Poster prize talk

Estimation of starting times of quasi-normal modes in ringdown gravitational waves with the Hilbert-Huang transform

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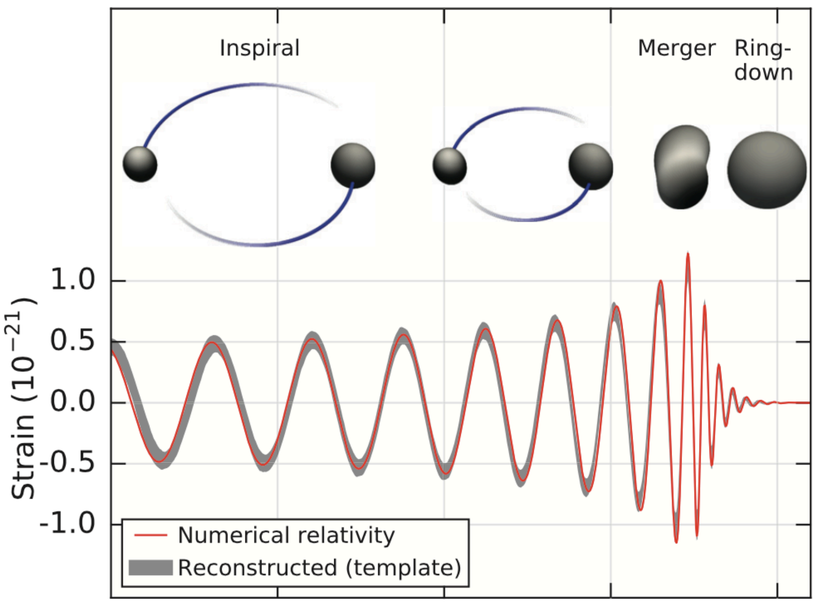
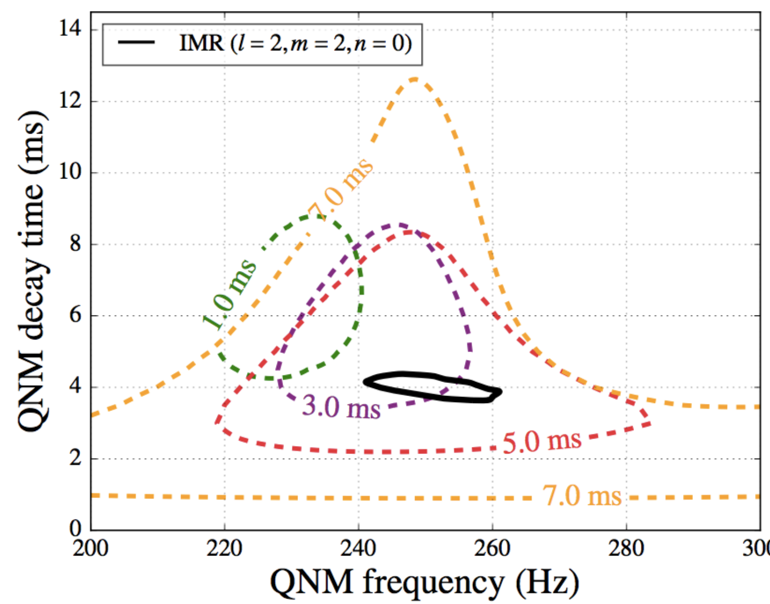
Introduction

We propose a method to estimate a starting time of QNM and QNM parameters by analyzing a GW from BBH.

- The estimation of the starting time is necessary to investigate a remnant BH of BBH coalescence.
- We are focusing on the Hilbert-Huang transform

