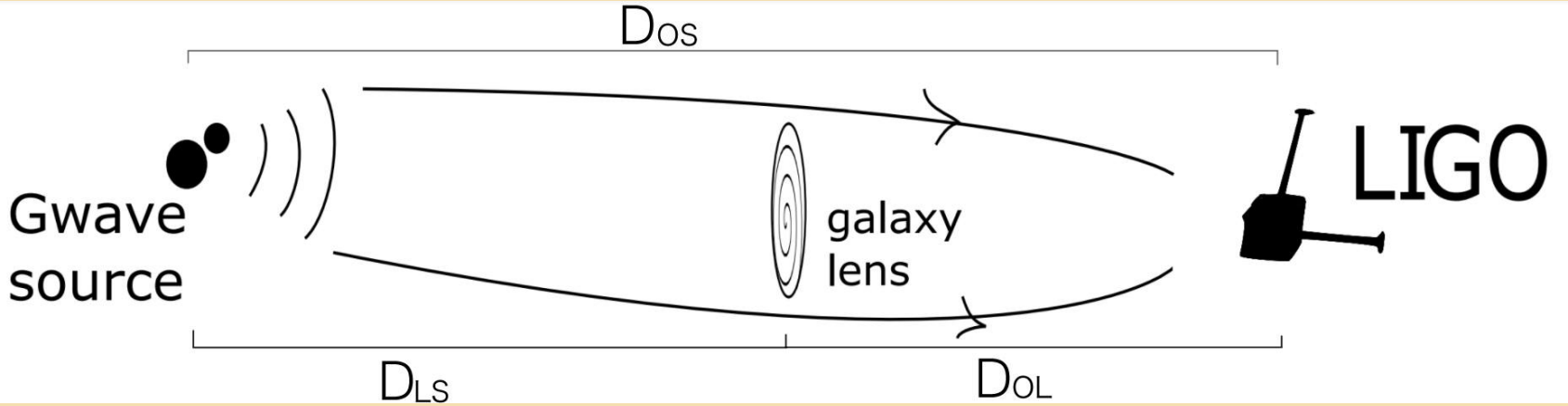
Gravitational Lensing of Gravitational Waves

Tjonnie G. F. Li (黎冠峰)
T. Broadhurst, K.K.Y Ng, K.W.K. Wong,
O.A Hannuksela, K.H. Lai, S. Sachdev

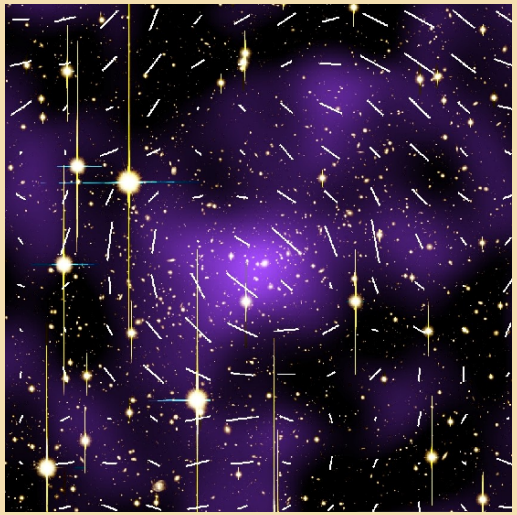
2017 Gravitational Waves Physics and Astronomy Workshop
Annecy, France

June 1, 2017

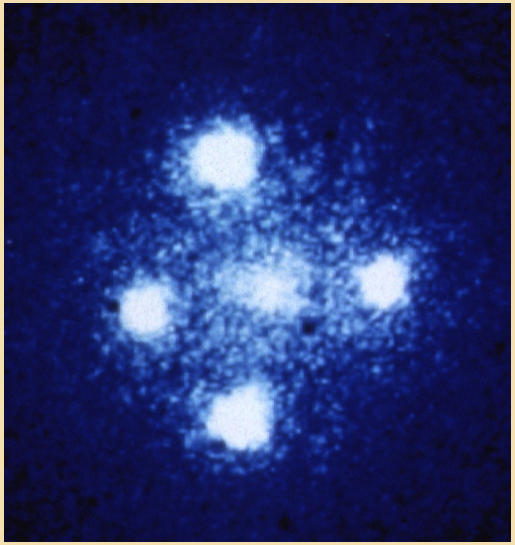
Gravitational Lensing



ESA/Hubble & NASA



Oguri et al. [1]



NASA, ESA & STScI

Effects of Gravitational Lensing

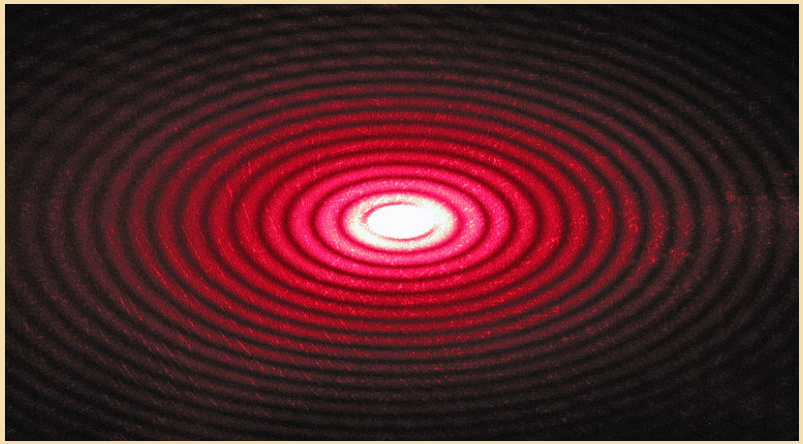
$$F = 4\pi M_L f_{\rm GW} = 2\pi \frac{R_S^L}{\lambda_{\rm GW}}$$

Geometric optics ($F \gg 1$)

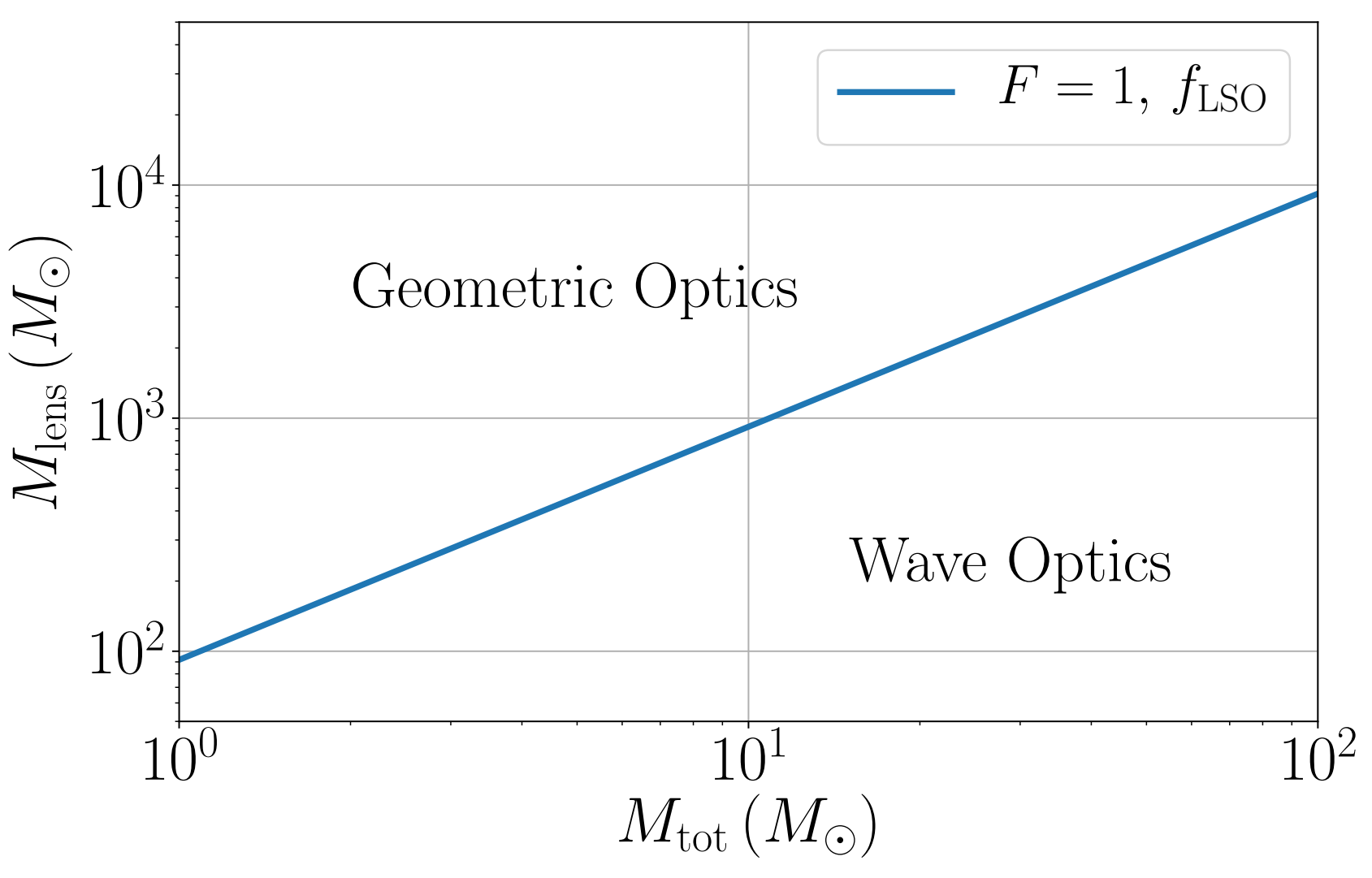
- (De)-magnification
- Multiple images
- Einstein rings

Wave Optics ($F \sim 1$)

- Scattering
- Dispersion (by matter)
- Diffraction



Regions of interest

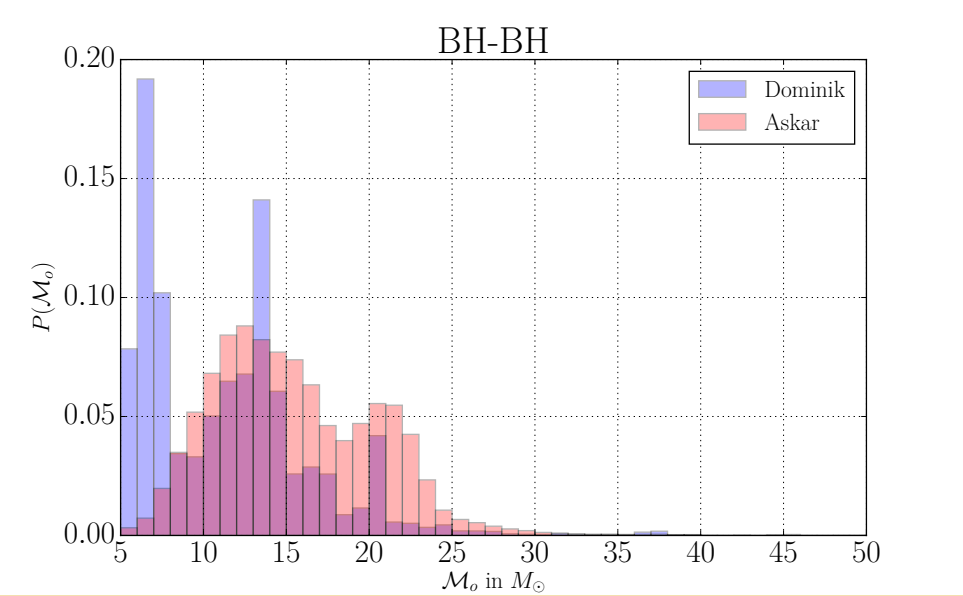


What is the rate of lensed gravitational-wave signals?

