Point source photometry and astrometry

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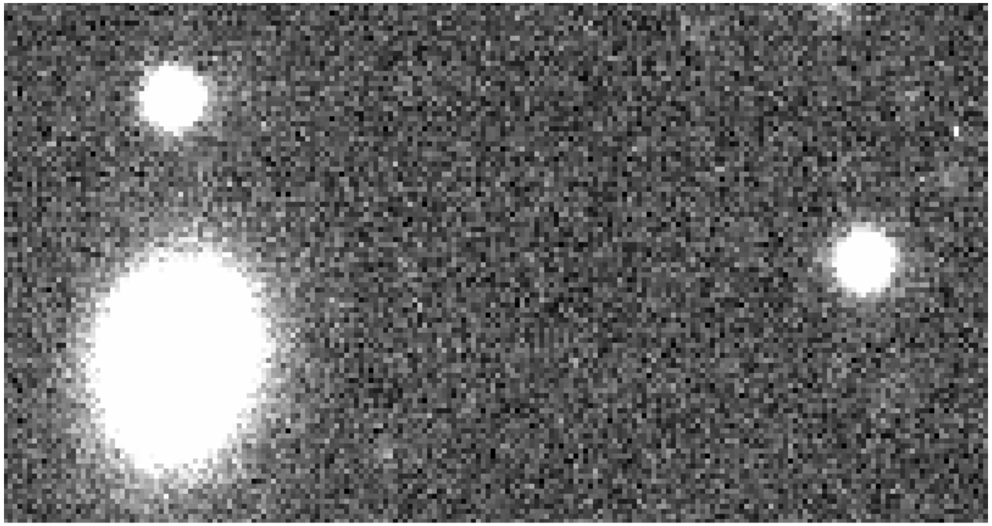
Photometry

- Photometry means measuring the light flux of astronomical objects.
- In practice, we measure ratios of fluxes.
- Sensors:
    - Eye (possibly through a telescope)
    - Photographic emulsions
    - Photoelectric tubes (typically photo-multipliers)
    - Semiconductor devices, in particular CCDs

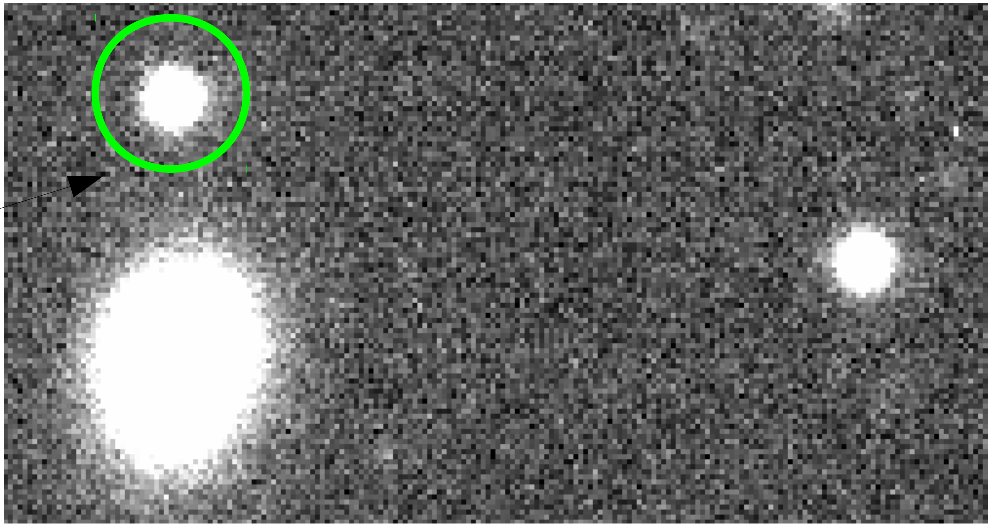
Photometry from pixelated images



- Preprocessing :
  - Make the response of pixels uniform: same signal means same amount of light
  - Estimate a sky background level across the image (ill-posed and difficult problem).

Photometry from pixelated images

$$f = \sum_{\text{pixels}} (\underbrace{I_p}_{\text{Pixel value}} - \underbrace{s}_{\text{Sky estimate}})$$



