A "No-hair" test of binary black hole nature

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GWPAW '17

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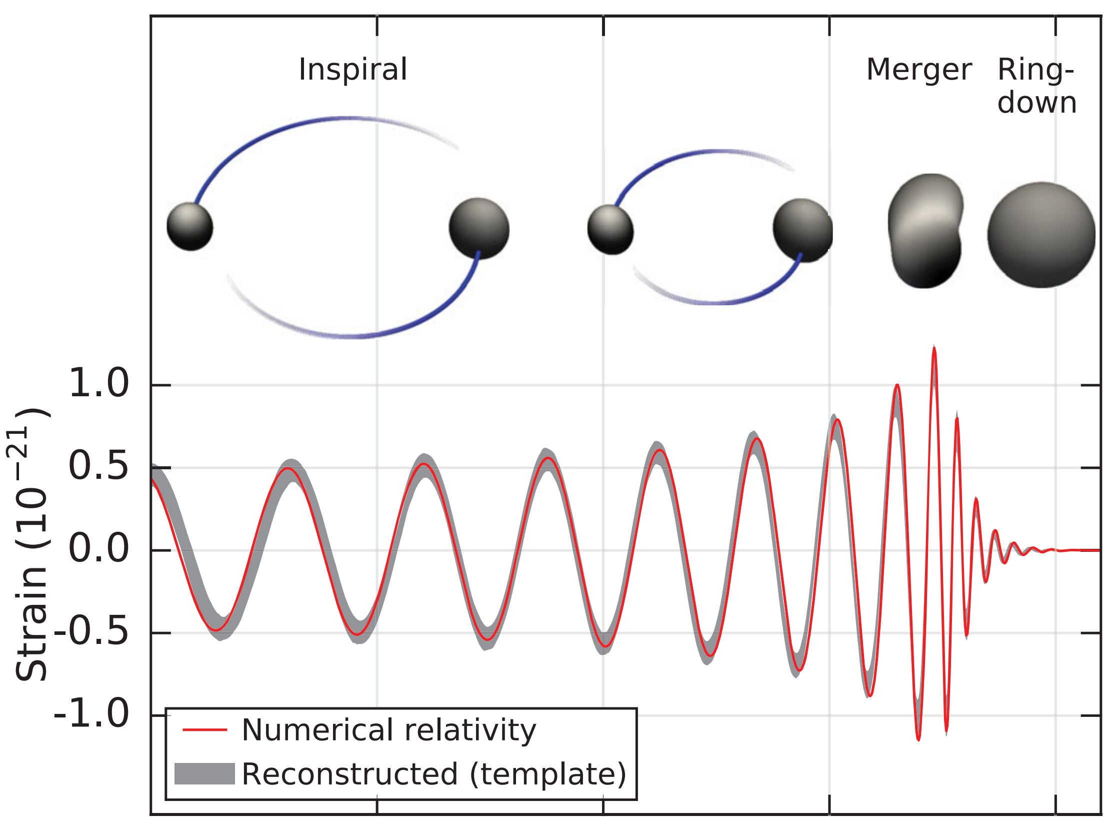
Reference: Krishnendu et al., arXiv:1701.06318