Rate of Lensed GW events

Perform Monte Carlo simulation to determine rate

- Masses from stellar evolution simulations
- Lensing optical depth $\tau(z) \propto 0.01\,(d_c/d_H)$
- Magnification universal for high magnification $P(\mu) \propto \mu^{-3}$
- Uniform distribution in orientation and volume

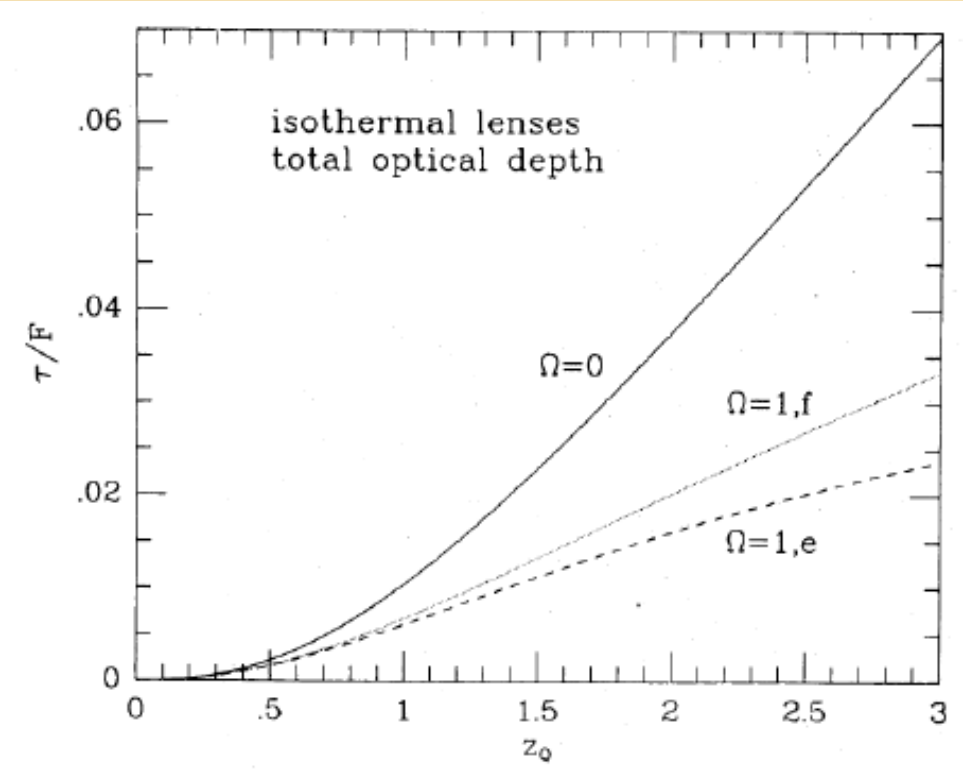


M. Dominik et al. *ApJ* 779, 72 (2013), A. Askar et al. *MNRAS* 464 (2017)

Rate of Lensed GW events

Perform Monte Carlo simulation to determine rate

- Masses from stellar evolution simulations
- Lensing optical depth $\tau(z) \propto 0.01\,(d_c/d_H)$
- Magnification universal for high magnification $P(\mu) \propto \mu^{-3}$
- Uniform distribution in orientation and volume

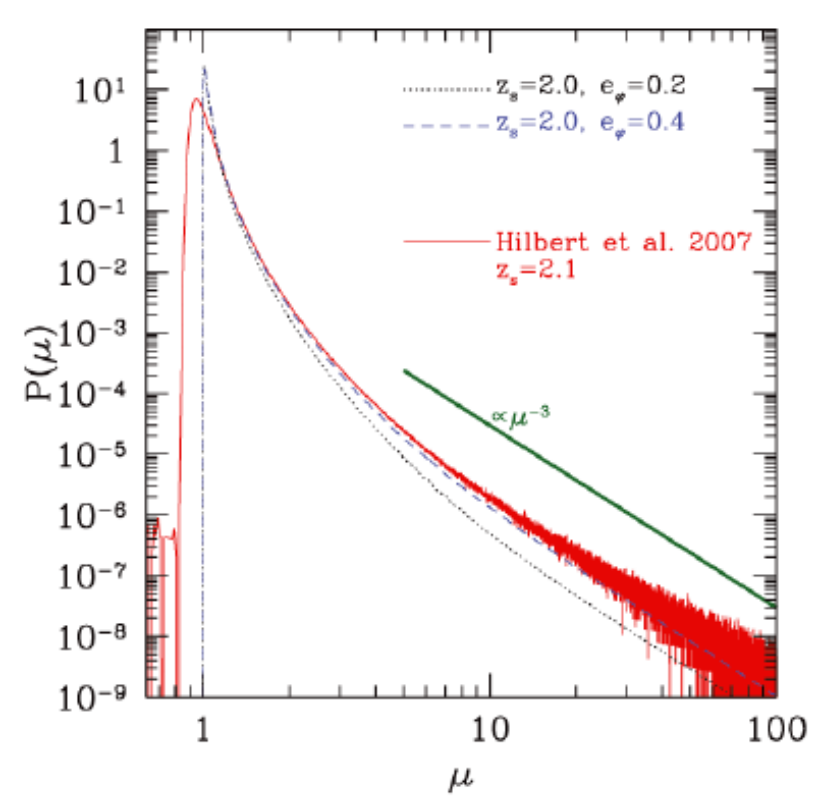


E. L. Turner et al. *ApJ* 284 (1984)

Rate of Lensed GW events

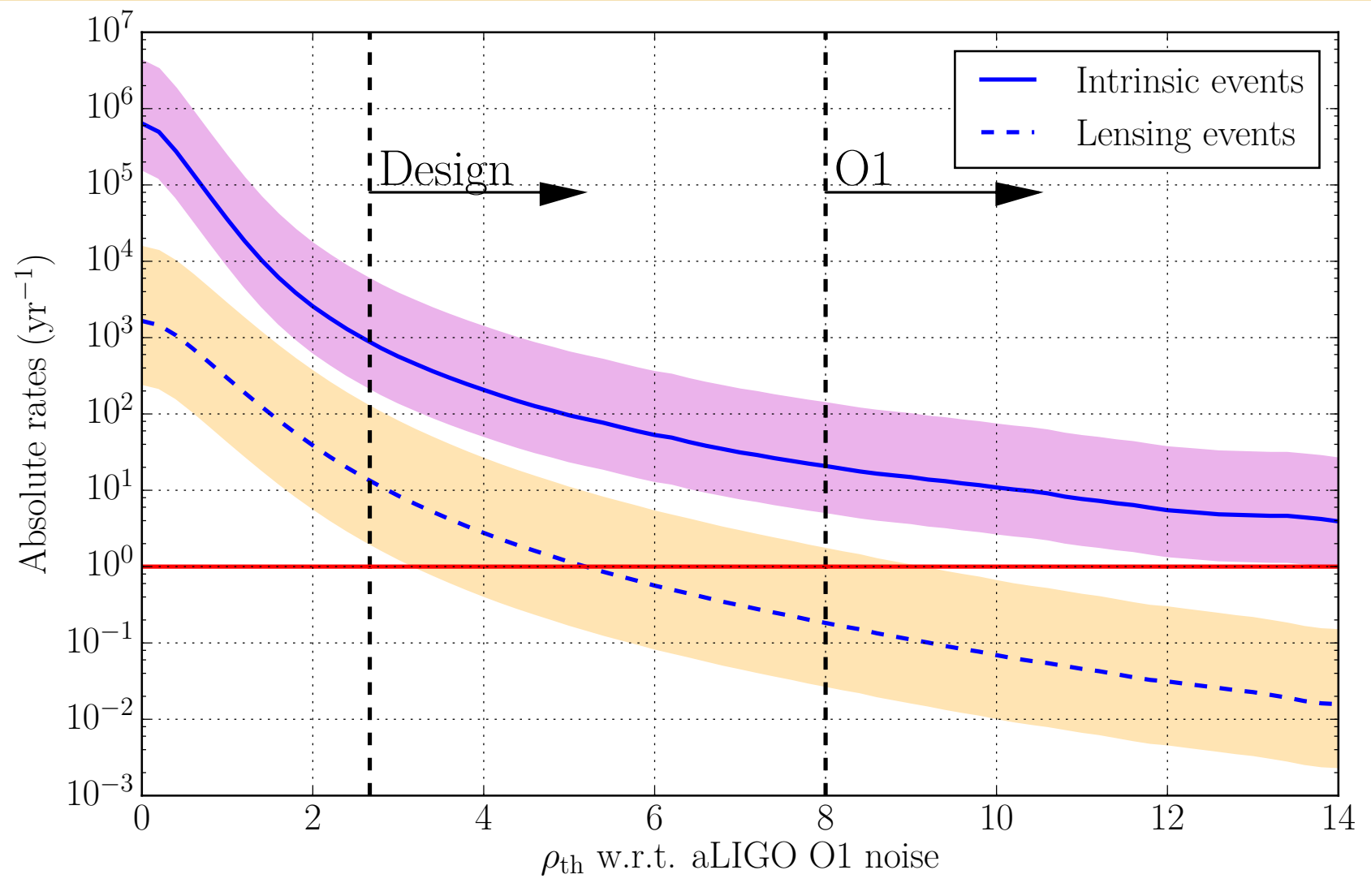
Perform Monte Carlo simulation to determine rate

- Masses from stellar evolution simulations
- Lensing optical depth $\tau(z) \propto 0.01\,(d_c/d_H)$
- Magnification universal for high magnification $P(\mu) \propto \mu^{-3}$
- Uniform distribution in orientation and volume



M. Lima et al. *MNRAS* 406 (2010)

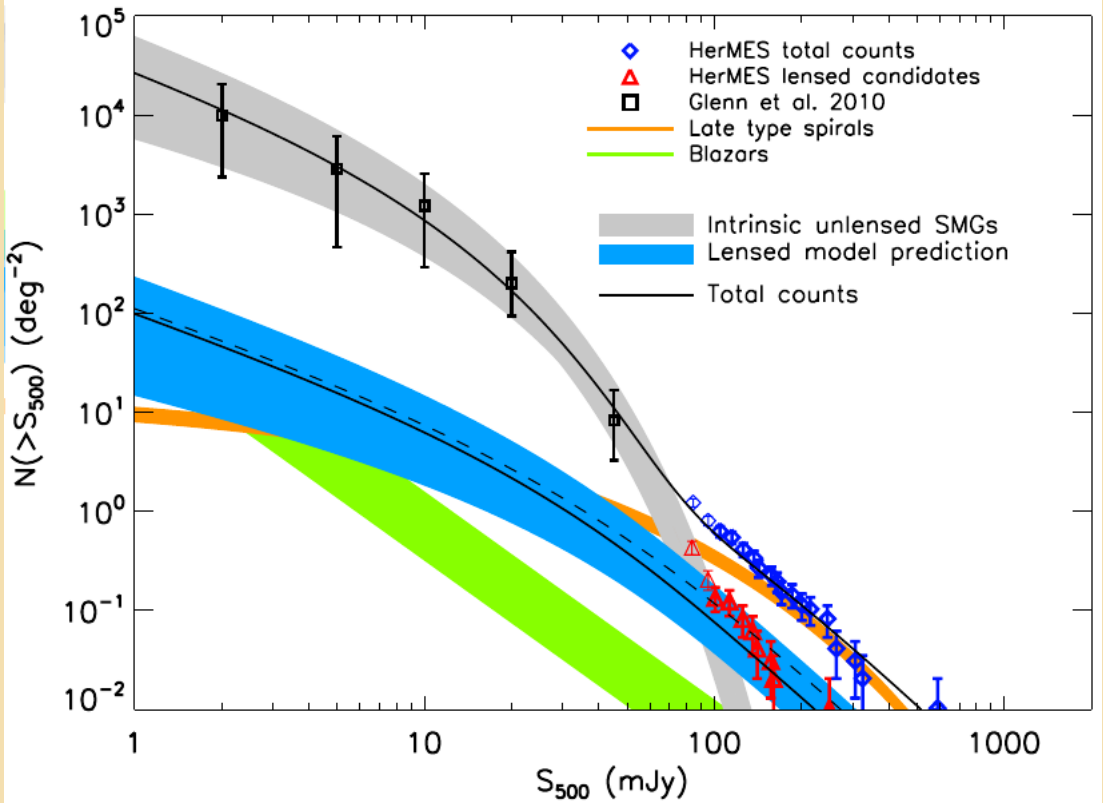
Rate of Lensed GW events



K. K. Y. Ng et al. *ArXiv e-prints* (2017)

Can we observe the population of lensed signals?

Magnification Bias



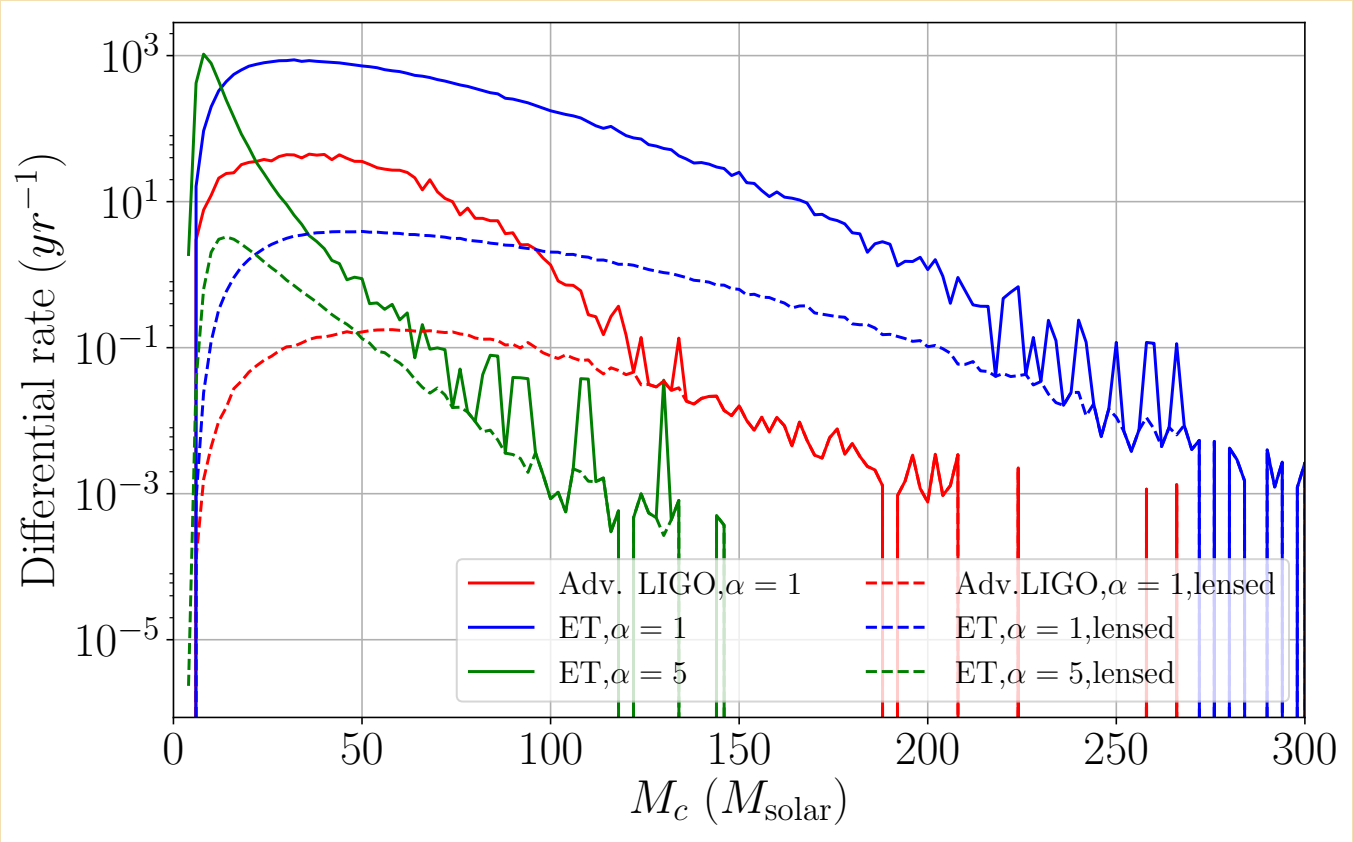
- Submillimeter galaxies
- $S_{500} > 80\,\mathrm{mJy}$
- Total count greater predicted without lensing
- Total count consistent with lensing