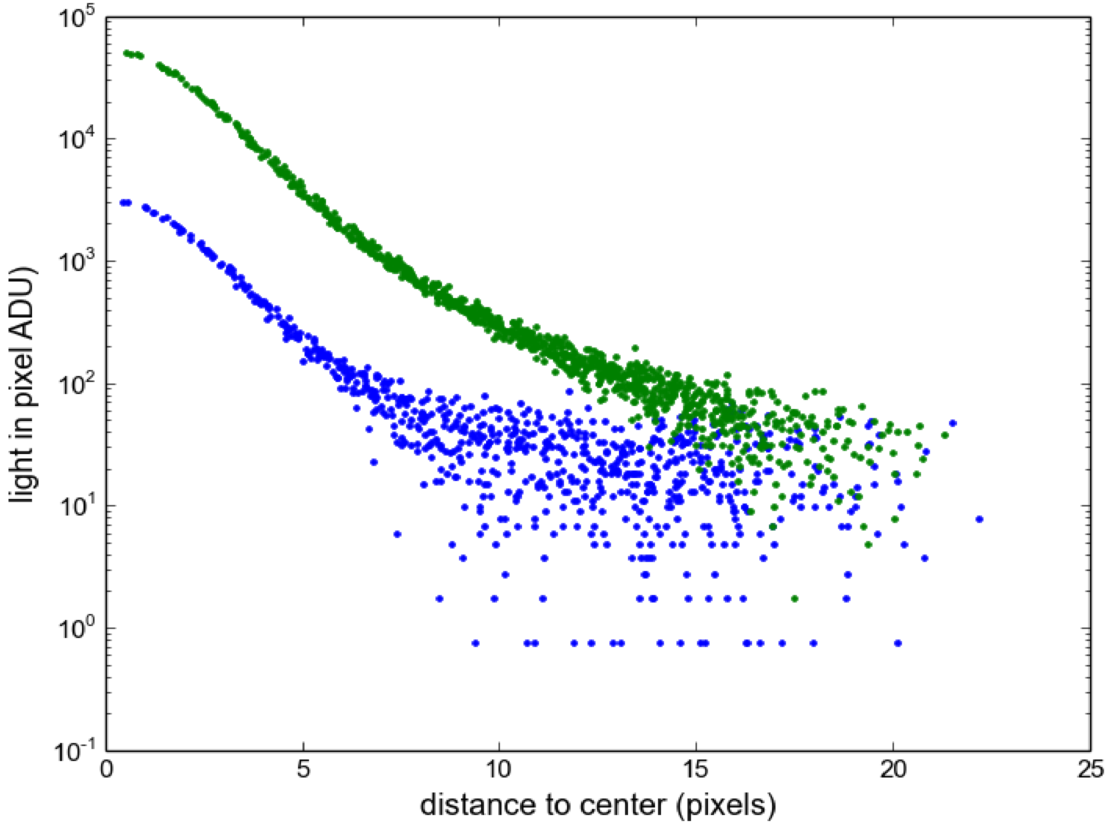
"Aperture photometry"

- The sum runs over pixels within an aperture.
- The larger the aperture:
  - The higher the fraction of the captured object flux
  - The larger the sky noise

Photometry of point sources

- A point source is a source which is (much) smaller than the angular resolution of the imaging system. Such a source is said to be “unresolved”
- Point sources:
  - Stars (Milky Way and nearby galaxies)
  - Supernovae (may be distant)
  - Quasars

Point sources have similar light profiles (in the same image)



Light profile around the position of two stars in the same image

- The 2-d light profile of point sources is called Point Spread Function
- It is attached to an image (for ground-based imaging).

Photometry of point sources

- Assume we know the PSF ψ . Then, what about measuring the flux of a point source via least-squares:

$$\chi^2 = \sum_{\text{pixels}} (f \times \psi(x_p - x_0, y_p - y_0) - I_p - s)^2 \, w_p$$

flux coordinates Image value sky pixel weight

- If the weights w_p are the inverse of the variance of I_p, then the least-squares flux estimator has the smallest possible variance (Cramer-Rao inequality)

Photometry of point sources

- Flux estimator :

$$\hat{f} = \frac{\sum_{\text{pixels}} \psi_p (I_p - s) w_p}{\sum_{\text{pixels}} \psi_p^2 w_p}$$

- Linear with respect to the (sky-subtracted) image ?
  - It depends … no if position is fitted as well.
- What is w_p ?
  - By definition, $Var(I_p)$. When dominated by shot noise, I_p is a proxy for $Var(I_p)$
  - We could consider $w_p = (f\psi_p + s)^{-1}$ (gain=1)
  - Or $w_p = (s)^{-1}$

Variance

$$\hat{f} = \frac{\sum_{\text{pixels}} \psi_p (I_p - s) w_p}{\sum_{\text{pixels}} \psi_p^2 w_p}$$

If estimating only flux
(and possibly position)

$$Var[\hat{f}] = \frac{\sum_{\text{pixels}} \psi_p^2 Var[I_p] w_p^2}{\left[\sum_{\text{pixels}} \psi_p^2 w_p\right]^2}$$

$$= \frac{\sum_{\text{pixels}} \psi_p^2 w_p}{\left[\sum_{\text{pixels}} \psi_p^2 w_p\right]^2}$$

$$= \frac{1}{\sum_{\text{pixels}} \psi_p^2 w_p}$$

$$Var[I_p] w_p = 1$$

High- and low-flux limits

$$\hat{f} = \frac{\sum_{\text{pixels}} \psi_p (I_p - s) w_p}{\sum_{\text{pixels}} \psi_p^2 w_p}$$

$$w_p \to s^{-1} \equiv w$$

low

high

$$w_p \to (f\psi_p)^{-1}$$

$$\hat{f} = \frac{\sum_{\text{pixels}} \psi_p (I_p - s)}{\sum_{\text{pixels}} \psi_p^2}$$

$$\hat{f} = \sum_{\text{pixels}} (I_p - s)$$

$$Var[\hat{f}] = \frac{1}{w \sum_{\text{pixels}} \psi_p^2}$$

$$Var[\hat{f}] = f$$

Noise equivalent area (faint sources)

$$Var[\hat{f}] = \frac{1}{w \sum_{\text{pixels}} \psi_p^2} = V_s A$$

Variance of the sky times an area.

$$1/w \equiv V_s$$

(Faint sources)

$$A \equiv \frac{1}{\sum_{\text{pixels}} \psi_p^2}$$

Noise equivalent area (unit: pixels)

For a Gaussian PSF, $A = 4\pi\sigma^2$

NEA : Area over which one integrates the sky noise when optimally measuring a point source

Technical details

- Modeling the PSF ?
  - Empirical methods. Pretty efficient these days.
  - Not discussed here.
- Sampling :
  - The PSF model should in general integrate over pixels.
  - For well-sampled images, the value at the pixel center is an accurate proxy.

