Phenomenological Status of Neutrino Mixing

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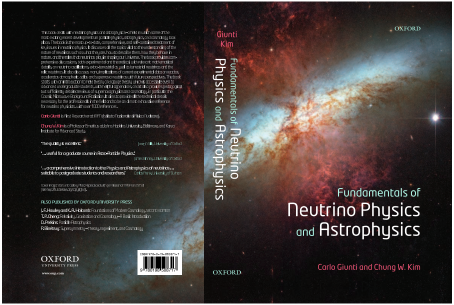

C. Giunti and C.W. Kim
Fundamentals of Neutrino Physics and Astrophysics
Oxford University Press
15 March 2007 – 728 pages

Outline

- Brief Introduction to Neutrino Masses and Mixing
- Three-Neutrino Mixing and Oscillations
- Absolute Scale of Neutrino Masses
- Conclusions

Brief Introduction to Neutrino Masses and Mixing

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  - Standard Model: Massless Neutrinos
  - Extension of the SM: Massive Neutrinos
  - Lepton Numbers
  - Neutrino Oscillations
- Three-Neutrino Mixing and Oscillations
- Absolute Scale of Neutrino Masses
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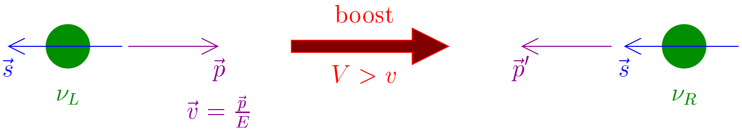
Standard Model: Massless Neutrinos



Standard Model: $\nu_L, \nu_L^c = (\nu^c)_R \quad \Longrightarrow$ no Dirac mass term

$$\mathcal{L}^{\mathrm{D}} \sim m^{\mathrm{D}} \overline{\nu_L} \nu_R \qquad \text{(no } \nu_R, (\nu^c)_L\text{)}$$

Majorana Neutrino: $\nu \equiv \nu^c$

$(\nu^c)_R \equiv \nu_R \quad \Longrightarrow$ Majorana mass term

$$\mathcal{L}^{\mathrm{M}} \sim m^{\mathrm{M}} \overline{\nu_L} \nu_L^c = m^{\mathrm{M}} \overline{\nu_L} (\nu^c)_R$$

Standard Model: Majorana mass term not allowed by $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$

(no Higgs triplet)

Extension of the SM: Massive Neutrinos

Standard Model can be extended with ν_R $(e_L, e_R;\ u_L, u_R;\ d_L, d_R;\ \ldots)$

$\nu_L + \nu_R \Rightarrow$ Dirac neutrino mass term $\mathcal{L}^{\mathrm{D}} \sim m^{\mathrm{D}} \overline{\nu_L} \nu_R \Rightarrow m^{\mathrm{D}} \lesssim 100\,\mathrm{GeV}$

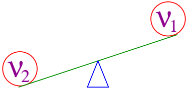
surprise: Majorana neutrino mass for ν_R is allowed! $\mathcal{L}_R^{\mathrm{M}} \sim m_R^{\mathrm{M}} \overline{(\nu^c)_L} \nu_R$

total neutrino mass term $\mathcal{L}^{\mathrm{D+M}} \sim \begin{pmatrix} \overline{\nu_L} & \overline{(\nu^c)_L} \end{pmatrix} \begin{pmatrix} 0 & m^{\mathrm{D}} \\ m^{\mathrm{D}} & m_R^{\mathrm{M}} \end{pmatrix} \begin{pmatrix} (\nu^c)_R \\ \nu_R \end{pmatrix}$

m_R^{M} can be arbitrarily large (not protected by SM symmetries)

$m_R^{\mathrm{M}} \sim$ scale of new physics beyond Standard Model $\Rightarrow m_R^{\mathrm{M}} \gg m^{\mathrm{D}}$

diagonalization of $\begin{pmatrix} 0 & m^{\mathrm{D}} \\ m^{\mathrm{D}} & m_R^{\mathrm{M}} \end{pmatrix} \Rightarrow m_1 \simeq \dfrac{(m^{\mathrm{D}})^2}{m_R^{\mathrm{M}}}, \quad m_2 \simeq m_R^{\mathrm{M}}$



see-saw mechanism

natural explanation of smallness of light neutrino masses

massive neutrinos are Majorana!

3-GEN $\Rightarrow$ effective low-energy 3-ν mixing

[Minkowski, PLB 67 (1977) 42]

[Yanagida (1979); Gell-Mann, Ramond, Slansky (1979); Mohapatra, Senjanovic, PRL 44 (1980) 912]

Lepton Numbers

Standard Model: Lepton numbers are conserved

	L_e	L_μ	L_τ
$(\nu_e\,,\,e^-)$	$+1$	0	0
$(\nu_\mu\,,\,\mu^-)$	0	$+1$	0
$(\nu_\tau\,,\,\tau^-)$	0	0	$+1$

	L_e	L_μ	L_τ
$((\nu^c)_e\,,\,e^+)$	-1	0	0
$((\nu^c)_\mu\,,\,\mu^+)$	0	-1	0
$((\nu^c)_\tau\,,\,\tau^+)$	0	0	-1

$$L = L_e + L_\mu + L_\tau$$

Dirac mass term $m^{\text{D}}\overline{\nu_L}\nu_R \to \begin{pmatrix}\overline{\nu_{eL}} & \overline{\nu_{\mu L}} & \overline{\nu_{\tau L}}\end{pmatrix} \begin{pmatrix} m^{\text{D}}_{ee} & m^{\text{D}}_{e\mu} & m^{\text{D}}_{e\tau} \\ m^{\text{D}}_{\mu e} & m^{\text{D}}_{\mu\mu} & m^{\text{D}}_{\mu\tau} \\ m^{\text{D}}_{\tau e} & m^{\text{D}}_{\tau\mu} & m^{\text{D}}_{\tau\tau} \end{pmatrix} \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix}$

L_e, L_μ, L_τ are not conserved, but L is conserved $L(\nu_{\alpha R}) = L(\nu_{\beta L}) \Rightarrow |\Delta L| = 0$

