EFFICIENT MULTI-BAND DEBLENDING

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THE CURRENT STATE

- Aperture fluxes do not work!
- Models for blended objects are unstable!
- Deblender must disambiguate galaxies vs stars vs moving objects
- Once detection is done: deblending is (as hard as) model fitting

A NEW DEBLENDER: TAKE 1

- General estimator $p(\theta \,|\, D) \propto p(D \,|\, \theta)\; p(\theta)$ $\theta = \{A_b, S_b : b \in \mathcal{B}\}$
- Several objects $D(x) = \sum_k^K D_k(x)$
- Huge space $p(D \,|\, \theta_1 \dots \theta_K) p(\theta_1 \dots \theta_K)$

$$p(D \,|\, \theta_1 \dots \theta_K) p(\theta_1) \dots p(\theta_K)$$

COLOR – MORPHOLOGY RELATIONS

Source: wikipedia.org

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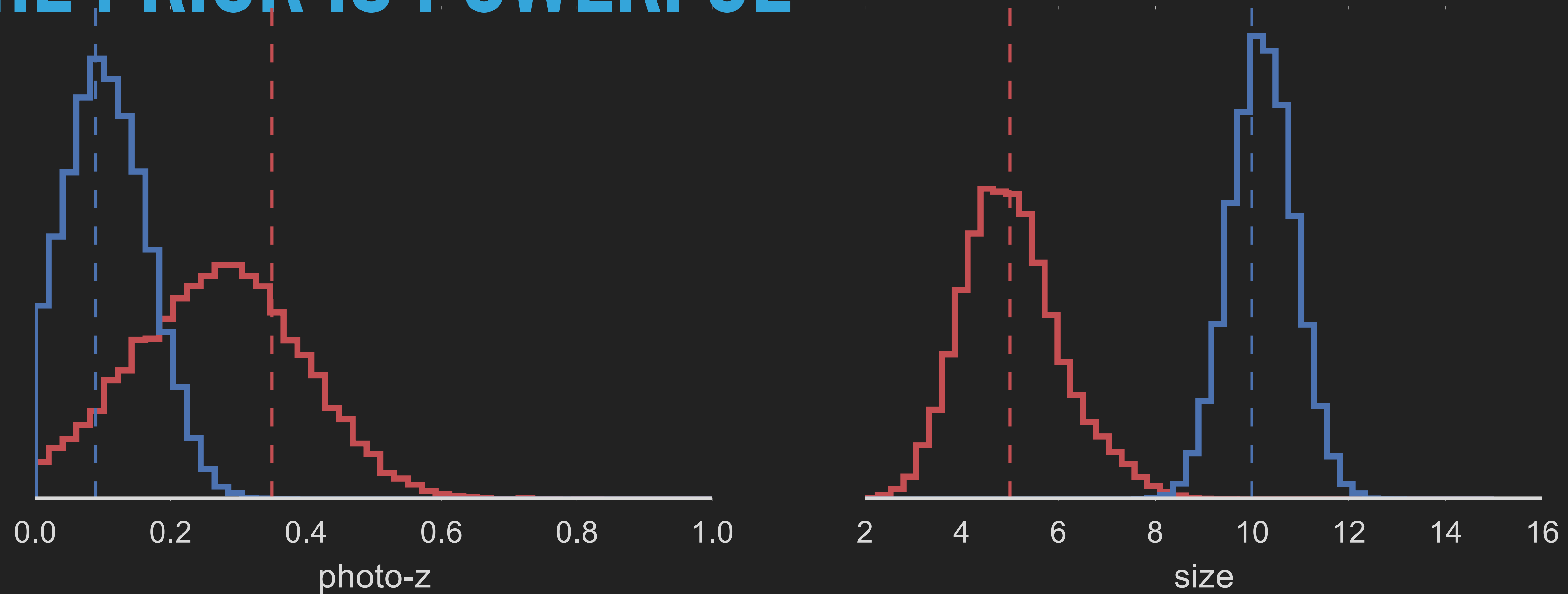
- Huge space $$p(D \,|\, \theta_1 \ldots \theta_K) p(\theta_1 \ldots \theta_K)$$ $$p(D \,|\, \theta_1 \ldots \theta_K) p(\theta_1) \ldots p(\theta_K)$$
- Finite template set $$p(\theta_k) = \sum_t^T p(\theta_k \,|\, t) p(t)$$
- In restframe $$p(\theta_k, z_k) = \sum_t^T p(\theta_k \,|\, z_k, t) p(z_k \,|\, t) p(t)$$
- Realistically $$p(\theta_k, z_k) = \sum_t^T \delta\big(\{A_{bk}(z_1, t)\}, \hat{S}_k(t)\big) p(z| \, t) p(t)$$