PSF photometry in a nutshell

- One needs the PSF (which usually varies smoothly with position).
- The flux estimator is non-linear if the position is fit from the same data.
  - Implies bias at low S/N.
- One should chose a scheme for the weights.
- One should wonder what are the flux biases caused by a “wrong” PSF.

PSF photometry in a nutshell

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Exercise 1

- One should chose a scheme for the weights.

Exercise 2

- One should wonder what are the flux biases caused by a “wrong” PSF.

Exercise 3

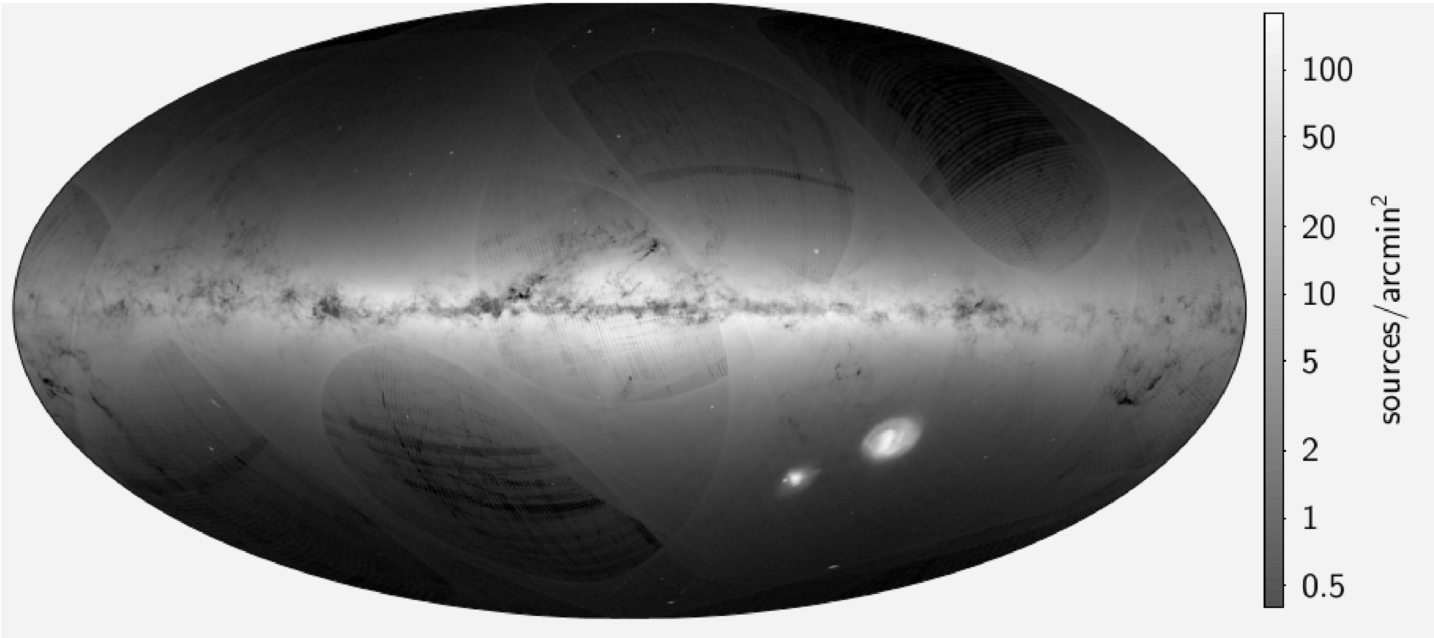
Astrometry (basics)

What is astrometry ?

- In principle, anything that has to do with measuring positions (and motions) on the sky of astrophysical objects.
- Now, the reference frame is defined in practice by the Gaia catalog. (For a while !)
- We are then concerned about relative astrometry
- It boils down to mapping position measurements in sensor coordinates on a global reference frame.

The Gaia catalog

- ~ 1.1 Billions sources
- Down to m ~20/21
- Position uncertainties well below the mas level
  - 1 mas ~= 5 nrad.



Gaia source density by Mark Taylor et al

~1 arcmin^{-1} at high Gal. latitude

Measuring positions in the image

- At least 3 methods:
  - Center of gravity.
  - Weighted center of gravity.
  - PSF positions for point sources.

Center of gravity (« aperture position »)

sky

$$X_0 = \frac{\sum_p (I_p - s) X_p}{\sum_p (I_p - s)} = \frac{\sum_p (I_p - s) X_p}{f}$$

Faint source,
Uniform noise $Var[I_p] = b$

$$Var[X_0] = \frac{b \sum_p (X_p - X_0)(X_p - X_0)^T}{f^2}$$

- $E[X_0]$ is well-defined when increasing the integration domain...
- But $Var[X_0]$ diverges.