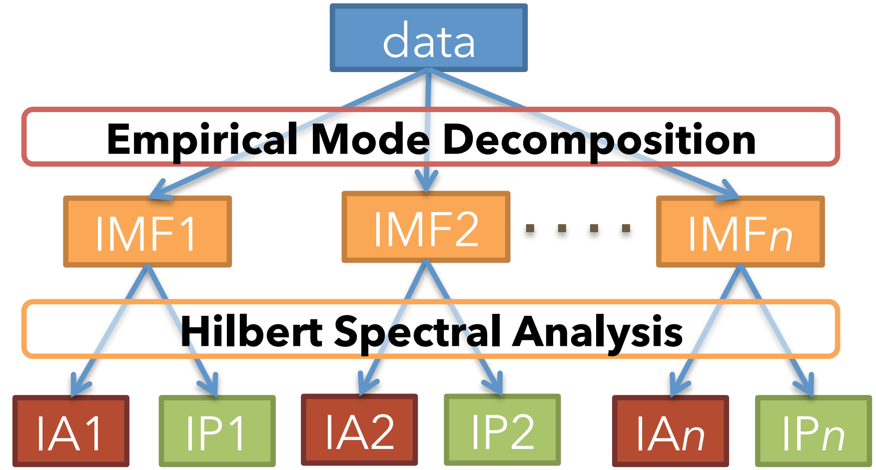




Hilbert-Huang transform, and QNM

Time-frequency analysis.

- decomposes data into intrinsic mode functions, which are zero-mean signals, or symmetric oscillations.
- extracts amplitude and phase of each IMF as **time series**.



IA, IP: instantaneous amplitude, phase

$$s(t) = \sum_{i=1}^{N_{\rm IMF}} \underbrace{a_i(t)}_{\rm IA} \cos\big(\underbrace{\phi_i(t)}_{\rm IP}\big) + r(t)$$

GW from QNM is

$$h_{\rm QNM}(t) = \Re\left[A_0 {\rm e}^{-{\rm i}\{\omega(t-t_0)+\varphi_0\}}\right] = A_0 {\rm e}^{\omega_{\rm I}(t-t0)} \cos\big(\omega_{\rm R}(t-t_0)+\varphi_0\big)$$

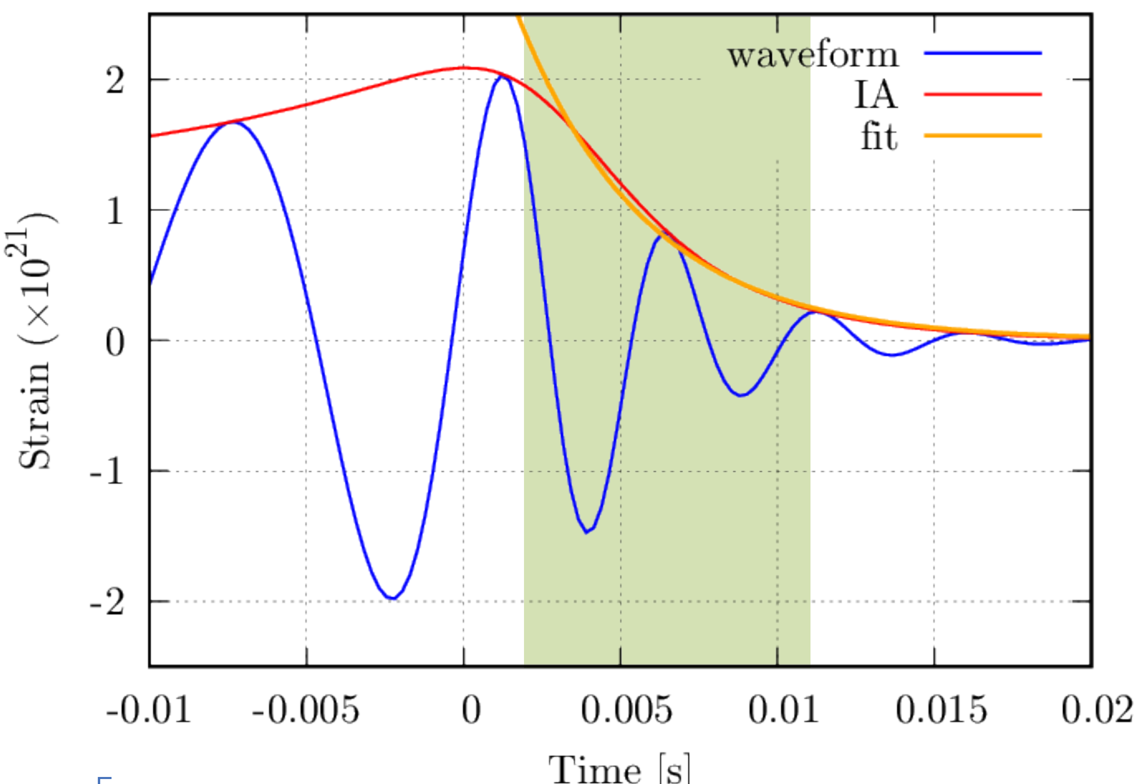
therefore, if a QNM is dominant in the j-th IMF,

$$\ln a_j(t) = \omega_{\rm I}(t-t_0) + \ln A_0, \quad \phi_j(t) = \omega_{\rm R}(t-t_0) + \varphi_0$$