GW observations and BH signatures

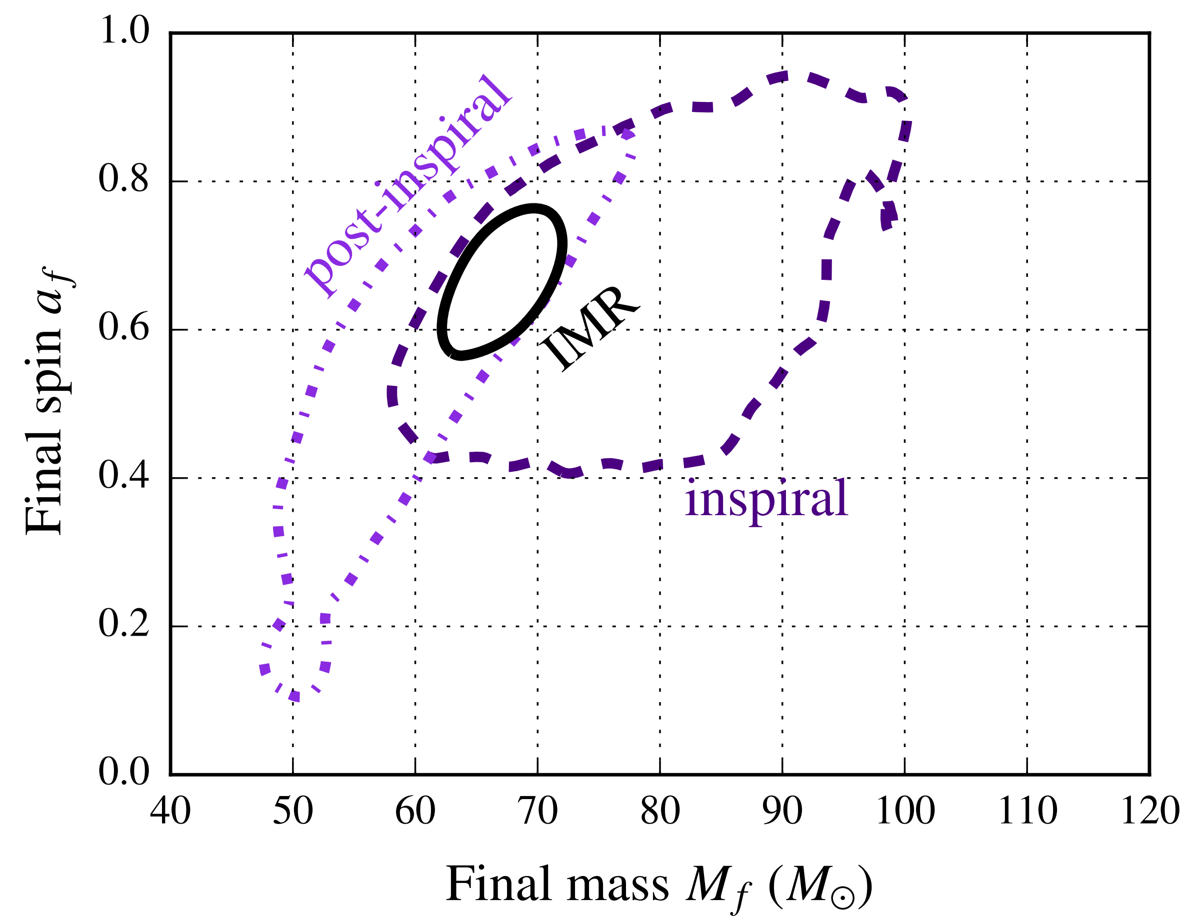
Twin GW events and their BBH nature

Waveform Reconstructions



LVC, Phys. Rev. Lett. 116, 061102 (2016)

