talks)**
- The extraction of **astrophysical QNMs (beyond l=m=2) may take some time (Einstein Telescope)**, but there is some uncertainty about how much …



<u>Context</u>: Quasinormal models resulting from the merger of stellar mass BHs, and learning as much as we can from post-merger (ringdown) signals …

The Quasi-normal Modes of Black Holes

Review and Recent Updates

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May 31st 2017