Testing gravity with binary black hole coalescences: results and prospects

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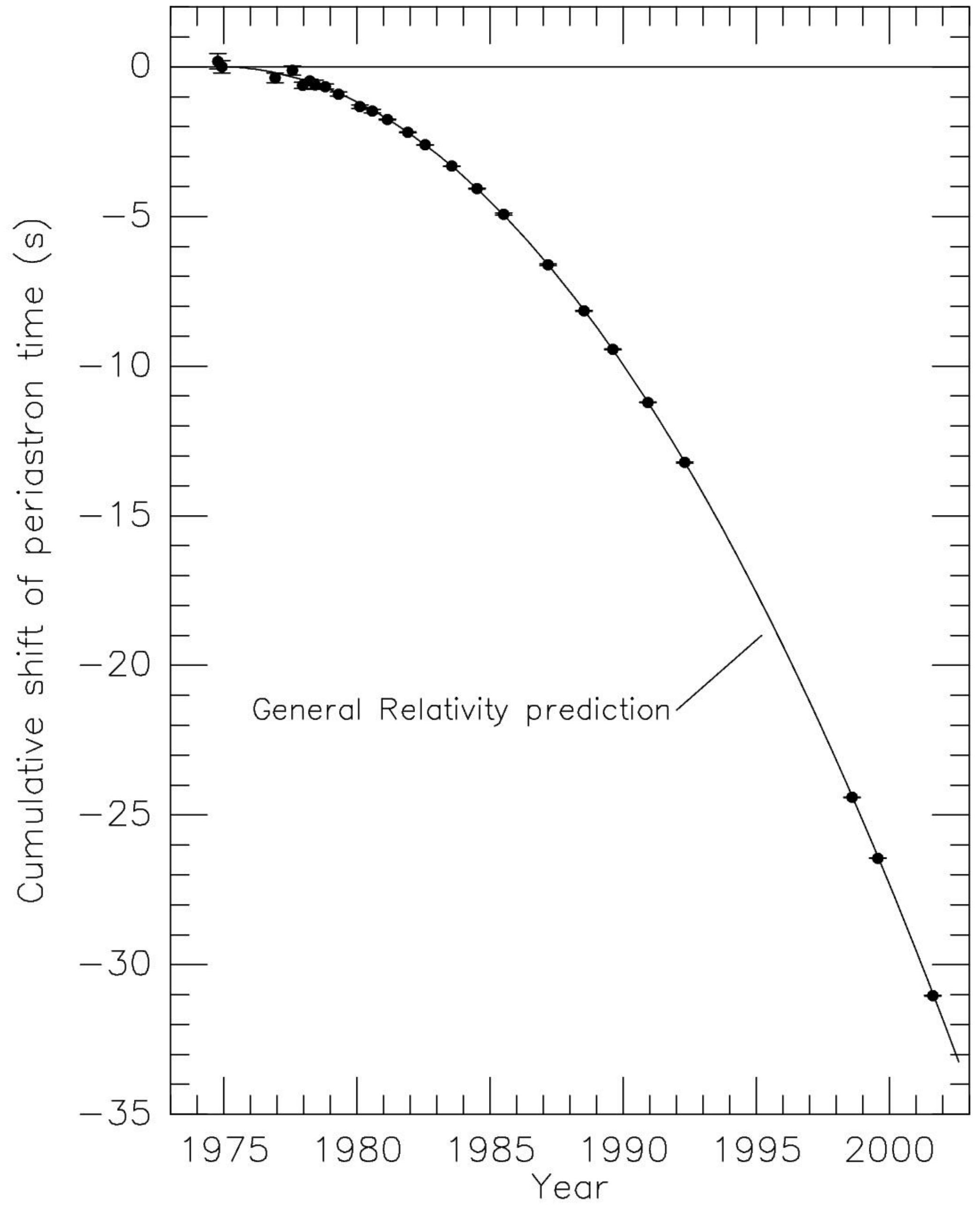
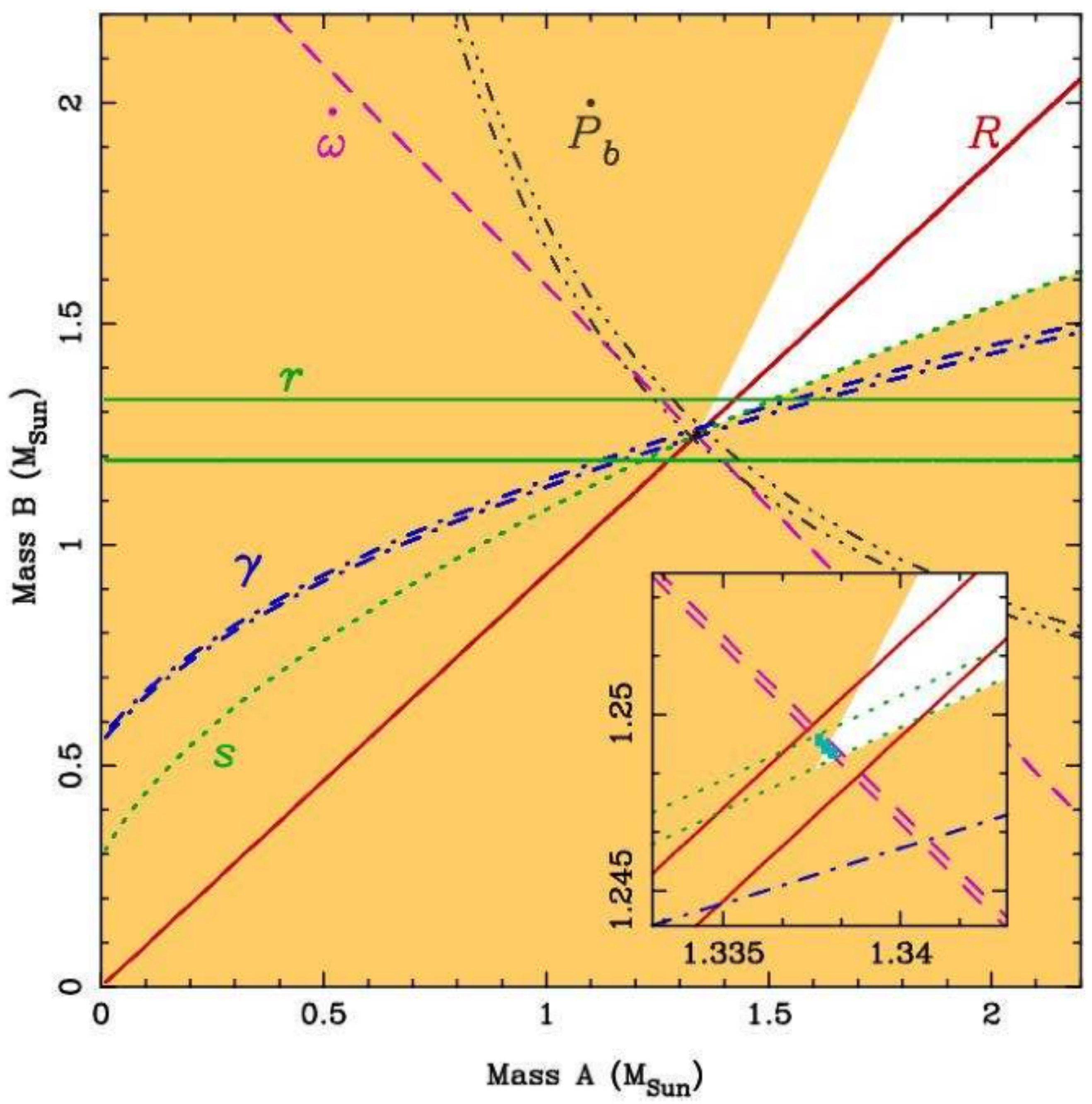
Why test general relativity?

- GR is non renormalisable
  - higher order terms in the action
- Dark matter & dark energy
  - signature of modified gravity?
- GR is extremely well tested in between these regimes (Will, arXiv:1403.7377, Psaltis, arXiv:0806.1531)



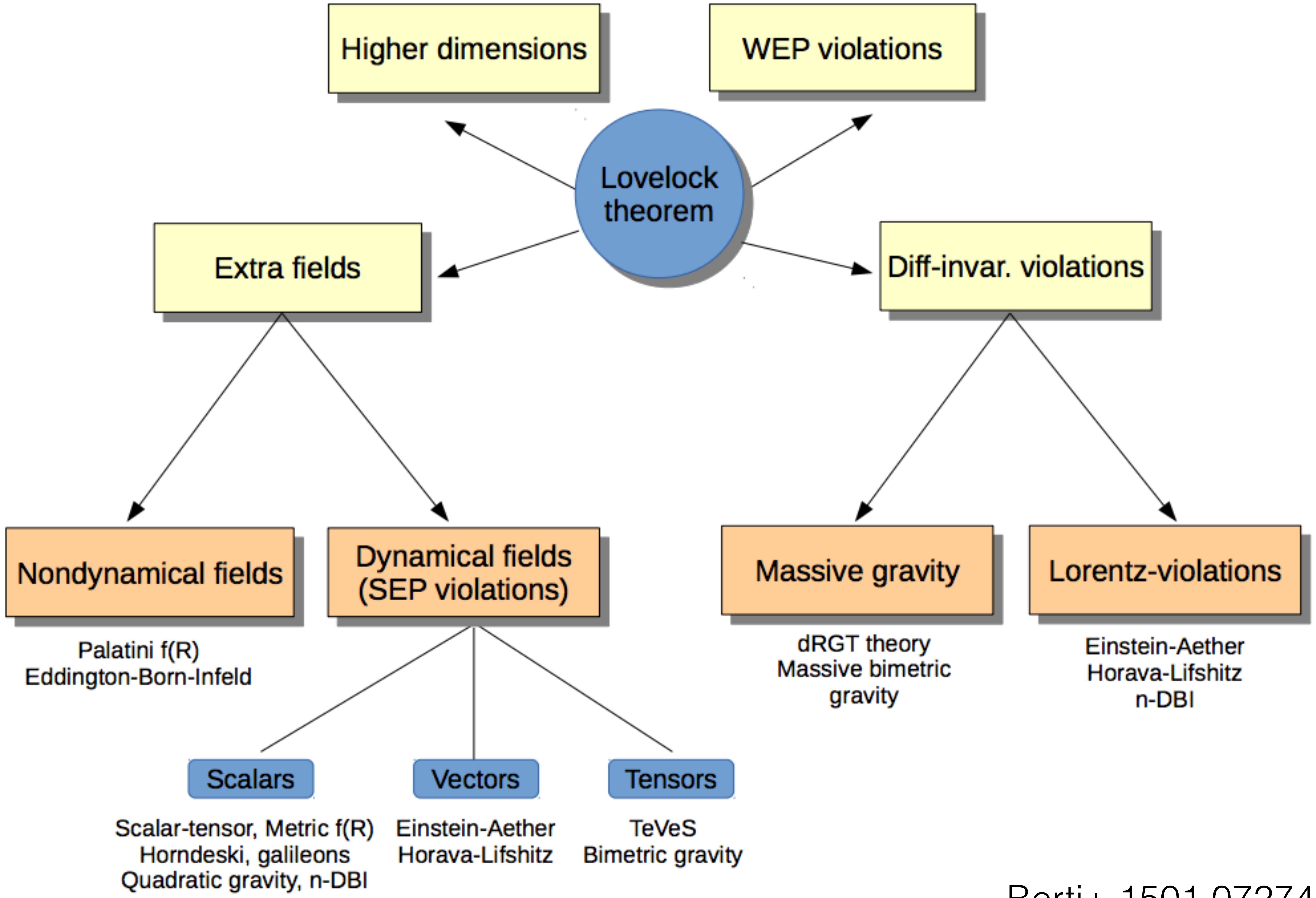


Weisberg & Taylor, arXiv:0407149
Kramer+,arXiv:0609417

What to test for?

- Alternative theories
  - Introduce extra degrees of freedom:
    - additional fields
    - higher-curvature terms
  - Challenge GR assumptions:
    - Lorentz invariance
    - Equivalence principle
- Need tests in the strong-field

Lovelock theorem: In 4D, the only divergence free symmetric rank-2 tensor constructed only by the metric and its derivatives up to 2nd order and preserving diffeomorphism invariance is the Einstein tensor plus a constant.



Berti+,1501.07274

Gravitational strong-field

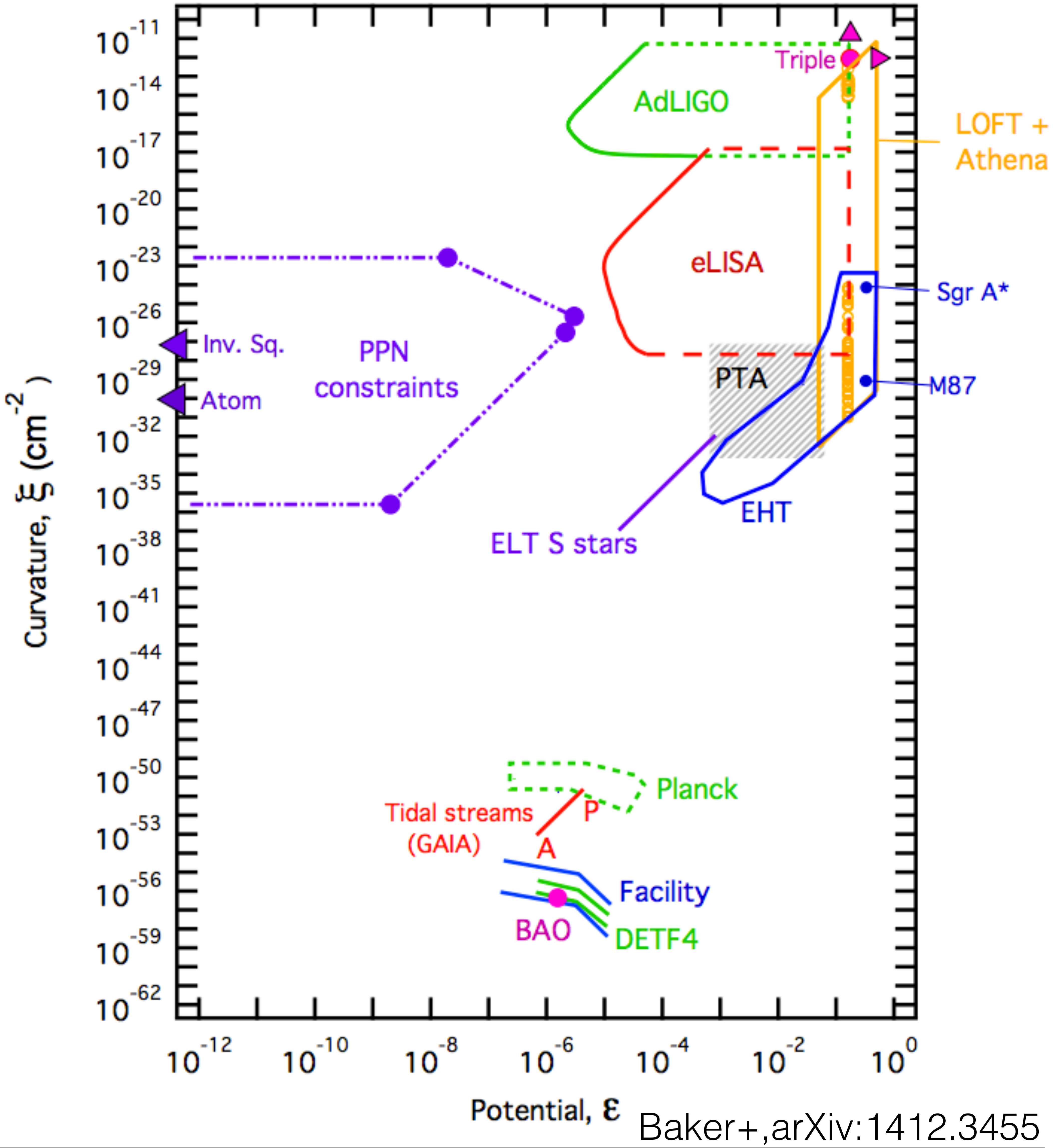
- Field strength

$$\epsilon = \frac{GM}{c^2 R}$$

- Curvature (Kretschmann scalar)