Our method

1. Calculate fitting errors of $\ln a_1(t)$, for every possible section

$$\mathrm{RMSE}(n_0, N) = \min_{b,c} \sqrt{\frac{1}{N} \sum_{n=n_0}^{n_0+N-1} \{\ln a_1[n] - (bt + c)\}^2}$$



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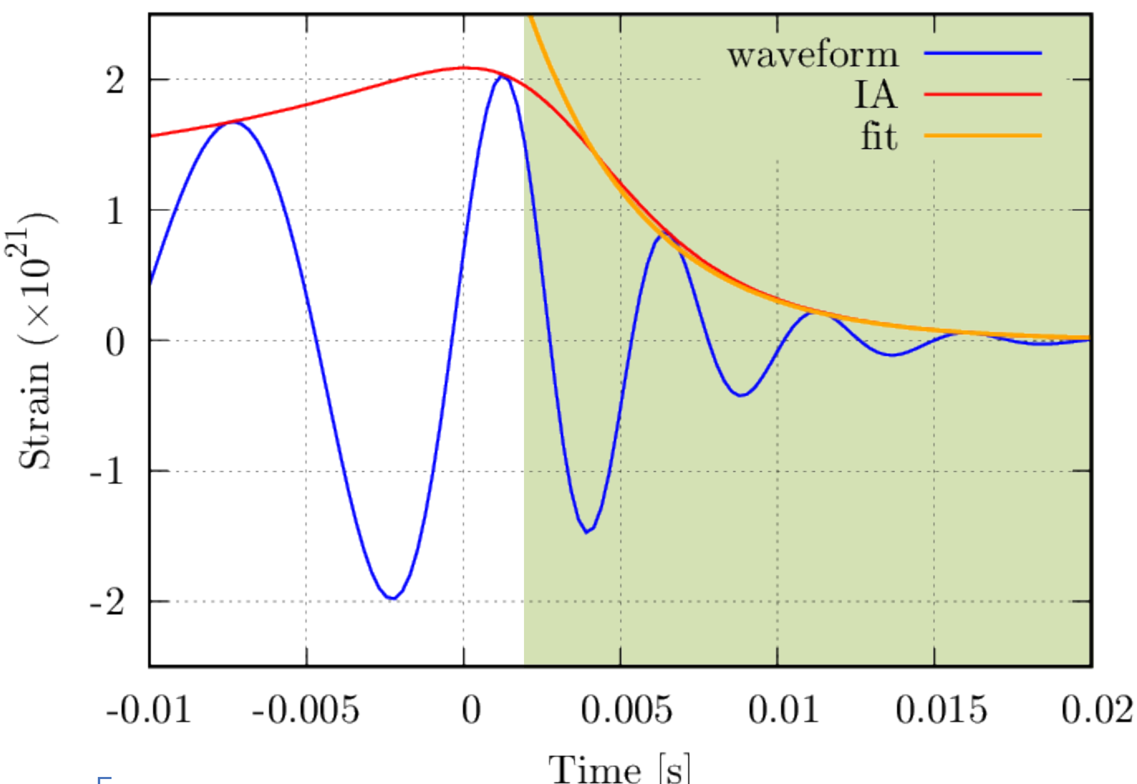