$\rightarrow$ the expectation of the unweighted second moments is ill-defined.

Weighted center of gravity

$$X_0 = \frac{\sum_p (I_p - s) W[X_p - X_0] X_p}{\sum_p W[X_p - X_0](I_p - s)}$$

W is in general an even function.
This is (only) an implicit definition of X_0

Faint source,
→ Uniform background $Var[I_p] = b$

$$Var[X_0] = \frac{b \sum_p W^2[X_p - X_0](X_p - X_0)(X_p - X_0)^T}{\left[\sum_p W[X_p - X_0](I_p - s)\right]^2}$$

The variance converges if W decays faster than $1/R^2$

Gaussian-weighted positions

$$W(X_p - X_0) \propto \exp\left[-\frac{1}{2}(X_p - X_0)^T \mathbf{w}_g (X_p - X_0)\right]$$

One van adapt w_g to describe the shape of the object

(SDSS, Berstein & Jarvis (2002))

The fit of position (e.g. Irwin, 1985) and w_g can be done iteratively by looking for the fixed point of:

$$X_0 = \frac{\sum_p X_p W[X_p](I_p - s)}{\sum_p W[X_p](I_p - s)}$$

$$\mathbf{w}_g^{-1} = 2\frac{\sum_p (X_p - X_0)(X_p - X_0)^T W[X_p](I_p - s)}{\sum_p W[X_p](I_p - s)}$$

$$\text{with} \quad W[X_p] \equiv \exp\left[-\frac{1}{2}(X_p - X_0)^T \mathbf{w}_g (X_p - X_0)\right]$$

Crafting a WCS

- WCS : World Coordinate System
- Standardization of encoding of the mapping from image coordinates to “more standard” coordinates
- Goes into the FITS headers.
- Examples of mappings:
  - « pixel » coordinates → RA, Dec
  - Position into a tabulated spectrum → wavelength.
  - Many more exotic possibilities

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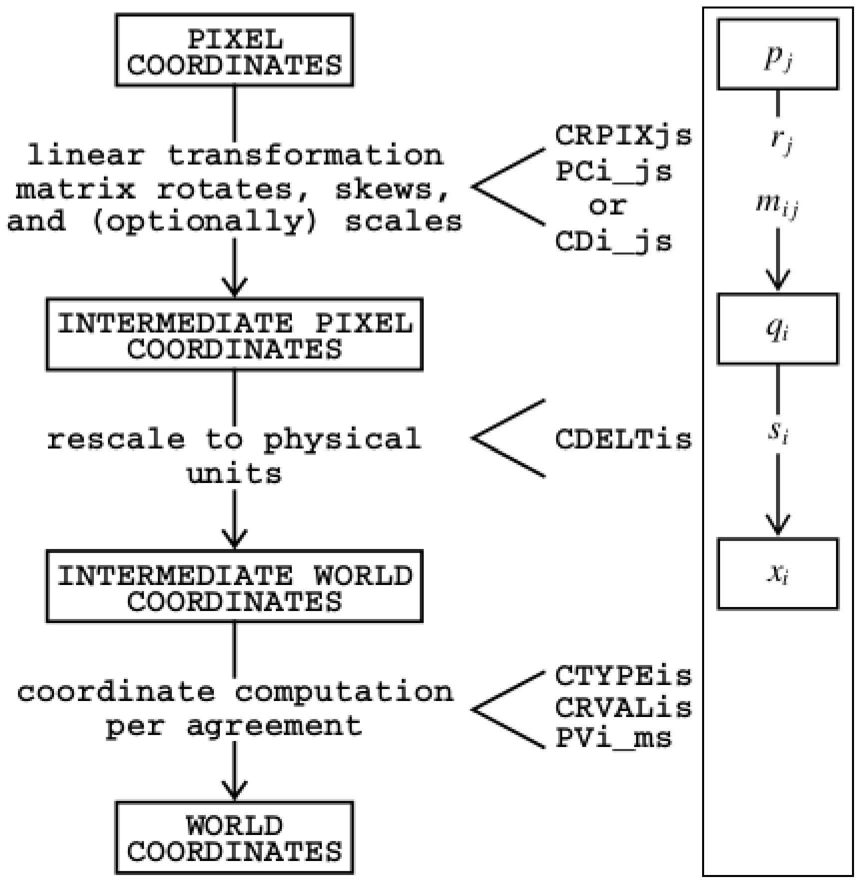
WCS : concepts

Greisen, E.W. & Calabretta, M.R. A&A 395, (2002)

"Redundant" convention :
There are different ways
To encode the same mapping

It is a de facto standard,
with less standard extensions.