$$\xi = (R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta})^{1/2}$$

- Gravitational waves from binary black holes are the optimal probes



Baker+,arXiv:1412.3455

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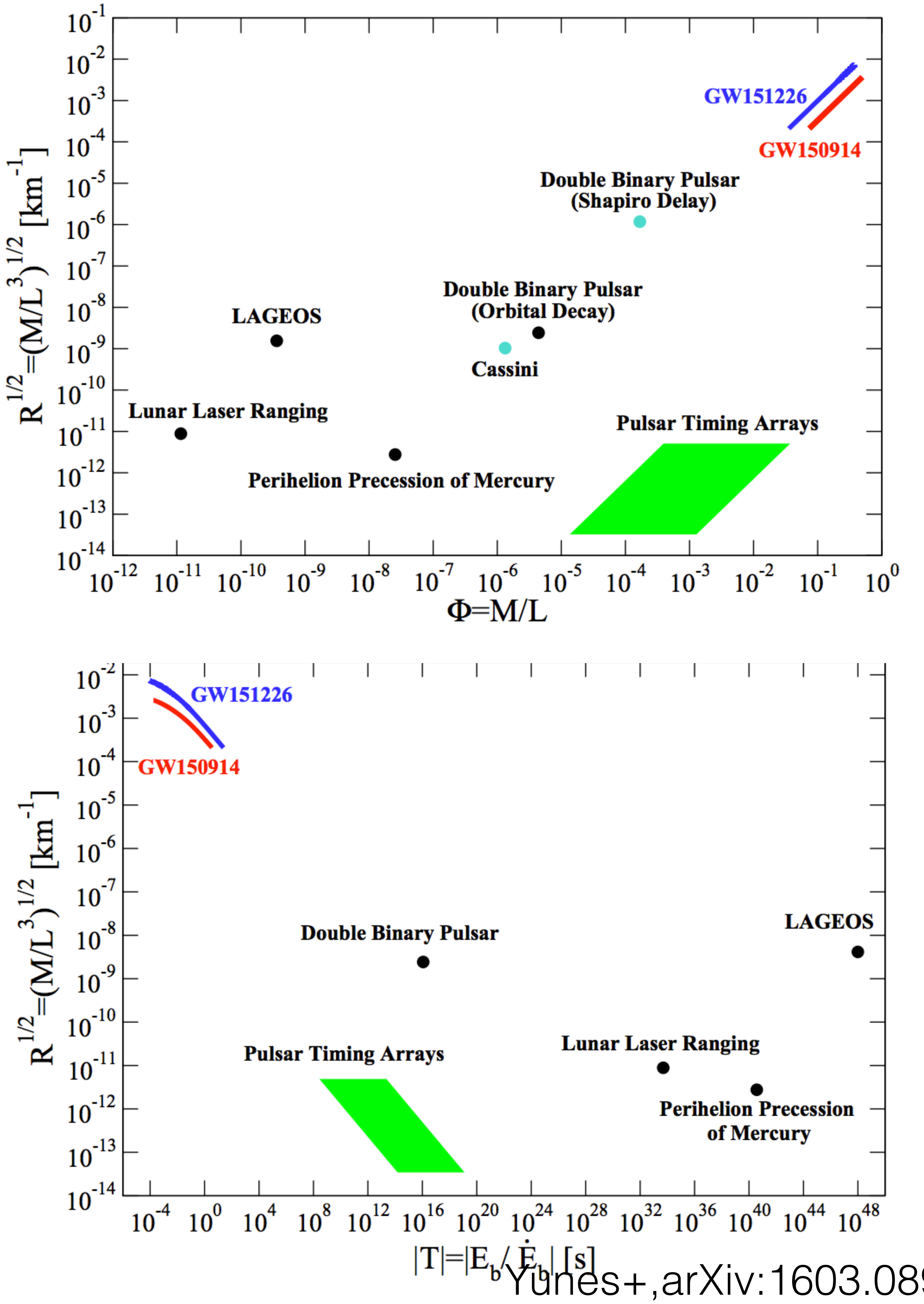
- Gravitational waves from binary black holes are the optimal probes

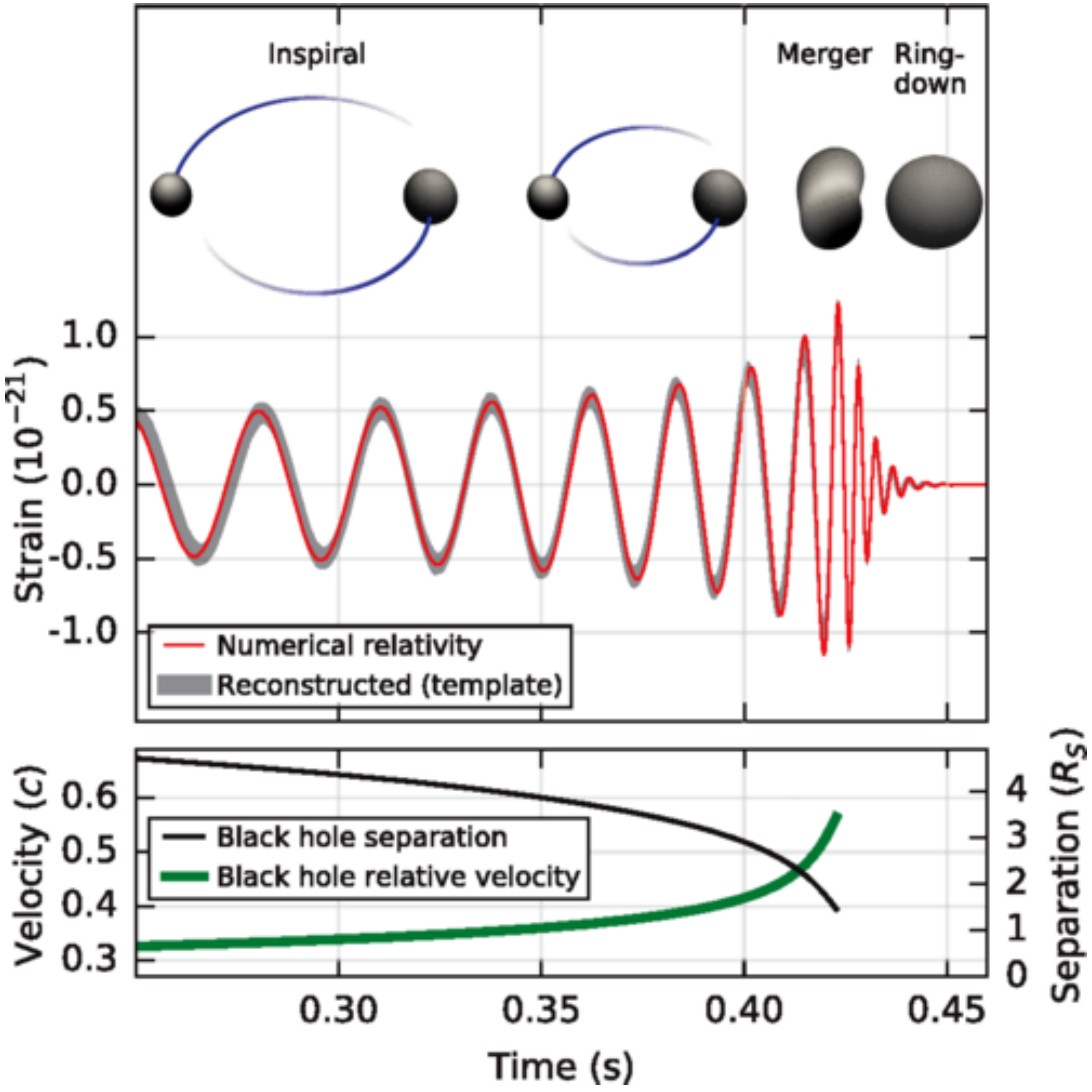
- Space-time is *dynamic*



Yunes+,arXiv:1603.08955

Gravitational waves

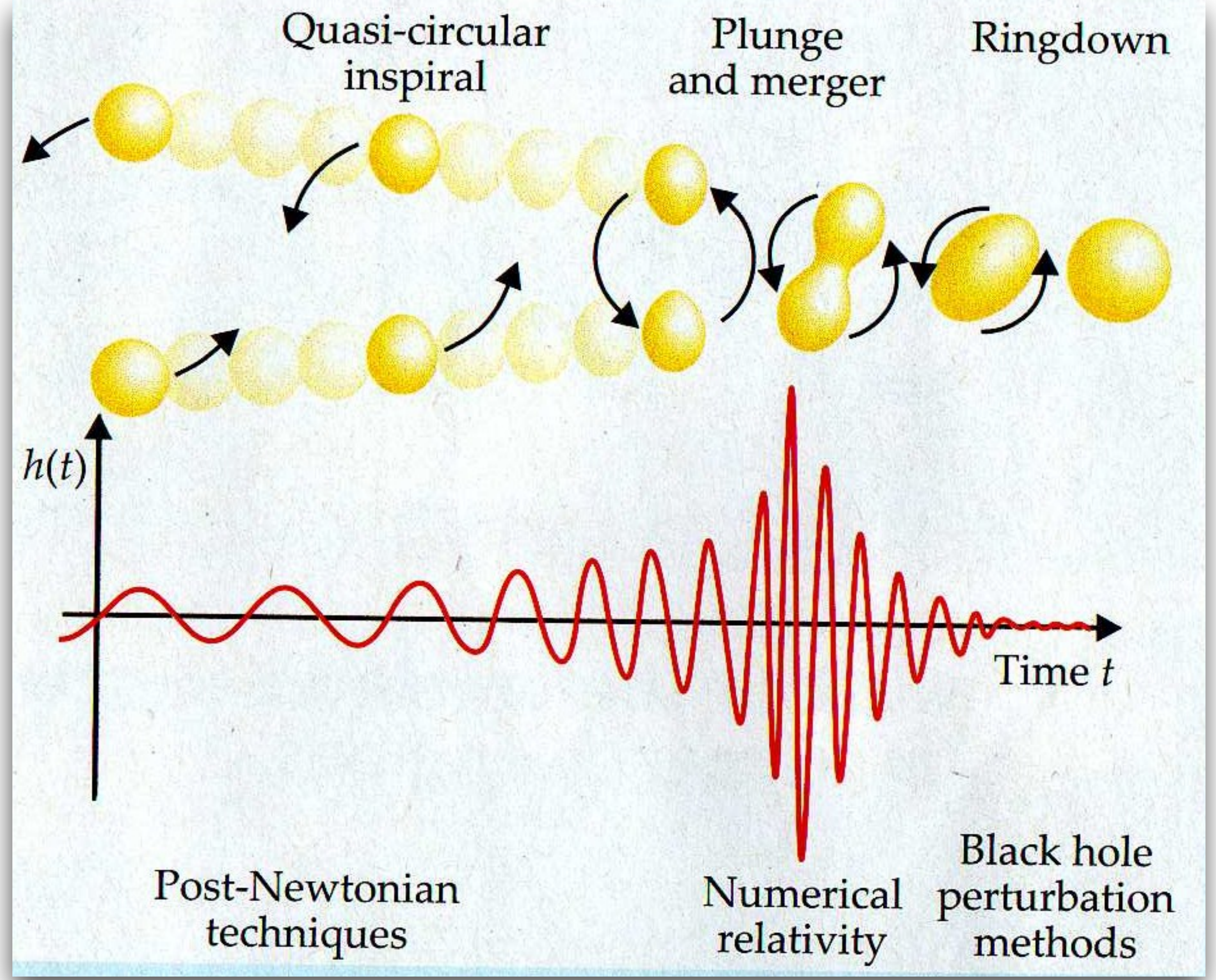
- In GR, gravitational waves (GW) are wave solutions to Einstein's equations generated from time varying mass quadrupoles and propagating at the speed of light
- The GW signal is a messenger carrying information about
  - binary dynamics and component nature
  - non-linear dynamics of space-time
  - final object nature
- Matching observed data with a solution to Einstein's equations allows inference of all of the above



LVC, arXiv:1602.03837

Gravitational wave solutions in GR

- Binary black holes solutions are constructed combining:
  - post-Newtonian theory in the weakly non-linear inspiral regime
  - direct numerical solution in the highly non-linear merger regime
  - perturbation theory in the ringdown regime

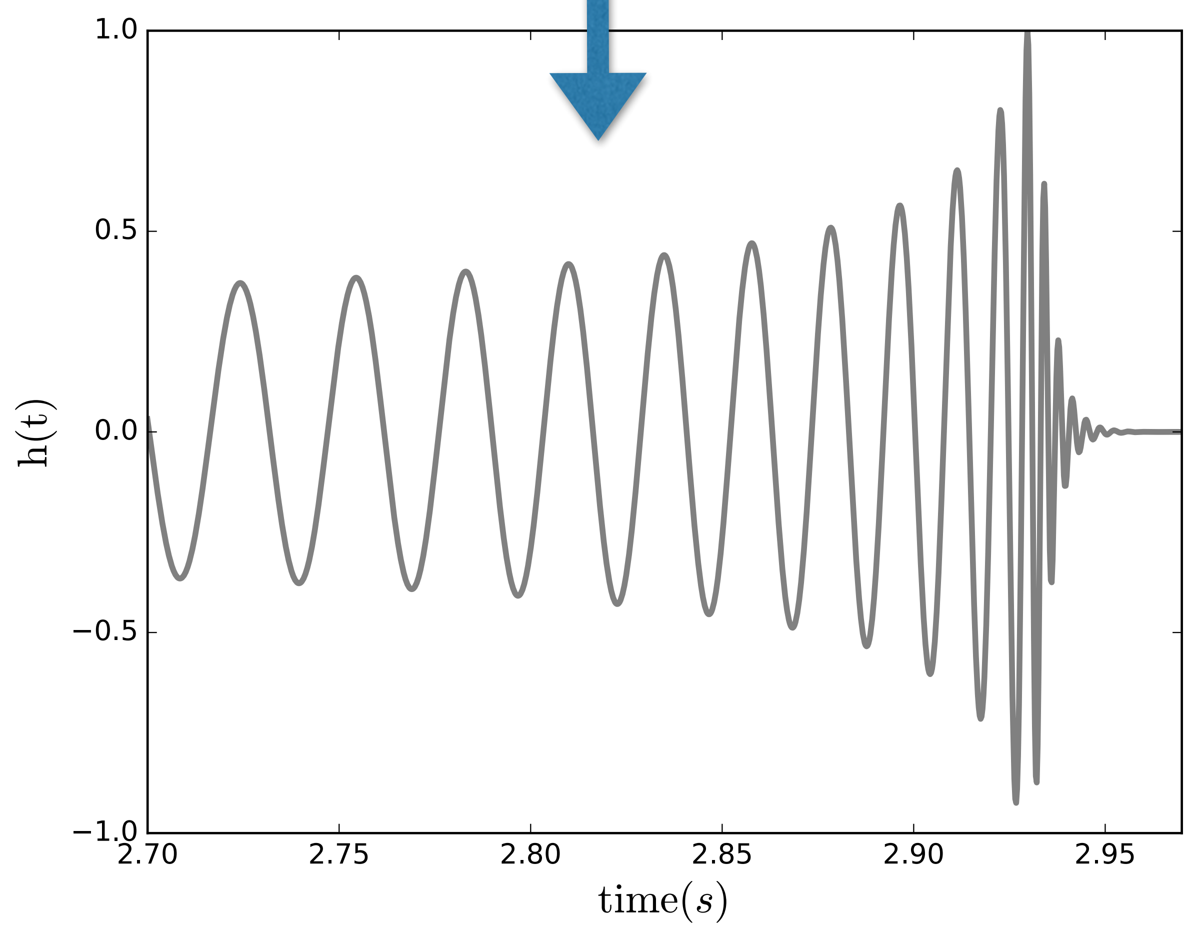


Strong-field GR solutions

- Accurate solutions obtained by direct integration
- Formulation and implementation highly non-trivial
- Computationally challenging
- Numerical solution used to inform and complement analytical formulations:
    - Effective one body (Buonanno & Damour, arXiv:9811091, Bohe+,arXiv:1611.03703)
    - Phenomenological (e.g. Khan+,arXiv:1508.07253)