Majorana mass term $m^{\text{M}}\overline{\nu_L}(\nu^c)_R \to \begin{pmatrix}\overline{\nu_{eL}} & \overline{\nu_{\mu L}} & \overline{\nu_{\tau L}}\end{pmatrix} \begin{pmatrix} m^{\text{M}}_{ee} & m^{\text{M}}_{e\mu} & m^{\text{M}}_{e\tau} \\ m^{\text{M}}_{\mu e} & m^{\text{M}}_{\mu\mu} & m^{\text{M}}_{\mu\tau} \\ m^{\text{M}}_{\tau e} & m^{\text{M}}_{\tau\mu} & m^{\text{M}}_{\tau\tau} \end{pmatrix} \begin{pmatrix} (\nu^c_e)_R \\ (\nu^c_\mu)_R \\ (\nu^c_\tau)_R \end{pmatrix}$

L, L_e, L_μ, L_τ are not conserved $L(\nu^c_\alpha) = -L(\nu_\beta) \Rightarrow |\Delta L| = 2$

Neutrino Oscillations

[Eliezer, Swift, NPB 105 (1976) 45] [Fritzsch, Minkowski, PLB 62 (1976) 72] [Bilenky, Pontecorvo, SJNP 24 (1976) 316]

[Bilenky, Pontecorvo, Nuovo Cim. Lett. 17 (1976) 569] [Bilenky, Pontecorvo, Phys. Rep. 41 (1978) 225]

Flavor Neutrino Production: $j^{\rho}_{W,\mathrm{L}} = 2 \sum_{\alpha=e,\mu,\tau} \overline{\nu_{\alpha L}} \, \gamma^{\rho} \, \ell_{\alpha L}$

$$\nu_{\alpha L} = \sum_k U_{\alpha k} \, \nu_{kL}$$

Fields $\overline{\nu_{\alpha L}} = \sum_k U^*_{\alpha k} \, \overline{\nu_{kL}} \implies |\nu_\alpha\rangle = \sum_k U^*_{\alpha k} \, |\nu_k\rangle$ States

$$|\nu_k(t,x)\rangle = e^{-iE_k t + ip_k x} \, |\nu_k\rangle \quad \Rightarrow \quad |\nu_\alpha(t,x)\rangle = \sum_k U^*_{\alpha k} \, e^{-iE_k t + ip_k x} \, |\nu_k\rangle$$

$$|\nu_k\rangle = \sum_{\beta=e,\mu,\tau} U_{\beta k} \, |\nu_\beta\rangle \quad \Rightarrow \quad |\nu_\alpha(t,x)\rangle = \sum_{\beta=e,\mu,\tau} \underbrace{\left(\sum_k U^*_{\alpha k} e^{-iE_k t + ip_k x} U_{\beta k} \right)}_{\mathcal{A}_{\nu_\alpha \to \nu_\beta}(t,x)} |\nu_\beta\rangle$$

Transition Probability

$$P_{\nu_\alpha \to \nu_\beta}(t,x) = \left| \langle \nu_\beta | \nu_\alpha(t,x) \rangle \right|^2 = \left| \mathcal{A}_{\nu_\alpha \to \nu_\beta}(t,x) \right|^2 = \left| \sum_k U^*_{\alpha k} e^{-iE_k t + ip_k x} U_{\beta k} \right|^2$$

ultra-relativistic neutrinos $\Longrightarrow$ $t \simeq x = L$ source-detector distance

$$E_k t - p_k x \simeq (E_k - p_k)\,L = \frac{E_k^2 - p_k^2}{E_k + p_k}\,L = \frac{m_k^2}{E_k + p_k}\,L \simeq \frac{m_k^2}{2E}\,L$$

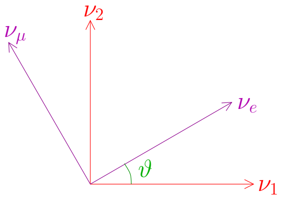
$$\begin{aligned} P_{\nu_\alpha\to\nu_\beta}(L,E) &= \left|\sum_k U_{\alpha k}^*\, e^{-i m_k^2 L/2E}\, U_{\beta k}\right|^2 \\ &= \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i\frac{\Delta m_{kj}^2 L}{2E}\right) \end{aligned}$$

$$\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$$

$$\begin{aligned} P_{\nu_\alpha\to\nu_\beta}(L/E) &= \delta_{\alpha\beta} - 4\sum_{k>j} \mathrm{Re}\Big[U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^*\Big]\sin^2\left(\frac{\Delta m_{kj}^2 L}{4E}\right) \\ &\quad + 2\sum_{k>j} \mathrm{Im}\Big[U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^*\Big]\sin\left(\frac{\Delta m_{kj}^2 L}{2E}\right) \end{aligned}$$

Two-Neutrino Mixing and Oscillations

$$|\nu_\alpha\rangle = \sum_{k=1}^{2} U_{\alpha k} \, |\nu_k\rangle \qquad (\alpha = e, \mu)$$



$$U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$$

$$|\nu_e\rangle = \cos\vartheta \, |\nu_1\rangle + \sin\vartheta \, |\nu_2\rangle$$
$$|\nu_\mu\rangle = -\sin\vartheta \, |\nu_1\rangle + \cos\vartheta \, |\nu_2\rangle$$

$$\Delta m^2 \equiv \Delta m^2_{21} \equiv m_2^2 - m_1^2$$

Transition Probability: $P_{\nu_e \to \nu_\mu} = P_{\nu_\mu \to \nu_e} = \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$

Survival Probabilities: $P_{\nu_e \to \nu_e} = P_{\nu_\mu \to \nu_\mu} = 1 - P_{\nu_e \to \nu_\mu}$

Three-Neutrino Mixing and Oscillations

- Brief Introduction to Neutrino Masses and Mixing
- Three-Neutrino Mixing and Oscillations
  - Experimental Evidences of Neutrino Oscillations
  - Three-Neutrino Mixing
  - Allowed Three-Neutrino Schemes
  - Mixing Matrix
- Absolute Scale of Neutrino Masses
- Conclusions

Experimental Evidences of Neutrino Oscillations

Solar
$\nu_e \to \nu_\mu, \nu_\tau$

Homestake
Kamiokande
GALLEX/GNO & SAGE
Super-Kamiokande
SNO
BOREXino

Reactor
$\bar{\nu}_e$ disappearance

(KamLAND)

$\to$
$$\Delta m^2_{\text{SOL}} \simeq (7.6 \pm 0.2) \times 10^{-5}\,\text{eV}^2$$
$$\tan^2\vartheta_{\text{SOL}} \simeq 0.47 \pm 0.06$$

Atmospheric
$\nu_\mu \to \nu_\tau$

Kamiokande
IMB
Super-Kamiokande
MACRO
Soudan-2

Accelerator
ν_μ disappearance

(K2K & MINOS)