A SIMPLE TEST CASE

- single Sersic-type galaxies, convolved with constant Gaussian
- SEDs and morphologies from late-type and early-type galaxy
- simple template redshifts

THE PRIOR IS POWERFUL



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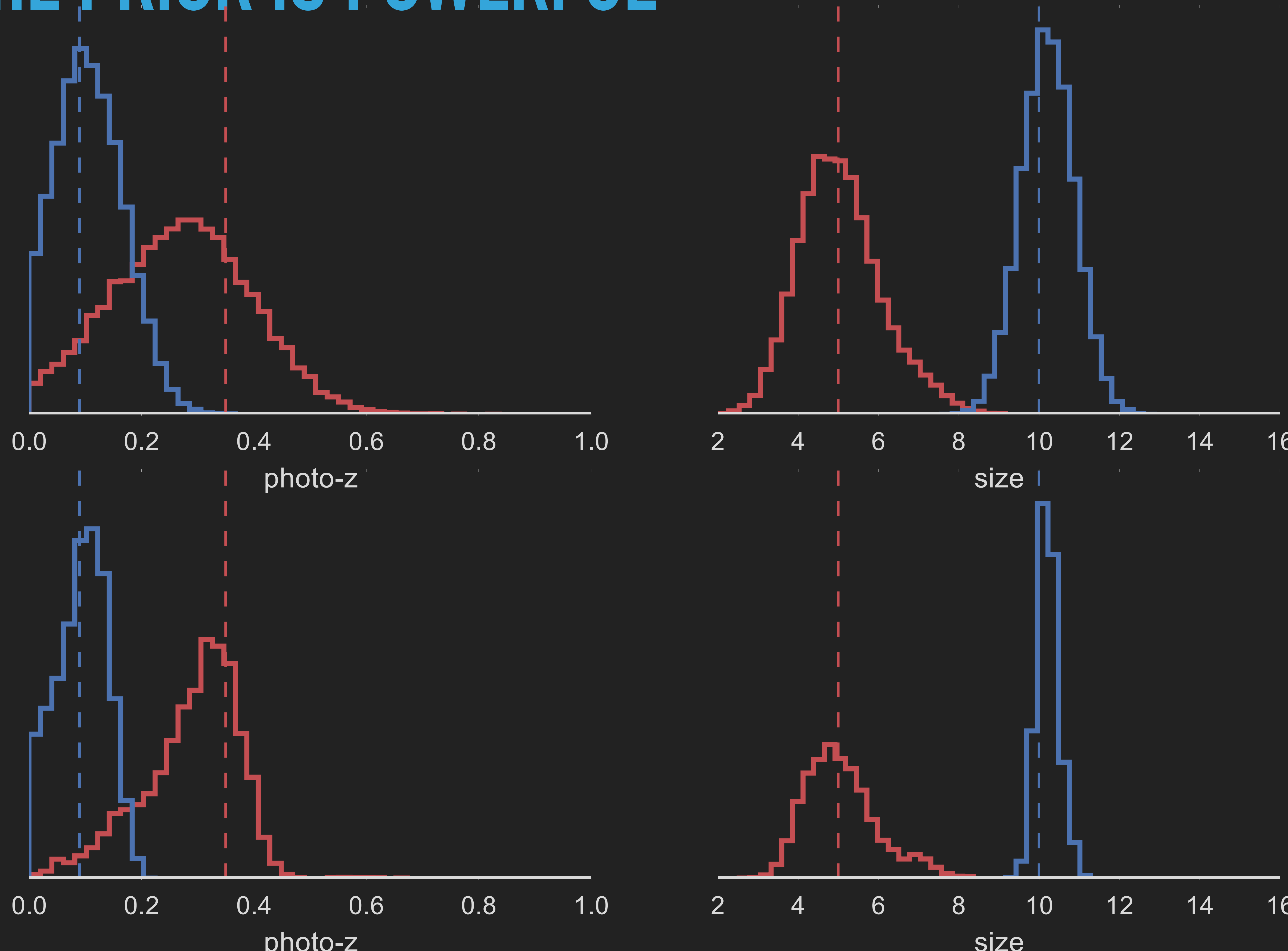
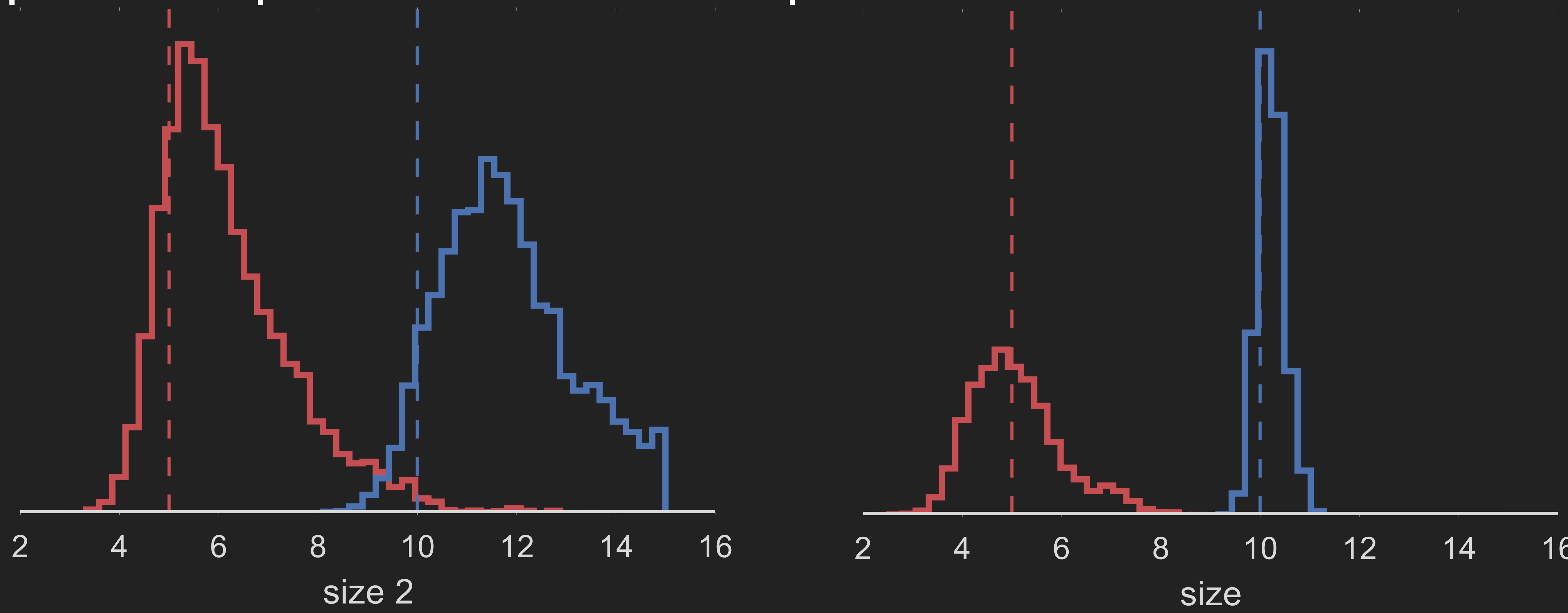