J. L. Wardlow et al. *ApJ* 762, 59 (2013)

Observe lensing through brightness distribution

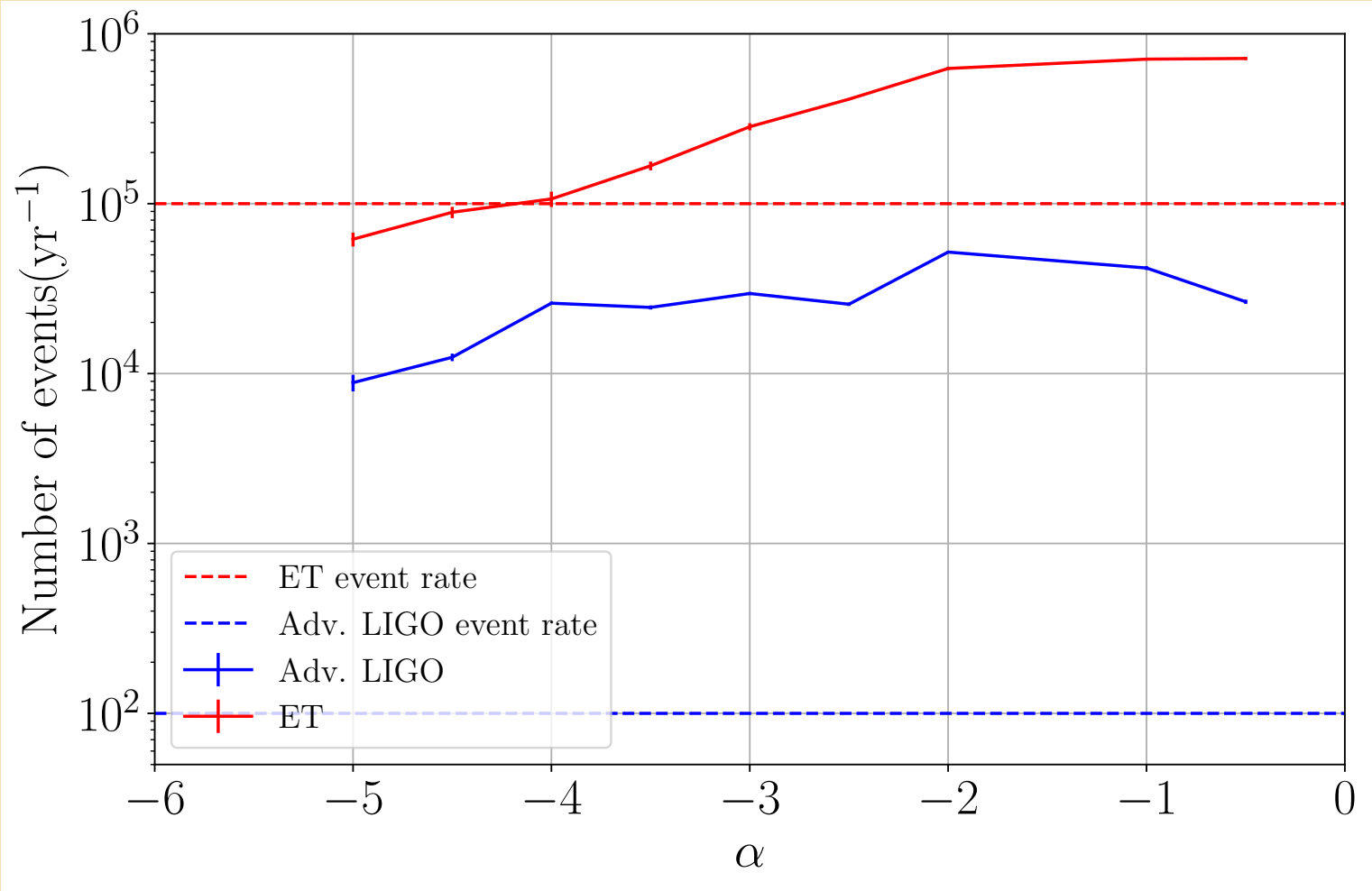
Lensing Bias for Gravitational Waves

$$P(m) \propto m^{-\alpha}$$



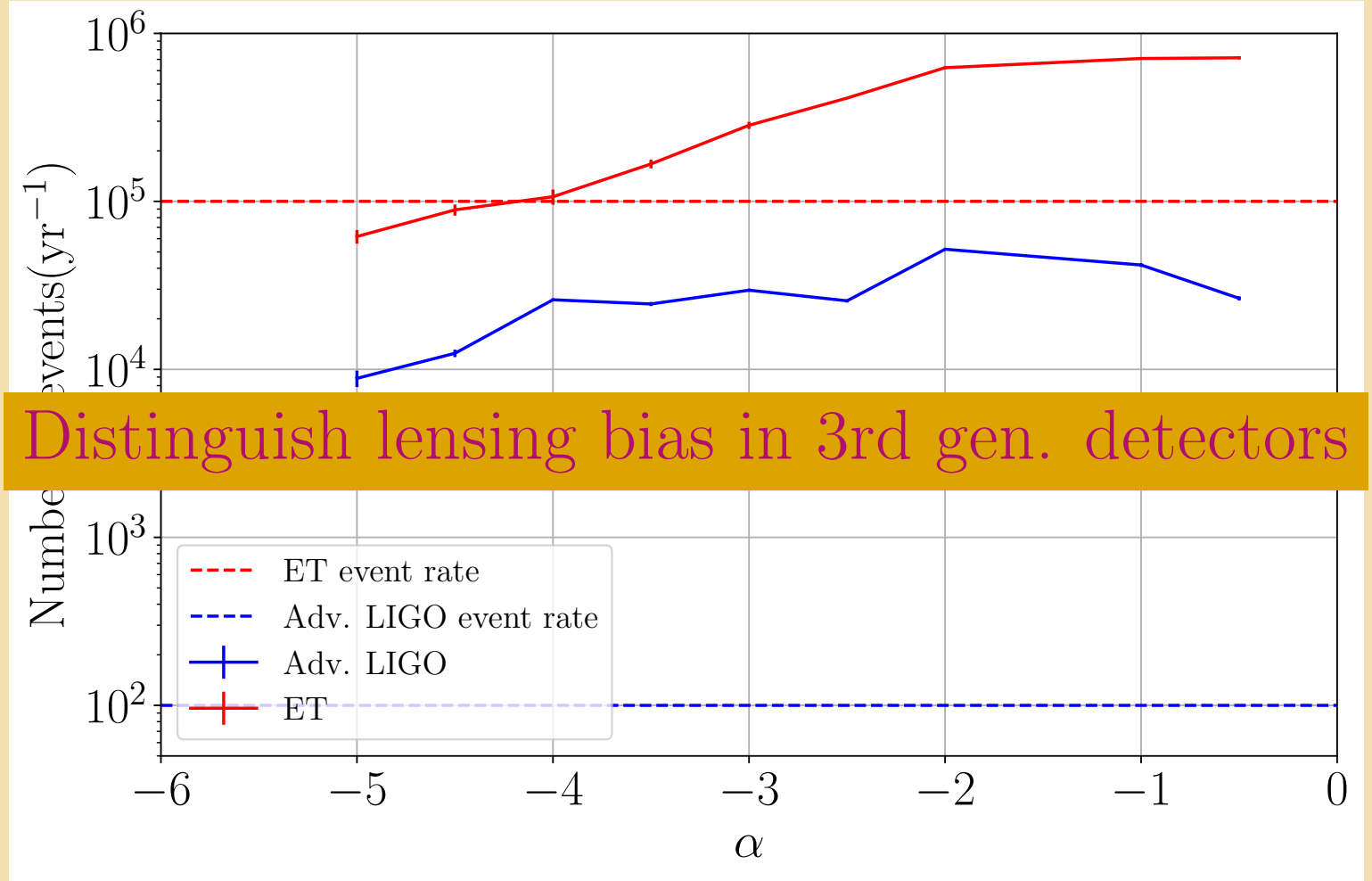
K. K. W. Wong *et al*, in preparation

Observing the Lensing Bias



K. K. W. Wong *et al*, in preparation

Observing the Lensing Bias

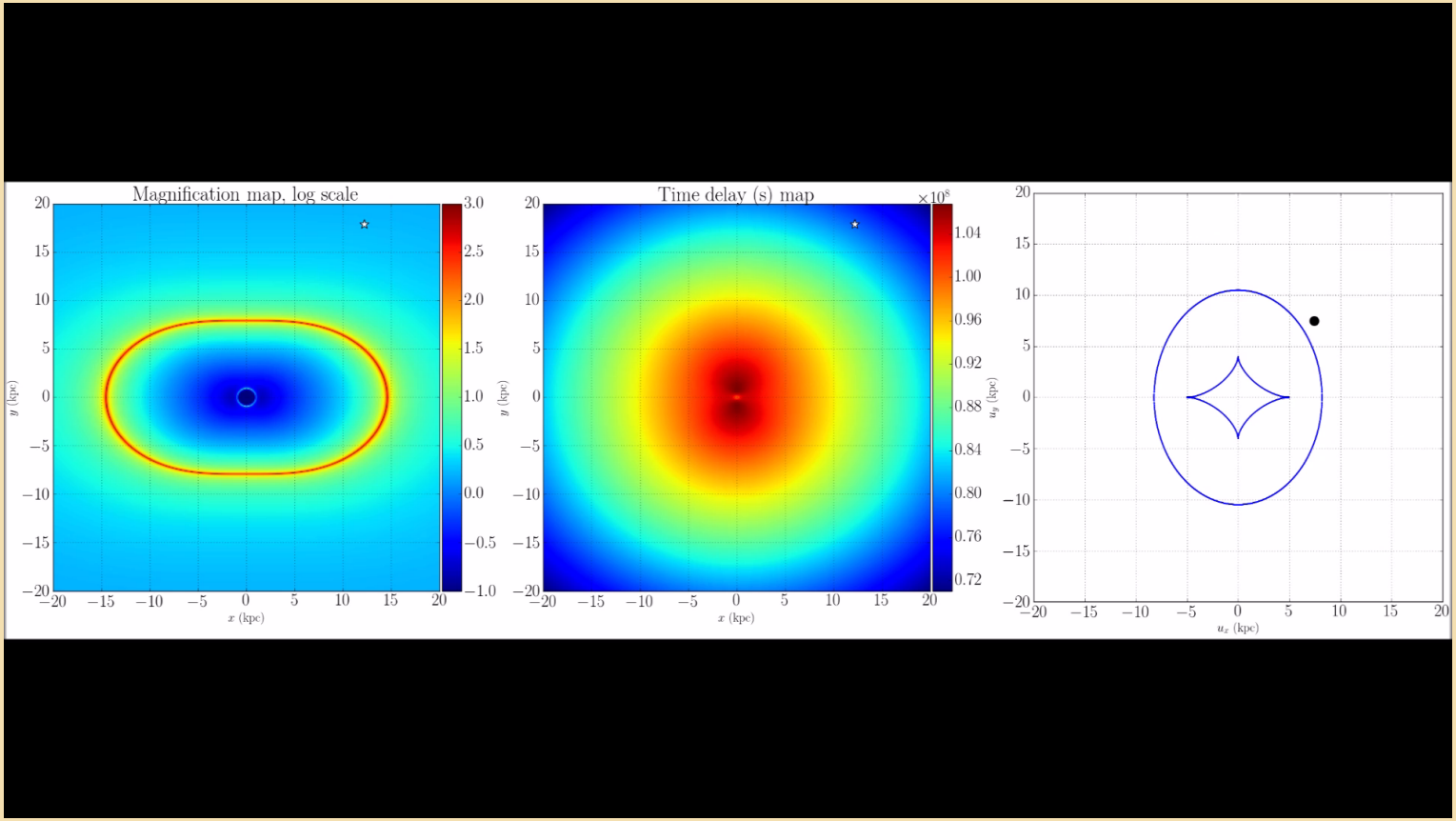


Distinguish lensing bias in 3rd gen. detectors