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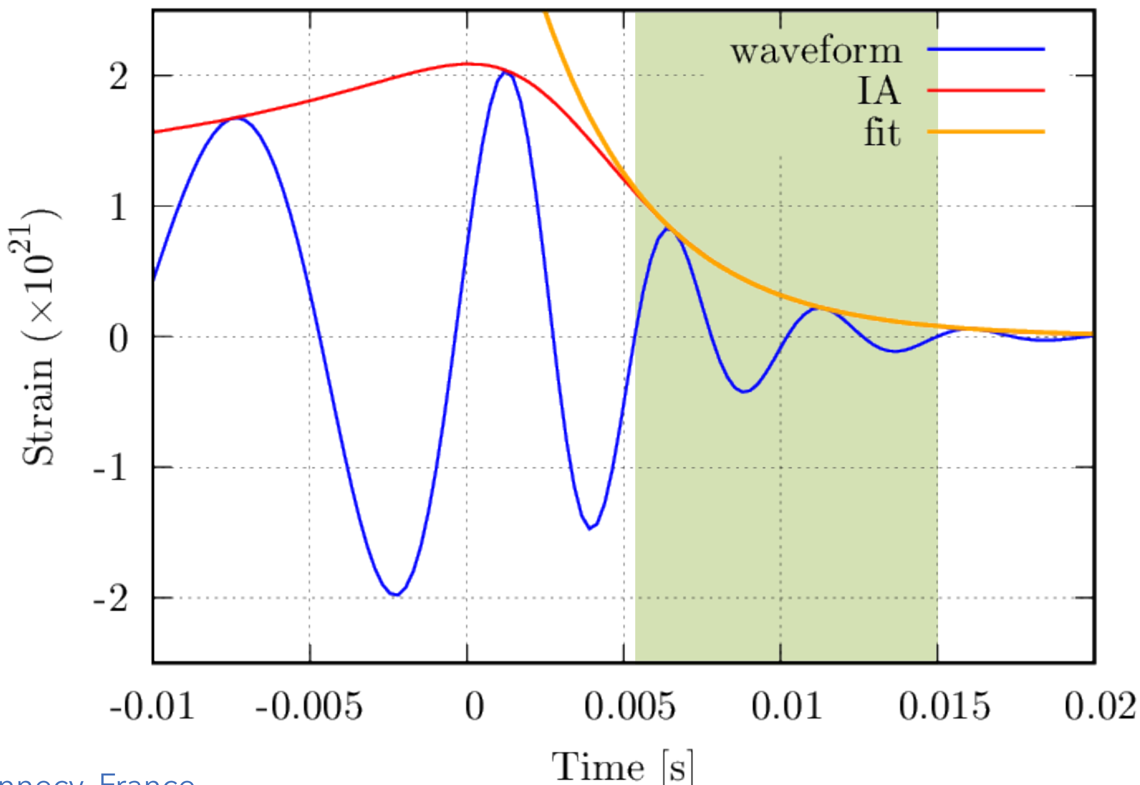
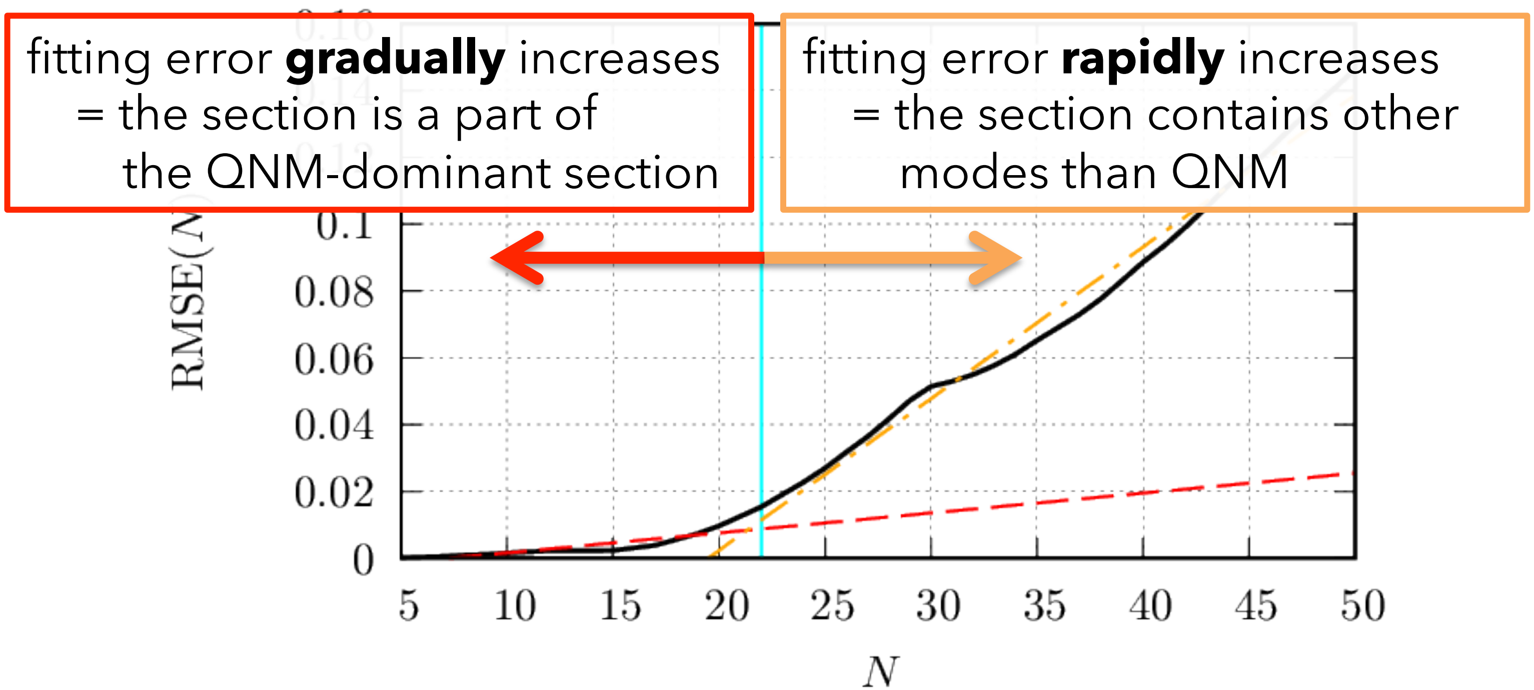