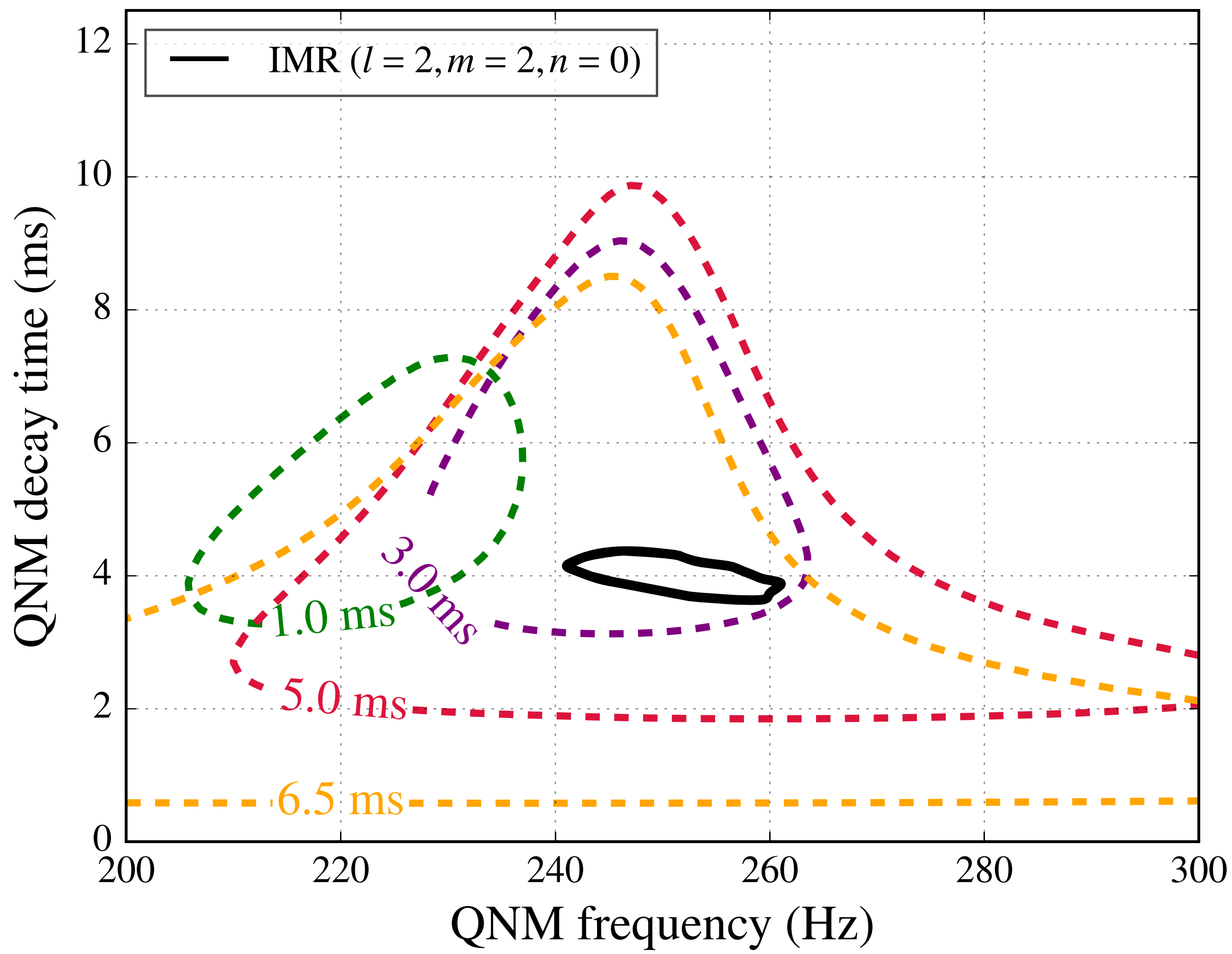
Waveform Consistency Tests



LVC, Phys. Rev. Lett. 116, 221101 (2016)

Is the final object a BH?

Observed quasi-normal mode spectrum of the remnant



LVC, Phys. Rev. Lett. 116, 221101 (2016)

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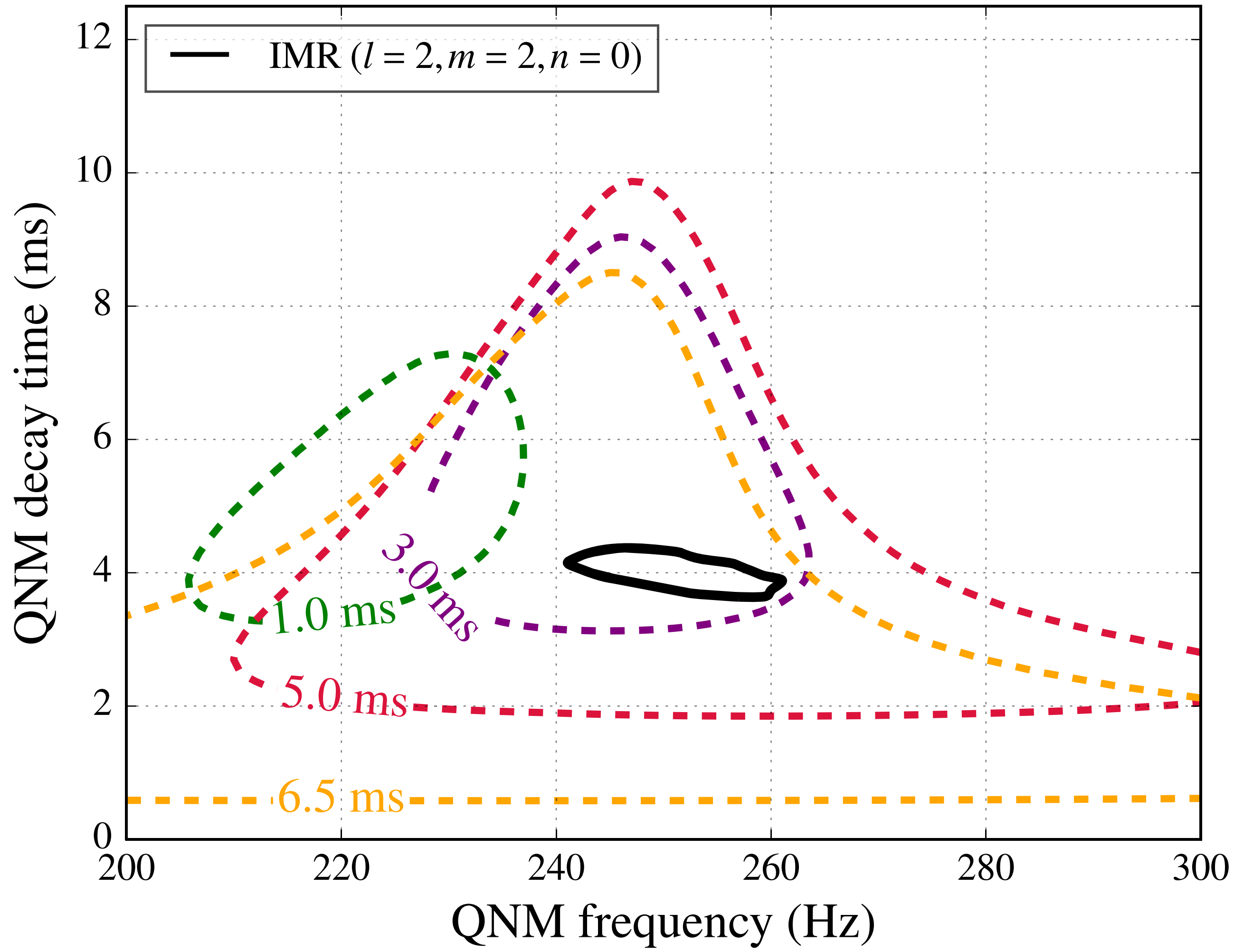