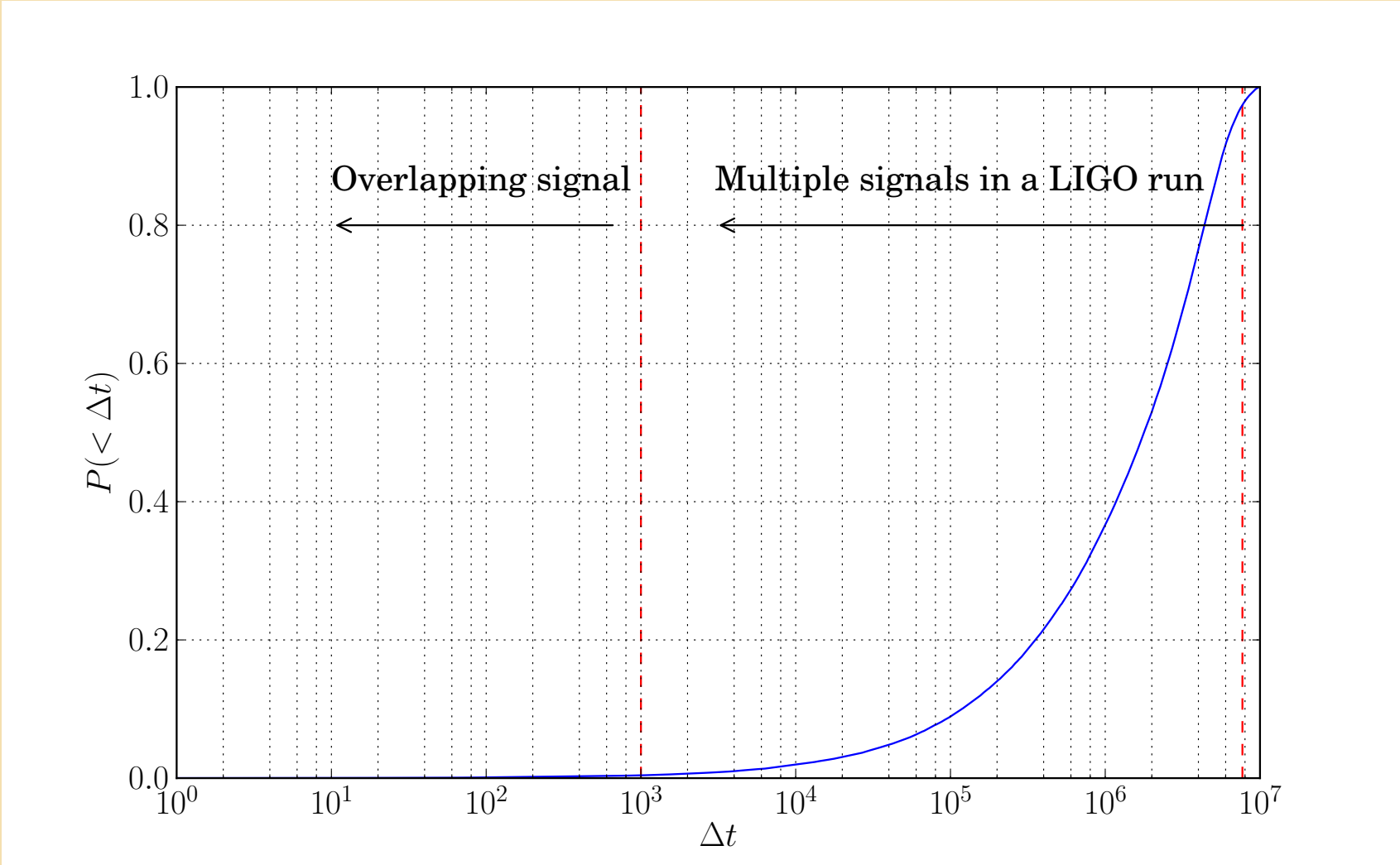
K. K. W. Wong *et al*, in preparation

Can we observe multiple images?

Multiple Images

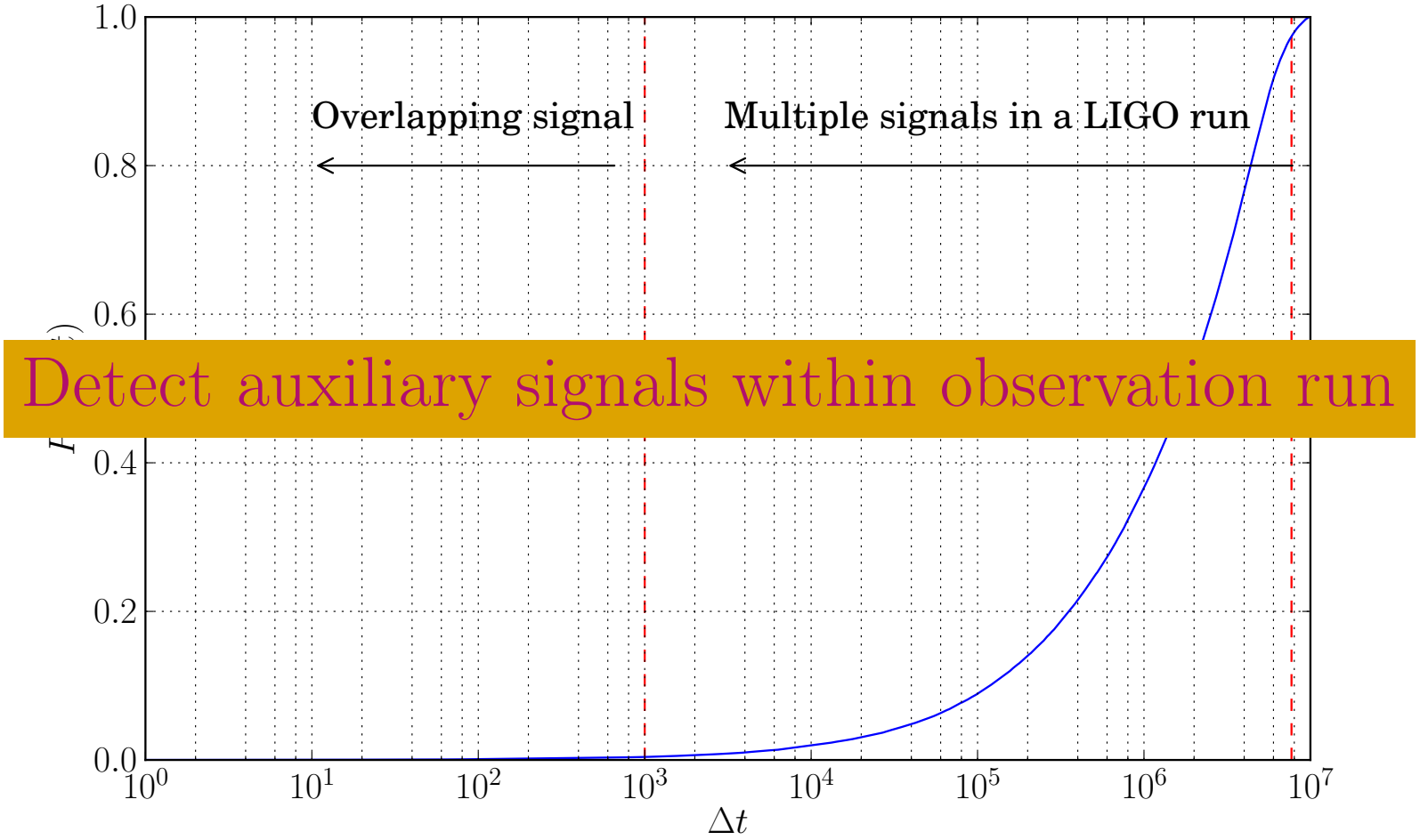


Multiple Images



K. K. W. Wong *et al*, in preparation

Multiple Images



Detect auxiliary signals within observation run

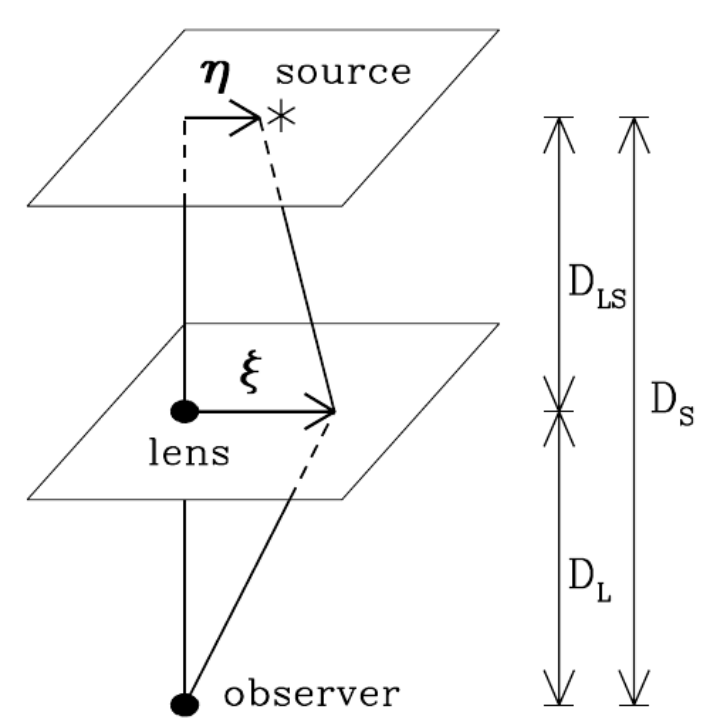
K. K. W. Wong *et al*, in preparation

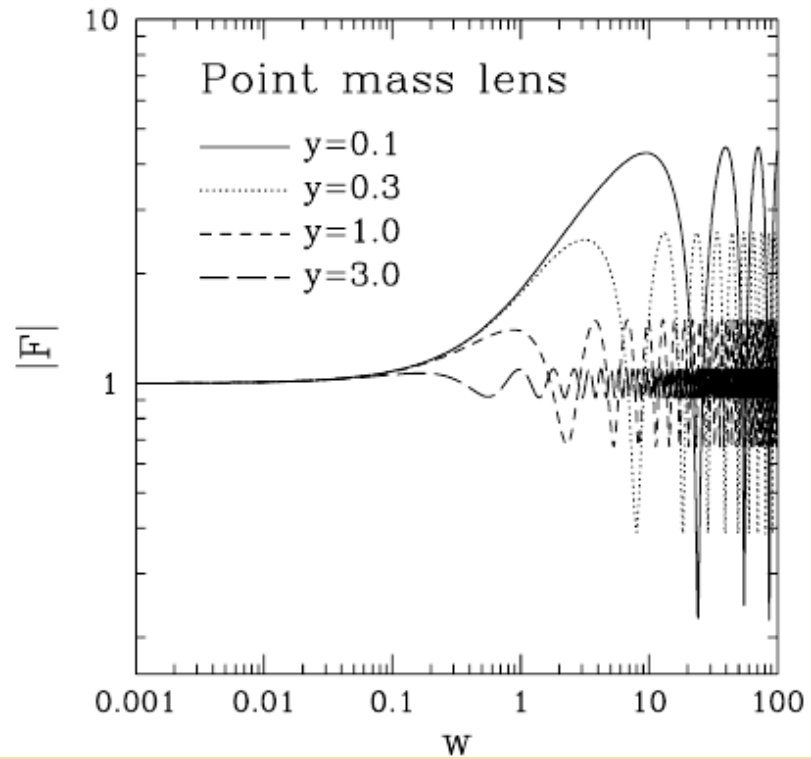
Can be observe diffraction effects?