$\to$
$$\Delta m^2_{\text{ATM}} \simeq (2.4 \pm 0.1) \times 10^{-3}\,\text{eV}^2$$
$$\sin^2\vartheta_{\text{ATM}} \simeq 0.50 \pm 0.07$$

Two scales of Δm^2: $\Delta m^2_{\text{ATM}} \simeq 30\, \Delta m^2_{\text{SOL}}$

Large mixings: $\vartheta_{\text{ATM}} \simeq 45^\circ$, $\quad \vartheta_{\text{SOL}} \simeq 34^\circ$

Three-Neutrino Mixing

$$\nu_{\alpha L} = \sum_{k=1}^{3} U_{\alpha k} \, \nu_{kL} \qquad (\alpha = e, \mu, \tau)$$

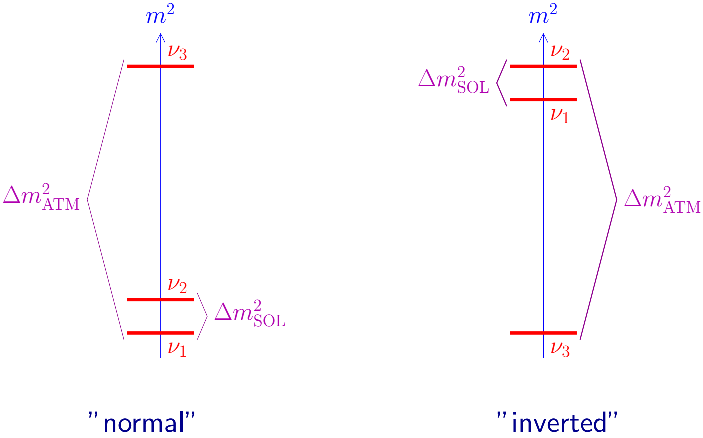
three flavor fields: ν_e, ν_μ, ν_τ

three massive fields: ν_1, ν_2, ν_3

$$\Delta m^2_{\text{SOL}} = \Delta m^2_{21} \simeq (7.6 \pm 0.2) \times 10^{-5} \, \text{eV}^2$$

$$\Delta m^2_{\text{ATM}} \simeq |\Delta m^2_{31}| \simeq |\Delta m^2_{32}| \simeq (2.4 \pm 0.1) \times 10^{-3} \, \text{eV}^2$$

Allowed Three-Neutrino Schemes



different signs of $\Delta m^2_{31} \simeq \Delta m^2_{32}$

absolute scale is not determined by neutrino oscillation data

Mixing Matrix

$$\Delta m^2_{21} \ll |\Delta m^2_{31}|$$

SOL →

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

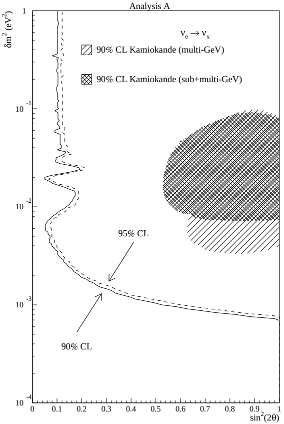
↑ ATM

CHOOZ: $\begin{cases} \Delta m^2_{\text{CHOOZ}} = \Delta m^2_{31} = \Delta m^2_{\text{ATM}} \\ \sin^2 2\vartheta_{\text{CHOOZ}} = 4|U_{e3}|^2(1 - |U_{e3}|^2) \end{cases}$

$$\Downarrow$$

$$|U_{e3}|^2 \lesssim 5 \times 10^{-2}$$

[Bilenky, Giunti, PLB 444 (1998) 379]



[CHOOZ, PLB 466 (1999) 415]

[Palo Verde, PRD 64 (2001) 112001]

SOLAR AND ATMOSPHERIC ν OSCILLATIONS ARE PRACTICALLY DECOUPLED!

TWO-NEUTRINO SOLAR and ATMOSPHERIC ν OSCILLATIONS ARE OK!

$$\sin^2\vartheta_{\text{SOL}} = \frac{|U_{e2}|^2}{1 - |U_{e3}|^2} \simeq |U_{e2}|^2 \qquad \sin^2\vartheta_{\text{ATM}} = |U_{\mu 3}|^2$$

[Bilenky, Giunti, PLB 444 (1998) 379]

[Guo, Xing, PRD 67 (2003) 053002]

$$U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\vartheta_{23} \simeq \vartheta_{\text{ATM}}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix}}_{\vartheta_{13} \simeq \vartheta_{\text{CHOOZ}}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\vartheta_{12} \simeq \vartheta_{\text{SOL}}} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}}_{\beta\beta_{0\nu}}$$

$$= \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{13}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{13}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{13}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{13}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{13}} & c_{23} c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

Global Analysis $\Longrightarrow \sin^2 \vartheta_{13} < 0.035 \quad (90\% \text{ C.L.})$

[Schwetz, Tortola, Valle, New J. Phys. 10 (2008) 113011]

$\sin^2 \vartheta_{13} = 0.016 \pm 0.010$ [Fogli, Lisi, Marrone, Palazzo, Rotunno, PRL 101 (2008) 141801]

$\sin^2 \vartheta_{13} = 0.014 \pm 0.008$ (Bayesian) [Ge, Giunti, Liu, hep-ph/0810.5443]

Bilarge Mixing

$$U \simeq \begin{pmatrix} c_{\vartheta_S} & s_{\vartheta_S} & 0 \\ -s_{\vartheta_S} c_{\vartheta_A} & c_{\vartheta_S} c_{\vartheta_A} & s_{\vartheta_A} \\ s_{\vartheta_S} s_{\vartheta_A} & -c_{\vartheta_S} s_{\vartheta_A} & c_{\vartheta_A} \end{pmatrix} \Longrightarrow \begin{cases} \nu_e = c_{\vartheta_S} \nu_1 + s_{\vartheta_S} \nu_2 \\ \nu_a^{(S)} = -s_{\vartheta_S} \nu_1 + c_{\vartheta_S} \nu_2 \\ \qquad = c_{\vartheta_A} \nu_\mu - s_{\vartheta_A} \nu_\tau \end{cases}$$

$$\sin^2 2\vartheta_A \simeq 1 \Longrightarrow \vartheta_A \simeq \frac{\pi}{4} \Longrightarrow U \simeq \begin{pmatrix} c_{\vartheta_S} & s_{\vartheta_S} & 0 \\ -s_{\vartheta_S}/\sqrt{2} & c_{\vartheta_S}/\sqrt{2} & 1/\sqrt{2} \\ s_{\vartheta_S}/\sqrt{2} & -c_{\vartheta_S}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

$$\text{Solar } \nu_e \to \nu_a^{(S)} \simeq \frac{1}{\sqrt{2}} (\nu_\mu - \nu_\tau)$$

$$\frac{\Phi_{CC}^{SNO}}{\Phi_{\nu_e}^{SSM}} \simeq \frac{1}{3} \Longrightarrow \Phi_{\nu_e} \simeq \Phi_{\nu_\mu} \simeq \Phi_{\nu_\tau} \text{ for } E \gtrsim 6\,\text{MeV}$$

$$\sin^2 \vartheta_S \simeq \frac{1}{3} \Longrightarrow U \simeq \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Tri-Bimaximal Mixing