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

few weeks later

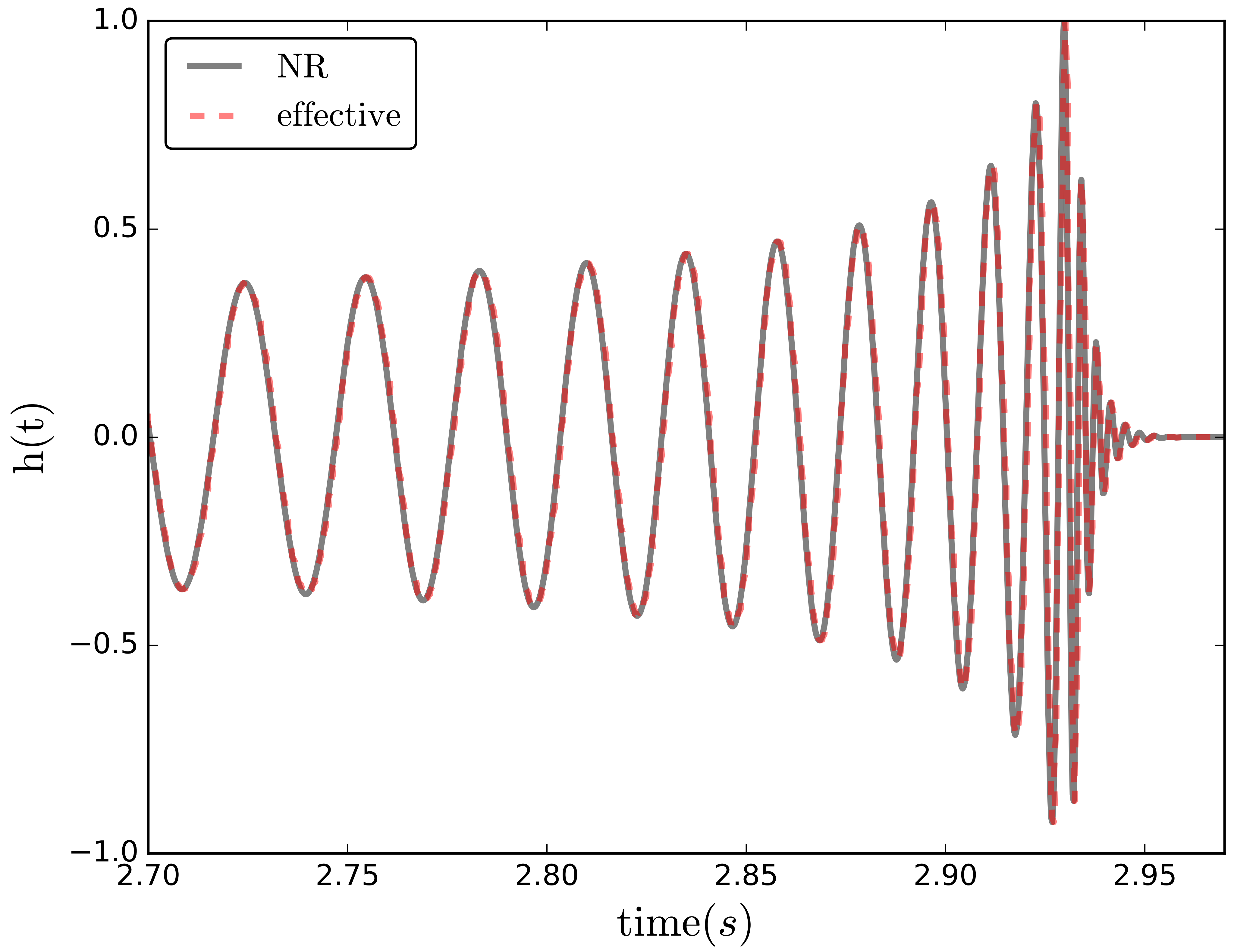


GW templates in GR

- Analytical, parametric description of GW solution in GR

$$h(f;\theta) = A(f;\theta)e^{i\Phi(f;\theta)}$$
$$\Phi(f;\theta) \equiv \Phi(f; m_1, m_2, \vec{s}_1, \vec{s}_2)$$

- Suitable for detection and parameter estimation analyses



GW in alternative gravity

- Alternative to GR can introduce extra-fields, curvature terms, challenge GR pillars, …
- Almost no full solution in non-GR known
- GW phase is modified:
  - non-GR action (extra fields, higher curvature, …): no full non-linear description, only post-Newtonian
  - Propagation (Lorentz violations, graviton mass, …): GR-like BBH dynamics, but modified GW propagation (see Samajdar's talk)
  - non-GR BHs (extra-fields, exotic objects):
    - tidal deformability
    - ringdown spectrum (see London, Cabero and Ghosh's talks)
    - Echoes (see Nielsen and Abedi talks)

Coalescence analysis

- The detector output is linear

$$d(t) = h(t;\theta) + n(t)$$

- where $h(t;\theta)$ is the gravitational wave strain and $n(t)$ is the noise time series

- The noise is a zero-mean stationary Gaussian stochastic process

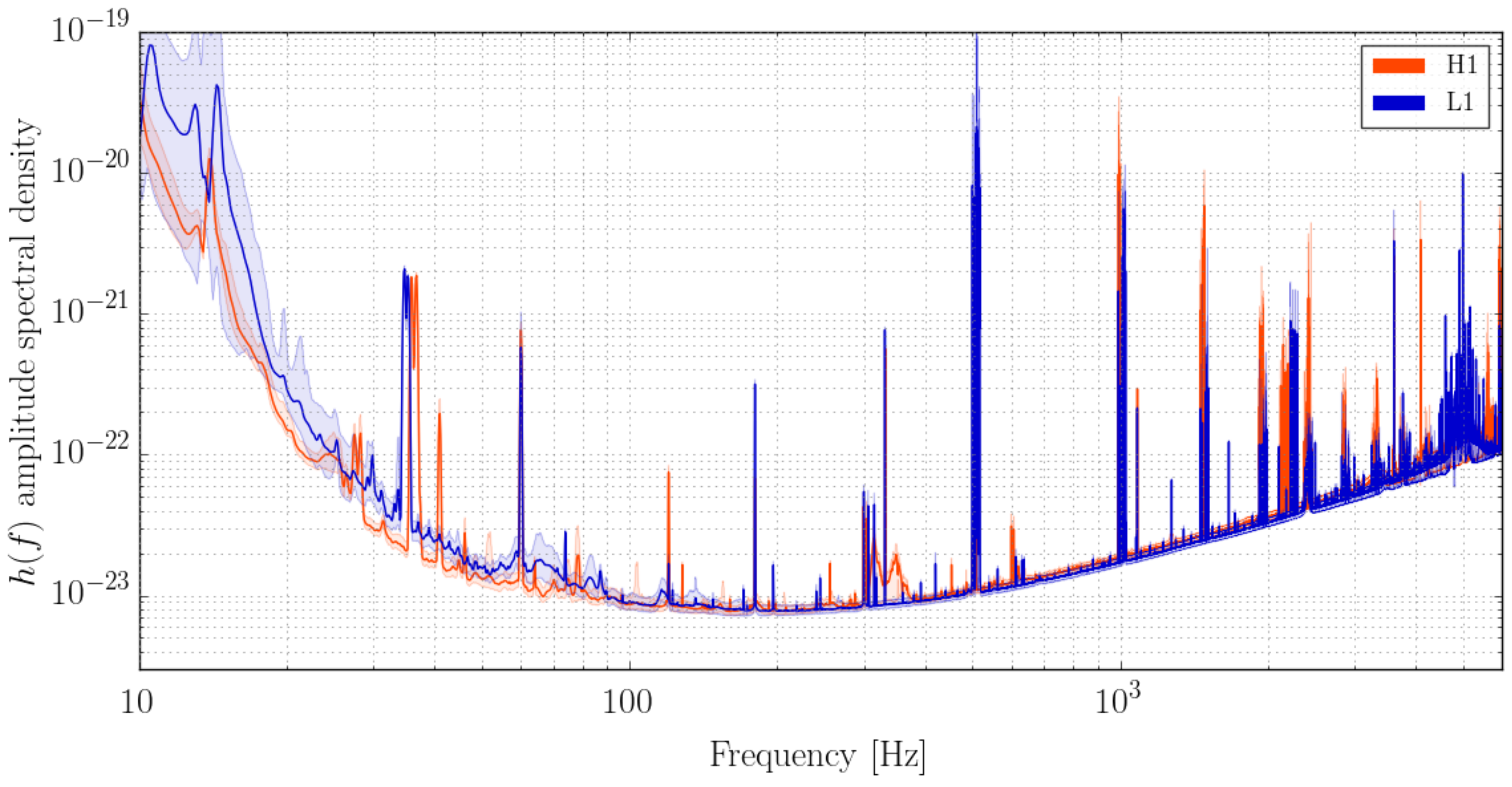
$$< n(f)n(f') >= \frac{1}{2}S(f)\delta(f-f')$$

- The probability of a given noise realisation

$$p(n) \sim e^{-(n|n)/2}$$

- The probability of a data realisation given a GW signal

$$p(d) \sim e^{-(d-h|d-h)/2}$$



$$(a|b) = 4\mathrm{Re}\left[\int df\, \frac{a^*(f)b(f) + a(f)b^*(f)}{S(f)}\right]$$

$$SNR = \sqrt{(d|h)}$$

Residuals

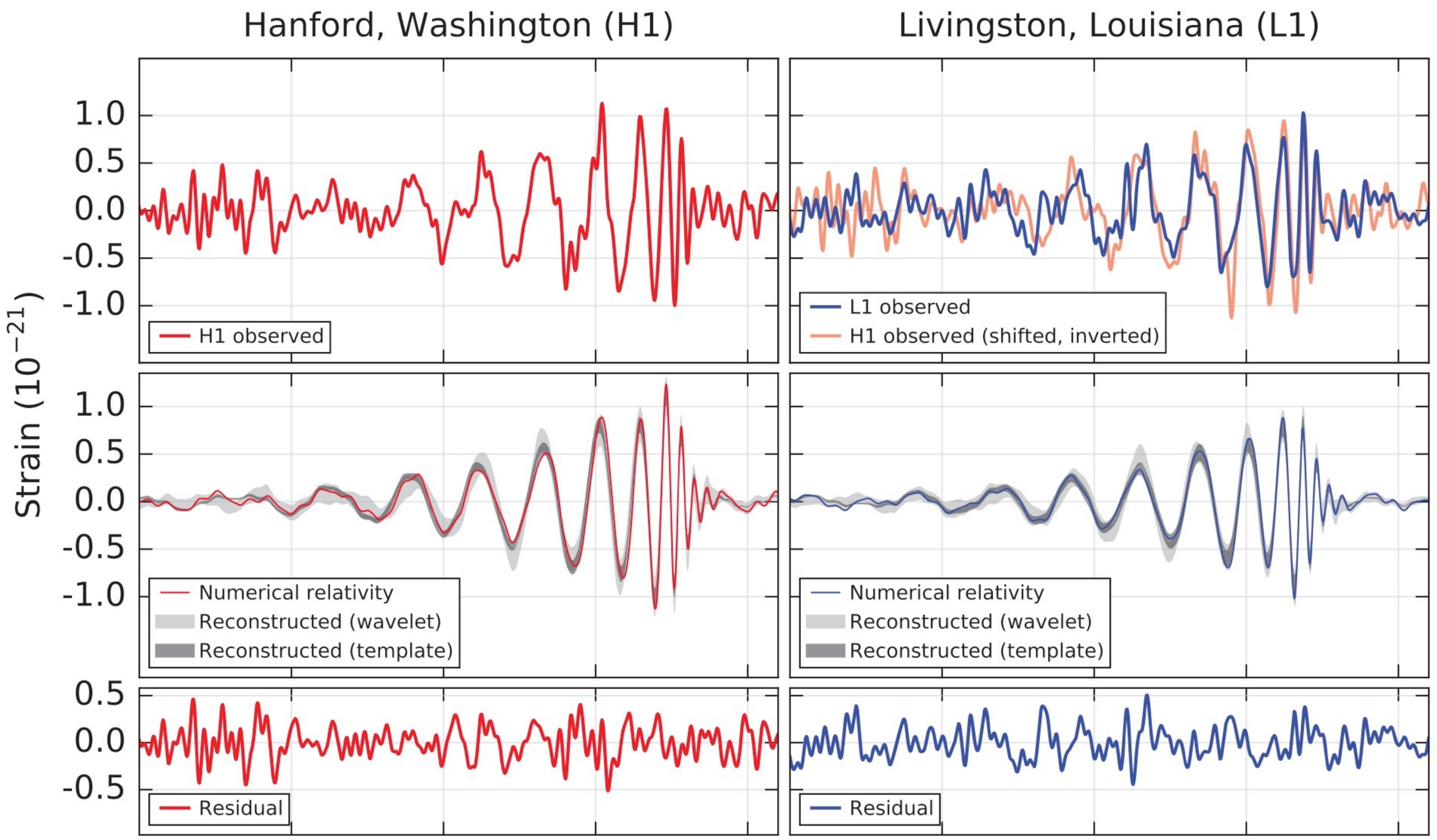
- After subtraction of the best fit GR waveform, the residuals must be consistent with Gaussian noise

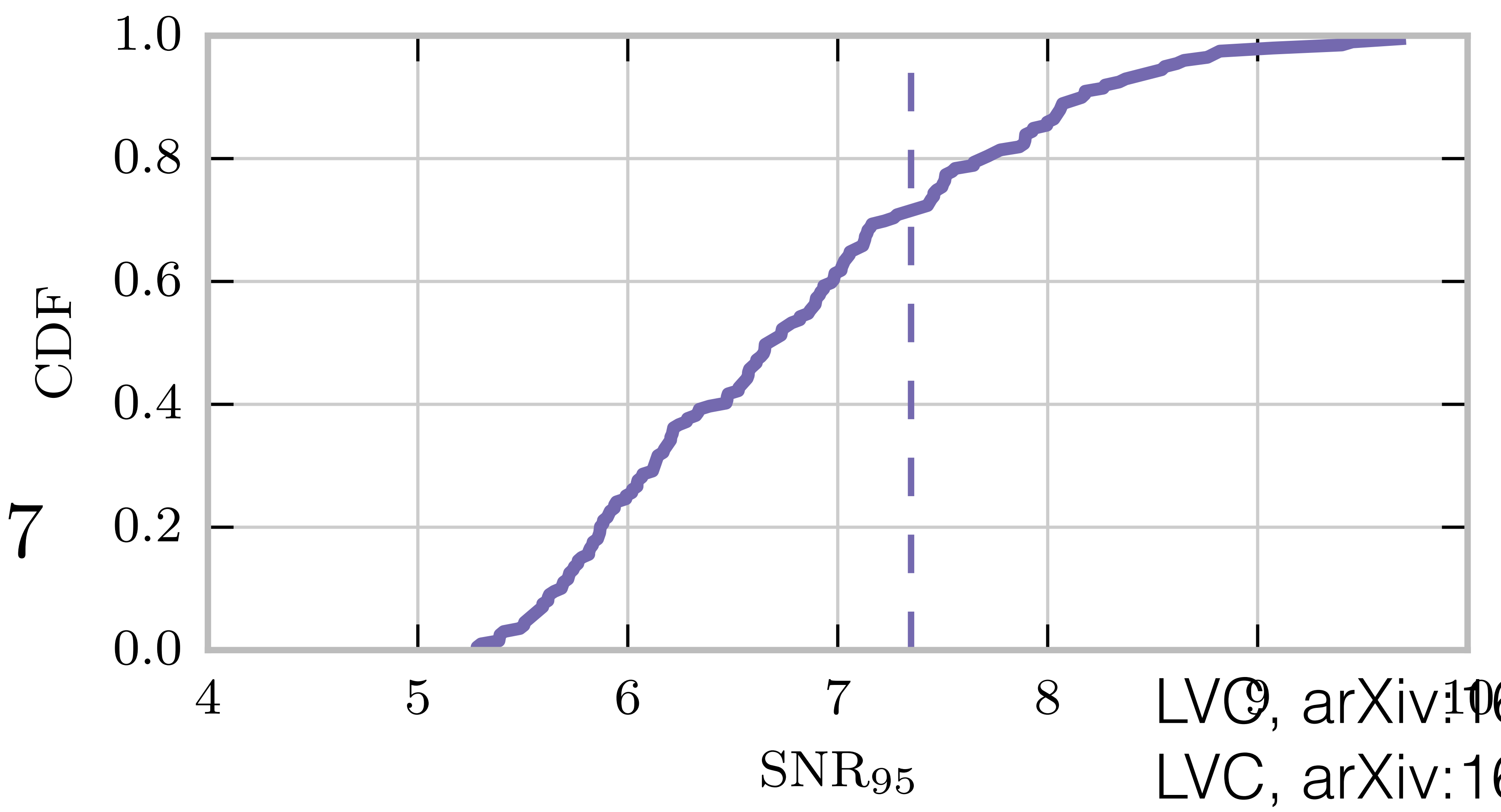
$$p(residuals) \sim p(n)$$

- Use un-modelled methods (Cornish & Littenberg, arXiv:1410.3835) to search for coherent power in the residuals
- GW150914 residual SNR < 7 at 95% confidence
- Match between GW150914 and the best GR template > 96%