Diffraction

$$h(f) \to h'(f) = F(f)\, h(f)$$

$$F(f) = \frac{D_S \zeta_0^2 (1+z_L)}{D_L D_{LS}} \frac{f}{i} \int d^2\mathbf{x} \, \exp\left[2\pi i f t_d(\mathbf{x}, \mathbf{y})\right]$$

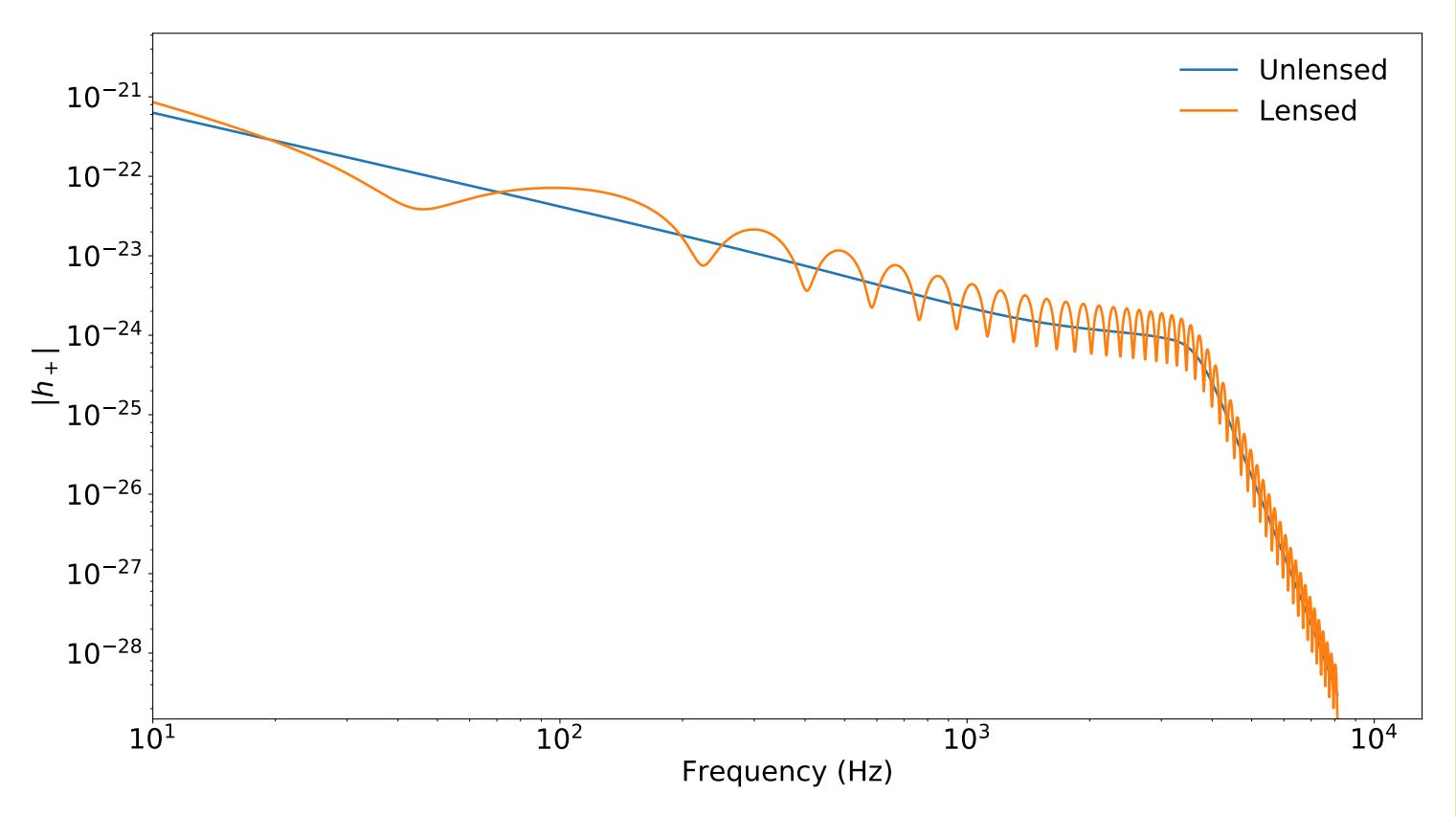




R. Takahashi et al. *ApJ* 595 (2003)

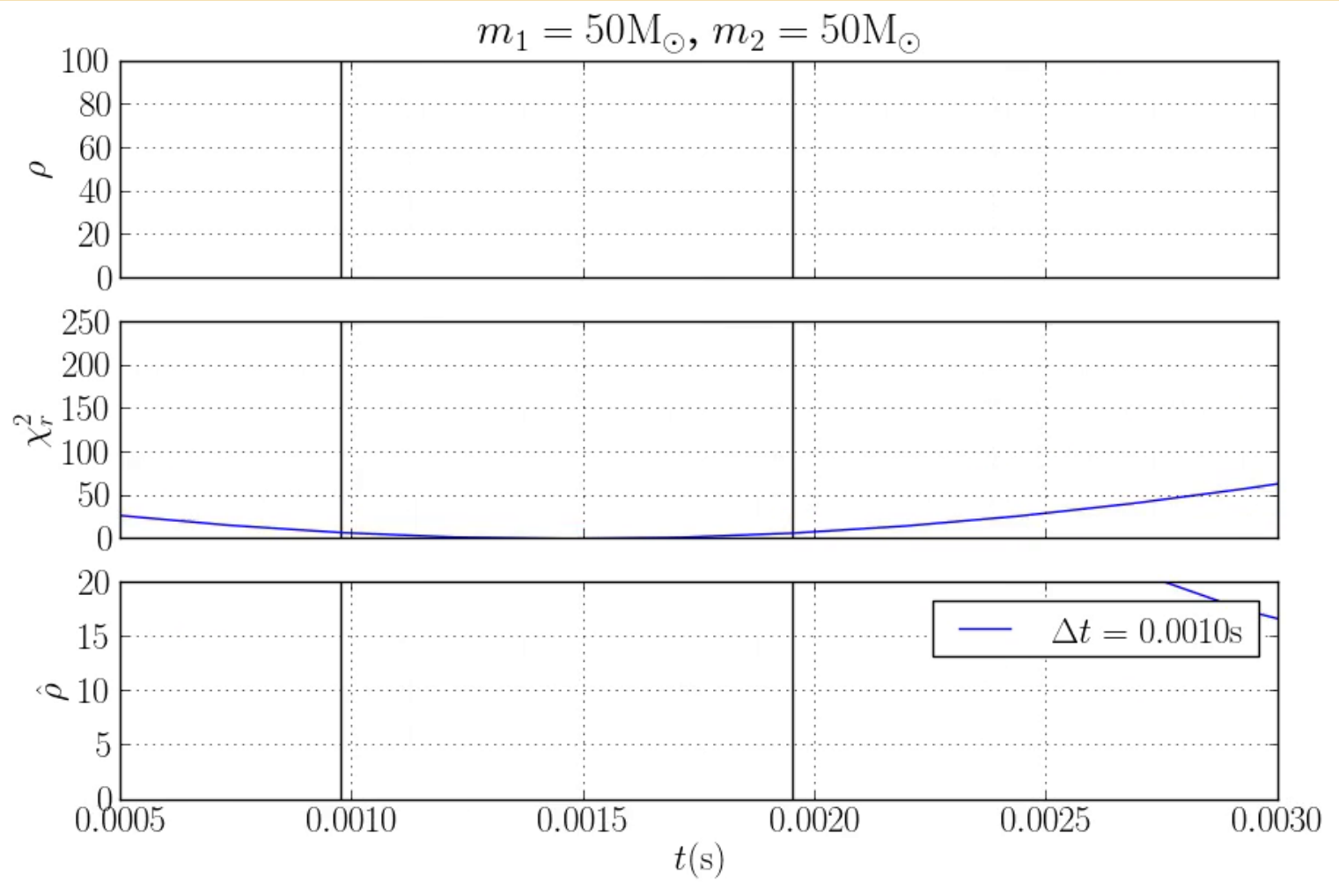
Diffraction – Waveform

IMR waveform lensed by a point mass lens



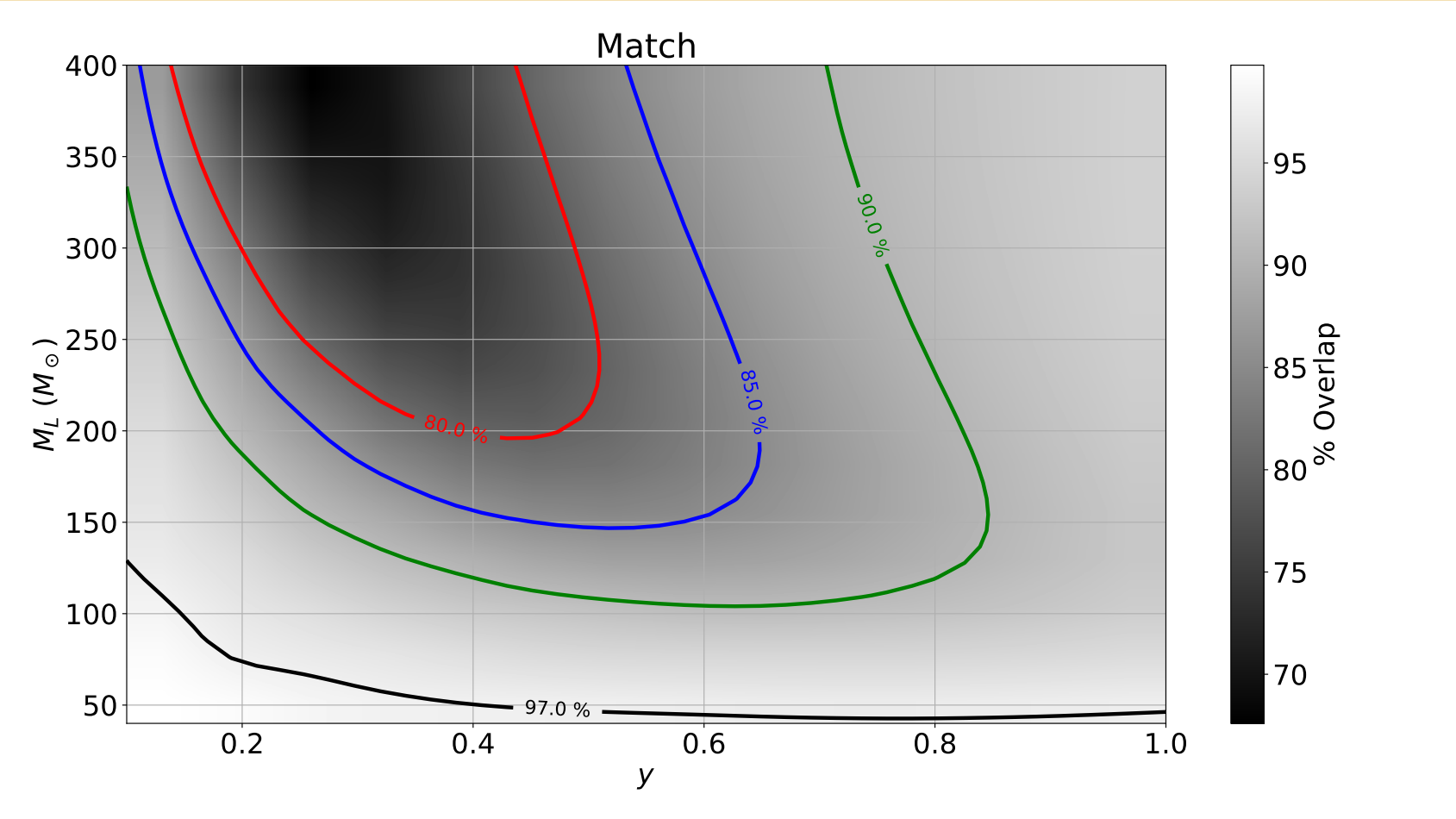
Significant modulation possible

Detecting Diffraction – Overlapping signals



Diffraction – Measurability

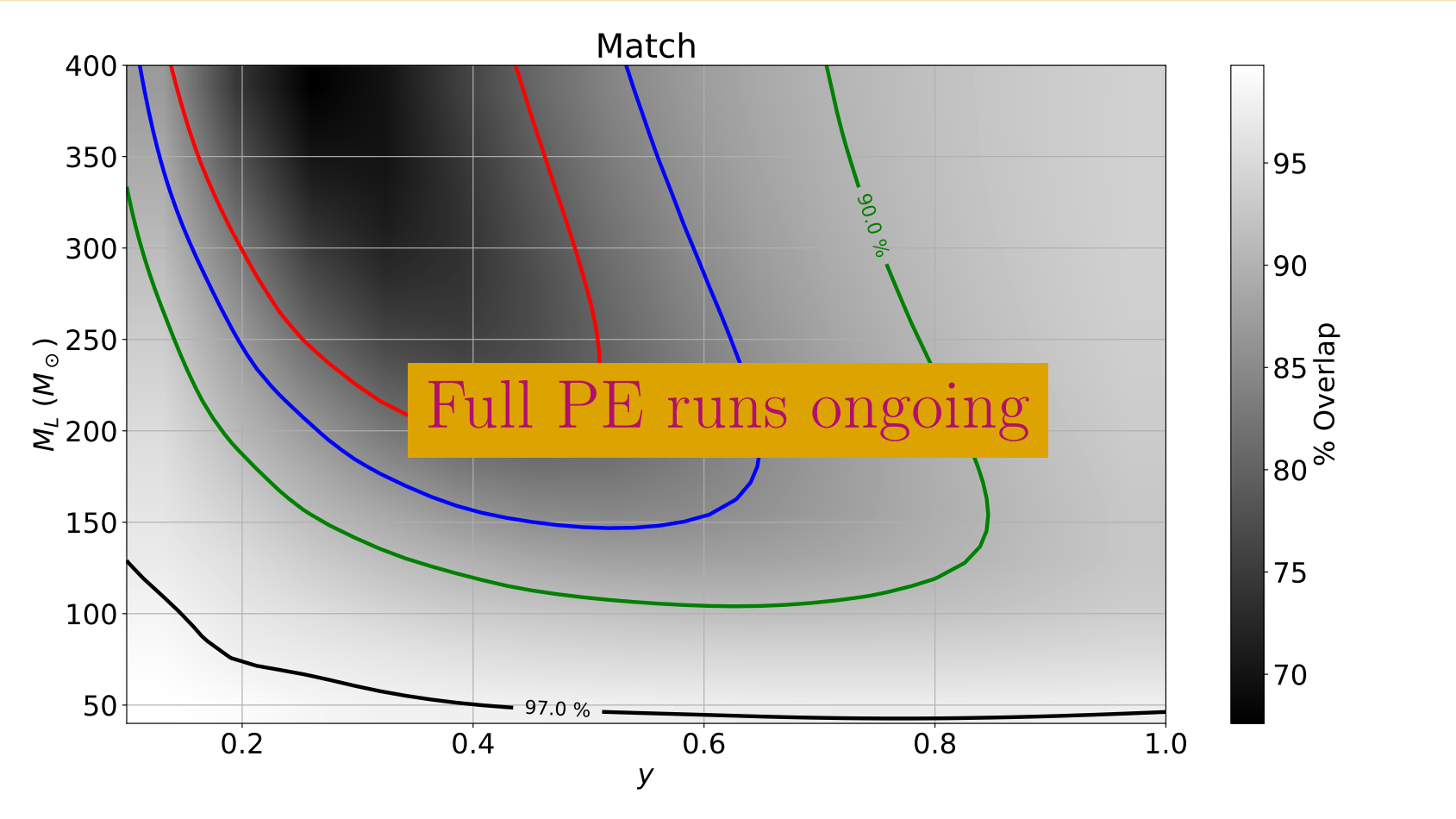
Matches deviate for $M_L \sim 10^2 - 10^4\, M_\odot$



Find evidence for IMBH?

