Advances in Astronomical Image Coaddition and Subtraction

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“Requirements”

- What do we want when adding/subtracting images?
  - Maximum sensitivity for all astrophysical measurements.
  - Reliable detection of sources/transients.
  - No human involvement.
- What we do not care about?
  - Image quality metrics (image SNR, sharpness, resolution, seeing, …)

Stages in the pipeline

- Image calibration (flat fielding, bias and gain).
- Finding an astrometric solution.
- Image alignment (shifting, rotating, removing distortion and resampling).
- PSF, background and zero-point estimation.
- Identifying bad pixels and particle hits.
- Image coaddition/subtraction
- Source/transient detection
- …

Notation and image model

- Image model:

$$M_j = P_j \otimes T + \epsilon_j$$

- M_j - j'th measurement
- T - True sky.
- P_j - j'th PSF
- ϵ_j - additive, white Gaussian noise.

Coaddition - commonly used methods

Weighted addition methods:
(Annis et al., 2014 Jiang et al., 2014)

$$S = \sum_j \alpha_j M_j$$

PSF homogenization
(Desai et al.,)

$$S = \sum_j \alpha_j K_j \otimes M_j$$

$$M_j \otimes K_j \approx M_{ref}$$

Speckle imaging methods:
Lucky imaging
Speckle interferometry

Problems with existing coaddition methods

- No argumentation or reasoning.
- Reduced sensitivity (5%-25% decrease in survey speed!).
- “No coaddition method is good for all applications”.
    - Trade-off between resolution and depth.
- Images with “bad” atmospheric conditions are discarded.
- Some coaddition methods involve regularized deconvolution (“PSF homogenization”)
    - Unstable, introduces spatial correlations and slow.
    - Unclear what further signal processing steps should follow.
    - Even less sensitive than weighted addition!

Our approach

- Go by the book:
  - Define the simplest statistical task.
  - Find it’s optimal statistic using Neyman-Pearson.
  - Extend the solution to all tasks (if possible).
  - Analyze the solution’s behavior when adding realistic complexities to the statistical model.
  - Apply corrections where needed.

Optimal source detection

- Statistical task - detecting point sources.

$$\mathcal{H}_0 : \; M_j = \epsilon_j$$

$$\mathcal{H}_1 : \; M_j = \delta_p \otimes P_j + \epsilon_j$$

$$S = \frac{\mathcal{P}(\{M\}|\mathcal{H}_1)}{\mathcal{P}(\{M\}|\mathcal{H}_0)} = \cdots = \sum_j \frac{\overleftarrow{P_j} \otimes M_j}{\sigma_j^2}$$

- S is the analogue of a match filtered image
  - Has correlated noise!
  - Does not fit our image model.

Sufficient statistic

- Statistical task - detecting point sources.

$$\mathcal{H}_0 : \; M_j = \epsilon_j$$

$$\mathcal{H}_1 \; : \; M_j = T(\theta) \otimes P_j + \epsilon_j$$

$$S = \frac{\mathcal{P}(\{M\}|\mathcal{H}_1)}{\mathcal{P}(\{M\}|\mathcal{H}_0)} = \cdots = \overleftarrow{T(\theta)} \otimes \sum_j \frac{\overleftarrow{P_j} \otimes M}{\sigma_j^2}$$

- Same trick works for **any** measurement.

- S is still not simple to use.

- Does not fit our image model.

Proper coaddition

- In fact, any two simple hypotheses about T could be tested using S.

- If S is a matched filtered image, can we find it's "original" image?

$$\hat{R} = \frac{\sum_j \frac{\overline{\hat{P}_j} \hat{M}_j}{\sigma_j^2}}{\sqrt{\sum_j \frac{|\hat{P}_j|^2}{\sigma_j^2}}} \qquad \hat{P}_r = \sqrt{\sum_j \frac{|\hat{P}_j|^2}{\sigma_j^2}}$$

*Kaiser

Properties of the new coadd image

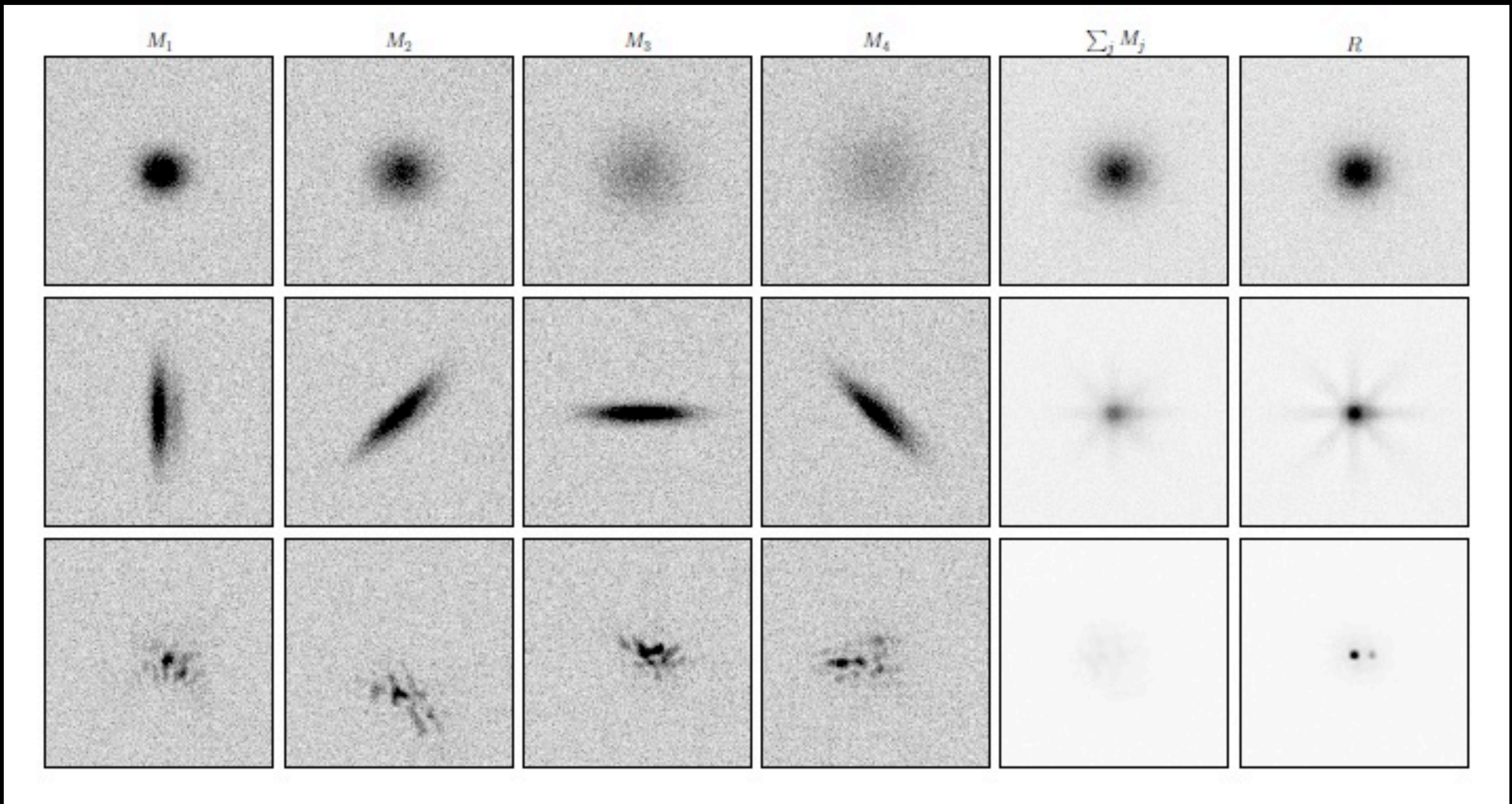
- Optimal for **all** decisions and measurements.
  - Assumes known PSFs and white Gaussian noise.
  - Sufficient statistic - Original data is redundant.
- 5%-25% more survey speed relative to weighted summation.
  - Even better relative to PSF homogenization.
- Numerically stable.
- Local - Can handle spatially changing PSF's.
- Indistinguishable from a regular image.