[Harrison, Perkins, Scott, PLB 530 (2002) 167]

Absolute Scale of Neutrino Masses

- Brief Introduction to Neutrino Masses and Mixing
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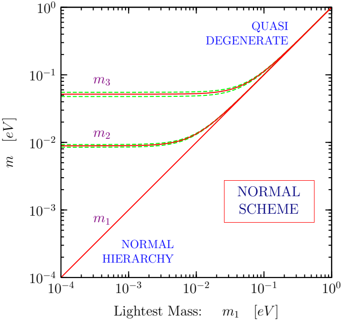
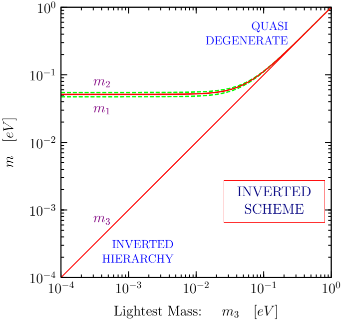
- Absolute Scale of Neutrino Masses
  - Mass Hierarchy or Degeneracy?
  - Tritium Beta-Decay
  - Cosmological Bound on Neutrino Masses
  - Neutrinoless Double-Beta Decay
- Conclusions

Mass Hierarchy or Degeneracy?

normal scheme



inverted scheme



$$m_2^2 = m_1^2 + \Delta m_{21}^2 = m_1^2 + \Delta m_{\text{SOL}}^2$$
$$m_3^2 = m_1^2 + \Delta m_{31}^2 = m_1^2 + \Delta m_{\text{ATM}}^2$$

$$m_1^2 = m_3^2 - \Delta m_{31}^2 = m_3^2 + \Delta m_{\text{ATM}}^2$$
$$m_2^2 = m_1^2 + \Delta m_{21}^2 \simeq m_3^2 + \Delta m_{\text{ATM}}^2$$

Quasi-Degenerate for $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \gg \sqrt{\Delta m_{\text{ATM}}^2} \simeq 5 \times 10^{-2}\,\text{eV}$

Tritium Beta-Decay

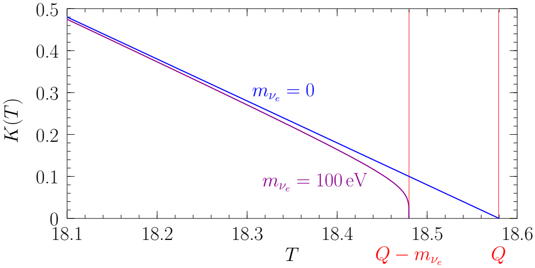
$^{3}\text{H} \to {}^{3}\text{He} + e^{-} + \bar{\nu}_e$

$$\frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE\, (Q-T) \sqrt{(Q-T)^2 - m_{\nu_e}^2}$$

$$Q = M_{^3\text{H}} - M_{^3\text{He}} - m_e = 18.58\,\text{keV}$$

Kurie plot

$$K(T) = \sqrt{\frac{d\Gamma/dT}{\frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE}} = \left[(Q-T)\sqrt{(Q-T)^2 - m_{\nu_e}^2}\right]^{1/2}$$



$m_{\nu_e} < 2.2\,\text{eV}$ (95% C.L.)

Mainz & Troitsk

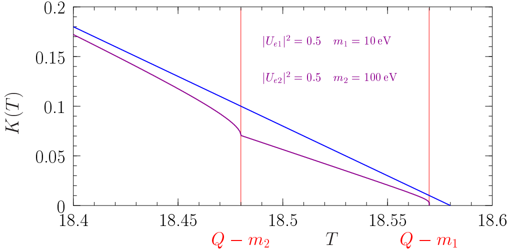
[Weinheimer, hep-ex/0210050]

future: KATRIN (start 2012)

[arXiv:0810.3281]

sensitivity: $m_{\nu_e} \simeq 0.2\,\text{eV}$ (3y)

Neutrino Mixing $\Longrightarrow K(T) = \left[(Q-T)\sum_k |U_{ek}|^2 \sqrt{(Q-T)^2 - m_k^2}\right]^{1/2}$



analysis of data is different from the no-mixing case: $2N-1$ parameters

$$\left(\sum_k |U_{ek}|^2 = 1\right)$$

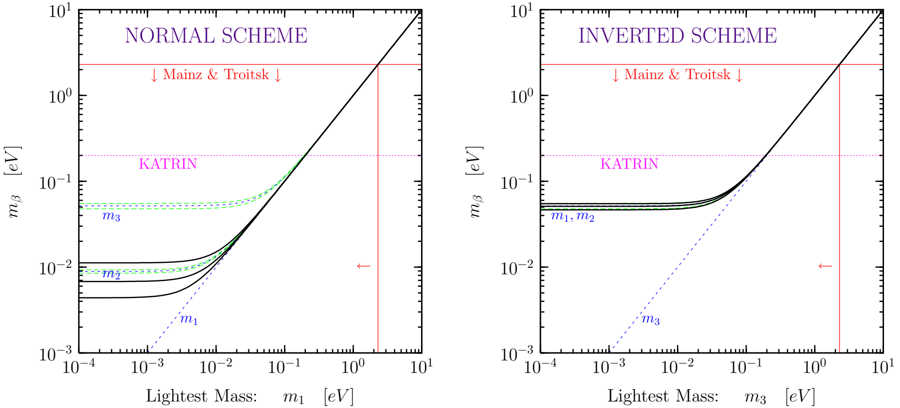
if experiment is not sensitive to masses ($m_k \ll Q - T$)

effective mass: $$m_\beta^2 = \sum_k |U_{ek}|^2 m_k^2$$

$$K^2 = (Q-T)^2 \sum_k |U_{ek}|^2 \sqrt{1 - \frac{m_k^2}{(Q-T)^2}} \simeq (Q-T)^2 \sum_k |U_{ek}|^2 \left[1 - \frac{1}{2}\frac{m_k^2}{(Q-T)^2}\right]$$

$$= (Q-T)^2 \left[1 - \frac{1}{2}\frac{m_\beta^2}{(Q-T)^2}\right] \simeq (Q-T)\sqrt{(Q-T)^2 - m_\beta^2}$$

$$m_\beta^2 = |U_{e1}|^2\, m_1^2 + |U_{e2}|^2\, m_2^2 + |U_{e3}|^2\, m_3^2$$



Quasi-Degenerate: $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \implies m_\beta^2 \simeq m_\nu^2 \sum_k |U_{ek}|^2 = m_\nu^2$