Diffraction – Measurability

Matches deviate for $M_L \sim 10^2 - 10^4\, M_\odot$



Find evidence for IMBH?

Concluding Remarks

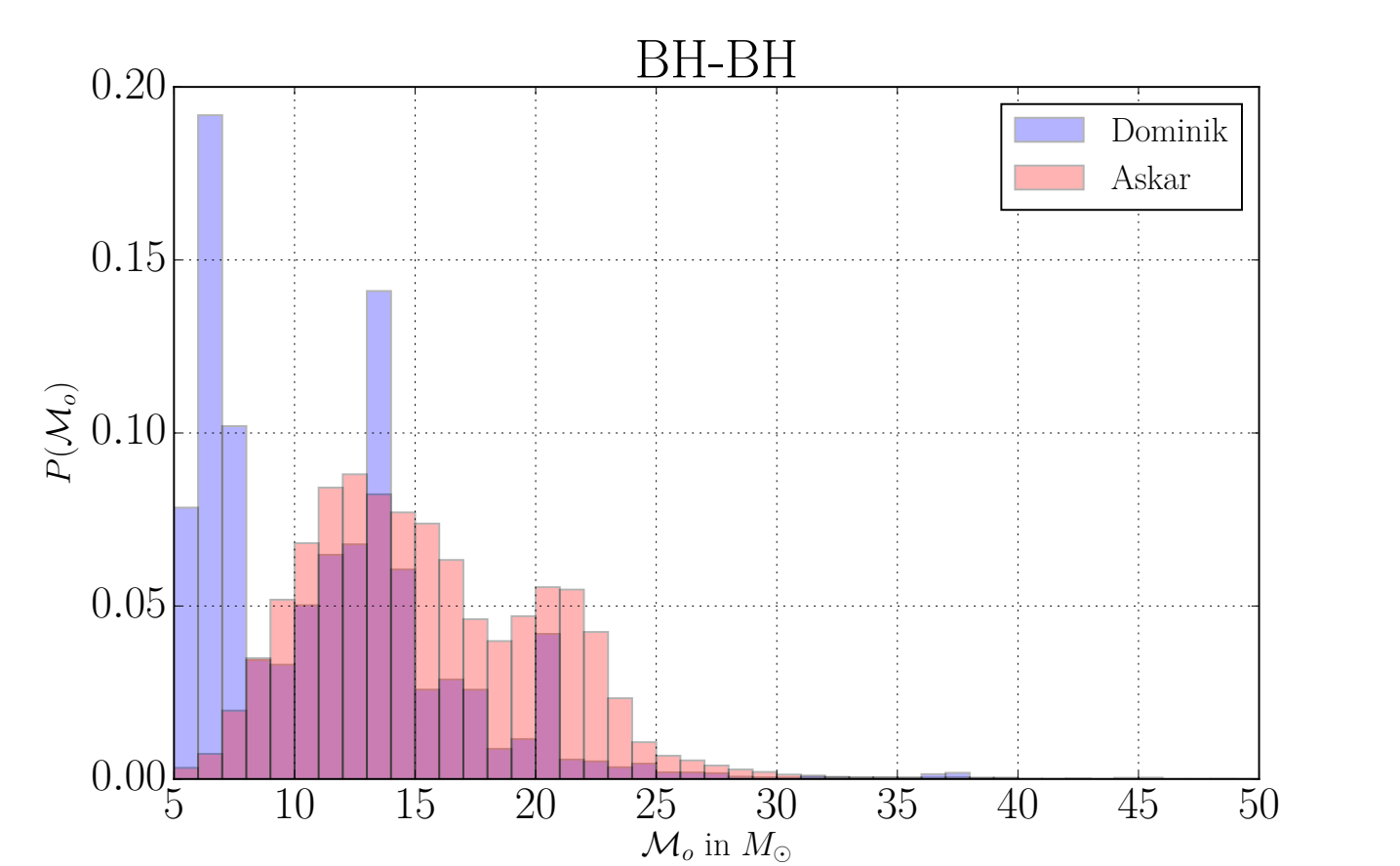
- Possible to see a lensed event in Advanced detector era
- Distinguish the lensed population in 3rd generation detectors.
- Multiple images within the same observation run
- Diffraction can cause missed events
- Diffraction can reveal the nature of the lens

- Test GR with lensed signals
- Measure cosmological parameters using lensed signals

Appendix

Mass Distribution

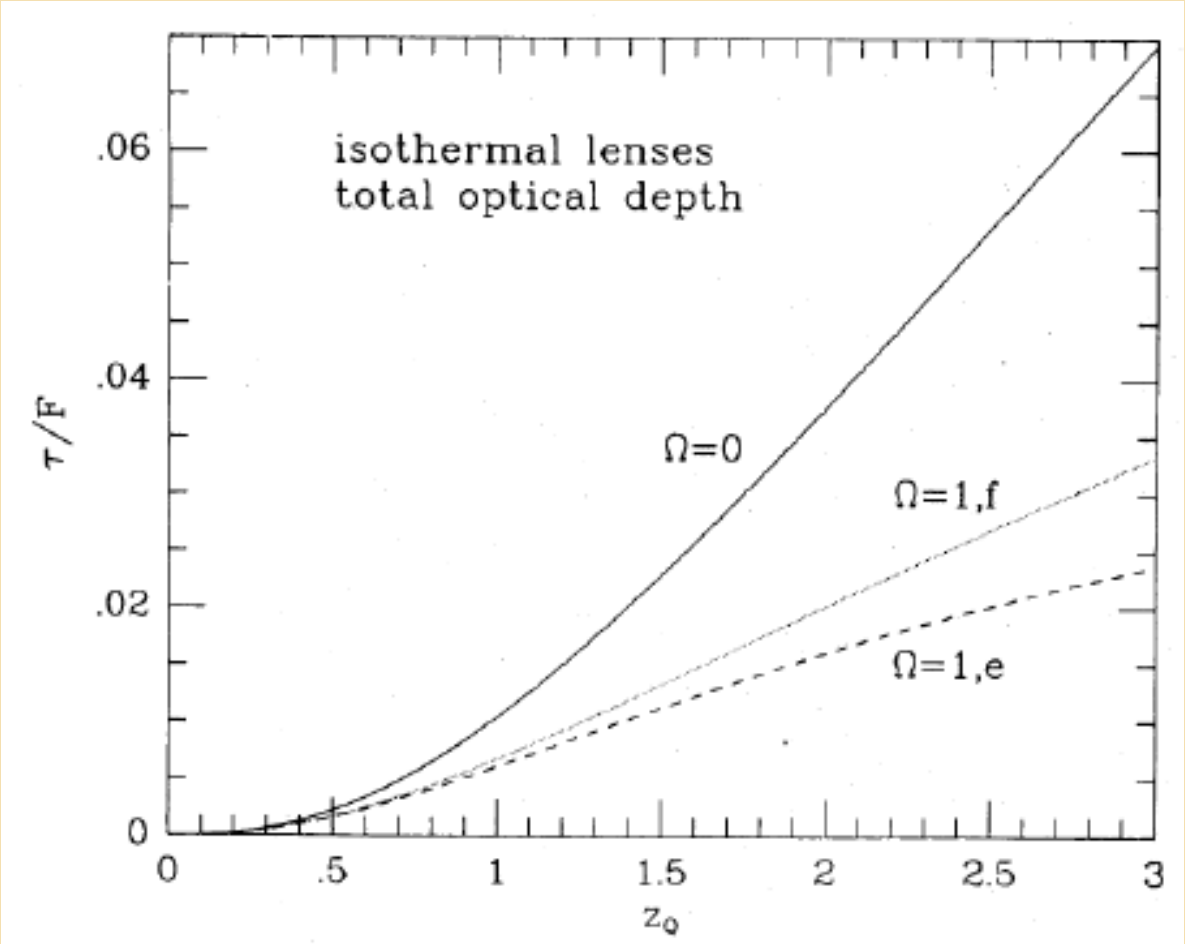
Masses from stellar evolution simulations



M. Dominik et al. *ApJ* 779, 72 (2013), A. Askar et al. *MNRAS* 464 (2017)

Optical Depth

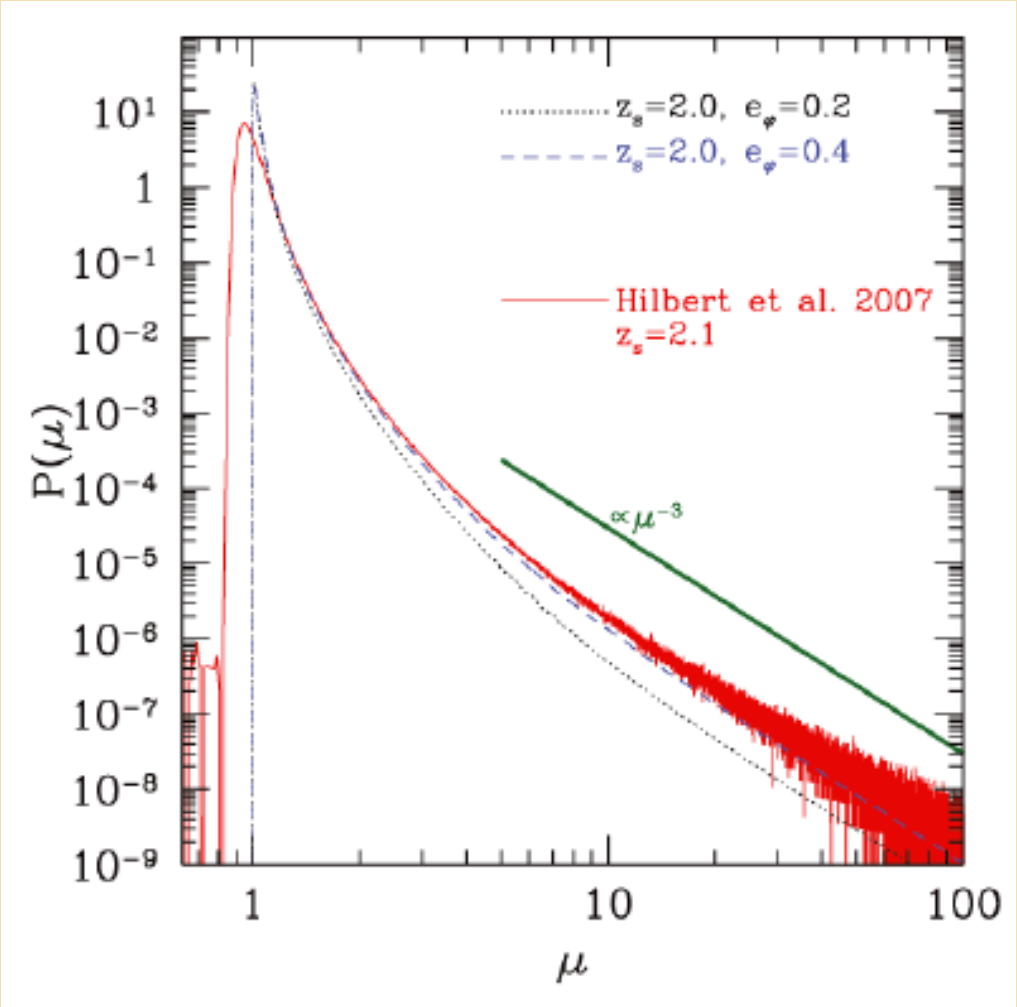
Lensing optical depth $\tau(z) \propto 0.01\,(d_c/d_H)$