LVC, Phys. Rev. Lett. 116, 221101 (2016)

- **Power in the ringdown signal.**
- **Uncertainties in predicting the start of the ringdown.**

What about the binary constituents?

Tidal deformability of the compact object

Induced quadrupole moment

External tidal field

$$Q_{ij} = -\lambda\, \epsilon_{ij}$$

Tidal love number

$$k_2 = \frac{3}{2} G \lambda R^{-5}$$

Dimensionless tidal love number: **0** for BHs

Damour & Nagar, 2009
Bennington & Poisson, 2009

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Higher order effects and hence difficult to measure.

Damour & Nagar, 2009
Bennington & Poisson, 2009

Cardoso+, arXiv: 1701.01116 (2017)
Maselli+, arXiv:1703.10612 (2017)

A “No-hair” Test for Compact Binaries

We propose a new way to test the BH nature
by measuring the spin-induced multipole moments of
a compact binary system.

The proposal

Measure spin-induced deformations of binary components

$$Q_2 = -\kappa\ M^3 \chi^2$$

$\kappa = 1$ for BHs

$\simeq 2 - 14$ for NS

$\simeq 10 - 150$ for Boson Stars

$- / +$ for Gravstars

$$S_3 = -\lambda\ M^4 \chi^3$$

$\lambda = 1$ for BHs

$\sim 4 - 30$ for NSs

$\sim 10 - 200$ for BSs

Poisson, PRD 57, 5287 (1998);
Laarakkers+, ApJ 512, 282 (1999);
Pappas+, PRL 108, 231103 (2012);
F. D. Ryan, PRD 55, 6081 (1997)
Uchikata+, CQG 32,085008(2015)

Gravitational Waveforms and spin deformations

- Post-Newtonian inspiral, spinning waveforms with 3.5PN phase and 2PN amplitude is now available.

- Spin-induced quadrupolar (spin-spin) corrections appear at 2PN and 3PN order in phase and at 2PN in amplitude : κ dependence

- Spin-induced octupolar (spin-spin-spin) corrections appear at the 3.5PN order in phase: λ dependence

Marsat, CQG 32, 085008 (2015);
Buonanno+, PRD 87, 044009 (2012);
Bohe+, CQG 32, 195010 (2015);
Arun+, PRD 79, 104023 (2009)
CKM+, PRD 93, 084054 (2016)
Krishnendu+, arXiv:1701.06318(2017)

What we did

Fisher Matrix approach to parameter estimation

— assumes Gaussian noise and high SNR

— gives a lower bound on errors / highly inexpensive

Parameter Space: $\{t_c, \phi_c, D_L, \iota, \mathcal{M}, \delta, \chi_1, \chi_2, \kappa_1, \kappa_2, \lambda_1, \lambda_2\}$

— alternatively, $\{t_c, \phi_c, D_L, \iota, \mathcal{M}, \delta, \chi_1, \chi_2, \kappa_s, \kappa_a, \lambda_s, \lambda_a\}$

Compact Binary

$$\kappa_s = (\kappa_1 + \kappa_2)/2; \kappa_a = (\kappa_1 - \kappa_2)/2$$

$$\lambda_s = (\lambda_1 + \lambda_2)/2; \lambda_a = (\lambda_1 - \lambda_2)/2$$

BBH

$$\kappa_s = 1; \kappa_a = 0$$

$$\lambda_s = 1; \lambda_a = 0$$

Large dimensionality and unreliable measurements

— We set $\kappa_a,\ \lambda_s,\ \lambda_a$ to their BBH values

What we did

New parameter space: $\{t_c, \phi_c, D_L, \iota, \mathcal{M}, \delta, \chi_1, \chi_2, \kappa_s\}$