$$FF = \sqrt{\frac{SNR^2_{det}}{SNR^2_{det} + SNR^2_{res}}}; \quad SNR_{det} = 25; \quad SNR_{res} \leq 7$$

$$FF \geq 0.96$$





LVC, arXiv:1602.03837
LVC, arXiv:1602.03841

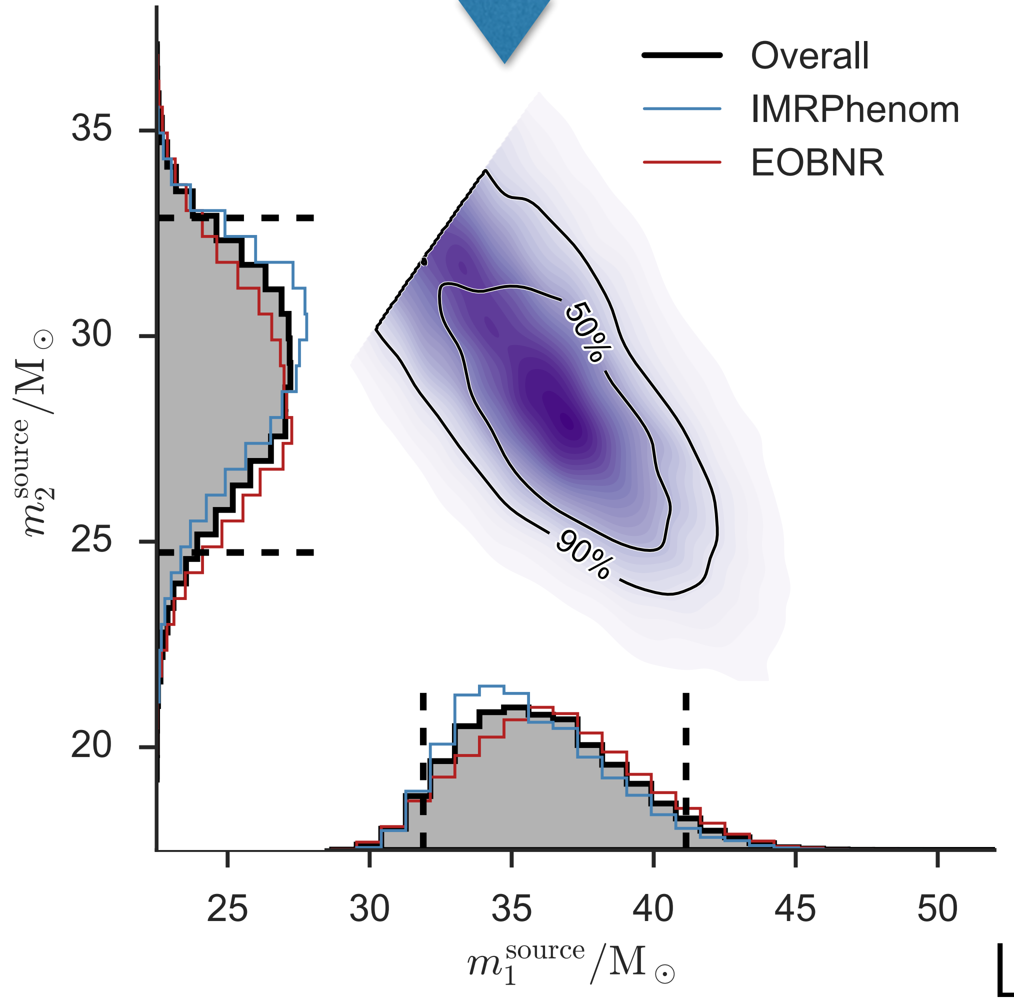
Coalescence analysis II

- The signal model $h(t;\theta)$ depends on a set of parameters θ
  - D=9 for non-spinning binaries: masses, orientation, sky location, reference time and phase, luminosity distance
  - D=15 in general: spin vectors
  - More parameters for extra physics (e.g. BH charges, tests of GR, tidal effects, etc…)
- Inference done via Bayes' theorem
- The problem is tackled using stochastic samplers (Veitch+,arXiv:1409.7215)

$$p(\theta|d,H,I) = p(\theta|H,I)\frac{p(d|\theta,H,I)}{p(d|H,I)}$$

posterior prior likelihood evidence

few days later



LVC,arXiv:1602.03840

Parametrised tests of GR

- GW waveforms are expressed in terms of effective series, for the Phenom family:

$$h(f;\theta) = A(f;\theta)e^{i\Phi(f;\theta)}$$

$$\Phi(f;\theta) = \sum_{k=0}^{7}(\varphi_k + \varphi_k^{(l)})f^{(k-5)/3} + \sum_{i\neq k}\varphi_i g(f)$$

post-Newtonian series   effective series

$$\varphi_j \equiv \varphi_j(m_1, m_2, \vec{s}_1, \vec{s}_2)$$

- Modified theories of gravity change the series (e.g. PPE: Yunes & Pretorius, arXiv:0909.3328, Cornish+,arXiv: 1105.2088)

- Perturb the GW phase around GR (Li+,arXiv:1110.0530, Agathos+,arXiv:1311.0420)

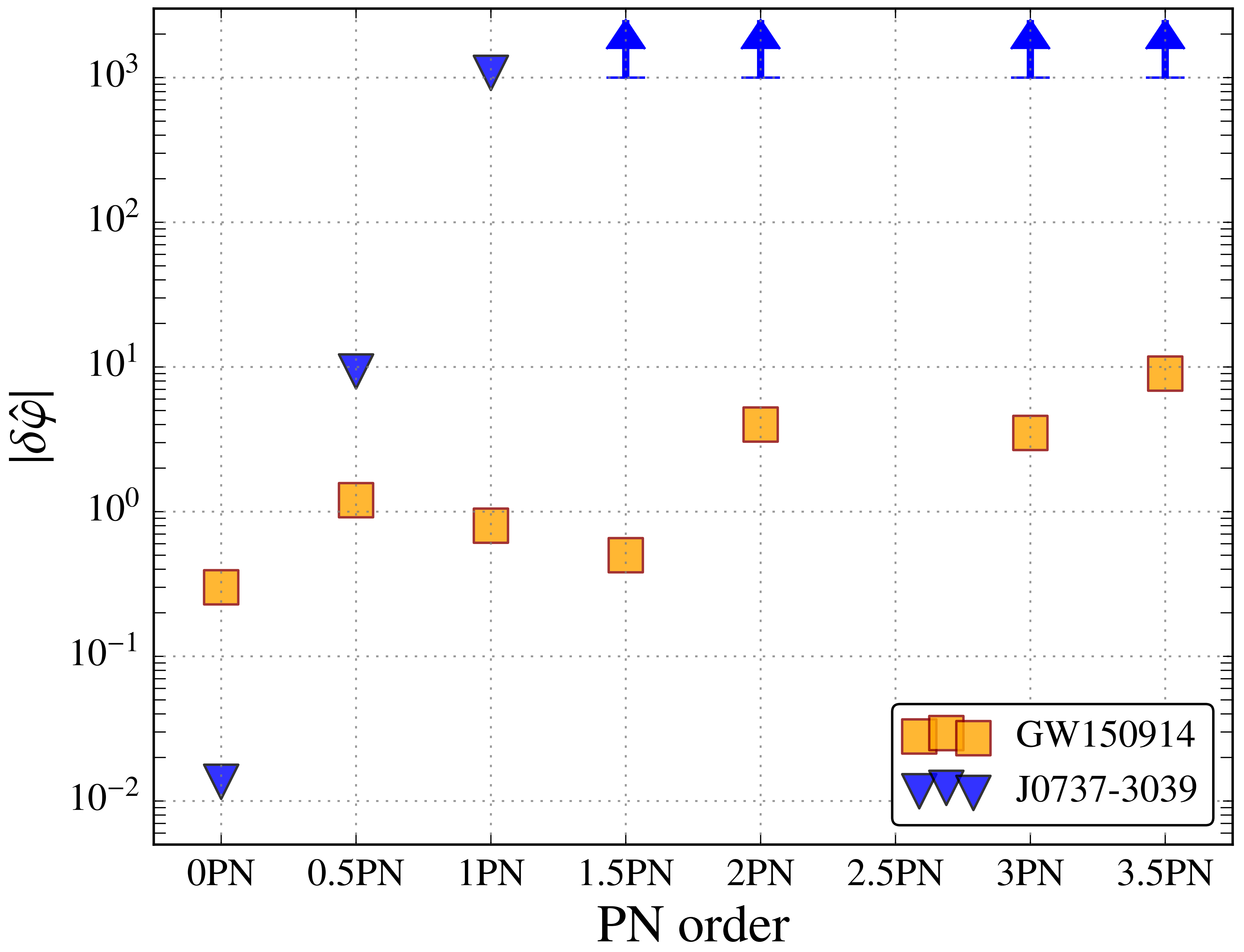
$$\hat{\varphi}_j \equiv \varphi_j^{GR}(1+\delta\hat{\varphi}_j) \qquad \delta\hat{\varphi}_j = 0 \iff \text{GR}$$

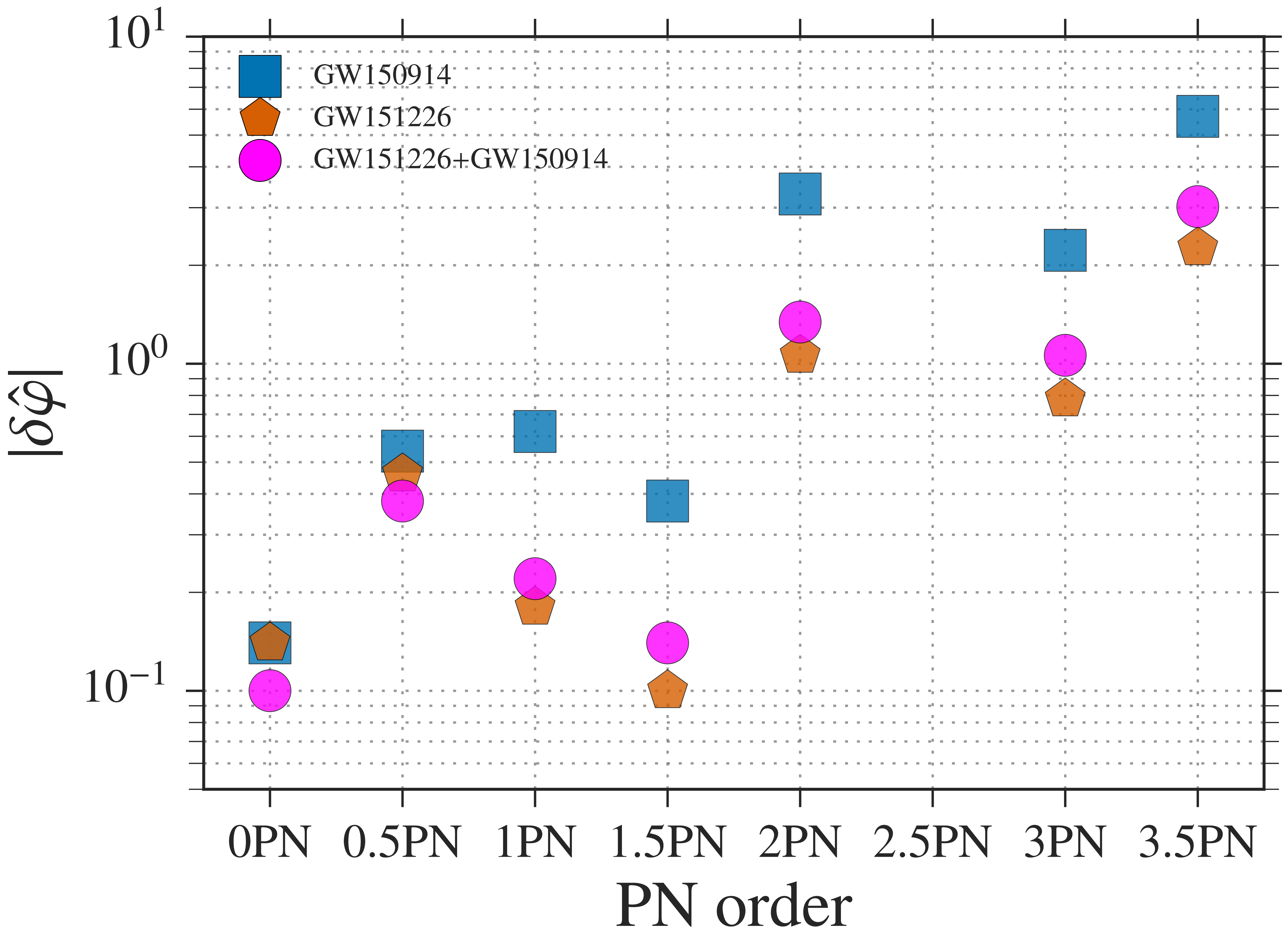
- Bound violations by computing posterior distributions for the $\delta\hat{\varphi}_j$ in concert with the physical parameters of the system

waveform regime	parameter	f–dependence	
early-inspiral regime	$\delta\hat{\varphi}_0$	$f^{-5/3}$	post-Newtonian
	$\delta\hat{\varphi}_1$	$f^{-4/3}$	
	$\delta\hat{\varphi}_2$	f^{-1}	
	$\delta\hat{\varphi}_3$	$f^{-2/3}$	
	$\delta\hat{\varphi}_4$	$f^{-1/3}$	
	$\delta\hat{\varphi}_{5l}$	$\log(f)$	
	$\delta\hat{\varphi}_6$	$f^{1/3}$	
	$\delta\hat{\varphi}_{6l}$	$f^{1/3}\log(f)$	
	$\delta\hat{\varphi}_7$	$f^{2/3}$	
intermediate regime	$\delta\hat{\beta}_2$	$\log f$	effective
	$\delta\hat{\beta}_3$	f^{-3}	
merger–ringdown regime	$\delta\hat{\alpha}_2$	f^{-1}	
	$\delta\hat{\alpha}_3$	$f^{3/4}$	
	$\delta\hat{\alpha}_4$	$\tan^{-1}(af+b)$	