E. L. Turner et al. *ApJ* 284 (1984)

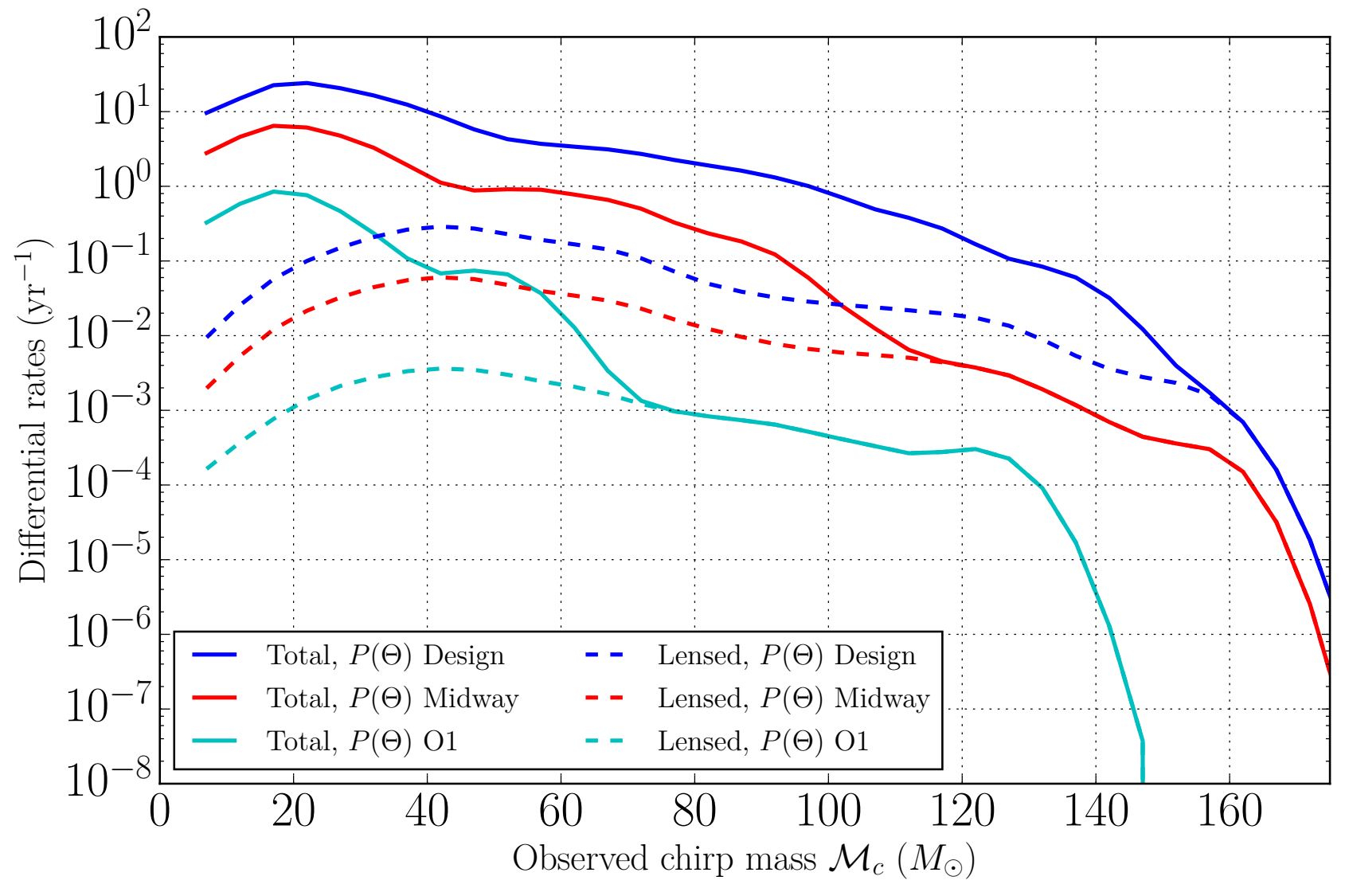
Magnification

Magnification universal for high magnification $P(\mu) \propto \mu^{-3}$



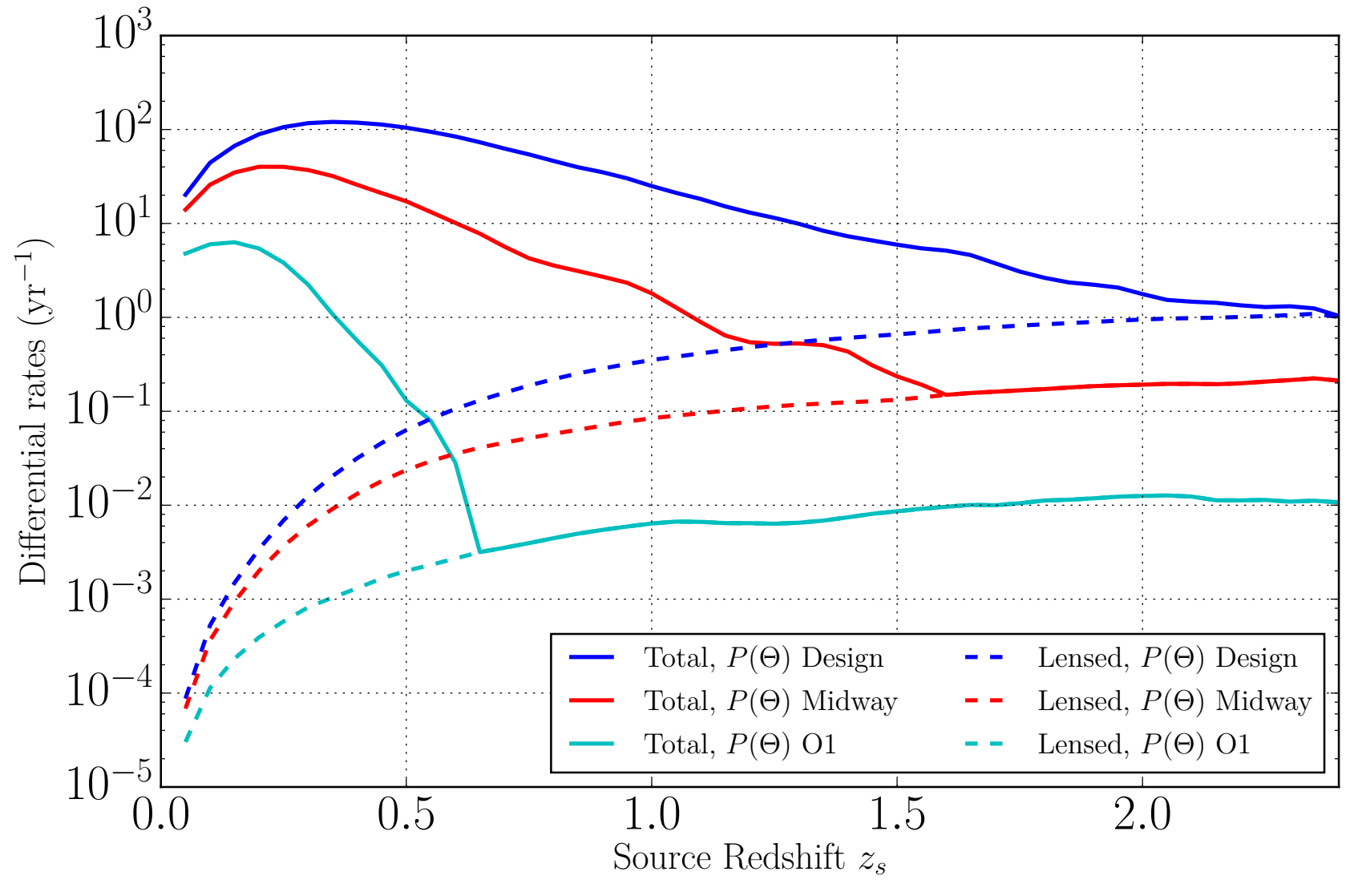
Lima et al. [5]

Rate of Lensed GW events – Chirp Mass



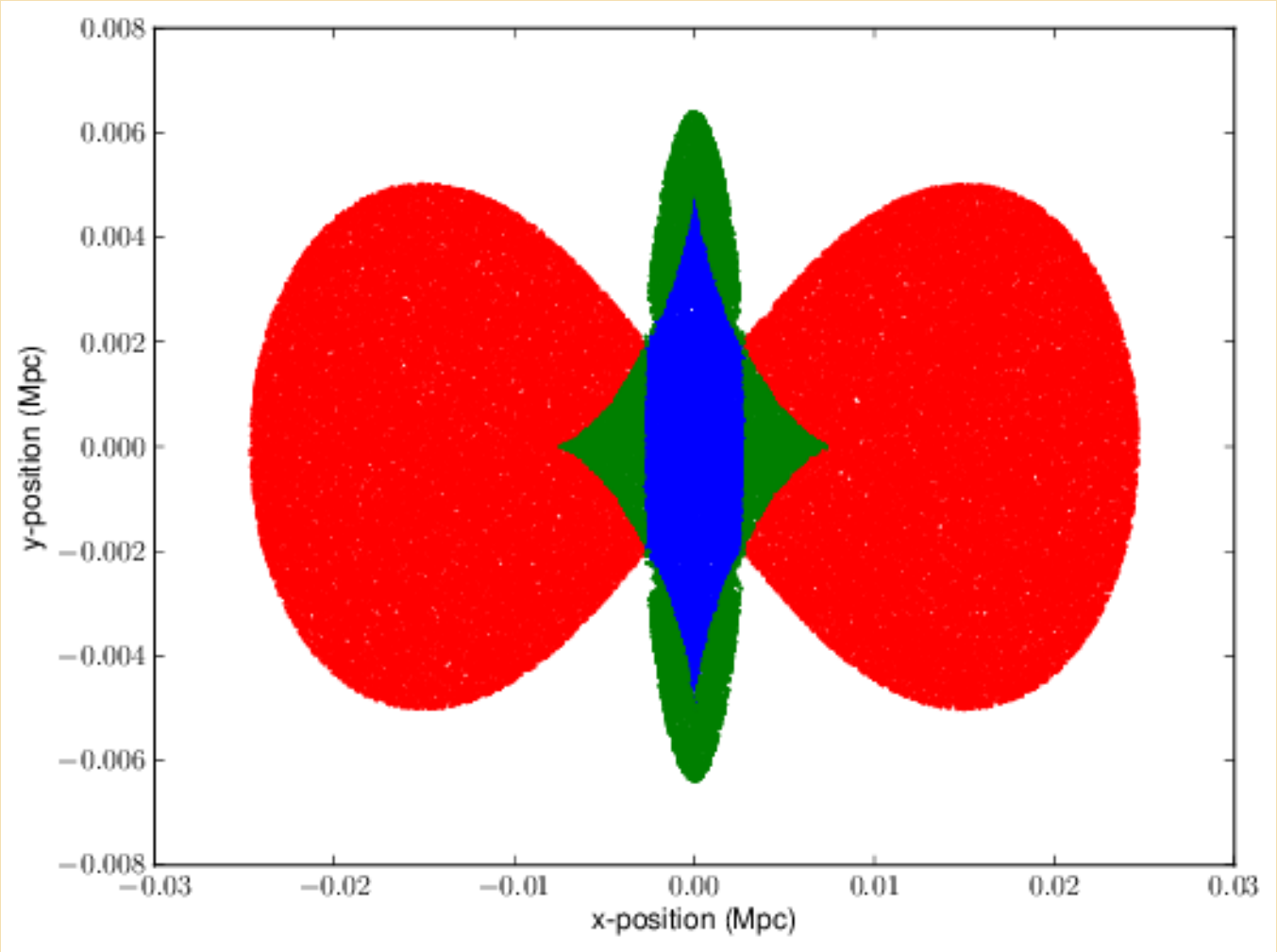
K. K. Y. Ng et al. *ArXiv e-prints* (2017)