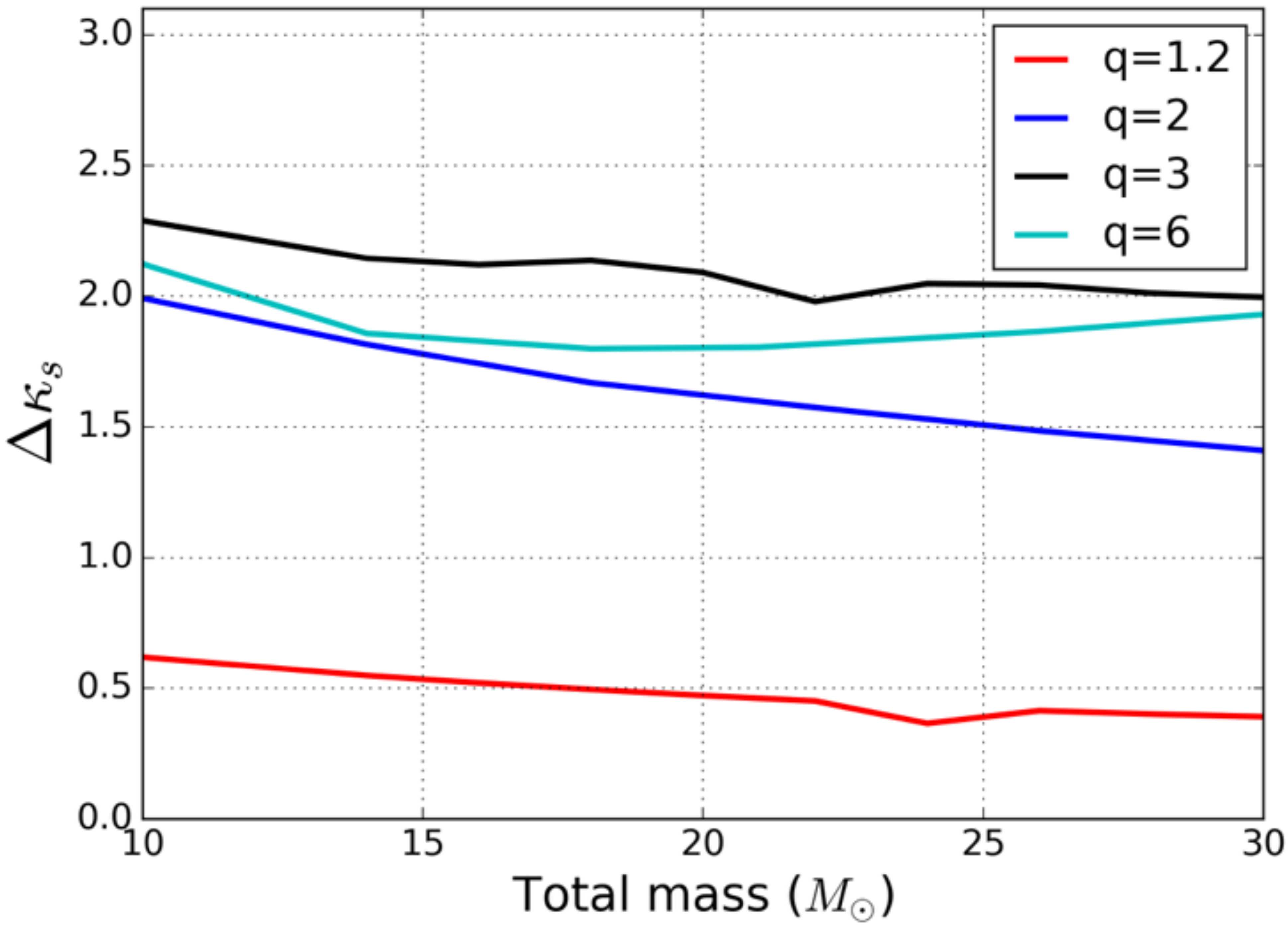
Note that $\kappa_s = 1$ for BBH

— accuracies with which it can be measured gives constrain on BBH nature

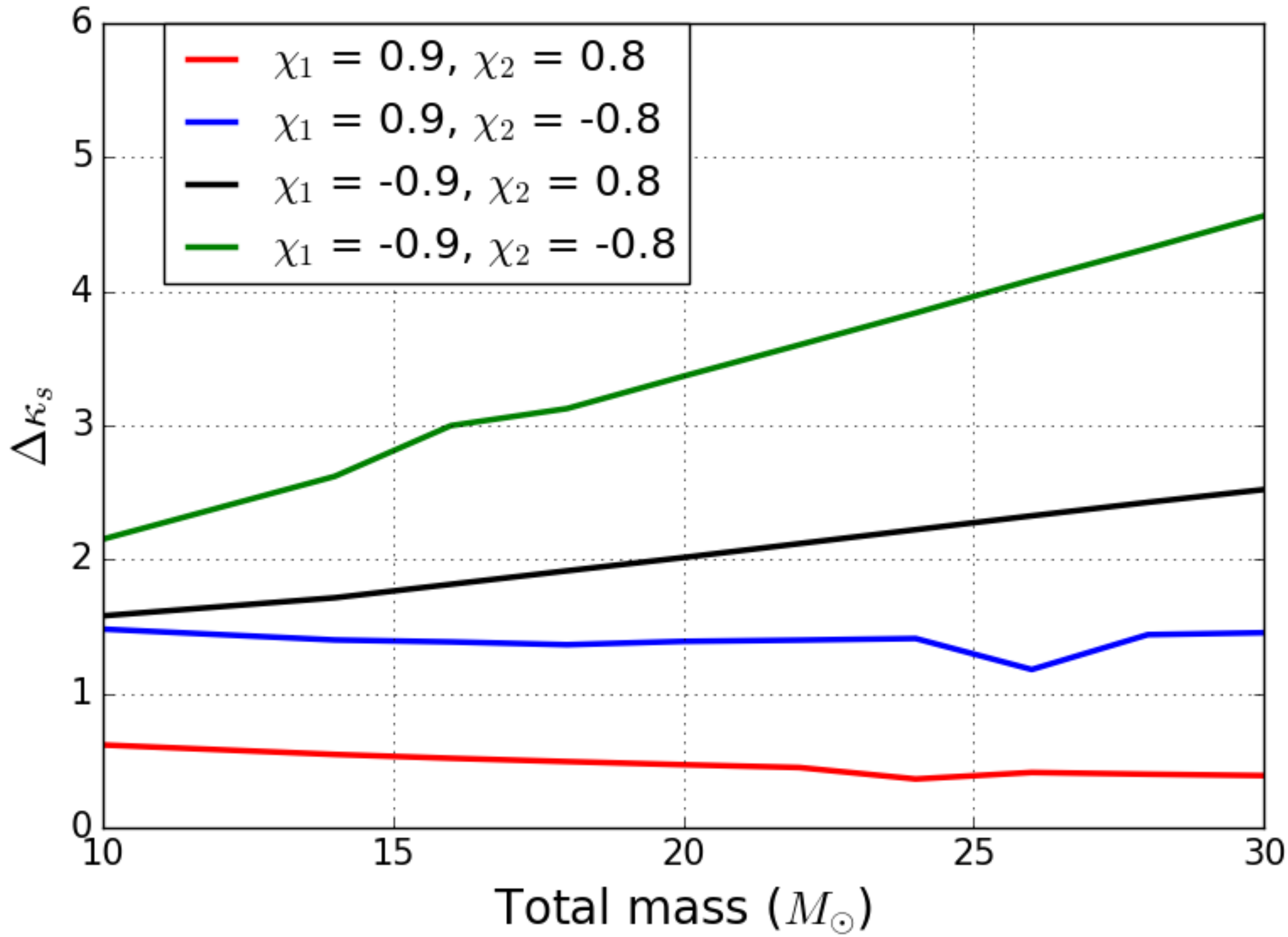
The choice $\kappa_a = 0$

— The test may be viewed as a Null test of the BBH nature of a compact binary

Effect of mass-ratio

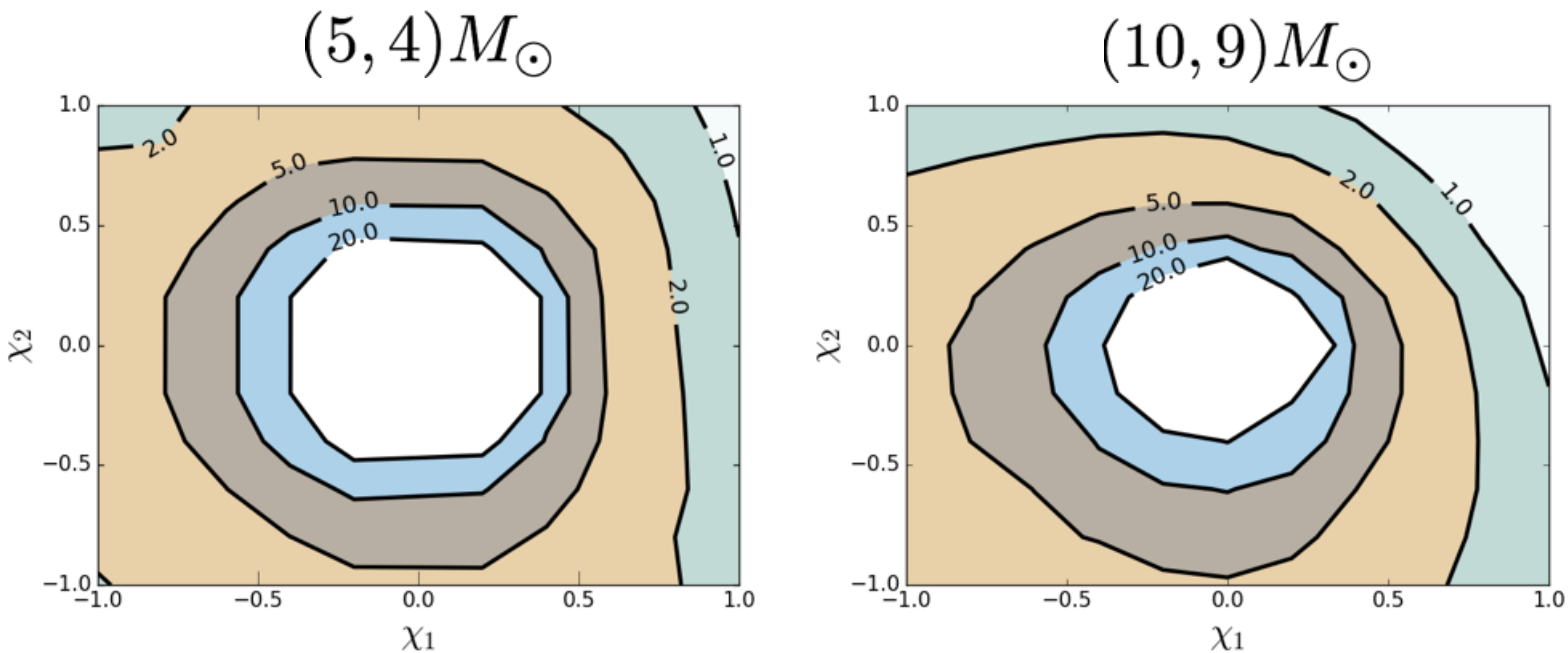


Effect of Spins



aLIGO

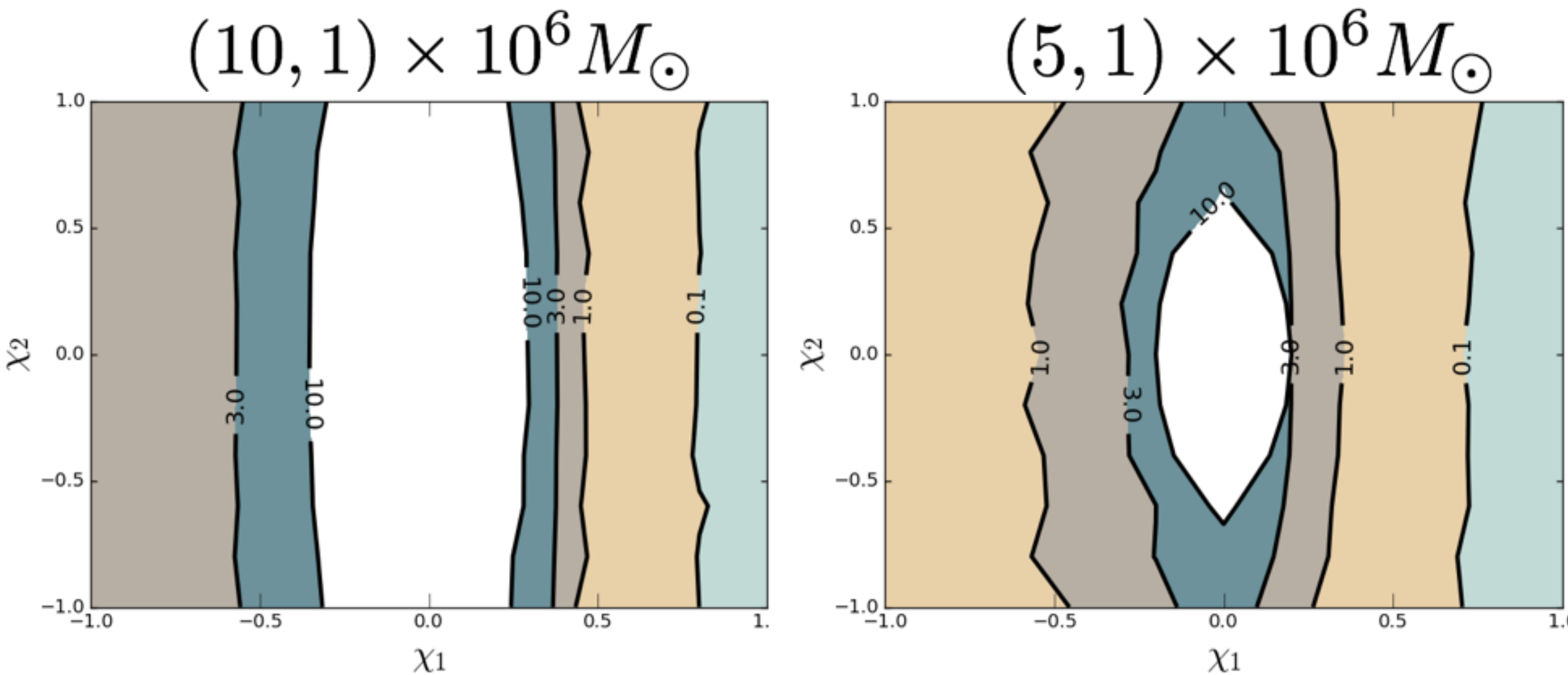
SNR=10; f_{low}=20 Hz



$1-\sigma$ error contours for κ_s

LISA

D_L=3 Gpc; f_{low}=10^{-4} Hz



$$1-\sigma \text{ error contours for } \kappa_s$$

Summary and Conclusions