Our method

2. Determine $\hat{n}_0(N)$, the optimal value of n_0 for each N, as

$$\hat{n}_0(N) = \underset{n_0}{\mathrm{argmin}}\, \mathrm{RMSE}(n_0, N)$$

3. Determine $\hat{N}$ as the transition of slope of $N - \mathrm{RMSE}(\hat{n}_0(N), N)$

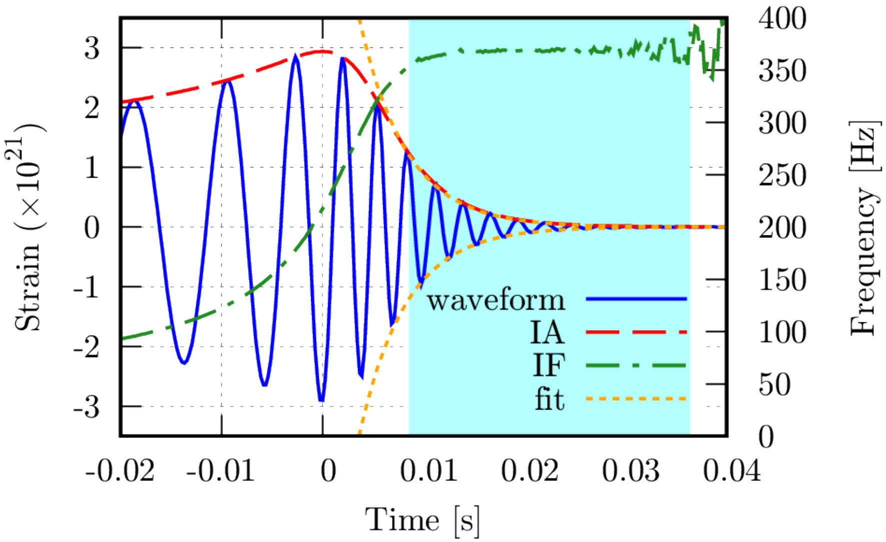


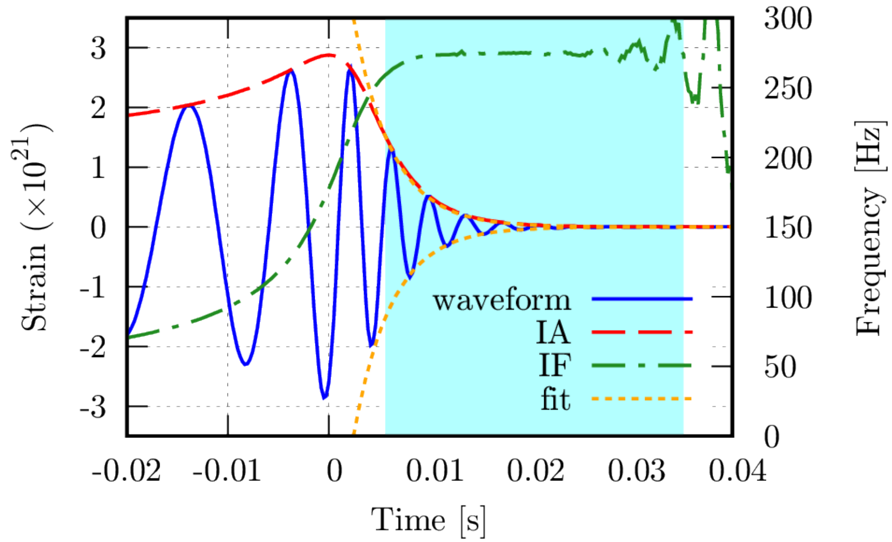
Then, we can get $\hat{t}_0 := t[\hat{n}_0(\hat{N})]$ and estimated QNM frequency.

Application to Simulated Waveforms