FUTURE: IF $m_\beta \lesssim 4 \times 10^{-2}\,\text{eV} \implies$ NORMAL HIERARCHY

Cosmological Bound on Neutrino Masses

neutrinos are in equilibrium in primeval plasma through weak interaction reactions

$\nu\bar{\nu} \leftrightarrows e^+e^- \quad \overset{(-)}{\nu} e \leftrightarrows \overset{(-)}{\nu} e \quad \overset{(-)}{\nu} N \leftrightarrows \overset{(-)}{\nu} N \quad \nu_e n \leftrightarrows p e^- \quad \bar{\nu}_e p \leftrightarrows n e^+ \quad n \leftrightarrows p e^- \bar{\nu}_e$

weak interactions freeze out

$$\Gamma_{\text{weak}} = N\sigma v \sim G_F^2 T^5 \sim T^2/M_P \sim \sqrt{G_N T^4} \sim \sqrt{G_N \rho} \sim H \Longrightarrow T_{\text{dec}} \sim 1\,\text{MeV}$$

neutrino decoupling

Relic Neutrinos: $T_\nu = \left(\frac{4}{11}\right)^{\frac{1}{3}} T_\gamma \simeq 1.945\,\text{K} \Longrightarrow k\,T_\nu \simeq 1.676 \times 10^{-4}\,\text{eV}$

$(T_\gamma = 2.725 \pm 0.001\,\text{K})$

number density: $n_f = \frac{3}{4}\frac{\zeta(3)}{\pi^2} g_f T_f^3 \Longrightarrow n_{\nu_k,\bar{\nu}_k} \simeq 0.1827\, T_\nu^3 \simeq 112\,\text{cm}^{-3}$

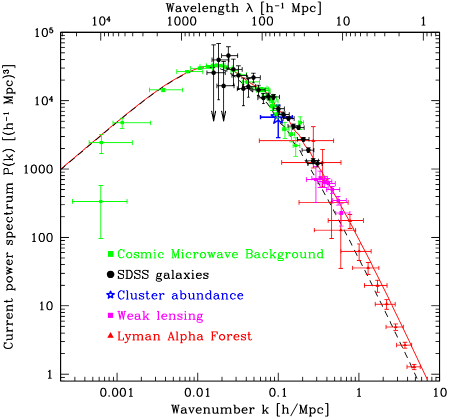
density contribution: $\Omega_k = \frac{n_{\nu_k,\bar{\nu}_k}\, m_k}{\rho_c} \simeq \frac{1}{h^2}\frac{m_k}{94.14\,\text{eV}} \Longrightarrow \boxed{\Omega_\nu h^2 = \frac{\sum_k m_k}{94.14\,\text{eV}}}$

$\left(\rho_c = \frac{3H^2}{8\pi G_N}\right)$

[Gershtein, Zeldovich, JETP Lett. 4 (1966) 120] [Cowsik, McClelland, PRL 29 (1972) 669]

$$h \sim 0.7, \quad \Omega_\nu \lesssim 0.3 \quad \Longrightarrow \quad \sum_k m_k \lesssim 14\,\text{eV}$$

Power Spectrum of Density Fluctuations



[Tegmark, hep-ph/0503257]

Solid Curve: flat ΛCDM model

$(\Omega_M^0 = 0.28\,,\ h = 0.72\,,\ \Omega_B^0/\Omega_M^0 = 0.16)$

Dashed Curve: $\sum_{k=1}^{3} m_k = 1\,\text{eV}$

hot dark matter
prevents early galaxy formation

$$\delta(\vec{x}) \equiv \frac{\rho(\vec{x}) - \overline{\rho}}{\overline{\rho}}$$

$$\langle \delta(\vec{x}_1)\delta(\vec{x}_2) \rangle = \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{x}}\, P(\vec{k})$$

small scale suppression

$$\frac{\Delta P(k)}{P(k)} \approx -8\,\frac{\Omega_\nu}{\Omega_m} \approx -0.8 \left(\frac{\sum_k m_k}{1\,\text{eV}}\right)\left(\frac{0.1}{\Omega_m h^2}\right)$$

for

$$k \gtrsim k_{nr} \approx 0.026 \sqrt{\frac{m_\nu}{1\,\text{eV}}}\sqrt{\Omega_m}\, h\,\text{Mpc}^{-1}$$

[Hu, Eisenstein, Tegmark, PRL 80 (1998) 5255]

WMAP (First Year), AJ SS 148 (2003) 175, astro-ph/0302209

CMB (WMAP, ...) + LSS (2dFGRS) + HST + SN-Ia $\Longrightarrow$ Flat ΛCDM

$T_0 = 13.7 \pm 0.2\,\text{Gyr}$ $\quad h = 0.71^{+0.04}_{-0.03}$

$\Omega_0 = 1.02 \pm 0.02$ $\quad \Omega_b = 0.044 \pm 0.004$ $\quad \Omega_m = 0.27 \pm 0.04$

$$\Omega_\nu h^2 < 0.0076 \quad \text{(95\% conf.)} \quad \Longrightarrow \quad \sum_{k=1}^{3} m_k < 0.71\,\text{eV}$$

WMAP (Five Years), astro-ph/0803.0547

CMB + HST + SN-Ia + BAO

$T_0 = 13.72 \pm 0.12\,\text{Gyr}$ $\quad h = 0.705 \pm 0.013$

$$-0.0179 < \Omega_0 - 1 < 0.0081 \quad \text{(95\% C.L.)}$$

$\Omega_b = 0.0456 \pm 0.0015$ $\quad \Omega_m = 0.274 \pm 0.013$

$$\sum_{k=1}^{3} m_k < 0.67\,\text{eV} \quad \text{(95\% C.L.)} \qquad N_{\text{eff}} = 4.4 \pm 1.5$$