At least two implementations :
libwcs
wcslib



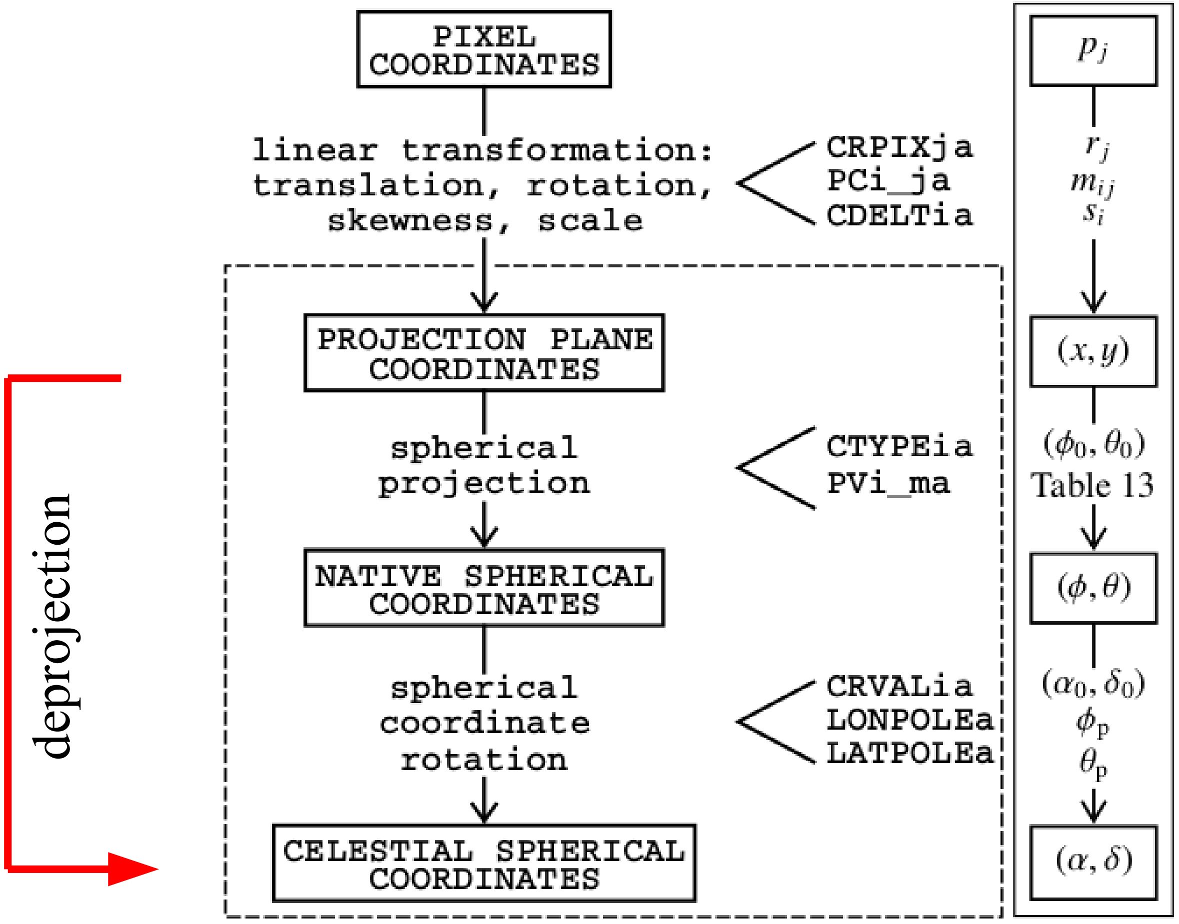
WCS : sidereal coordinates (1)

Calabretta, M.R., & Greisen, E.W., (2002), A & A, 395, 1077-1122

FITS keywords:

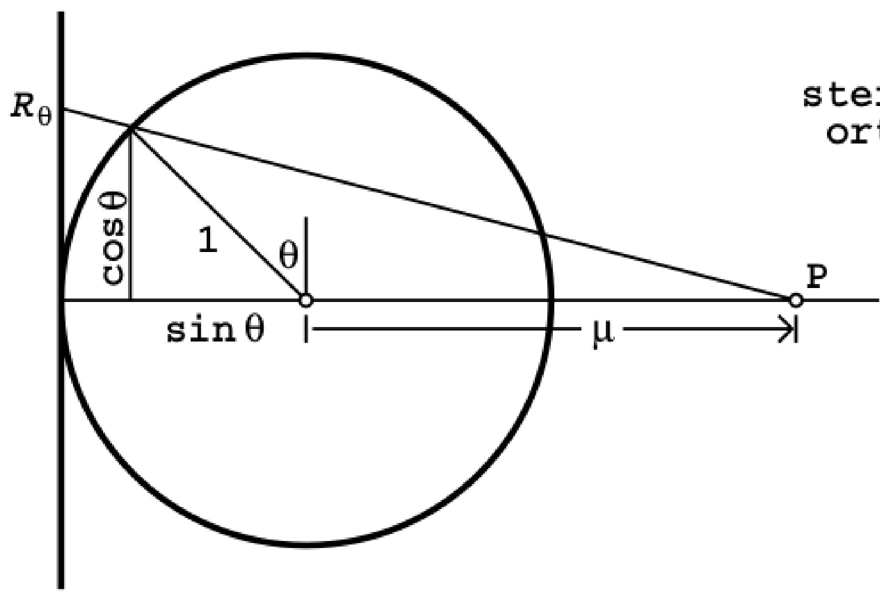
CRPIXi
CDi_j or CDELTi

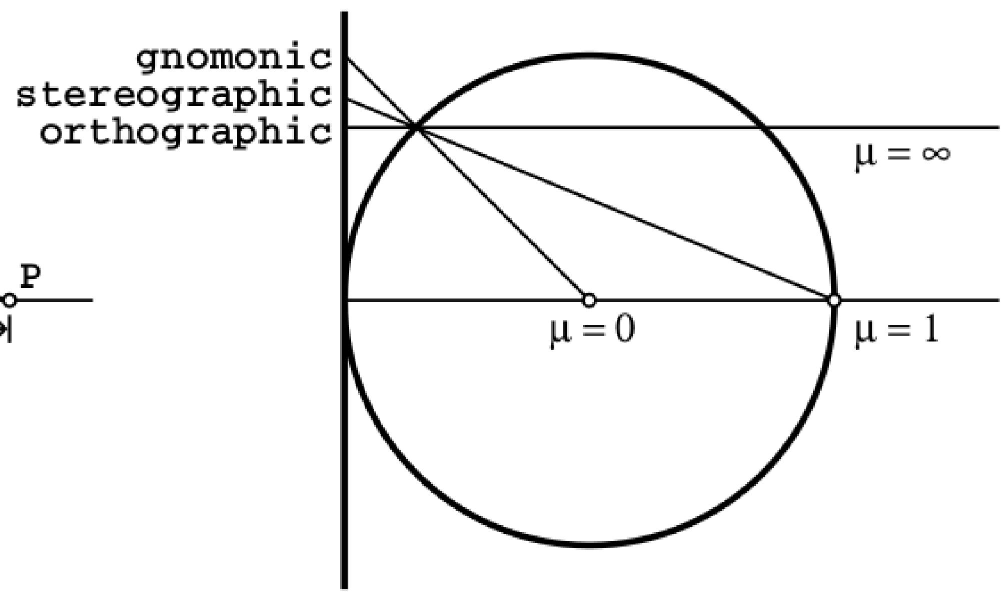
CTYPEi
CRVALi



WCS : sidereal coordinates (2)

There are dozens of projections (originally developed for geography).
The most common : “gnomonic” projection





Fits keywords for gnomonic:

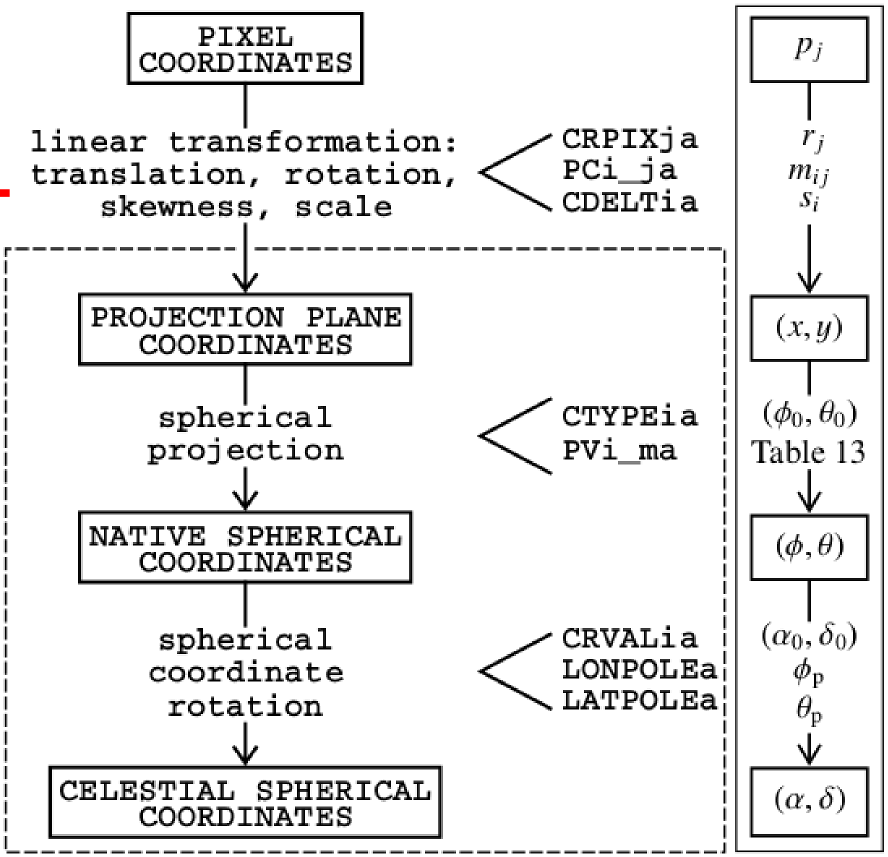
CTYPE1 = RA___TAN
CTYPE2 = DEC__TAN

Matching catalogs

- I won't discuss that in detail.
- Most of the implementations rely on “Hough transforms". They differ in the number of parameters they try to match.
- Further reading:
  - Astro-ph/9907229
  - Documentation of Scamp and astrometry.net
- As a rule, efficient algorithms find the right matching several times.