Results

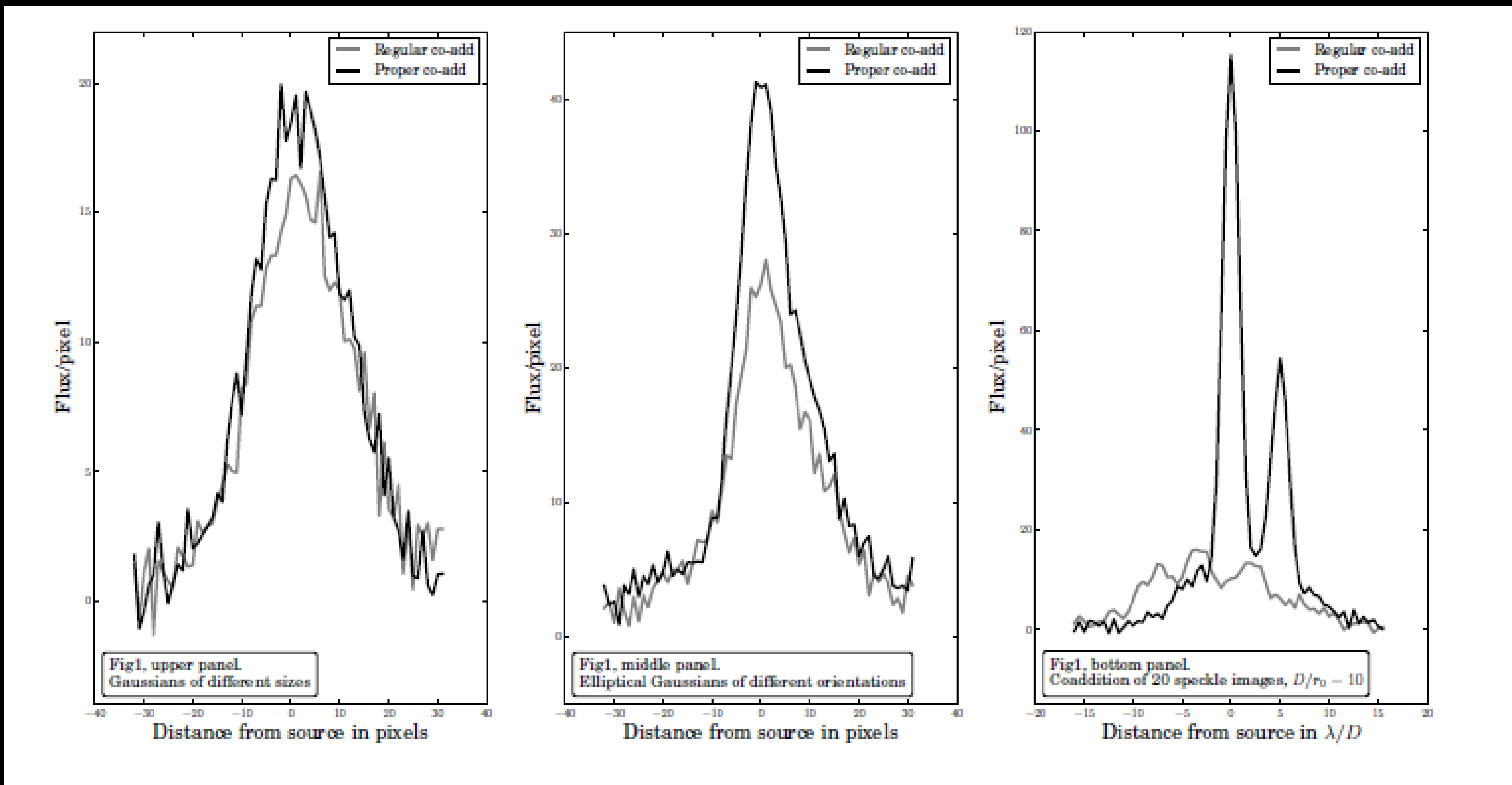
Simulated images of a binary star

Direct coadd

Proper coadd



Results


Regular co-add
Proper co-add
Flux/pixel
Distance from source in pixels
Fig1, upper panel.
Gaussians of different sizes
Regular co-add
Proper co-add
Flux/pixel
Distance from source in pixels
Fig1, middle panel.
Elliptical Gaussians of different orientations
Regular co-add
Proper co-add
Flux/pixel
Distance from source in λ/D
Fig1, bottom panel.
Coaddition of 20 speckle images, $D/r_0 = 10$

Future prospects - coaddition

- One coadd (per color) to summarize a survey
  - Makes all sky surveys compact and distributable for everyone.
  - provides 5%-25% more survey speed.
- Coaddition of speckle images.
- Proper super resolution (coaddition of under-sampled images).
- Application to high contrast imaging

Advancements in Image Subtraction

Subtraction - Notation

- Image model:

$$R = P_r \otimes T + \epsilon_r$$

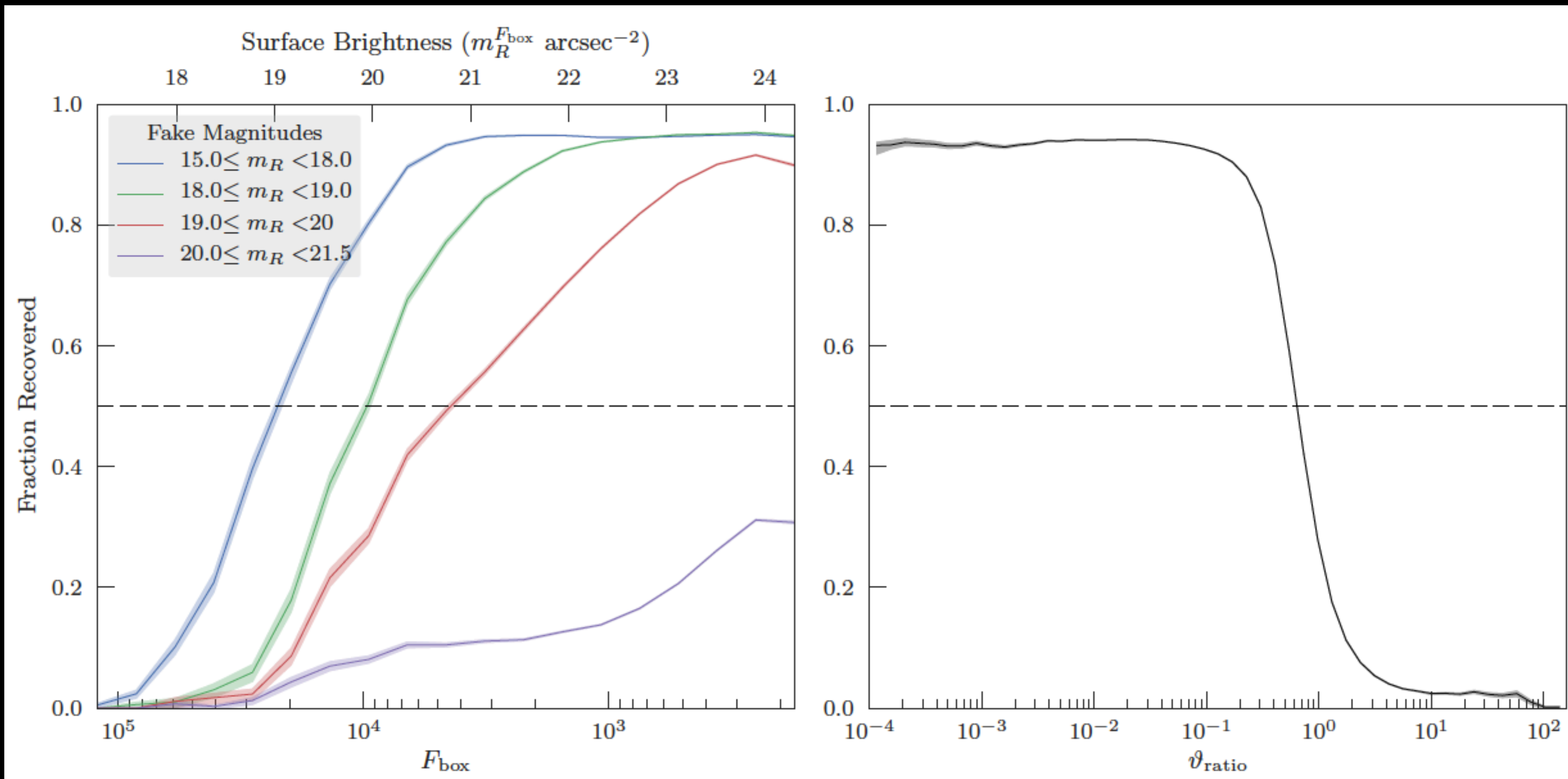
$$N = P_n \otimes T + \epsilon_n$$

- R - Reference image
- N - New image
- Noise is white and Gaussian
- No assumption on T.

Problems with current image subtraction algorithms

- No argumentation or reasoning.
    - Reduced sensitivity
    - Unclear what further signal processing should be applied
- False positives.
    - Machine learning sifting of millions of candidates per day
    - Human scanning for final sifting stage.
        - No automatic followup + inevitable 1 hour latency.
- Numerically unstable.
- Slow (may be a serious constraint for large surveys).

An unconditional surrender



Frohmaier et al. (2017)

REAL-TIME RECOVERY EFFICIENCIES AND PERFORMANCE OF THE PALOMAR TRANSIENT FACTORY'S TRANSIENT DISCOVERY PIPELINE

Ratio between:
Flux in a PSF box and
Flux of the transient

Existing methods for image subtraction

- Phillips & Davis (95) $$\widehat{D_{Phillips}} = \hat{N} - \frac{\hat{P}_n}{\hat{P}_r}\hat{R}$$
- Allard & Lupton (98) $$D_{AL} = N - K \otimes R$$
- Bramich (2000)
- Gal-Yam et al. (06). $$D_{GY} = P_r \otimes N - P_n \otimes R$$

Optimal transient detection

- Stating the hypotheses:

$$\mathcal{H}_0 : \; N = T \otimes P_n + \epsilon_n$$

$$\mathcal{H}_1 : \; N = (T + \delta(q)) \otimes P_n \epsilon_n$$

- Applying Neyman-Pearson:

$$S = \frac{\mathcal{P}(R, N|\mathcal{H}_1)}{\mathcal{P}(R, N|\mathcal{H}_0)} = \frac{\mathcal{P}(N|R, \mathcal{H}_1)}{\mathcal{P}(N|R, \mathcal{H})_0} \cancel{\frac{\mathcal{P}(R|\mathcal{H}_1)}{\mathcal{P}(R|\mathcal{H}_0)}} = \frac{\mathcal{P}(N|R, \mathcal{H}_1)}{\mathcal{P}(N|R, \mathcal{H})_0}$$

$$\hat{S} = \frac{|\hat{P}_r|^2 \overline{\hat{P}_n} \hat{N} - |\hat{P}_n|^2 \overline{\hat{P}_r} \hat{R}}{\sigma_n^2 |\hat{P}_r|^2 + \sigma_r^2 |\hat{P}_n|^2}$$