Fogli, Lisi, Marrone, Melchiorri, Palazzo, Rotunno, Serra, Silk, Slosar

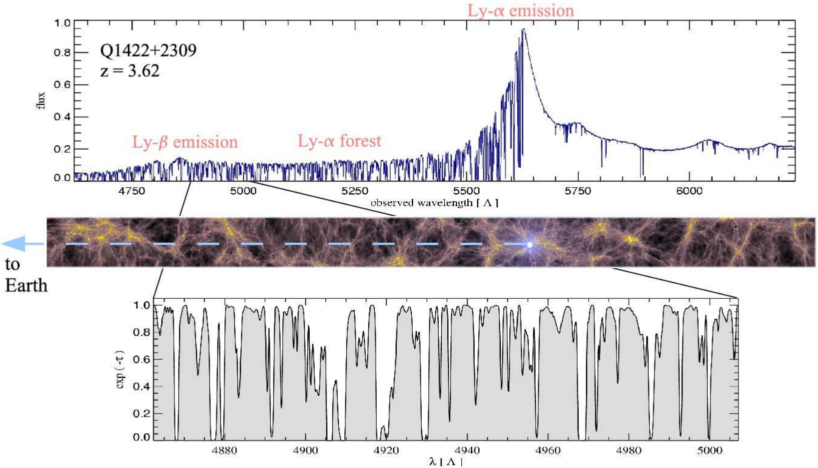
[PRD 78 (2008) 033010, hep-ph/0805.2517]

Flat ΛCDM

Case	Cosmological data set	Σ (at 2σ)
1	CMB	< 1.19 eV
2	CMB + LSS	< 0.71 eV
3	CMB + HST + SN-Ia	< 0.75 eV
4	CMB + HST + SN-Ia + BAO	< 0.60 eV
5	CMB + HST + SN-Ia + BAO + Lyα	< 0.19 eV

2σ (95% C.L.) constraints on the sum of ν masses Σ.

Lyman-alpha Forest



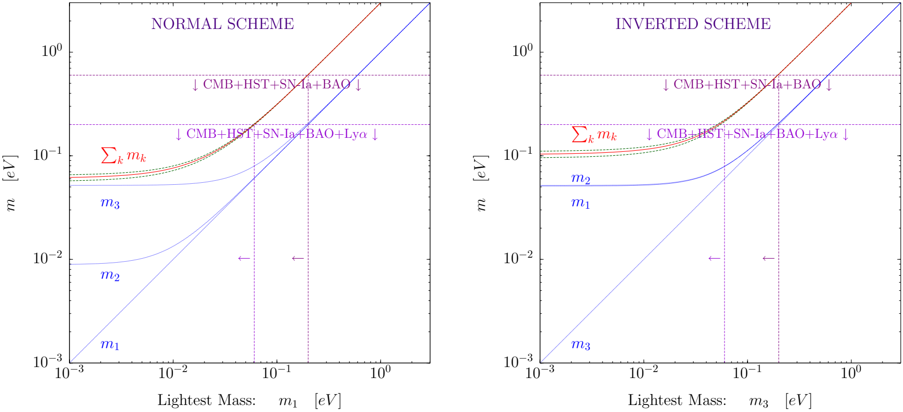
[Springel, Frenk, White, astro-ph/0604561]

Rest-frame Lyman α, β, γ wavelengths: $\lambda_\alpha^0 = 1215.67$ Å, $\lambda_\beta^0 = 1025.72$ Å, $\lambda_\gamma^0 = 972.54$ Å

Lyman-α forest: The region in which only Lyα photons can be absorbed: $[(1+z_q)\lambda_\beta^0, (1+z_q)\lambda_\alpha^0]$

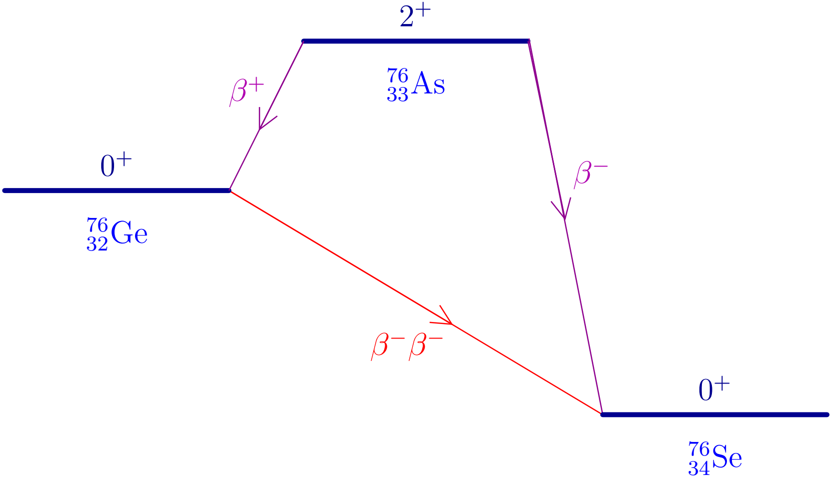
$$\sum_{k=1}^{3} m_k \lesssim 0.6\,\text{eV} \qquad (\sim 2\sigma) \qquad \text{CMB} + \text{HST} + \text{SN-Ia} + \text{BAO}$$

$$\sum_{k=1}^{3} m_k \lesssim 0.2\,\text{eV} \qquad (\sim 2\sigma) \qquad \text{CMB} + \text{HST} + \text{SN-Ia} + \text{BAO} + \text{Ly}\alpha$$



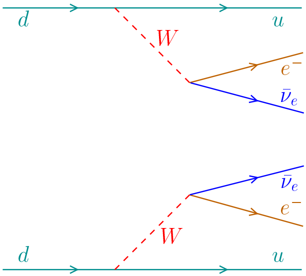
FUTURE: IF $\sum_{k=1}^{3} m_k \lesssim 9 \times 10^{-2}\,\text{eV} \Longrightarrow$ NORMAL HIERARCHY

Neutrinoless Double-Beta Decay



Effective Majorana Neutrino Mass: $m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$