We applied our method to numerical relativity waveforms by the SXS project.

<u>**Examples:**</u>





In the estimated QNM-dominant section, the amplitudes are well fitted by exponentially decaying curve, and the frequencies become constant with time.

Application to Simulated Waveforms

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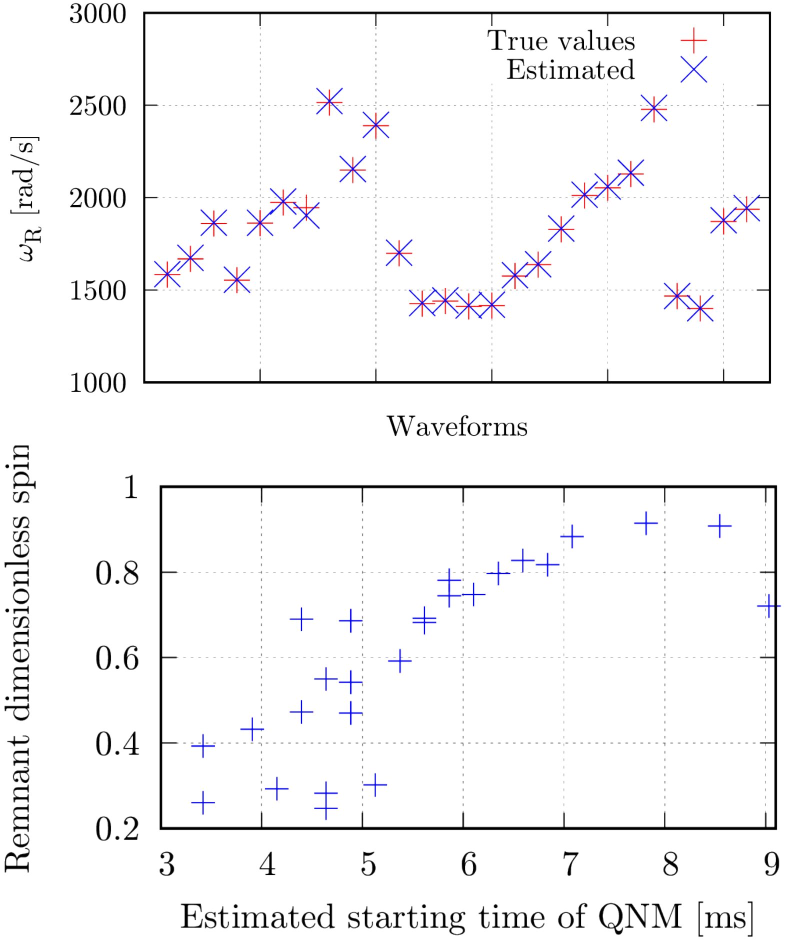
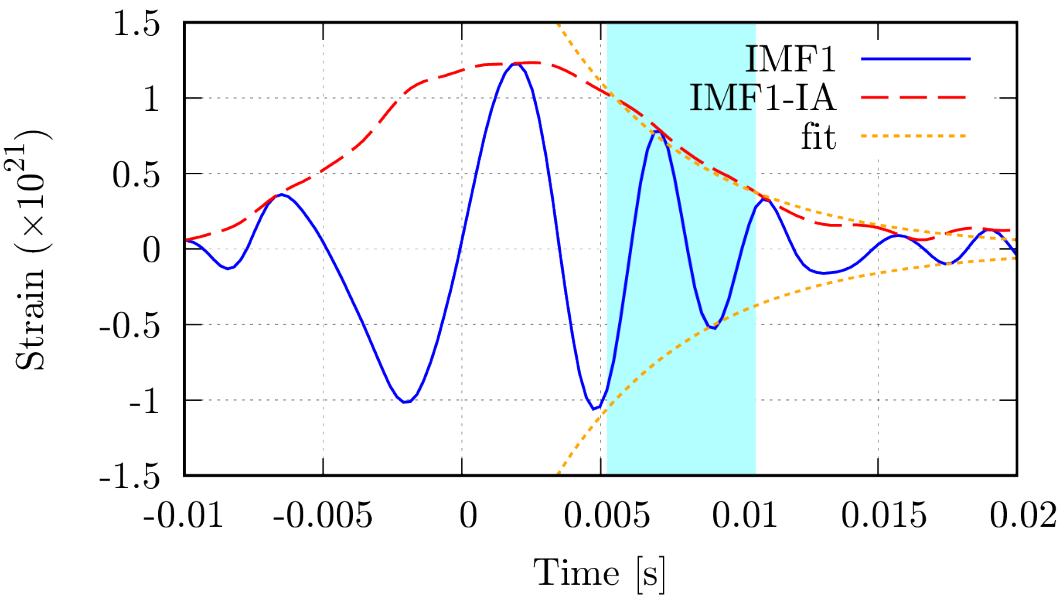