Rate of Lensed GW events – Redshift

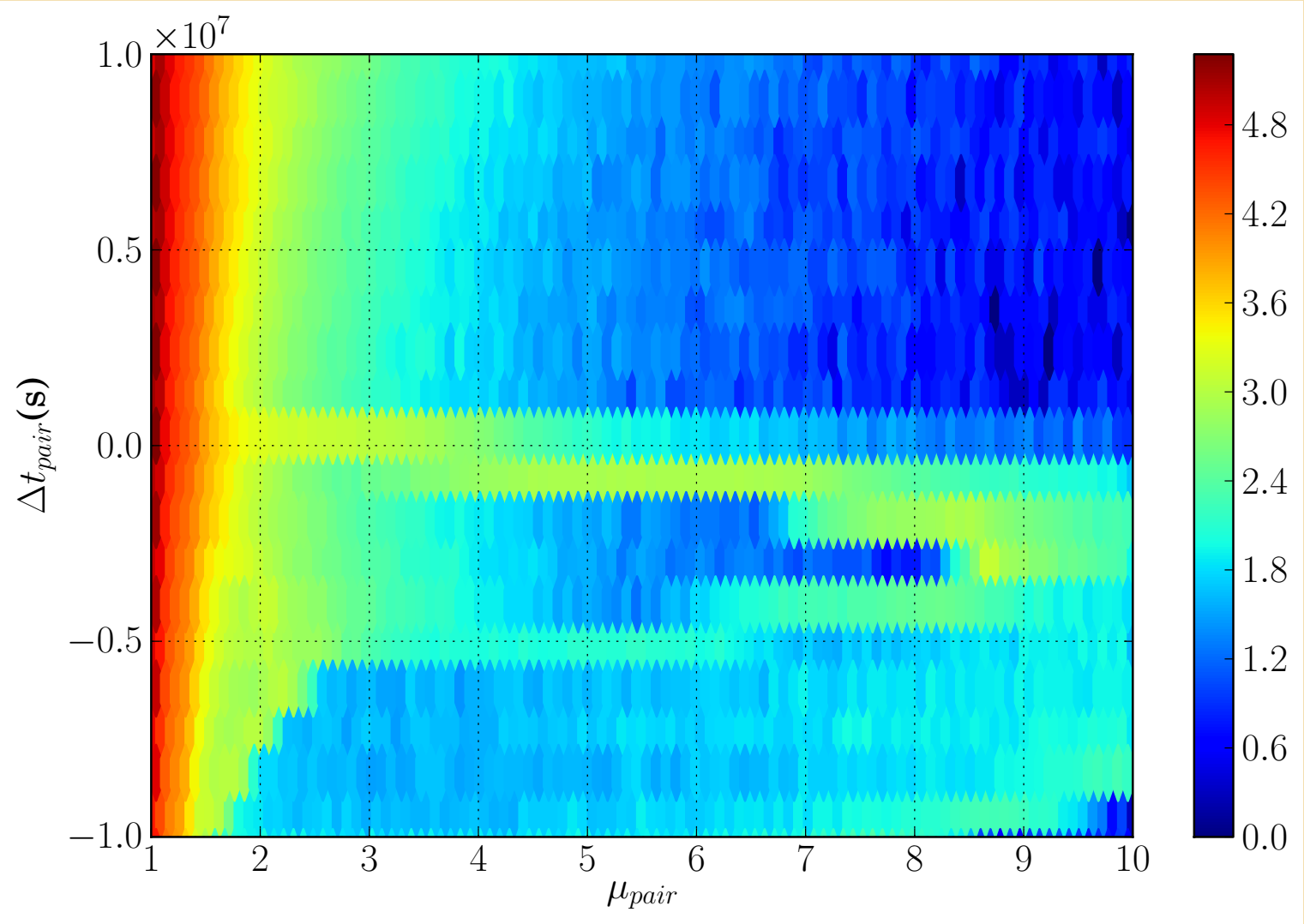


K. K. Y. Ng et al. *ArXiv e-prints* (2017)

Multiple Images



Multiple Images



Diffraction – Theory

- Thin-lens approximation with surface density $\Sigma(\xi)$

$$F(f) = \frac{D_S \zeta_0^2 (1+z_L)}{D_L D_{LS}} \frac{f}{i} \int d^2\mathbf{x} \exp\left[2\pi i f t_d(\mathbf{x},\mathbf{y})\right] \tag{1}$$

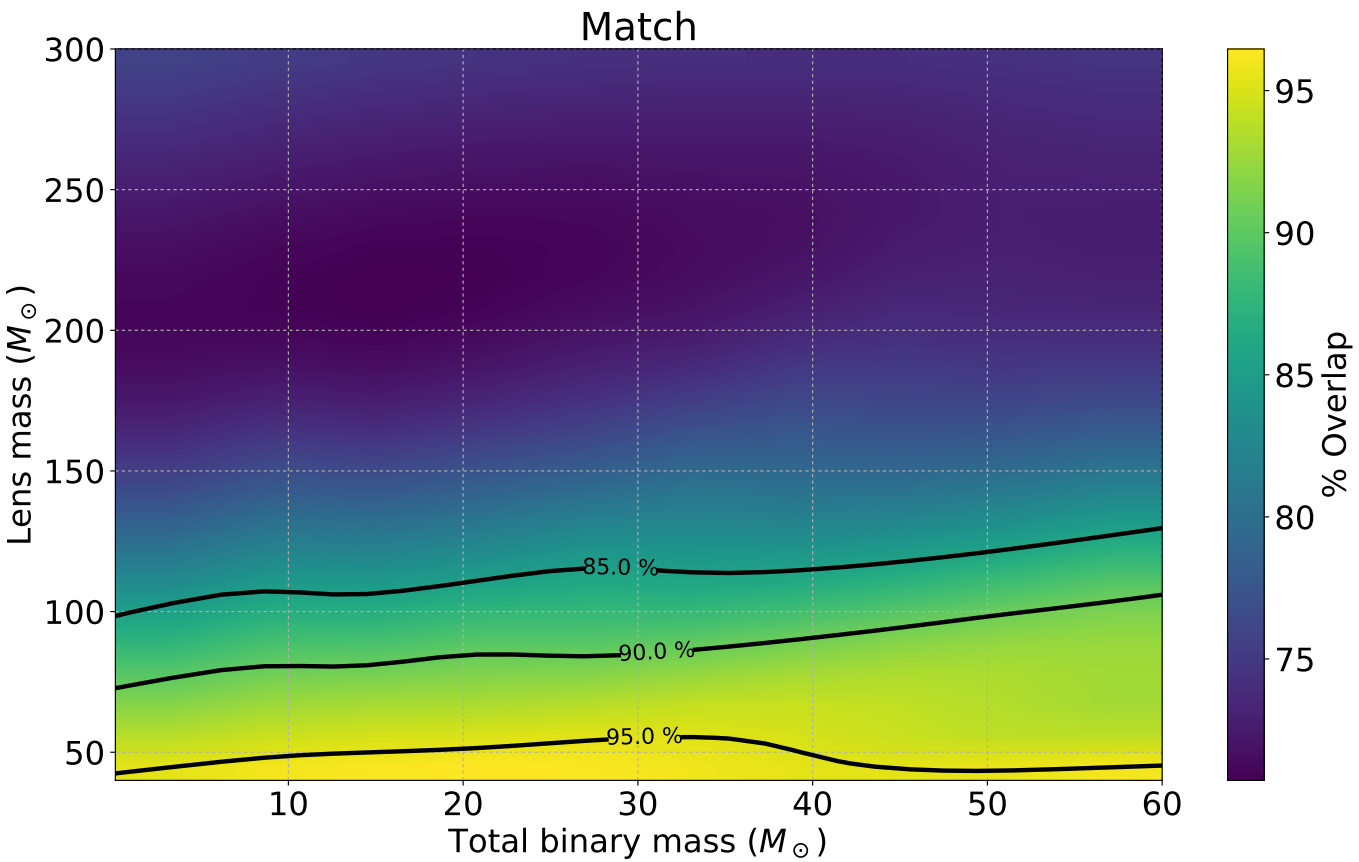
  - $\mathbf{x} = \xi/\xi_0$, $\mathbf{y} = \eta D_L/\xi_0 D_S$
- t_d is the arrival time at the observer from the source

$$t_d(\mathbf{x},\mathbf{y}) = \frac{D_S \xi_0^2}{D_L D_{LS}}(1+z_L)\left[\frac{1}{2}\left|\mathbf{x}-\mathbf{y}\right|^2 - \psi(\mathbf{x}) + \phi_m(\mathbf{y})\right] \tag{2}$$

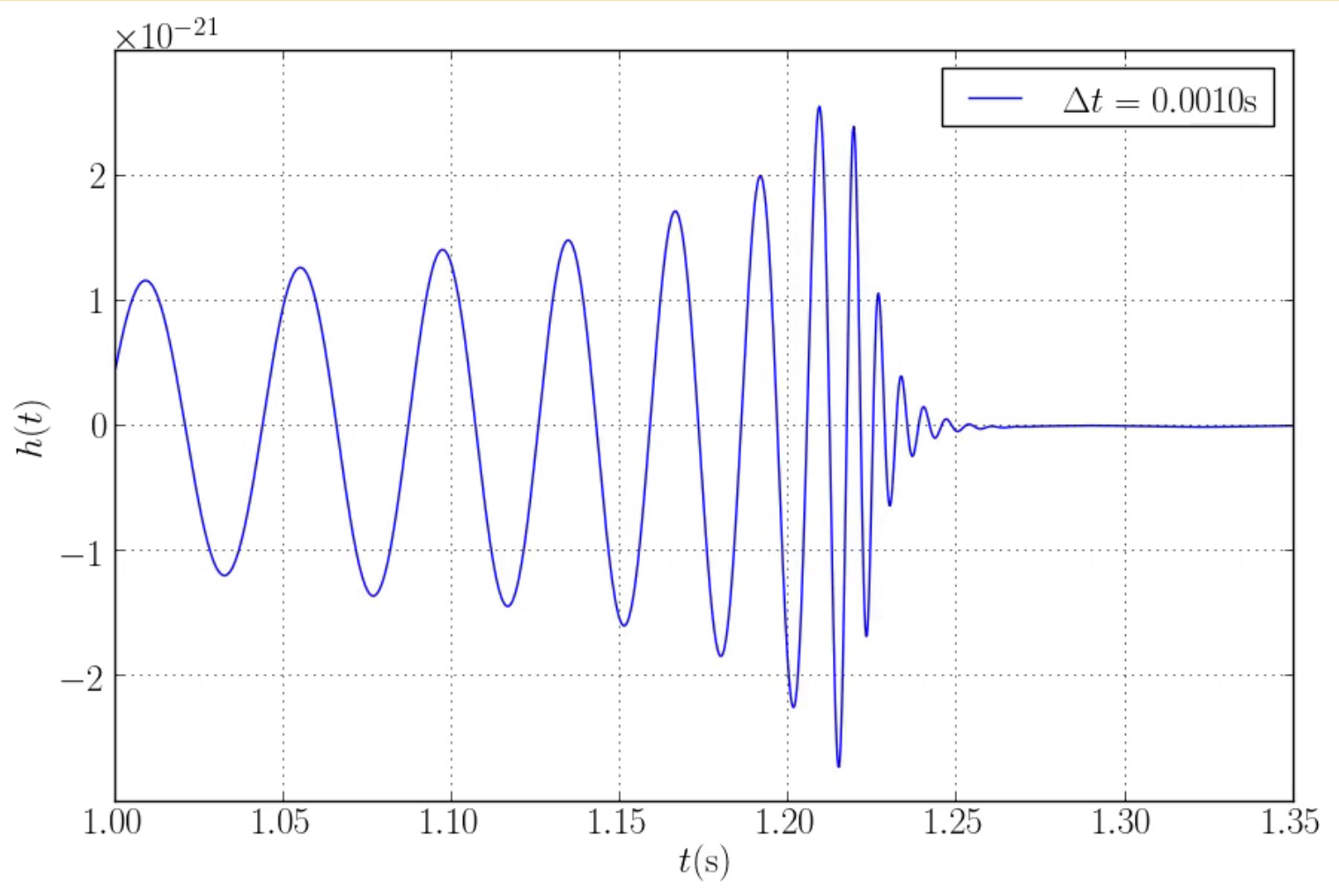
- Deflection potential $\psi(\mathbf{x})$ is given by

$$\nabla_x^2 \psi = \frac{2\Sigma}{\Sigma_{\mathrm{cr}}} \tag{3}$$

Diffraction – Measurability



Detecting Diffraction – Overlapping signals



Abstract

Gravitational lensing phenomena are widespread in electromagnetic astrophysics, and in principle may also be uncovered with gravitational waves. We examine gravitational wave events in the limit of geometric optics, where we expect to see multiple signals from the same event with different arrival times and amplitudes, and wave optics, where we expect to see effects such as interference and diffraction. We estimate the rate of lensed signals in the Advanced LIGO era, and discuss the strategies to identify them. Moreover, we investigate the physics that we can extract from lensed gravitational-wave signals such as the particulars of the lens.

References I

[1] M. Oguri et al. "Direct measurement of dark matter halo ellipticity from two-dimensional lensing shear maps of 25 massive clusters". *MNRAS* 405 (July 2010), pp. 2215–2230. arXiv: 1004.4214.

[2] M. Dominik et al. "Double Compact Objects. II. Cosmological Merger Rates". *ApJ* 779, 72 (Dec. 2013), p. 72. arXiv: 1308.1546 [astro-ph.HE].

[3] A. Askar et al. "MOCCA-SURVEY Database - I. Coalescing binary black holes originating from globular clusters". *MNRAS* 464 (Jan. 2017), pp. L36–L40. arXiv: 1608.02520 [astro-ph.HE].

[4] E. L. Turner et al. "The statistics of gravitational lenses - The distributions of image angular separations and lens redshifts". *ApJ* 284 (Sept. 1984), pp. 1–22.

References II

[5] M. Lima et al. "Lensing magnification: implications for counts of submillimetre galaxies and SZ clusters". *MNRAS* 406 (Aug. 2010), pp. 2352–2372. arXiv: 0907.4387.

[6] K. K. Y. Ng et al. "Precise LIGO Lensing Rate Predictions for Binary Black Holes". *ArXiv e-prints* (Mar. 2017). arXiv: 1703.06319.

[7] J. L. Wardlow et al. "HerMES: Candidate Gravitationally Lensed Galaxies and Lensing Statistics at Submillimeter Wavelengths". *ApJ* 762, 59 (Jan. 2013), p. 59. arXiv: 1205.3778.

[8] R. Takahashi et al. "Wave Effects in the Gravitational Lensing of Gravitational Waves from Chirping Binaries". *ApJ* 595 (Oct. 2003), pp. 1039–1051. eprint: astro-ph/0305055.