Two-Neutrino Double-β Decay: $\Delta L = 0$

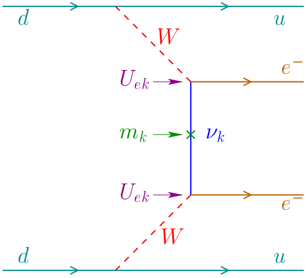


$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$$

$$(T_{1/2}^{2\nu})^{-1} = G_{2\nu} \, |\mathcal{M}_{2\nu}|^2$$

second order weak interaction process
in the Standard Model

Neutrinoless Double-β Decay: $\Delta L = 2$



$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^-$$

$$(T_{1/2}^{0\nu})^{-1} = G_{0\nu} \, |\mathcal{M}_{0\nu}|^2 \, |m_{\beta\beta}|^2$$

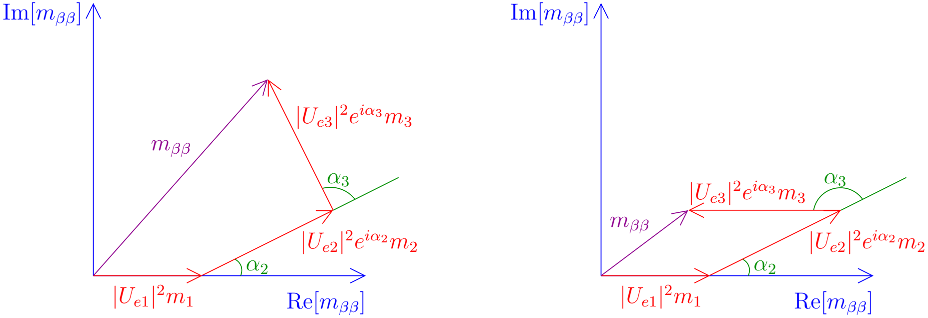
effective Majorana mass $\quad m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$

Effective Majorana Neutrino Mass

$$m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k \qquad \text{complex } U_{ek} \Rightarrow \text{possible cancellations}$$

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_2} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_3} \, m_3$$

$$\alpha_2 = 2\lambda_2 \qquad \alpha_3 = 2\,(\lambda_3 - \delta_{13})$$



Experimental Bounds

CUORICINO (^{130}Te) [PRC 78 (2008) 035502]

$T^{0\nu}_{1/2} > 3 \times 10^{24}\,\text{y} \quad (90\%\ \text{C.L.})$ $\Longrightarrow$ $|m_{\beta\beta}| \lesssim 0.19 - 0.68\,\text{eV}$

Heidelberg-Moscow (^{76}Ge) [EPJA 12 (2001) 147]

$T^{0\nu}_{1/2} > 1.9 \times 10^{25}\,\text{y} \quad (90\%\ \text{C.L.})$ $\Longrightarrow$ $|m_{\beta\beta}| \lesssim 0.32 - 1.0\,\text{eV}$

IGEX (^{76}Ge) [PRD 65 (2002) 092007]

$T^{0\nu}_{1/2} > 1.57 \times 10^{25}\,\text{y} \quad (90\%\ \text{C.L.})$ $\Longrightarrow$ $|m_{\beta\beta}| \lesssim 0.33 - 1.35\,\text{eV}$

NEMO 3 (^{100}Mo) [PRL 95 (2005) 182302]

$T^{0\nu}_{1/2} > 4.6 \times 10^{23}\,\text{y} \quad (90\%\ \text{C.L.})$ $\Longrightarrow$ $|m_{\beta\beta}| \lesssim 0.7 - 2.8\,\text{eV}$

FUTURE EXPERIMENTS

COBRA, XMASS, CAMEO, CANDLES

$|m_{\beta\beta}| \sim \text{few}\,10^{-1}\,\text{eV}$

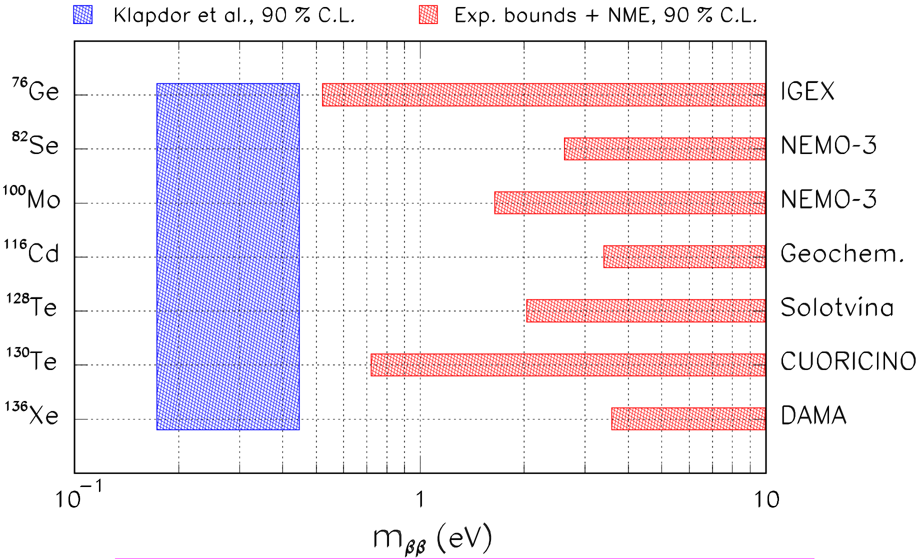
EXO, MOON, Super-NEMO, CUORE, Majorana, GEM, GERDA

$|m_{\beta\beta}| \sim \text{few}\,10^{-2}\,\text{eV}$

Nuclear Matrix Element

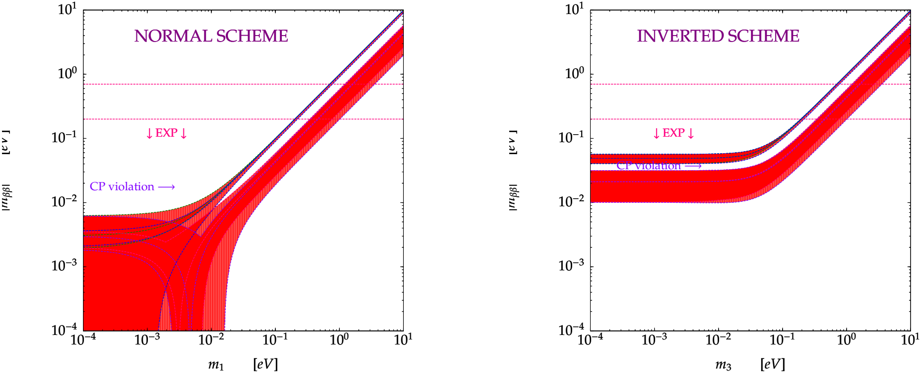
Faessler, Fogli, Lisi, Rodin, Rotunno, Simkovic

[hep-ph/0810.5733]