Proper image subtraction

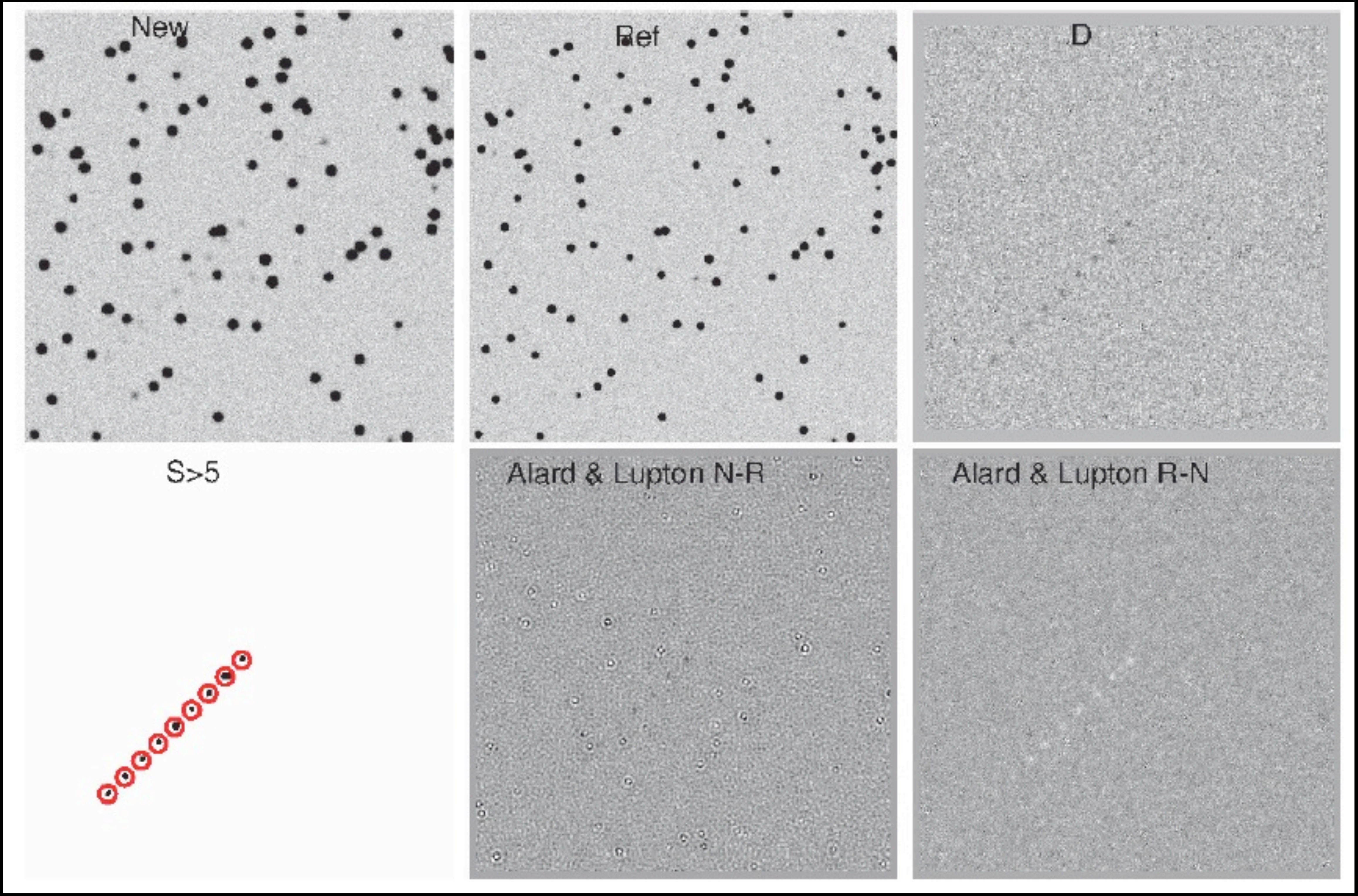
- What if we want to identify all types of transients?
  - Including defects and cosmic rays

$$\hat{D} = \frac{\hat{P}_r \hat{N} - \hat{P}_n \hat{R}}{\sqrt{\sigma_n^2 |\hat{P}_r|^2 + \sigma_r^2 |\hat{P}_n|^2}} \qquad \hat{P_D} = \frac{\hat{P}_r \hat{P}_n}{\sqrt{\sigma_n^2 |\hat{P}_r|^2 + \sigma_r^2 |\hat{P}_n|^2}}$$

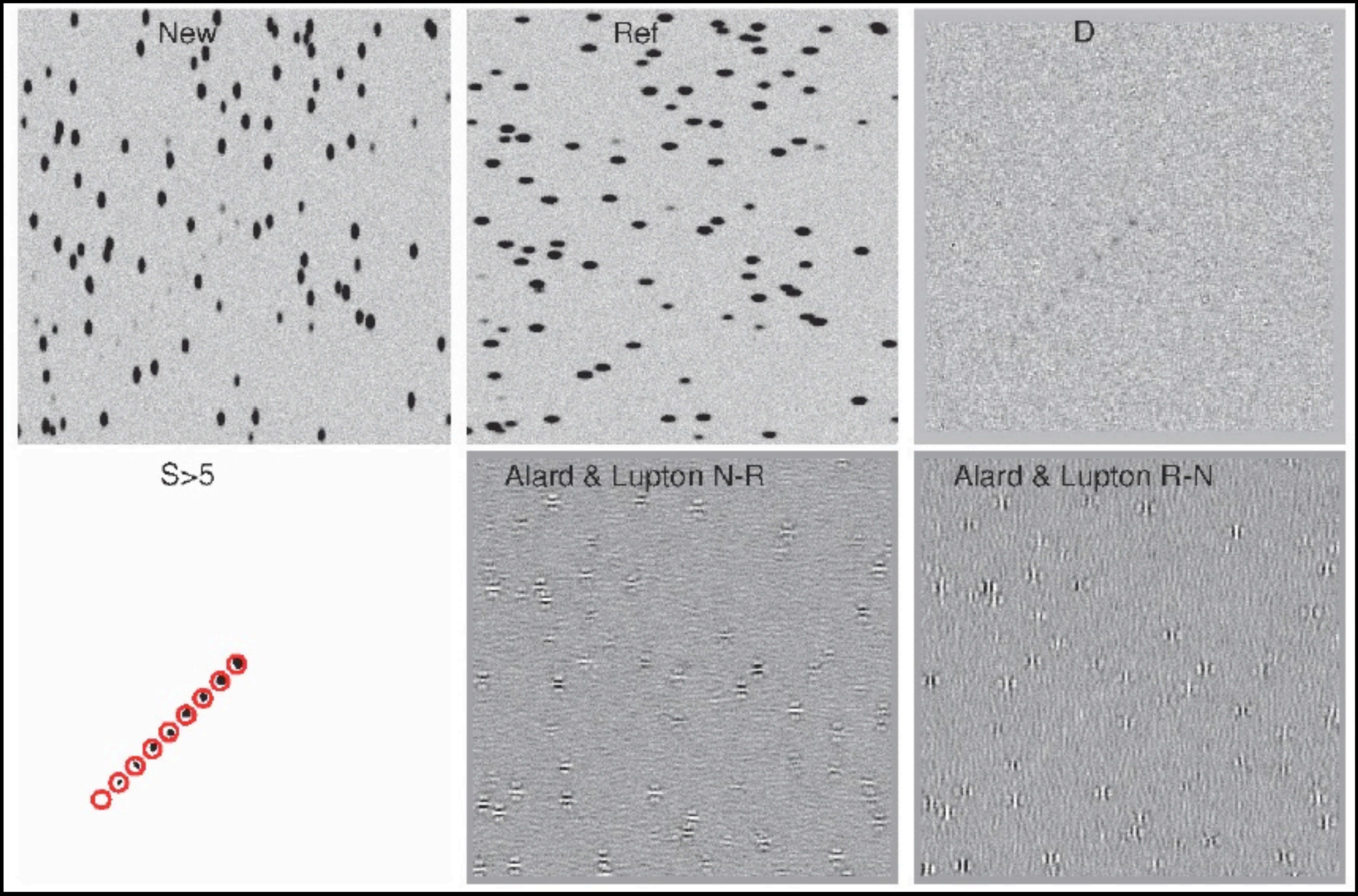
Properties of proper image subtraction

- Optimally sensitive (with a **rigorous mathematical proof**)
- Convolution kernels are **local (no problem with spatially varying PSFs).**
- **5%-50% more sensitive** than past methods**.**
- **Reliable significance and error bars** (this in itself increases sensitivity).
- Closed form (and symmetric to N,R interchange).
    - can correct for source noise and astrometric residuals)
- Fast.
- Sufficient for testing/measuring **any difference** between the images.
    - Can optimally measure all astrometric shifts of point sources.
- Numerically stable