LVC, arXiv:1602.03841

Post-Newtonian constraints



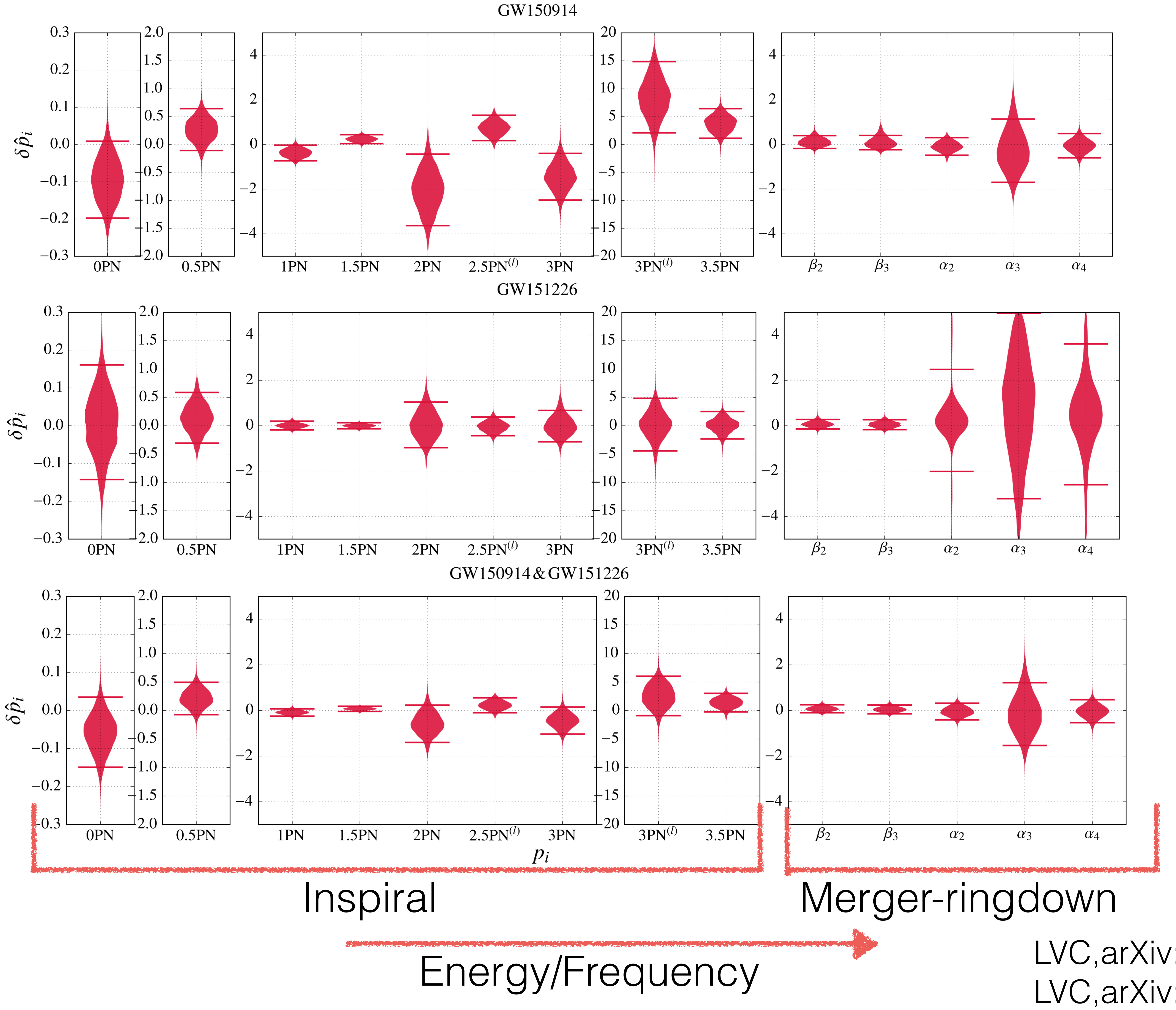


- Constraints not achievable by any other means
- Can be mapped to the space of specific theories (e.g. Yunes+,arXiv: 1603.08955)

LVC,arXiv:1602.03841
LVC,arXiv:1606.04855

Constraints on space-time dynamics

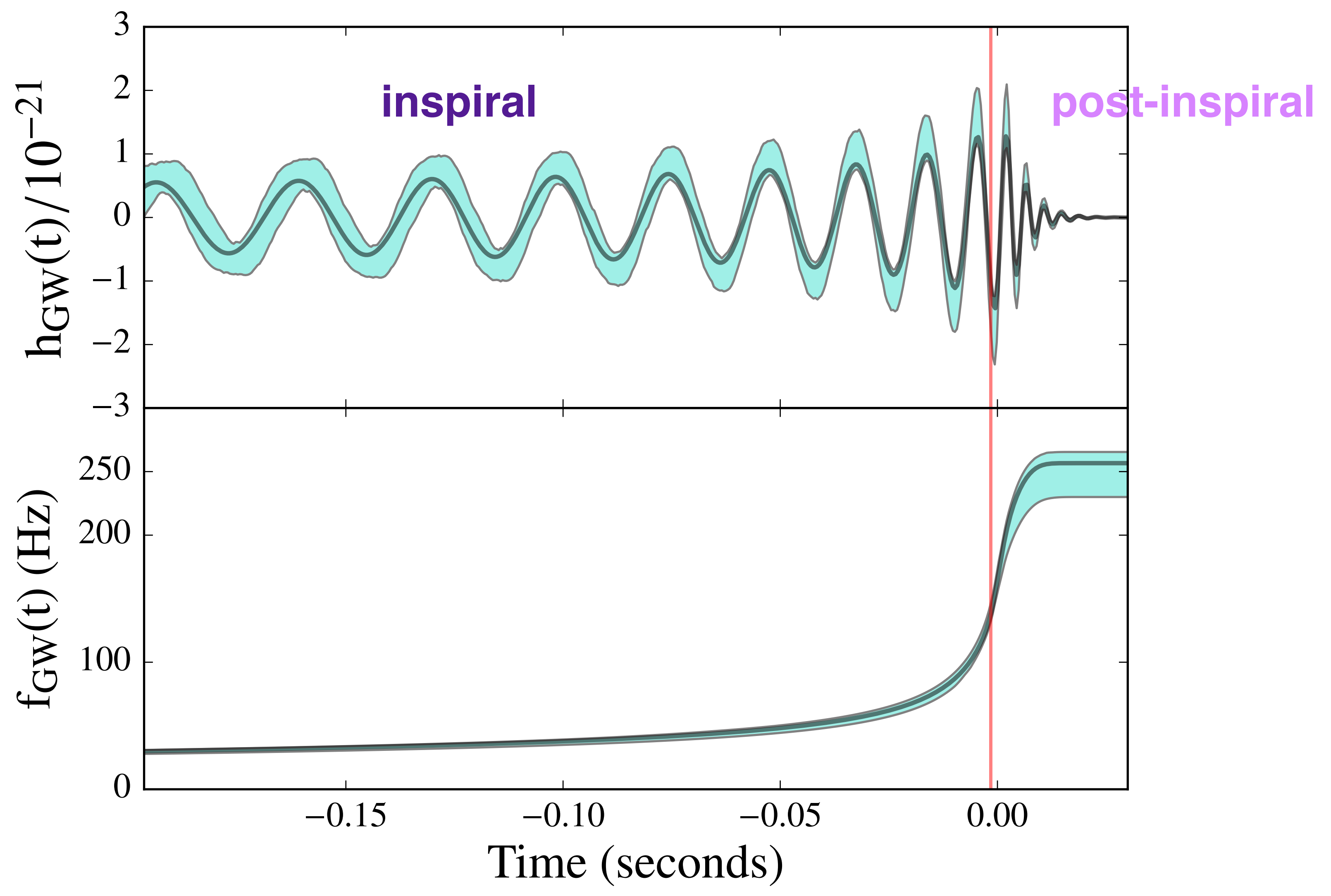
- Only constraints on space-time dynamics
- Posterior distributions for $\delta\hat{\varphi}_j$ show no evidence for violations of GR

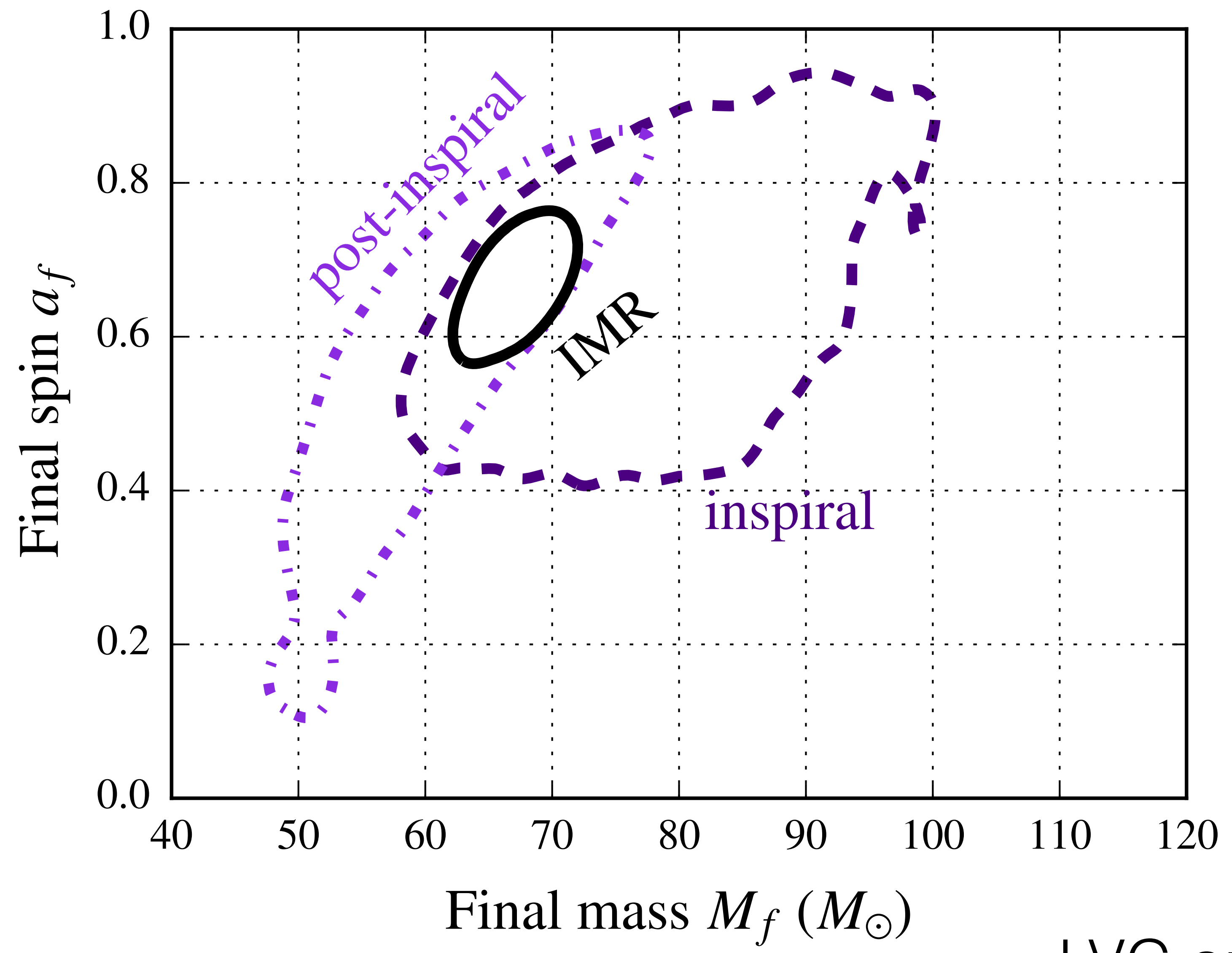


LVC,arXiv:1602.03841
LVC,arXiv:1606.04856

Reconstructed waveform consistency

- If GR is verified, the recovered GR waveform must be self-consistent
- Numerical solution provide predictions for spin and mass of remnant starting from the parents ones (e.g. Healy+, arXiv:1406.7295)
- Verify self-consistency by comparing final mass and spin predicted from the “inspiral” with the ones inferred from the “post-inspiral” (Ghosh+,arXiv:1602.02453)





LVC,arXiv:1602.03841

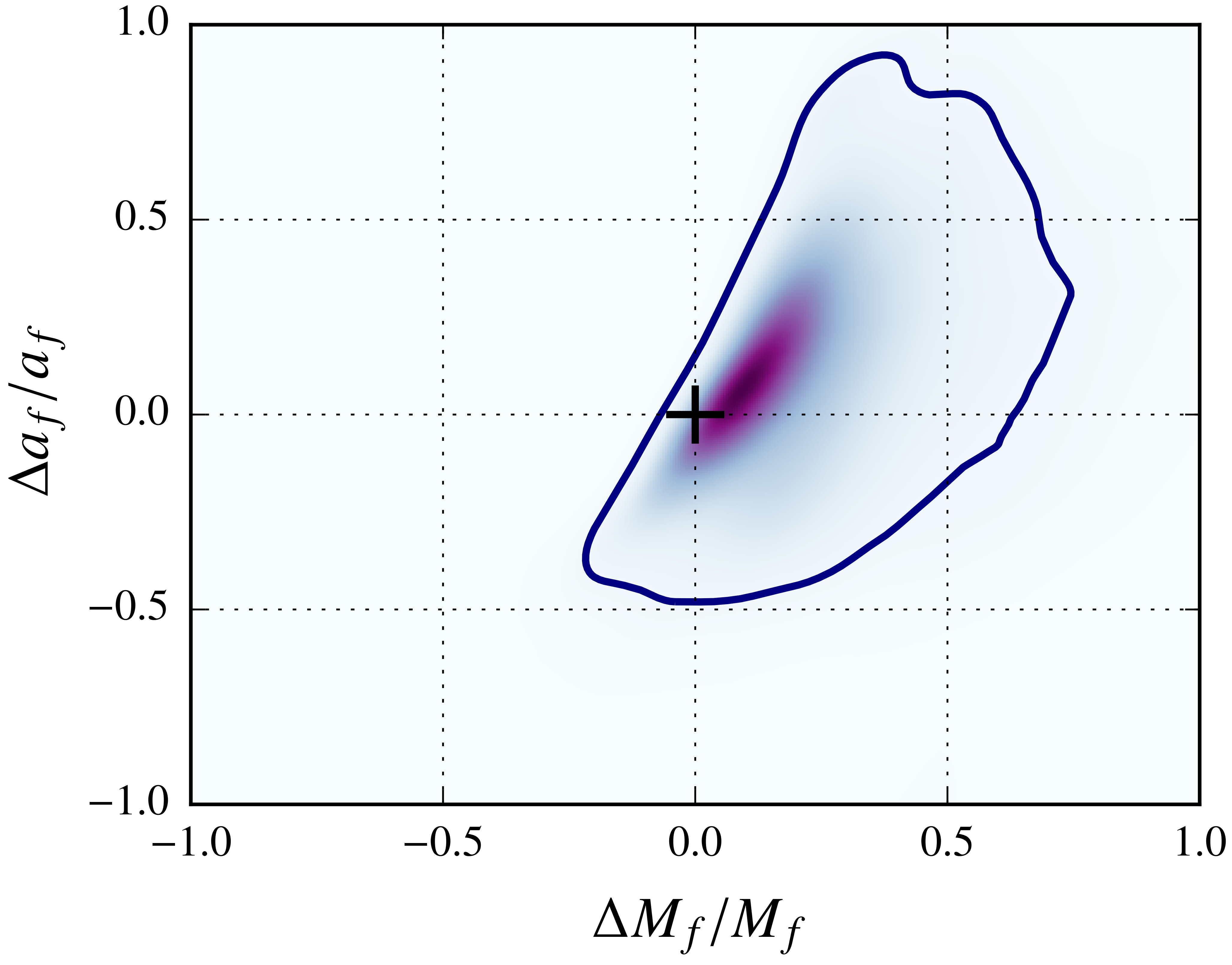
Reconstructed waveform consistency

- Re-parametrise in terms of relative differences

$$\begin{cases} \Delta M_f / M_f = 0 \\ \Delta a_f / a_f = 0 \end{cases} \iff GR$$