Some remarks:

- The relative errors of estimated values of ω are **less than 1%** on average.
- We found a correlation between the starting time of QNM and the remnant spin ($r = 0.764$)

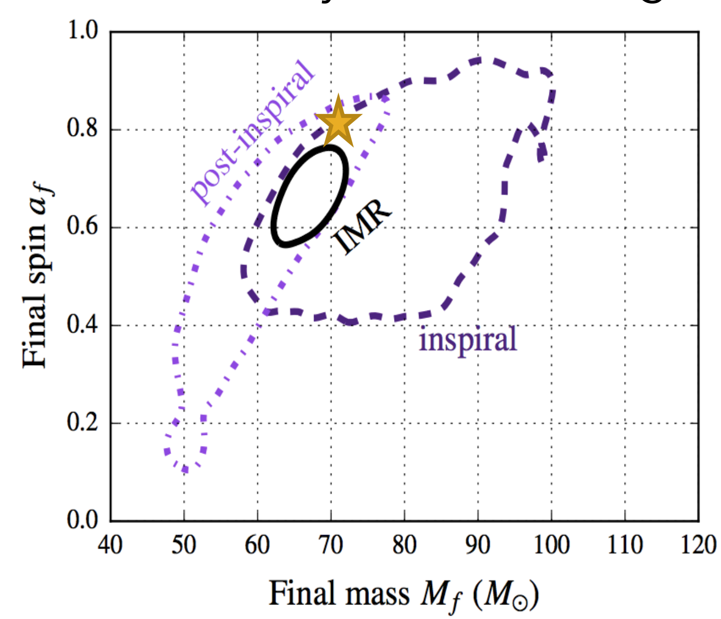
Application to GW150914

We also applied it to the observed data of GW150914



t_0/ms	rem. spin	rem. mass/M_{sun}
5.223	$0.8151^{+0.092}_{-0.096}$	$71.65^{+0.77}_{-0.75}$