Simulations - 1



Simulations - 2



Correcting for source noise and astrometric noise

- Can separate the transient detection score to the “New” part and “reference” part

$$S_n = N \otimes k_n \,,\; S_r = R \otimes k_r$$

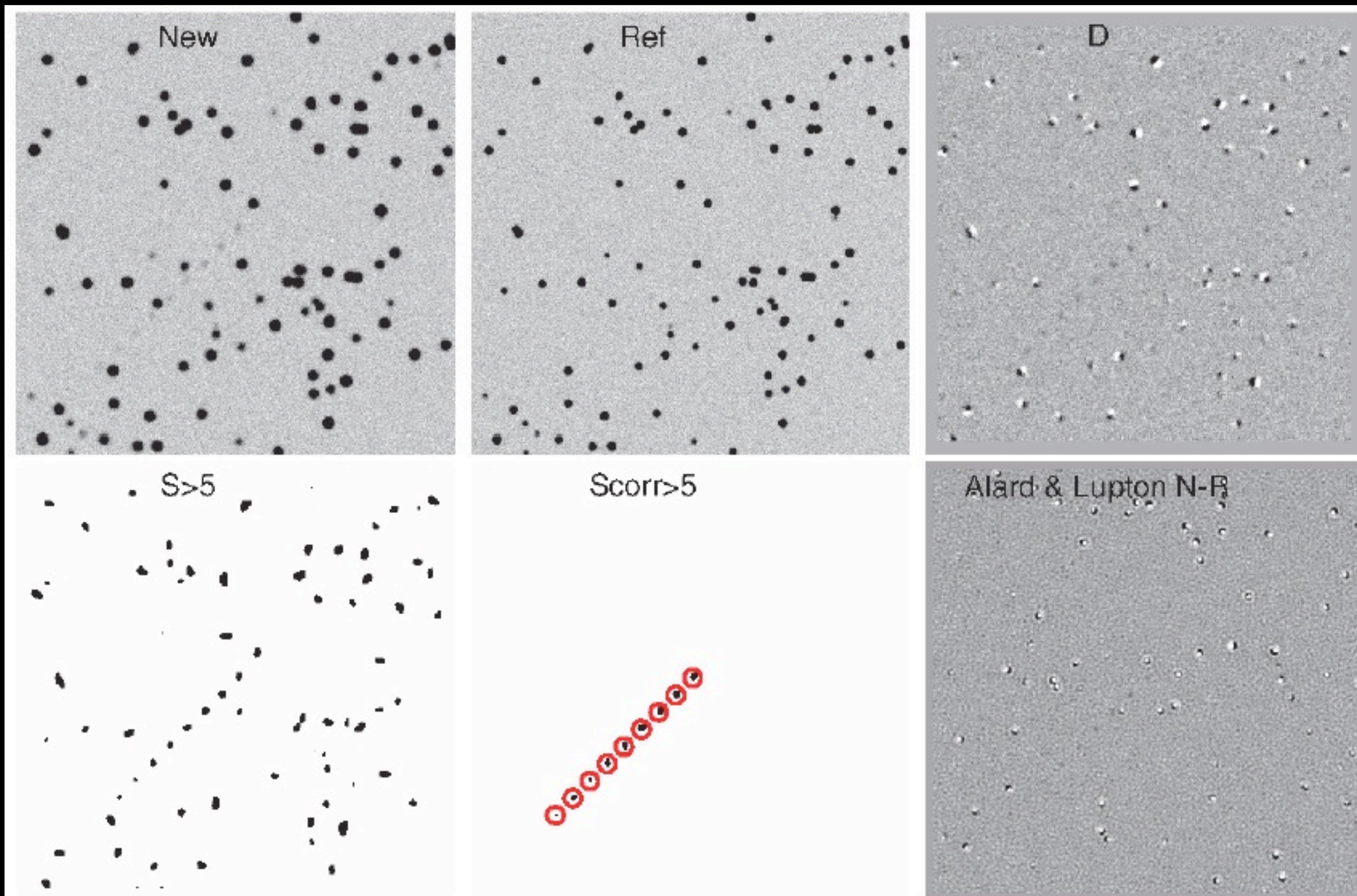
- Can bound the influence of source noise using the point-wise variance maps of N and R.

$$V_{S_n} = V(N) \otimes k_n^2 \,,\, V_{S_r} = V(R) \otimes k_r$$

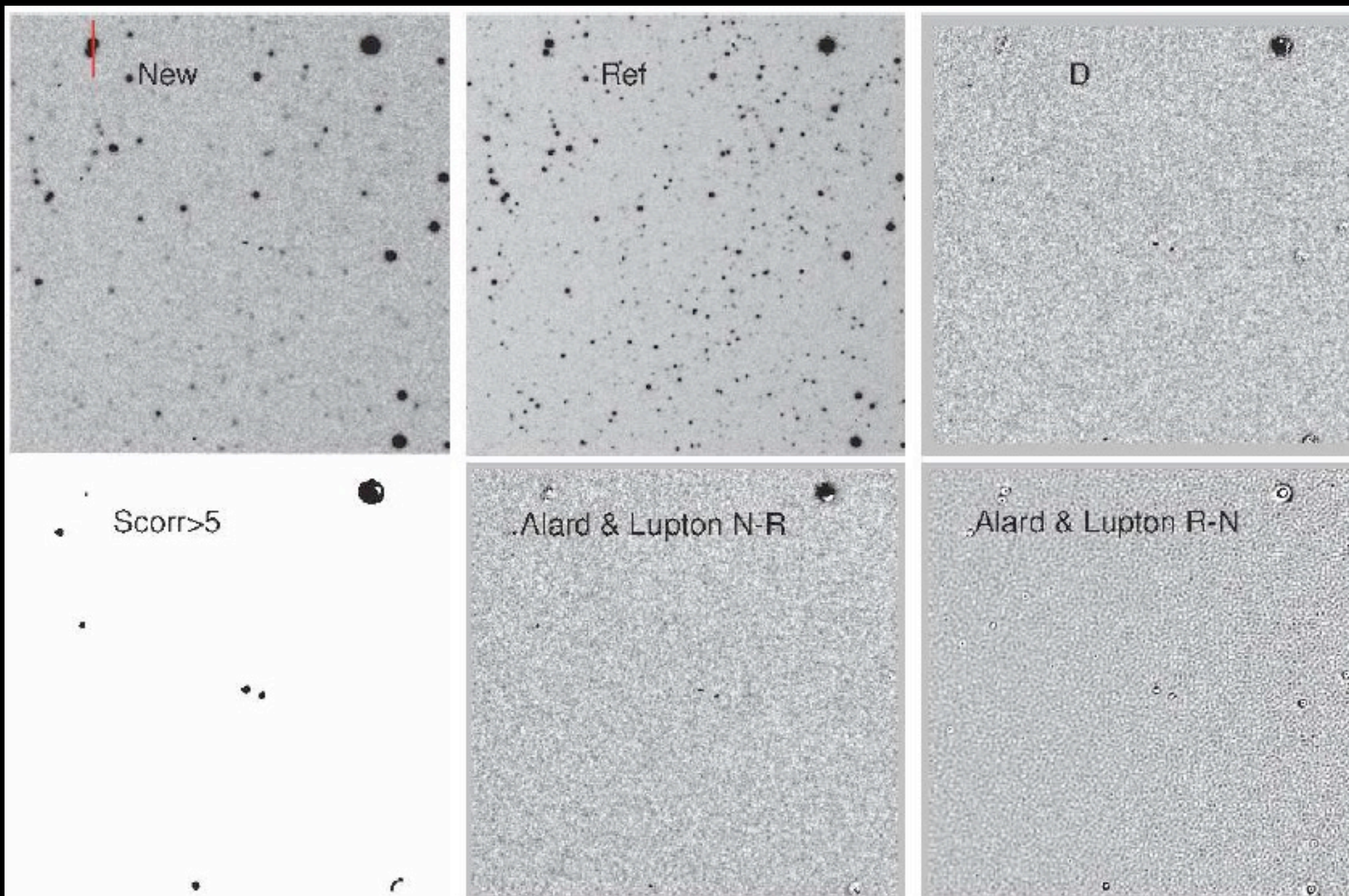
- Can bound the influence of astrometric noise using their pixel derivatives