- Final object has properties consistent with Kerr BH

- No violations of GR observed



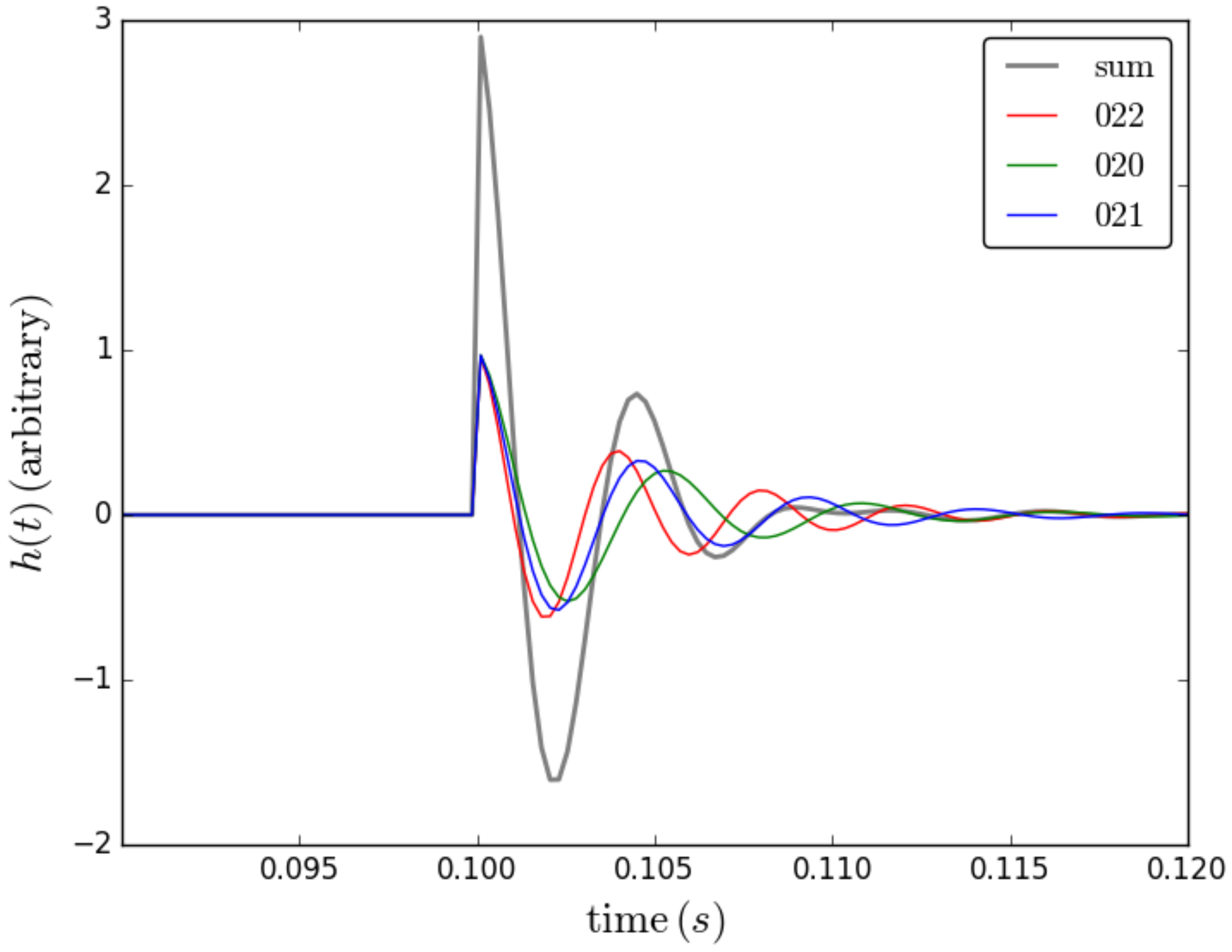
LVC,arXiv:1602.03841

Tests on the nature of the final object

- Ringdown signal for GR BHs is well understood

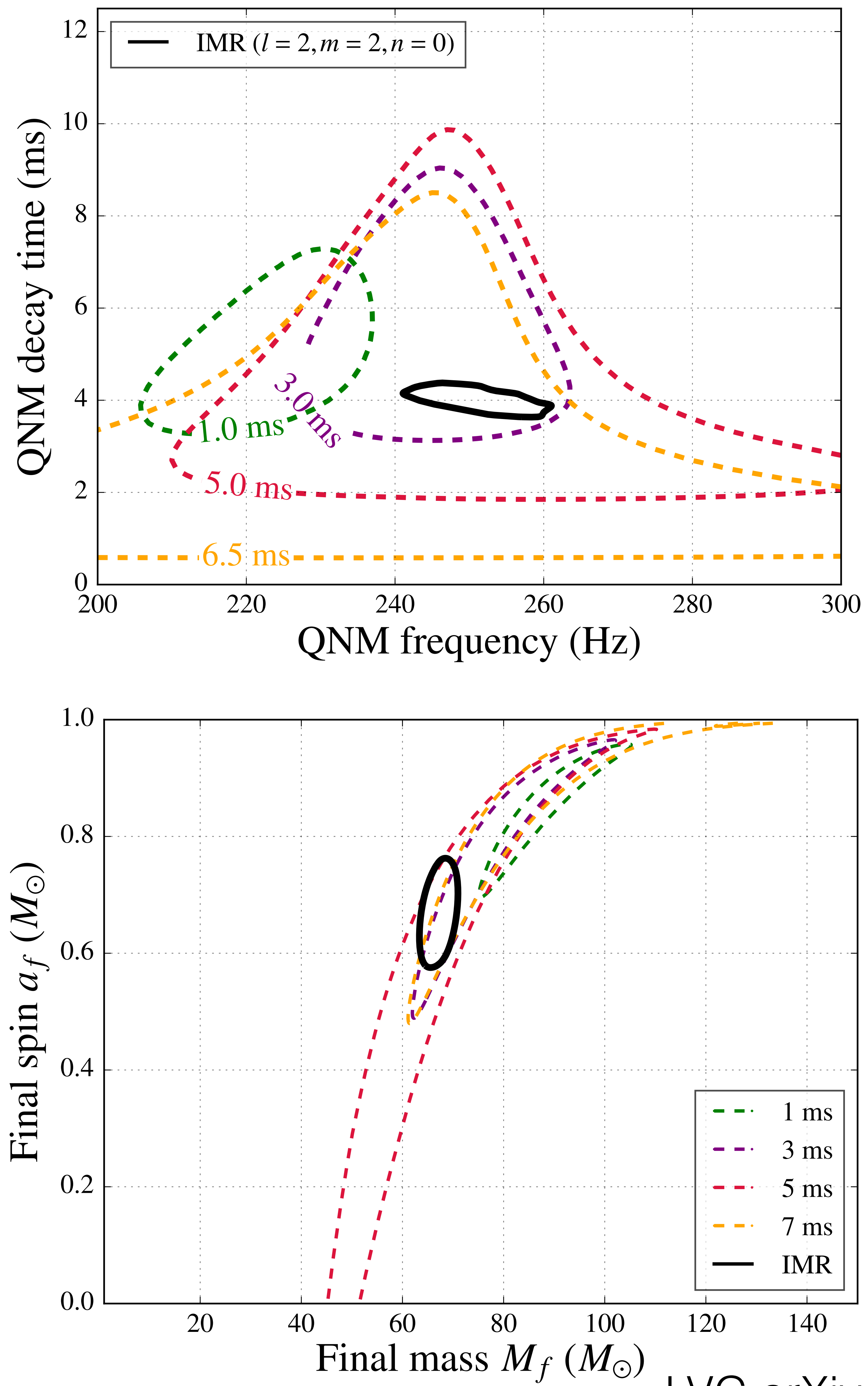
$$h(t) = \sum_{nlm} A_{nlm} e^{-\frac{t-t_0}{\tau_{nlm}}} \cos(\omega_{nlm}(t - t_0) + \varphi_{nlm})$$

- Central frequencies ω_{nlm} and decay times τ_{nlm} are functions of BH mass and spin only (the "no-hair" theorem, Berti+, arXiv:0512160)



Tests on the nature of the final object

- Multiple modes detection allows tests of BH nature and “no-hair” theorem (e.g. Gossan+, arXiv:1111.5819, Meidam+,arXiv:1406.3201, see also Cabero and Ghosh talks)
- Inference critically dependent on starting time t_0
- Single mode detection in GW150914, consistent with GR solution



LVC,arXiv:1602.03841

Propagation tests: massive gravity

- Families of alternative theories modify the propagation of GW (see Samajdar's talk)
- Massive gravity (e.g. Will, arXiv:9709011)

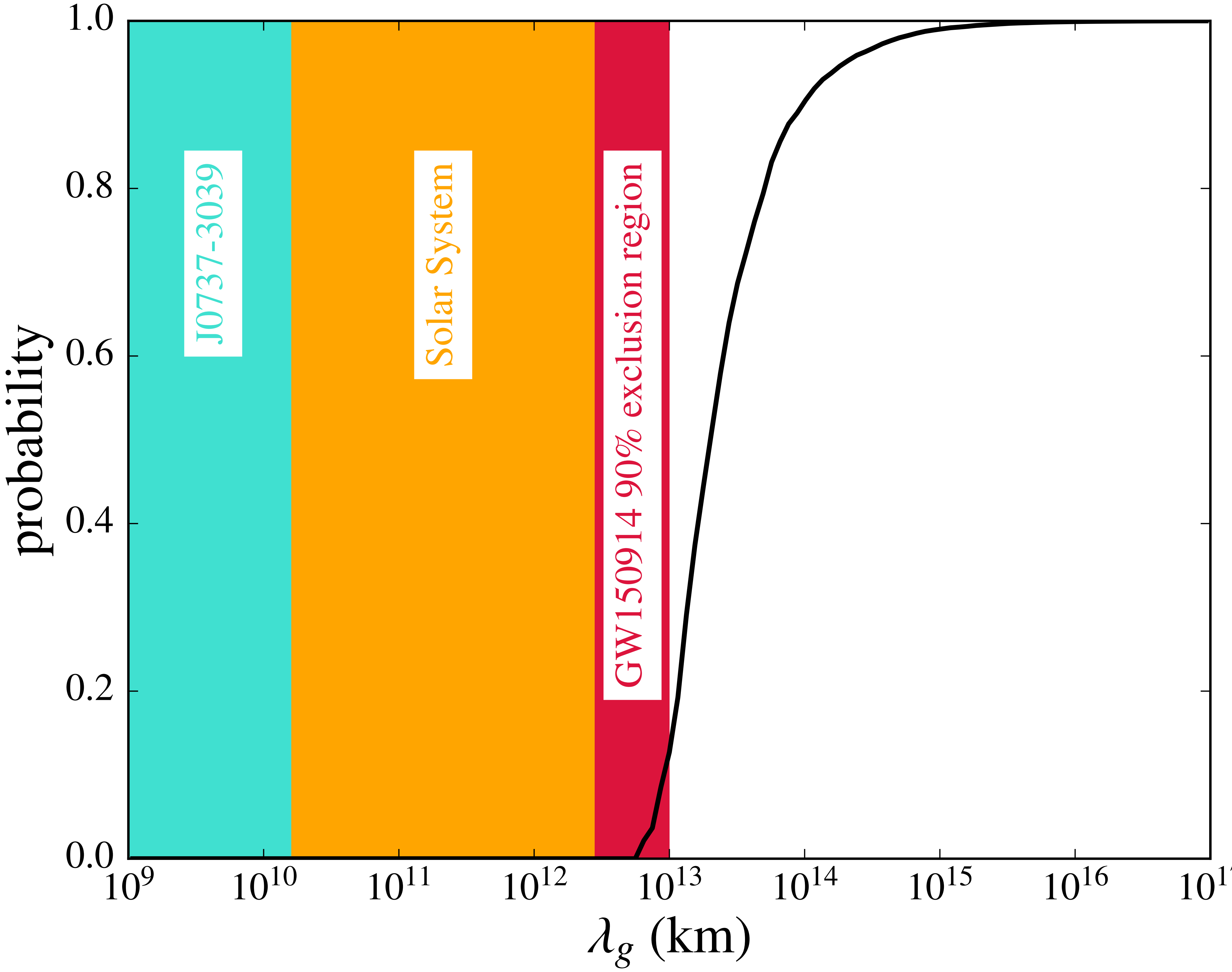
$$E^2 = p^2 v_g^2 + m_g^2 c^4$$

$$v_g^2/c^2 \simeq 1 - \frac{h^2 c^2}{\lambda_g^2 E^2} \qquad \lambda_g = \frac{h}{m_g c}$$

- GW phase affected

$$\Delta\Phi = -\frac{\pi^2 D M}{\lambda_g^2 (1+z)}$$

- GW constrains gravitons Compton wavelength



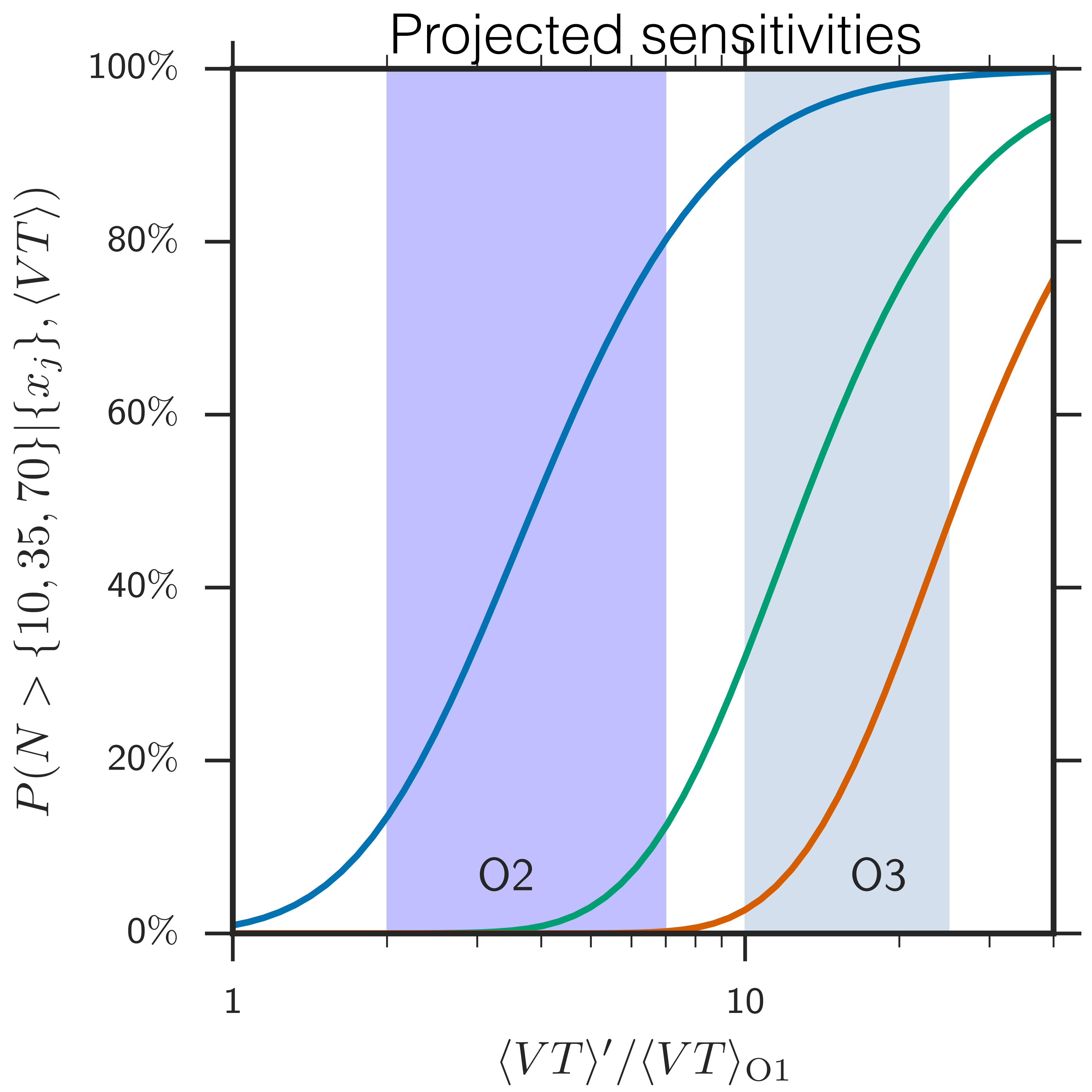
$$\lambda_g \geq 10^{13} \mathrm{km}\ (90\%)$$