Even with a moderate SNR of 10, observations of aLIGO can be used to measure spin-induced quadrupolar deformations for a narrow range of parameter space.

$$\kappa_s \leq 1$$

For configurations where the measurement accuracies are >100% these estimates can be used to put bounds on expected values for BH mimickers such as BSs and GSs.

LISA observations on the other hand can be used to measure these deformations for a wide range of parameter space with moderate to high spins.

Additional slides

Waveforms and spin deformations

Inspiral waveform (SPA)

$$\tilde{h}(f) = \frac{M^2}{D_L}\sqrt{\frac{5\pi}{48\eta}}\sum_{n=0}^{4}\sum_{k=1}^{6} V_k^{n-7/2}\, C_k^{(n)}\, e^{i(k\Psi_{\rm SPA}(f/k)-\pi/4)}$$

Amplitude corrections from kth harmonic at nth PN order

$$\Psi_{\rm SPA}(f) = 2\pi f t_{\rm c} - \phi_{\rm c} + \left\{\frac{3}{128\eta\, v^5}\left[\psi_{\rm NS} + \psi_{\rm SO} + \psi_{\rm SS} + \psi_{\rm SSS}\right]\right\}_{v=V_1(f)}$$

2PN amplitude & 4PN phase

Waveforms and spin deformations

Spin deformations in amplitude

$$\mathcal{C}_2^{(4)} \equiv \mathcal{C}_2^{(4)}(F_+, F_\times, \cos(\iota), \eta, \chi_1, \chi_2, \kappa_s, \kappa_a)$$

Spin deformations in phase

$$\psi_{\rm Spin} \equiv \psi_{\rm SO} + \psi_{\rm SS} + \psi_{\rm SSS} = v^3 \left[\mathcal{P}_3 + \mathcal{P}_4\, v + \mathcal{P}_5\, v^2 + \mathcal{P}_6\, v^3 + \mathcal{P}_7 v^4 + \mathcal{P}_8 v^5 + \cdots\right]$$

SS: $\mathcal{P}_{\{4,6\}} \equiv \mathcal{P}_{\{4,6\}}(M, \eta, \chi_1, \chi_2, \kappa_s, \kappa_a)$

SSS

$$\mathcal{P}_7 \equiv \mathcal{P}_7(M, \eta, \chi_1, \chi_2, \lambda_s, \lambda_a)$$

Parameter Estimation

Fisher Information Matrix

$$\Gamma_{ab} = 4 \int_{f_s}^{2 f_{\rm lso}} \frac{{\rm Re}(\tilde{h}_a^*(f)\tilde{h}_b(f))}{S_h(f)}$$

$$\Sigma^{ab} \equiv \langle \Delta\theta^a \Delta\theta^b \rangle = (\Gamma^{-1})^{ab}$$

Covariance Matrix

Measurement Errors

$$\sigma_a = \left\langle (\Delta\theta^a)^2 \right\rangle^{1/2} = \sqrt{\Sigma^{aa}}$$