$$V_a = \frac{\partial S_n}{\partial x}\sigma_{a_x}^2 + \frac{\partial S_n}{\partial y}\sigma_{a_y}^2$$

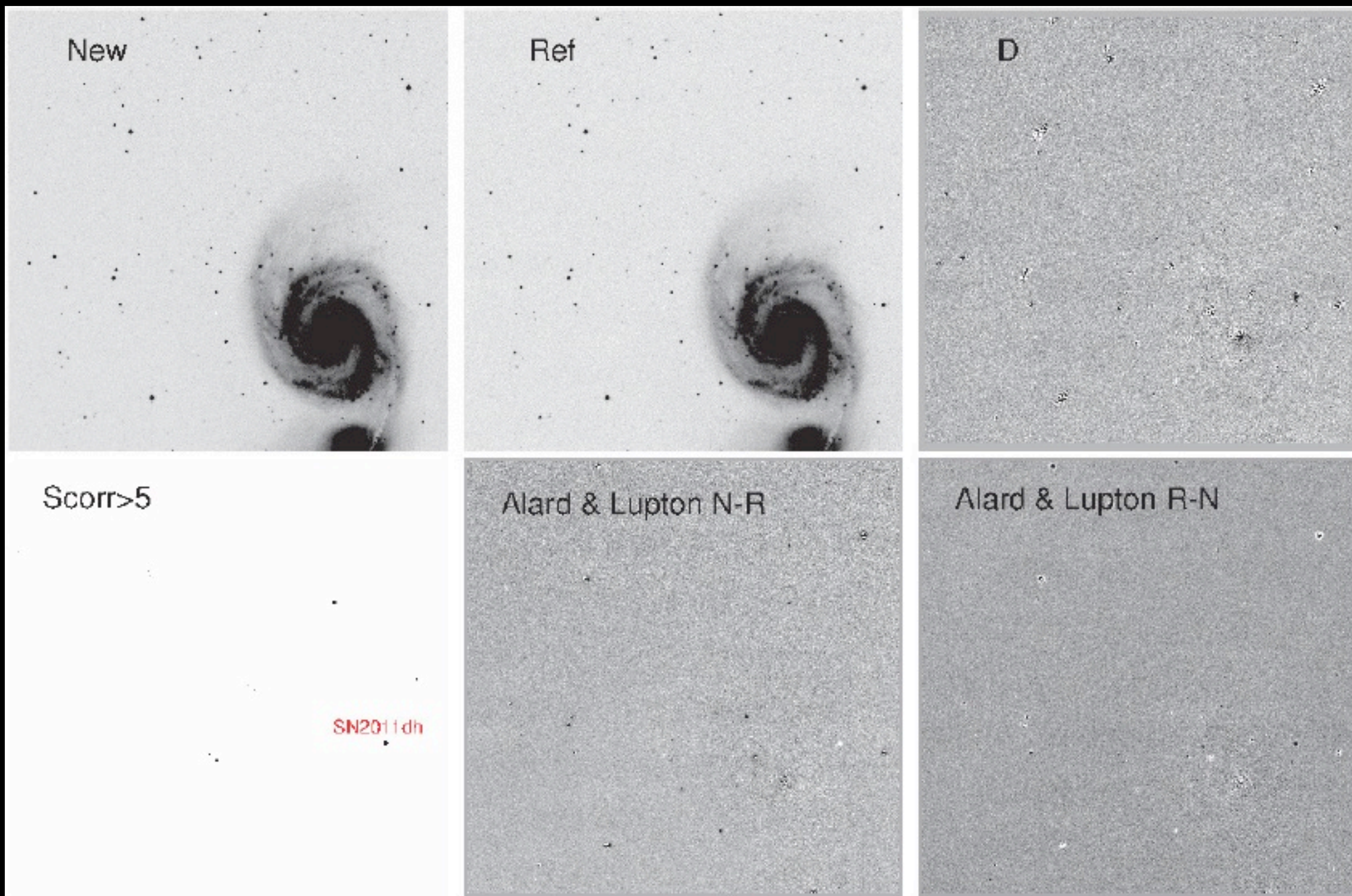
Correcting astrometric noise



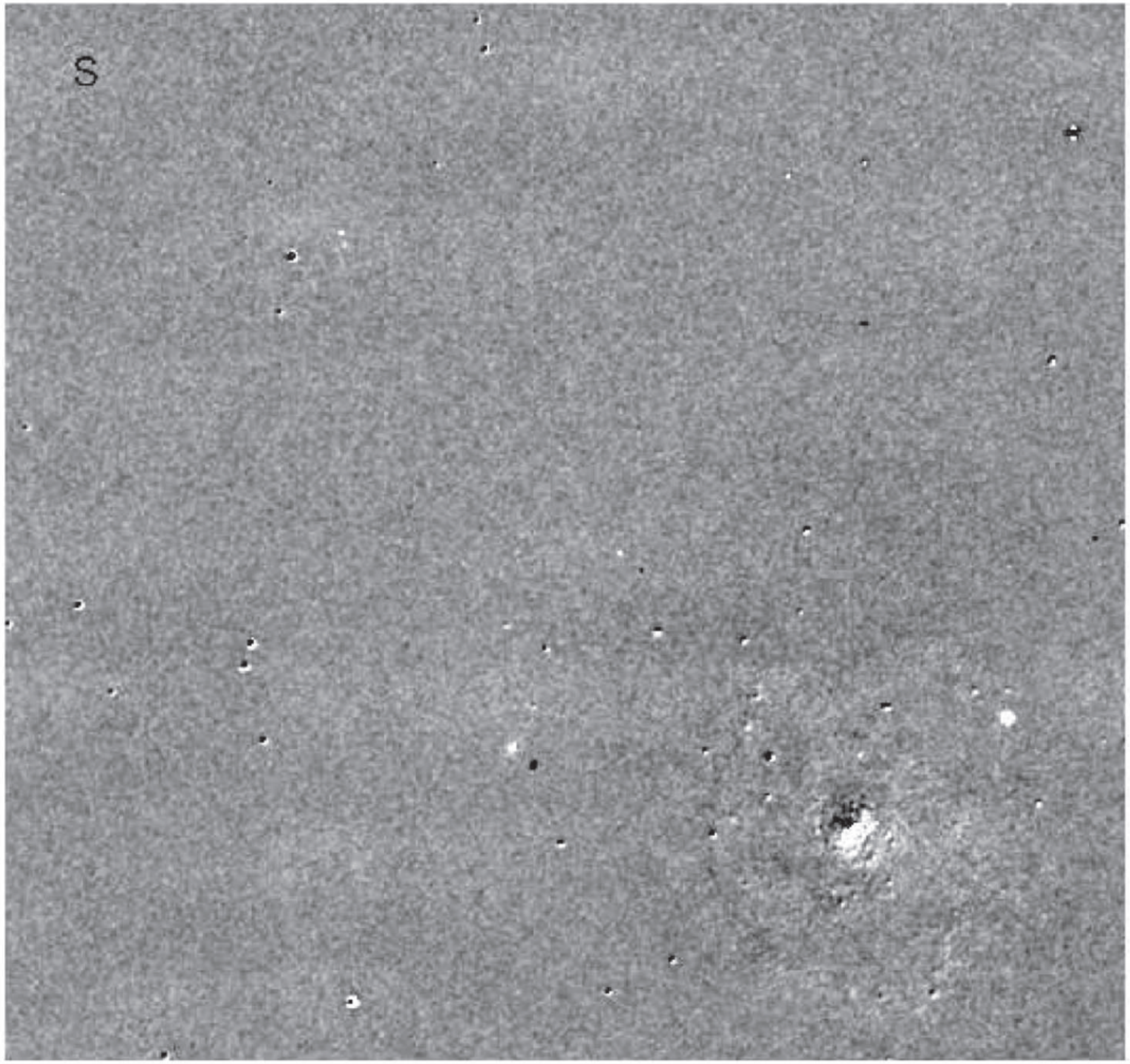
Real data - I

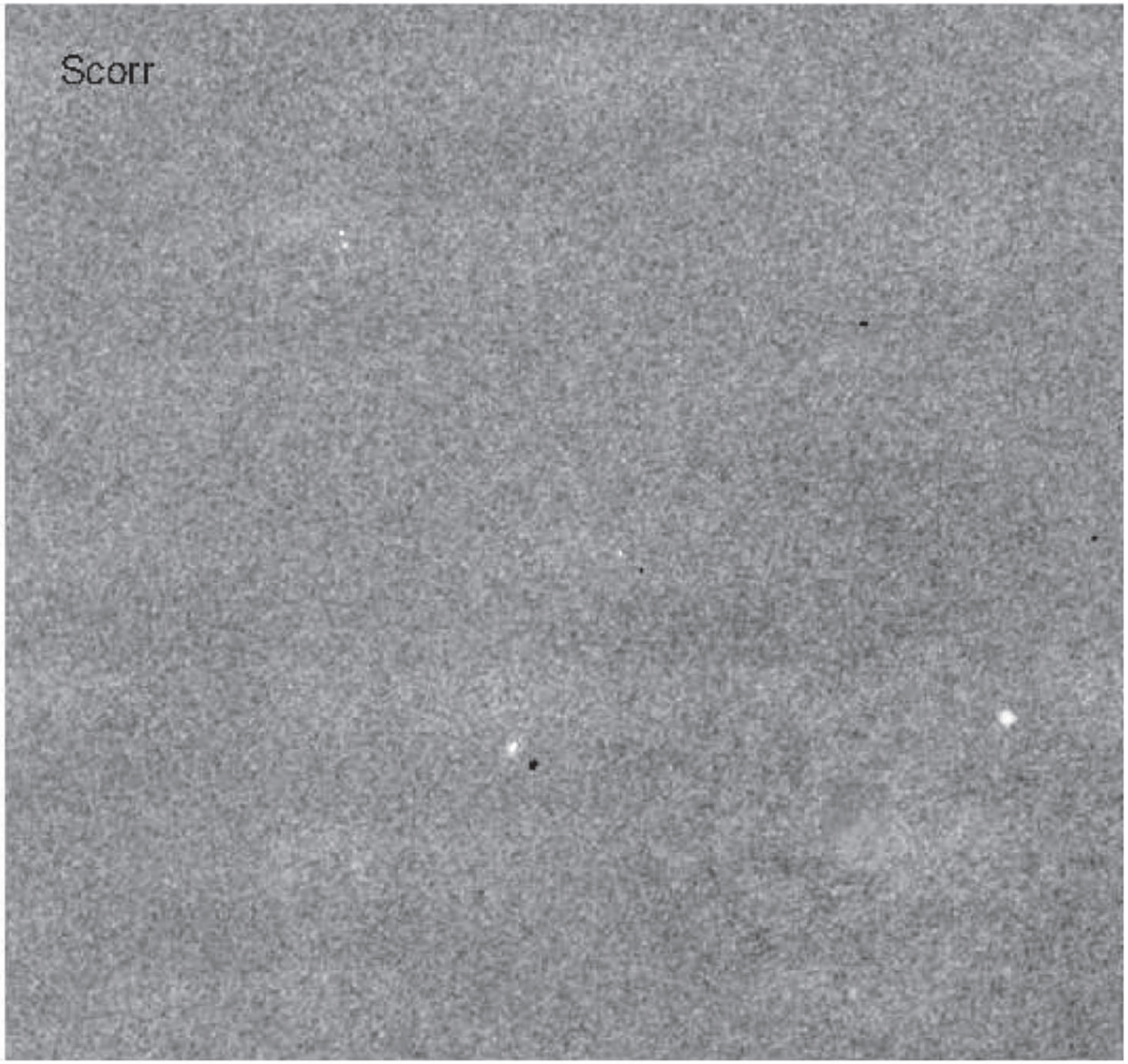


Real data - II



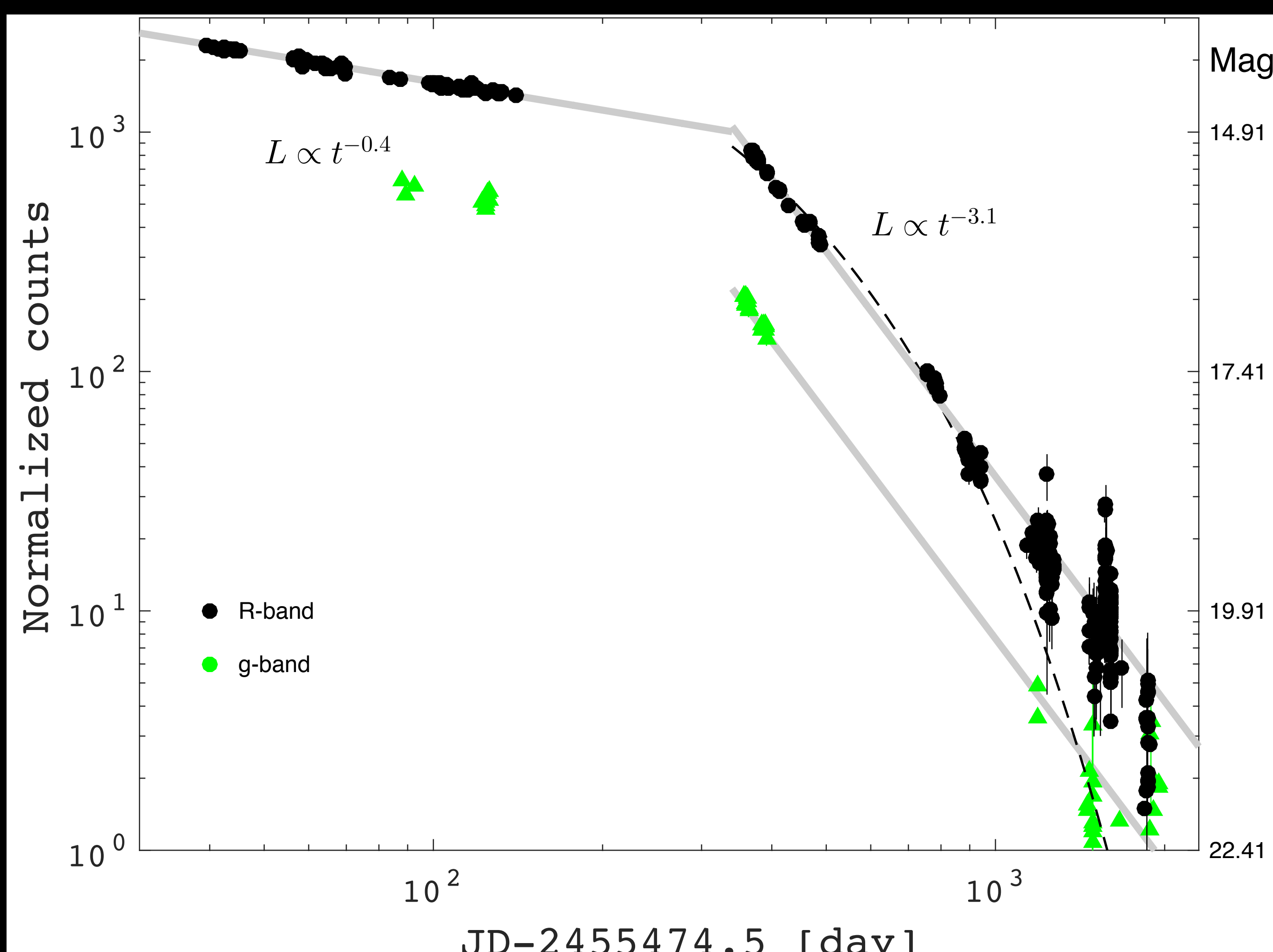