$$m_g \leq 1.2 \times 10^{-22} \mathrm{eV/c^2}\ (90\%)$$

LVC, arXiv:1602.03841

Near future

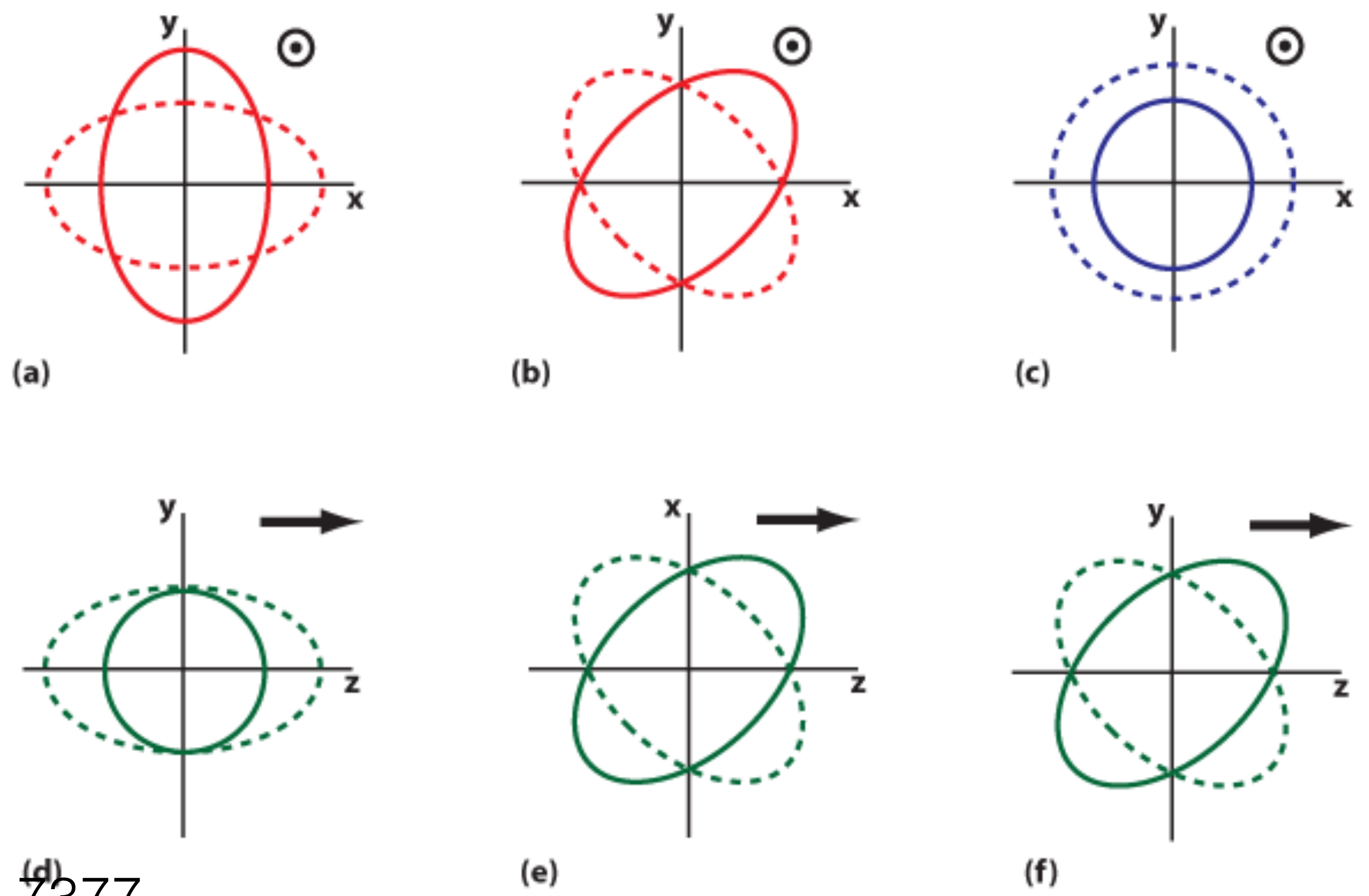
- Measured rate grants several more BBH detections
- Rate of BBH mergers 9–260 Gpc^{-3} yr^{-1}
- High SNR sources
- More detectors



LVC,arXiv:1606.04856

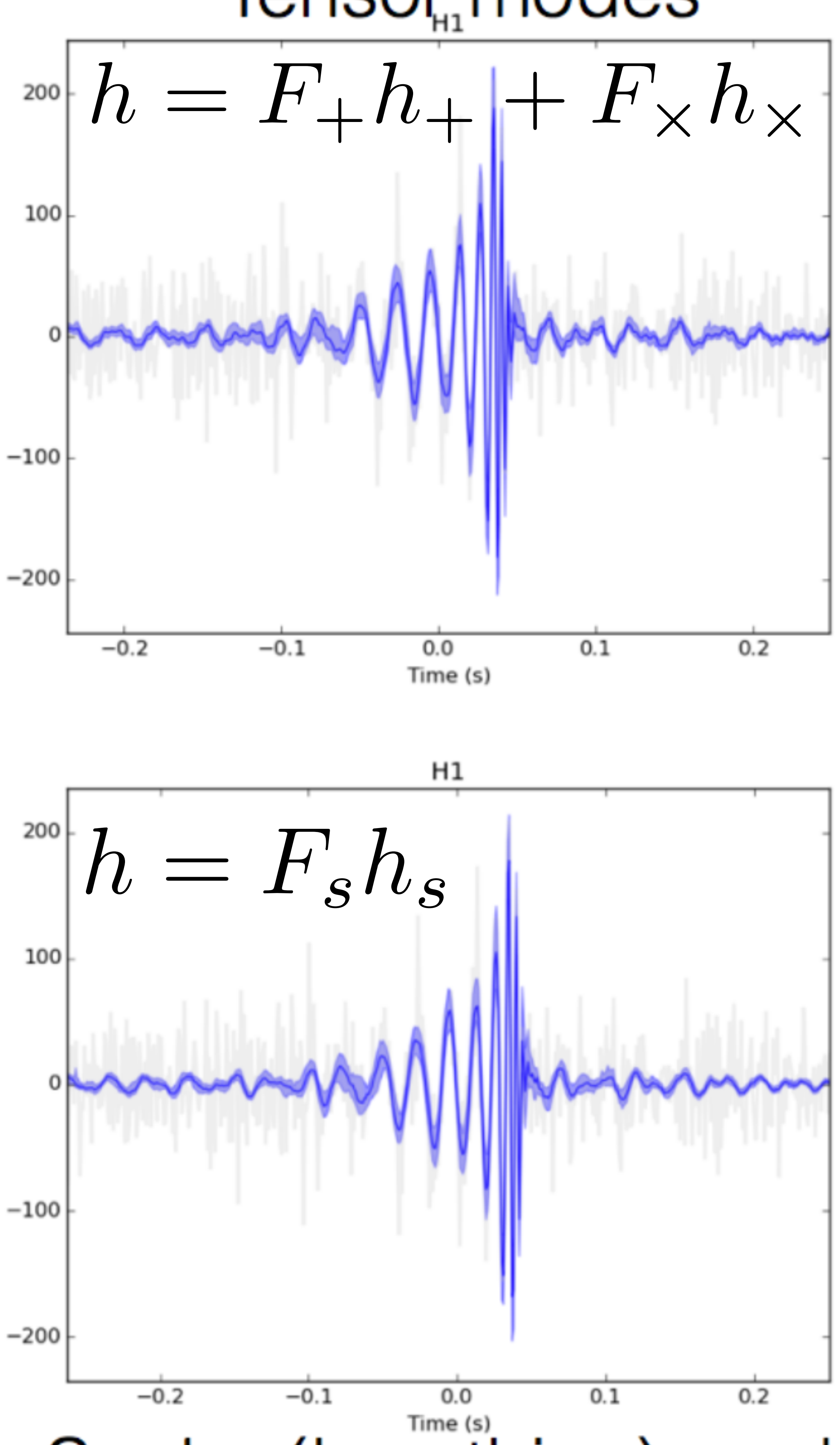
Additional polarisation states

- The presence of additional polarisation states is a general feature in extensions of general relativity
- Detection of non-tensor polarisations is a smoking gun for violations of GR
- More than 2 detectors or EM counterpart necessary