Fitting the WCS

$$\begin{pmatrix} x_{TP} \\ y_{TP} \end{pmatrix} = \begin{pmatrix} CD1\_1 & CD1\_2 \\ CD2\_1 & CD2\_2 \end{pmatrix} \begin{pmatrix} x_p - CRPIX1 \\ y_p - CRPIX2 \end{pmatrix}$$



For small angles,
the deprojection writes :

$$\begin{pmatrix} \alpha \\ \delta \end{pmatrix} = \begin{pmatrix} 1/\cos\delta & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_{TP} \\ y_{TP} \end{pmatrix} + \begin{pmatrix} CRVAL1 \\ CRVAL2 \end{pmatrix}$$

Then one cannot fit

- The CD (4)
- The CRPIX (2)
- The CRVAL (2)

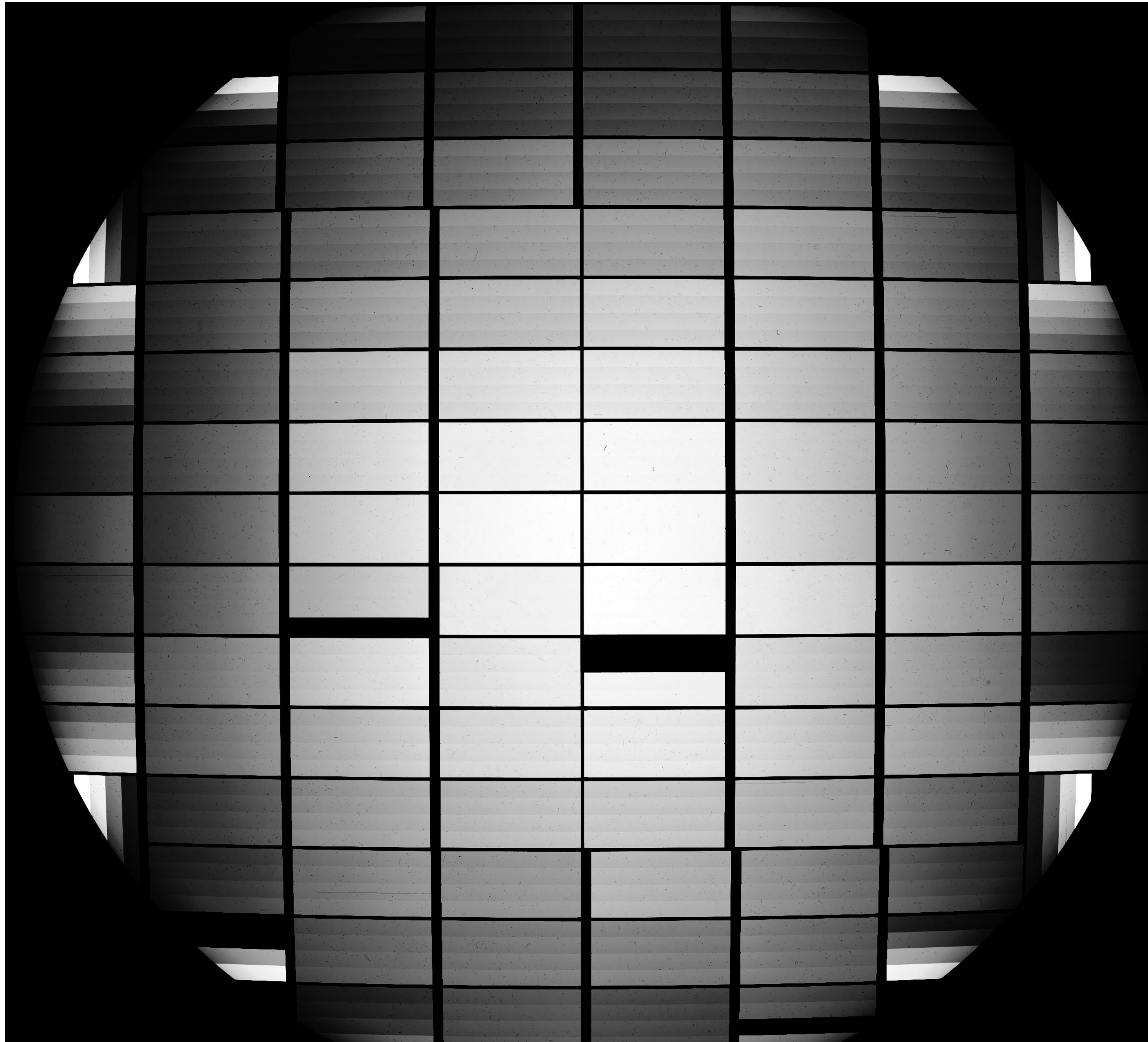
In practice, one usually fixes the projection point (CRVAL{1,2})

Distortions

- The minimal WCS model imposes a linear relation between pixel and tangent plane coordinates:
    - The image of a straight line is a straight line.
- In general wide-field correctors produce sizable distortions.