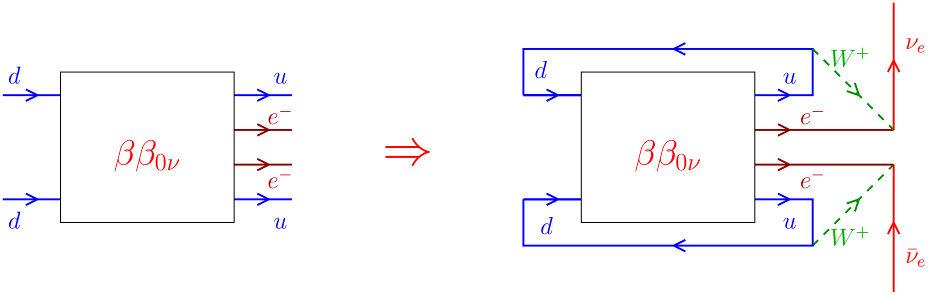
Bounds from Neutrino Oscillations

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_{21}} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_{31}} \, m_3$$



FUTURE: IF $|m_{\beta\beta}| \lesssim 10^{-2}\,\text{eV} \Longrightarrow$ NORMAL HIERARCHY

$\beta\beta_{0\nu}$ Decay $\Leftrightarrow$ Majorana Neutrino Mass



[Schechter, Valle, PRD 25 (1982) 2951] [Takasugi, PLB 149 (1984) 372]

Majorana Mass Term

$$\mathcal{L}_L^{\mathrm{M}} = -\frac{1}{2}\, m \left(\overline{\nu_L^c}\, \nu_L + \overline{\nu_L}\, \nu_L^c \right) = \frac{1}{2}\, m \left(\nu_L^T\, \mathcal{C}^\dagger\, \nu_L + \nu_L^\dagger\, \mathcal{C}\, \nu_L^* \right)$$

two conditions:
- u, d, e are massive
- standard left-handed weak interaction exists

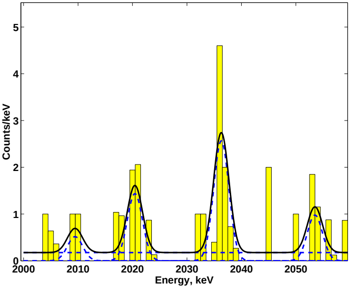
cancellation with other diagrams is very unlikely

(no symmetry, unstable under perturbative expansion)

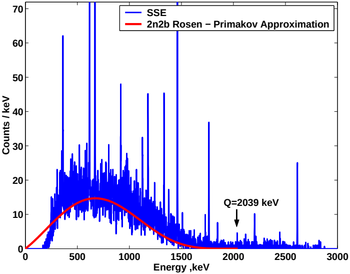
Experimental Positive Indication

[Klapdor et al., MPLA 16 (2001) 2409; FP 32 (2002) 1181; NIMA 522 (2004) 371; PLB 586 (2004) 198]

$T_{1/2}^{0\nu\,\mathrm{bf}} = 1.19 \times 10^{25}\,\mathrm{y}$ $\quad T_{1/2}^{0\nu} = (0.69 - 4.18) \times 10^{25}\,\mathrm{y}\,(3\sigma)$ $\quad 4.2\sigma$ evidence



pulse-shape selected spectrum



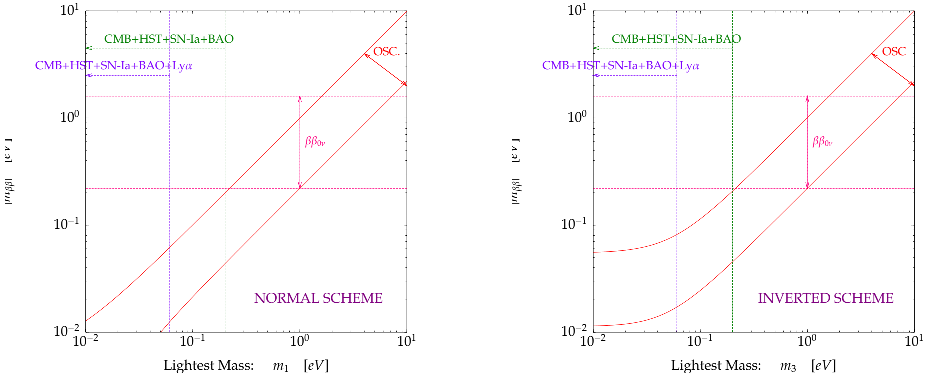
3.8σ evidence [PLB 586 (2004) 198]

the indication must be checked by other experiments

$$1.35 \lesssim |\mathcal{M}_{0\nu}| \lesssim 4.12 \quad \Longrightarrow \quad 0.22\,\mathrm{eV} \lesssim |m_{\beta\beta}| \lesssim 1.6\,\mathrm{eV}$$

if confirmed, very exciting (Majorana ν and large mass scale)

Indication of $\beta\beta_{0\nu}$ Decay: $0.22\,\text{eV} \lesssim |m_{\beta\beta}| \lesssim 1.6\,\text{eV}$ $(\sim 3\sigma$ range)



tension among

oscillation data – CMB+HST+SN-Ia+BAO(+Lyα) – $\beta\beta_{0\nu}$ signal

Conclusions

$\nu_e \to \nu_\mu, \nu_\tau$ with $\Delta m^2_{\text{SOL}} \simeq 8.3 \times 10^{-5}\,\text{eV}^2$ (solar ν, KamLAND)

$\nu_\mu \to \nu_\tau$ with $\Delta m^2_{\text{ATM}} \simeq 2.4 \times 10^{-3}\,\text{eV}^2$ (atm. ν, K2K, MINOS)

$\Downarrow$

Bilarge 3ν-Mixing with $|U_{e3}|^2 \ll 1$ (CHOOZ)

β Decay, Cosmology, $\beta\beta_{0\nu}$ Decay $\Longrightarrow m_\nu \lesssim 1\,\text{eV}$

FUTURE

Theory: Why lepton mixing $\neq$ quark mixing?
(Due to Majorana nature of ν's?)
Why only $|U_{e3}|^2 \ll 1$?
Continue improvement of $\mathcal{M}_{0\nu}$ uncertainties.

Exp.: Measure $|U_{e3}| > 0 \Rightarrow$ CP viol., matter effects, mass hierarchy
Check $\beta\beta_{0\nu}$ signal at Quasi-Degenerate mass scale
Improve β Decay, Cosmology, $\beta\beta_{0\nu}$ Decay measurements