Will, arXiv:1403.7377

Tensor modes

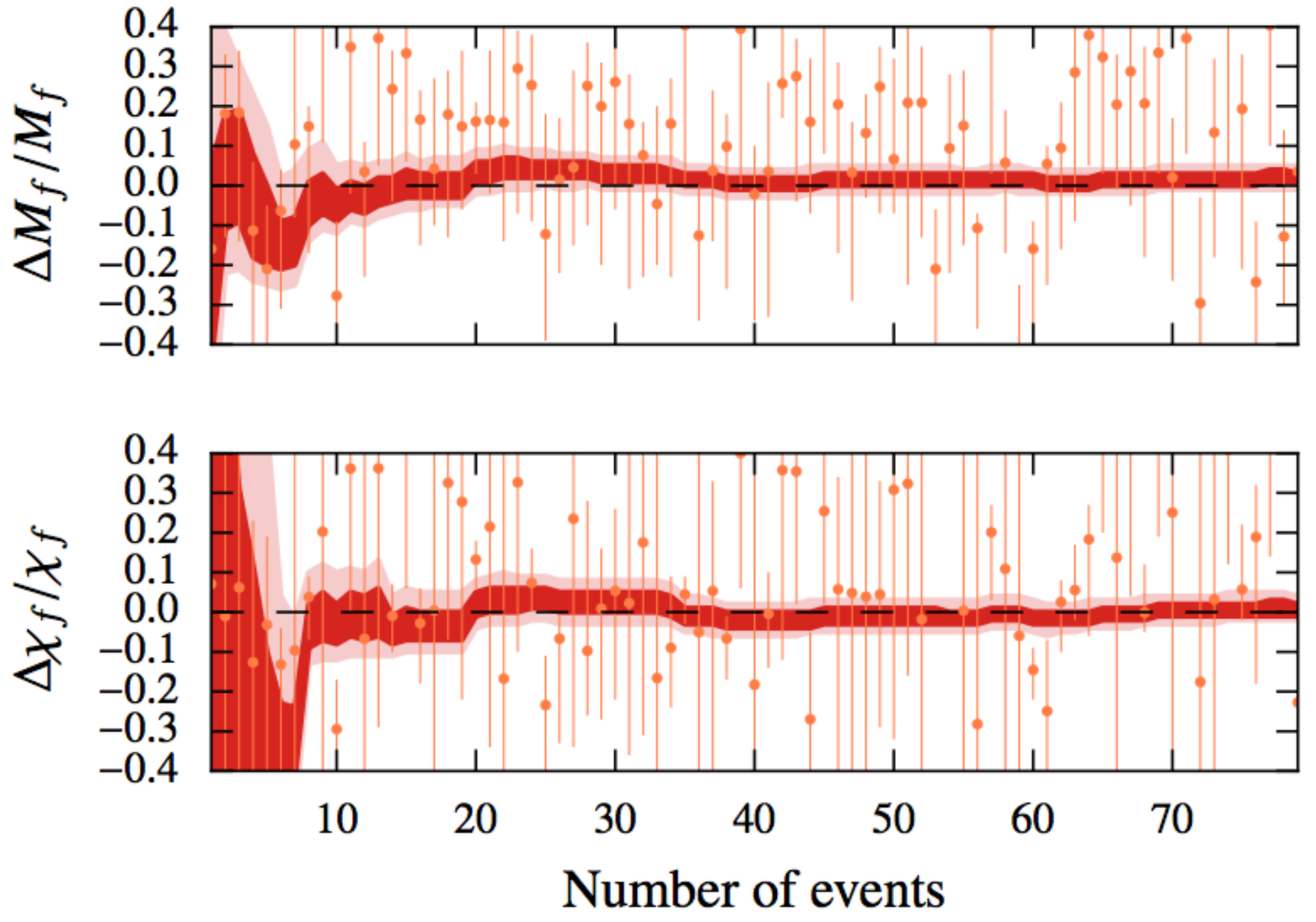


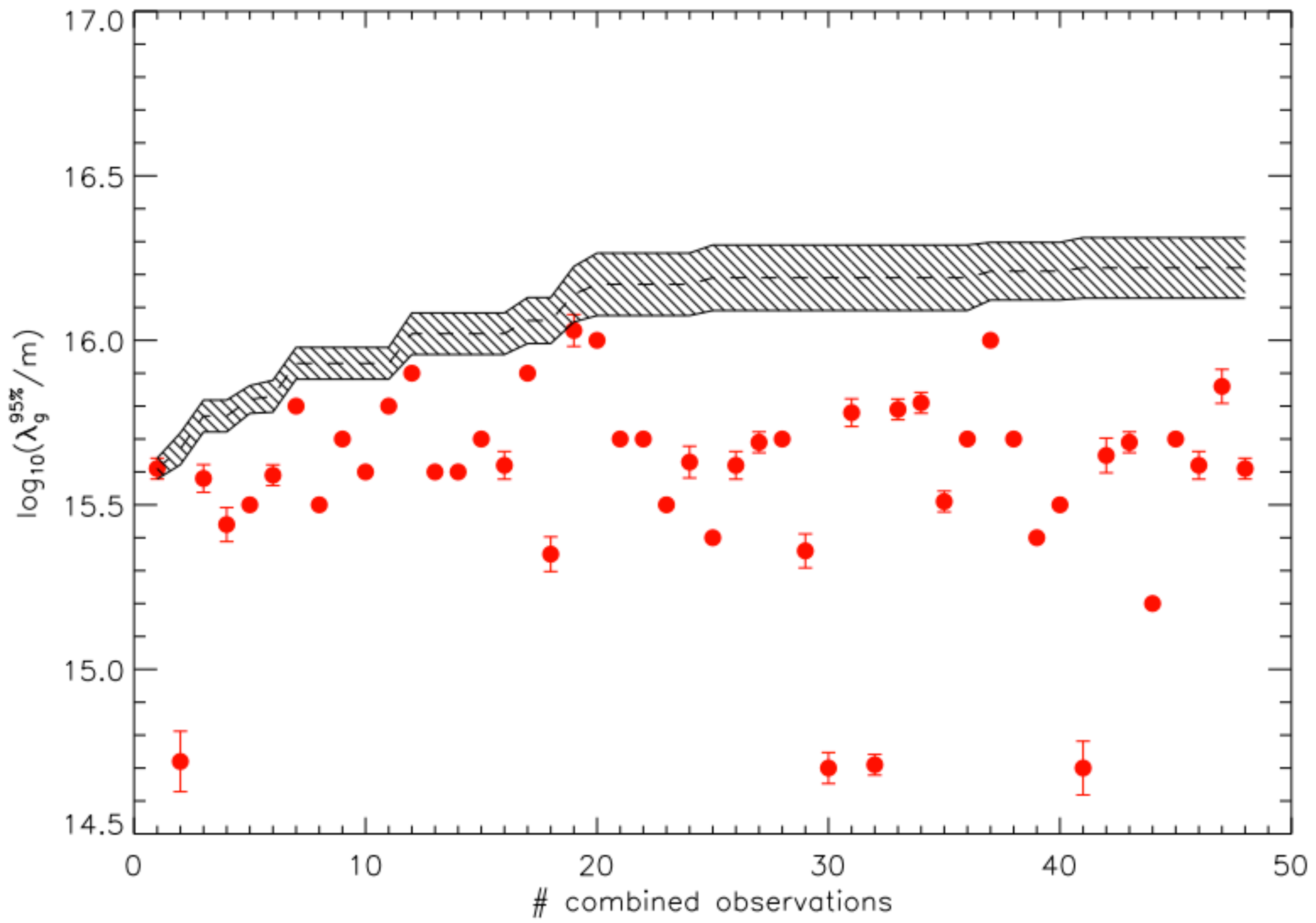
Scalar (breathing) mode

LVC,arXiv:1602.03841

Improved constraints

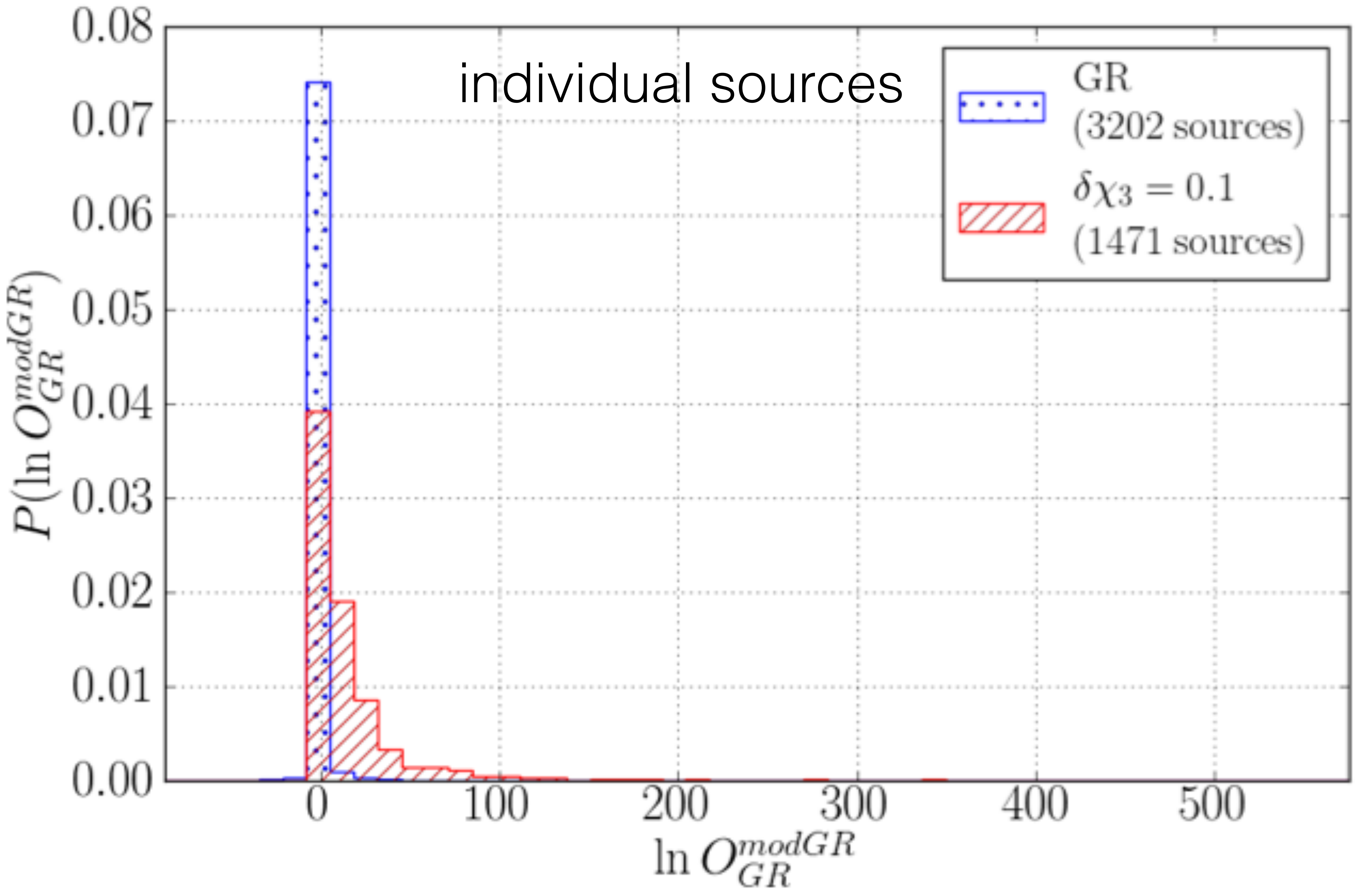
- Improved constraints on space-time dynamics
- Improved constraints from waveform consistency test (Ghosh+,arXiv:1602.02453)
- Improved constraints from propagation of GW on graviton Compton wavelength (Del Pozzo+,arXiv:1101.1391)

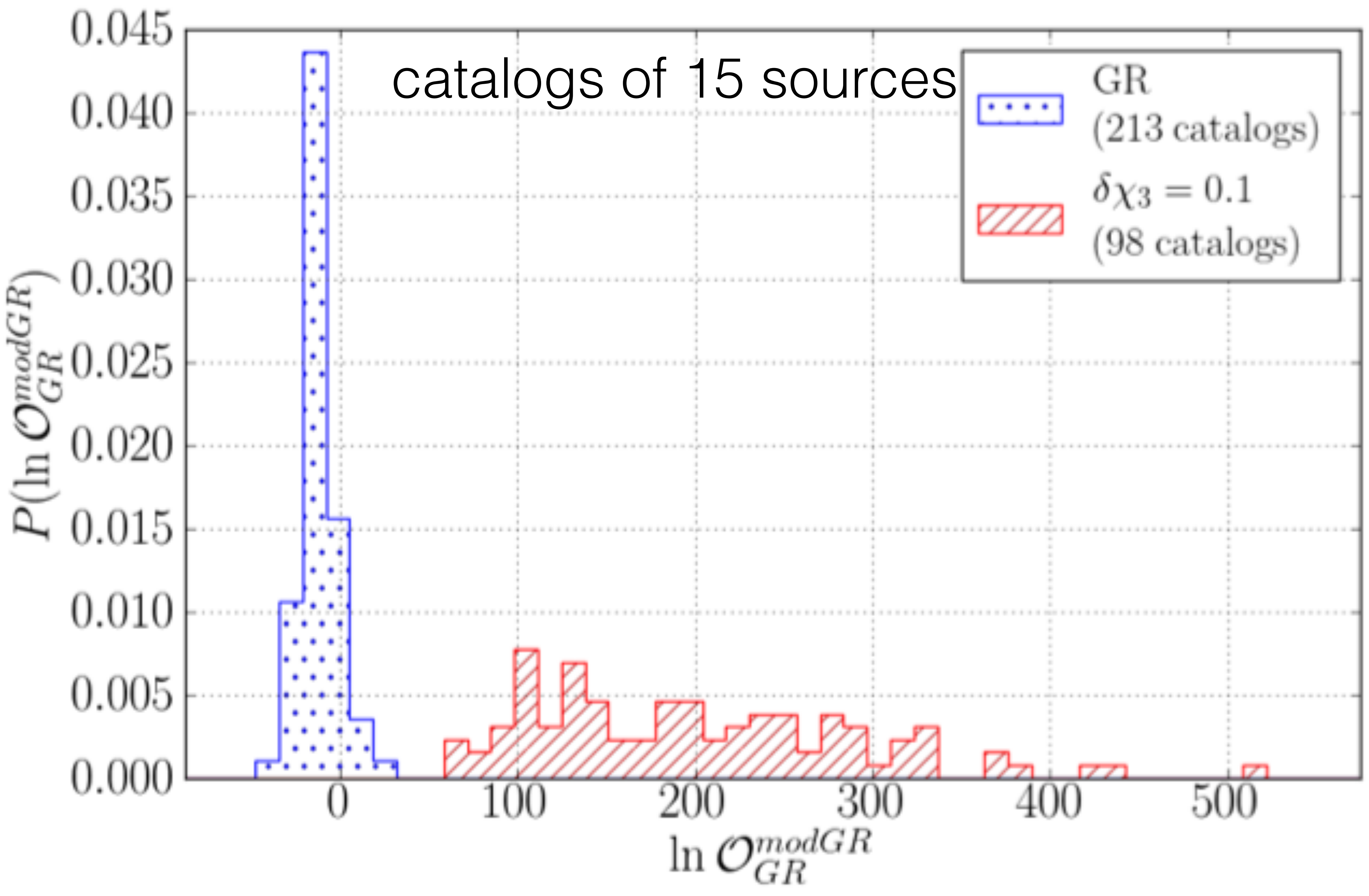




Prospects for detection of GR violations

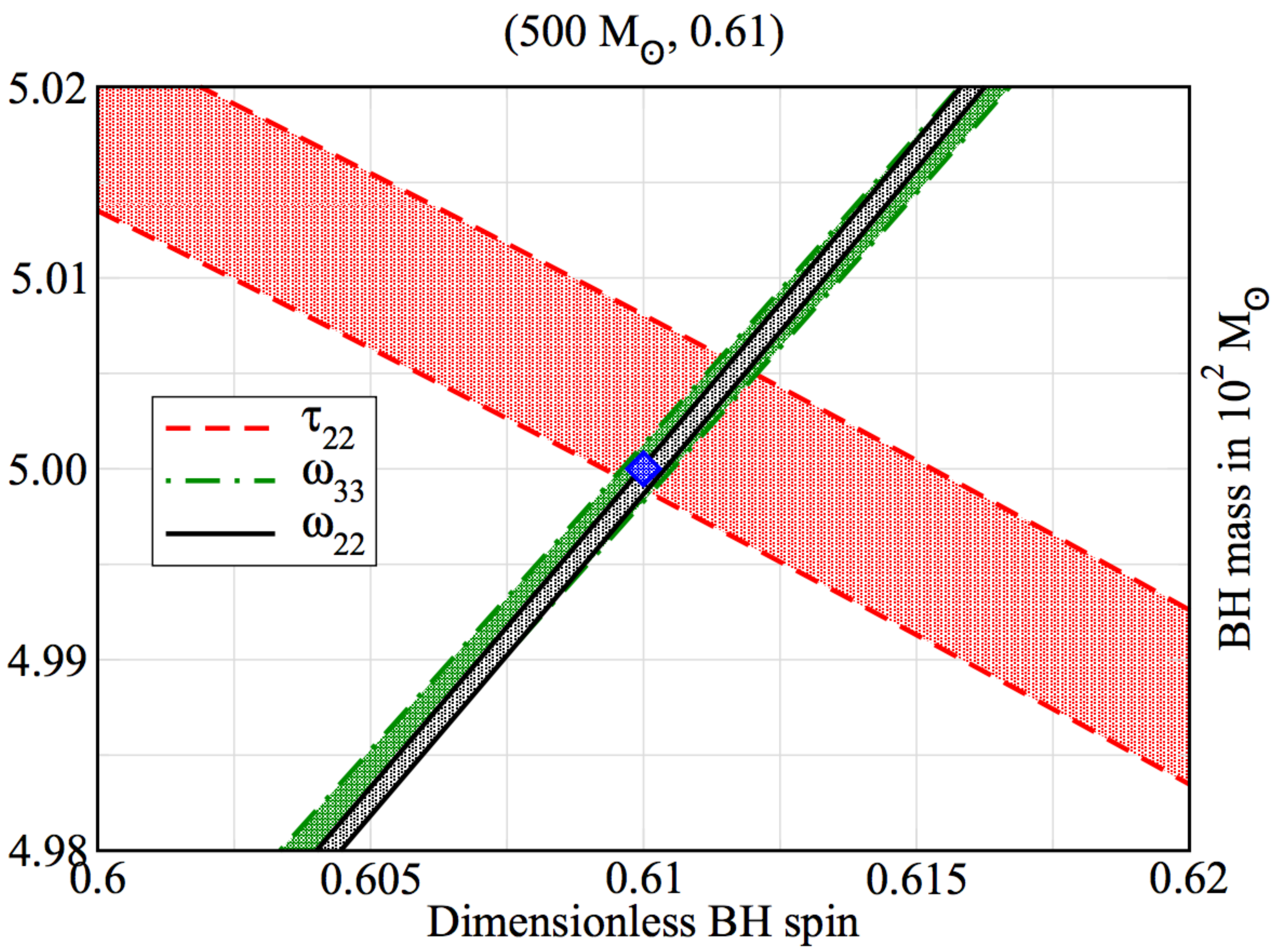
- Detection of small GR violations using Bayesian odds ratio (Li+,arXiv:1110.0530, Agathos+,arXiv:1311.0420)





"No hair" theorem

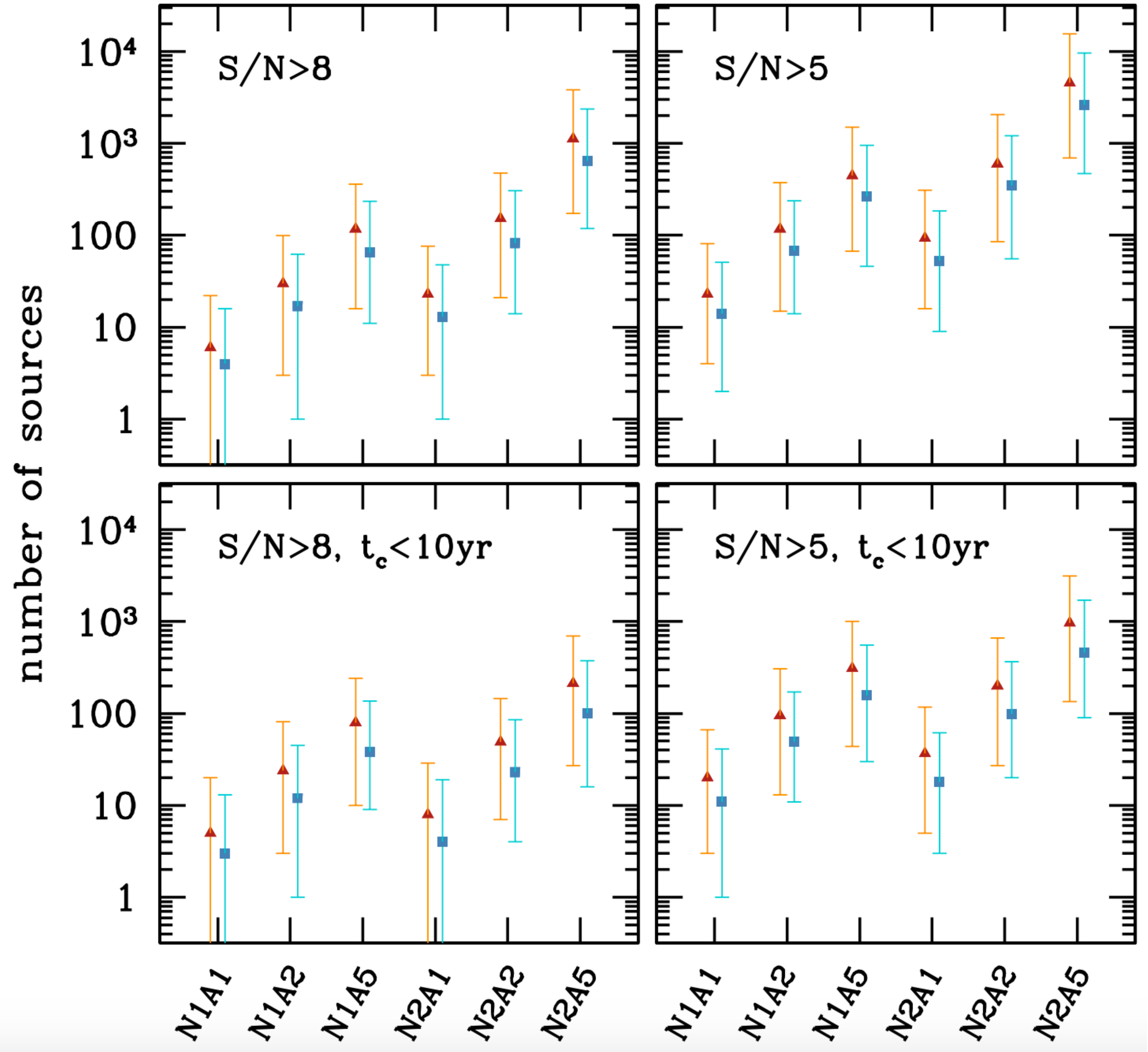
- Detection of more than one ringdown quasi-normal mode (QNM) allows independent determinations of the remnant mass and spin
- Constrain variations around expected GR solutions
- "no-hair theorem" test
- Second law of BH dynamics



Gossan+,arXiv:1111.5819

Far future

- LISA will observe O(100) of BBH systems
- Synergy LIGO+LISA
- Improved GR tests (e.g. Vitale, arXiv:1605.01037)
- Strong dipole radiation constraints (e.g. Barausse+, arXiv:1603.04075)



Sesana,arXiv:1602.06951

Summary

- The era of GW astrophysics is officially open
- First glimpse at space-time extreme regimes:
  - **BBHs behave just like GR predicts**
- Just the beginning:
  - many more detections in the future
  - improved sensitivities
  - multi-wavelength studies
- Look forward to a prolific season in gravitational physics