HSC:
mapping CCDs
on a tangent plane

Straight lines
get curved

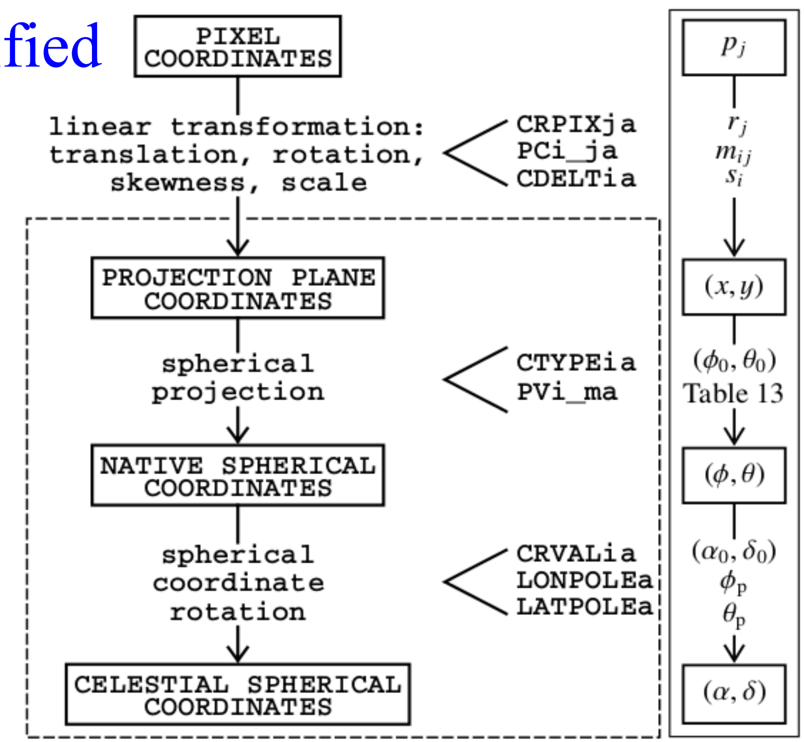


Encoding distortions

SIP standard

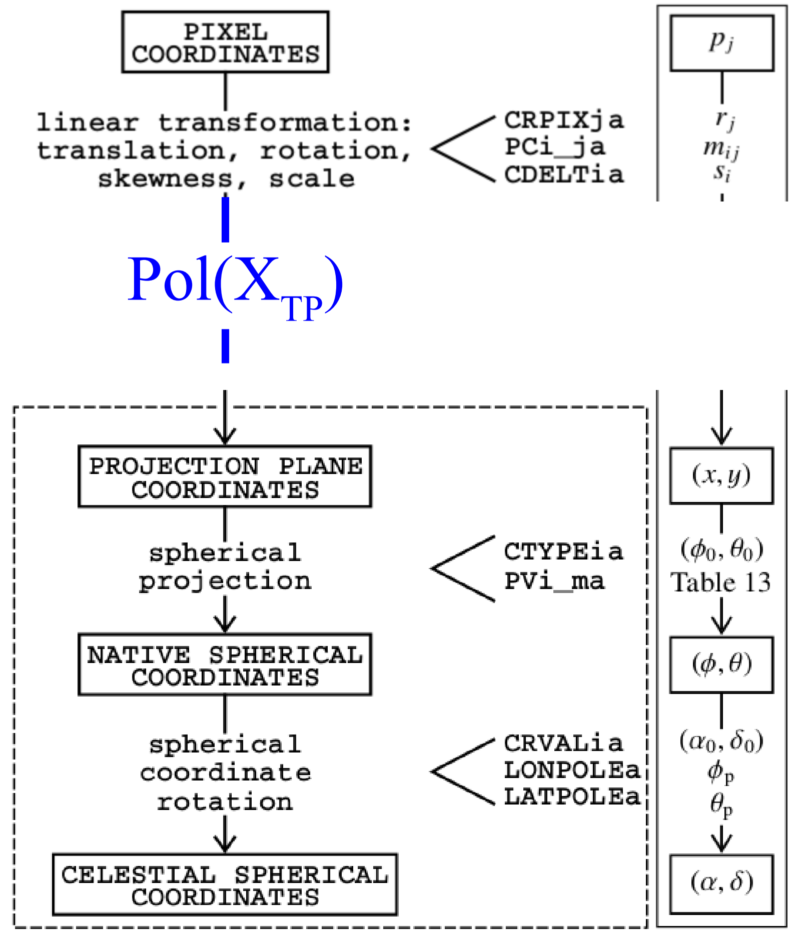
$Pol(X_{CCD})$

modified



"DV" standard

$Pol(X_{TP})$



How trustable are WCS's ?

- If the header contains distortion terms, then, the WCS is probably accurate, provided the software decodes those distortions.
- If the Cdi_j matrix contains 0's, the WCS was most probably never fitted.
- Ds9 allows one to overlay catalogs on images using the image WCS.

Exercises (on PSF photometry)

?

?

$$\hat{f} = \frac{\sum_p (I_p - s)\psi(X_p - X_0) w_p}{\sum_p \psi^2(X_p - X_0) w_p}$$

?

Exercises on PSF photometry

- 3 exercises. Sample code at: http://supernovae.in2p3.fr/PSF
- Exercise 1: Is there any flux bias at low S/N ?
- Exercise 2: How do I cook up V_p in the flux estimator ?

$$\hat{f} = \frac{\sum_{\mathrm{pixels}} \psi_p (I_p - s)/V_p}{\sum_{\mathrm{pixels}} \psi_p^2 / V_p}$$

- Exercise 3 : My PSF is slightly “wrong”. What happens to the flux estimate?
- The idea is to produce plots that answer the questions (or others!) in the README file.