Real data - II continued



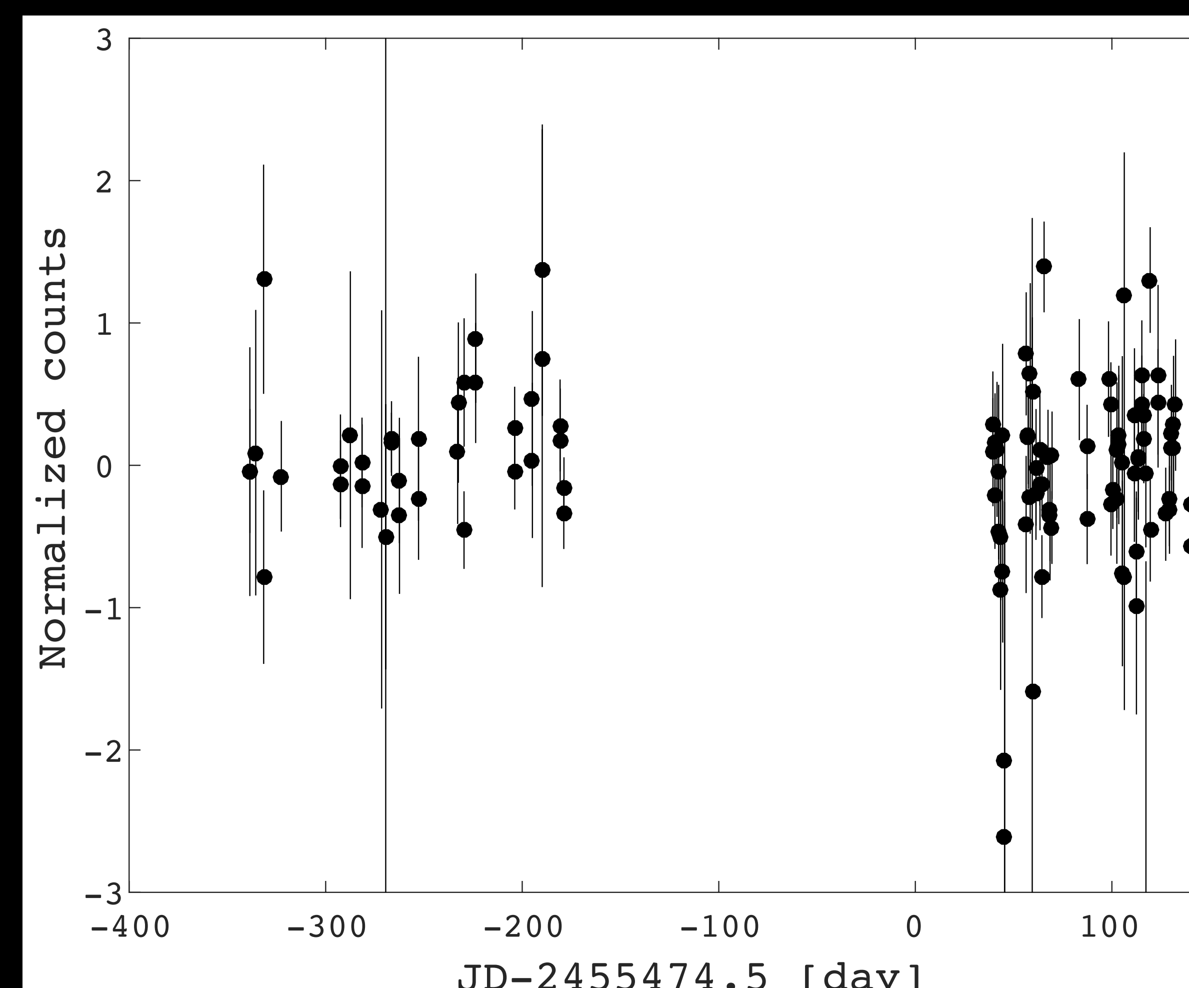


Real data - III

Subtracting a (very) bright galaxy



SN 2010jl light curve



Some random point

Future prospects - subtraction

- Robust, automatic transient detection and followup
  - Will be used by ZTF (Thanks to Brad Cenko, Eran Ofek and Frank Masci).
  - Will be used by BlackGem (Thanks to Paul Vreeswijk).
  - Hopefully, will be used by LSST (being implemented by David Reiss).
- Increased survey speed
- Detecting high dynamic range transients
  - How many faint sources we missed so far?
  - TDEs?
  - Collapsars?
- Differential astrometry (solving for relative astrometry using image subtraction)

Questions?