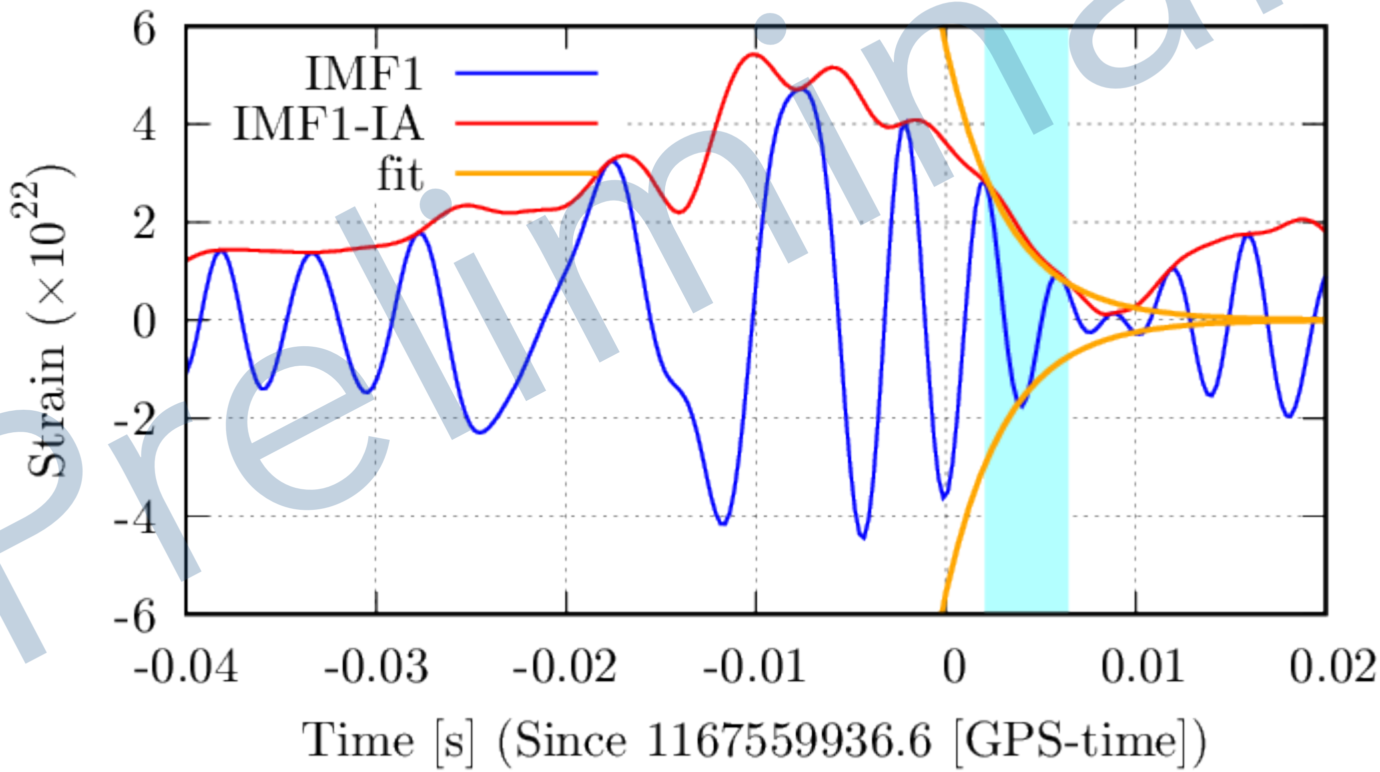
cf.) result by LIGO and Virgo



Although there are some systematic errors, the results are **consistent** with those given by LIGO and Virgo and a **reasonable starting time** is estimated.

Application to **GW170104** *[Preliminary]*

	t_0/ms	f_c/Hz	τ/ms	χ_{rem}	M_{rem}/M_{sun}
our method	2.05	298.54	2.71	0.41	47.9
LVC				$0.64^{+0.09}_{-0.20}$	$48.7^{+5.7}_{-4.6}$



Preliminary

Summary

We propose a method to estimate a starting time of QNM in a ringdown gravitational wave.

- We confirmed that it works properly for pure waveforms
- Although it is affected by noises, it can estimate some reasonable starting time for GW150914 and GW170104.

We are planning to make the method more robust to noise

- constructing a new mode-decomposing method specific to extraction of QNM from observed data

Acknowledgements

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