BUT ...

- this requires a correct photo-z estimator
- mistakes cannot be corrected later
- rest-frame properties are redshift-dependent
- spectral template libraries are incomplete



A NEW DEBLENDER: TAKE 2

- General estimator $p(\theta \,|\, D) \propto p(D \,|\, \theta)\; p(\theta) \qquad \theta = \{A_b, S_b : b \in \mathcal{B}\}$
- Several objects $D(x) = \sum_k^K D_k(x)$
- Huge space $p(D \,|\, \theta_1 \ldots \theta_K) p(\theta_1 \ldots \theta_K)$

  $p(D \,|\, \theta_1 \ldots \theta_K) p(\theta_1) \ldots p(\theta_K)$
- Finite template set $p(\theta_k, z_k) \cancel{= \sum_t^T} \delta\big(\{A_{bk}(z_1, t)\}, \hat{S}_k(t)\big) p(z | \, t) p(t)$
- Instead $p(\theta_k) = p(\{A_{bk}\}, \{S_{bk}\}) \stackrel{?}{=} p(\{A_{bk}\}) p(\{S_{bk}\})$

BUT . . .

- requires a viable morphological parameterization
- S/G is not obvious
- requires trans-dimensional sampling / reversible jump methods

A NEW DEBLENDER: TAKE 3

- Color should be useful
- Star/Galaxy separation is not obvious: non-parametric
- Objects are somehow “compact”, mostly symmetric

A NEW DEBLENDER: TAKE 3

- Color should be useful, photo-z are dangerous
- Star/Galaxy separation is not obvious: non-parametric
- Objects are somehow "compact", mostly symmetric

$$\text{scene} = \sum_k \text{SED}_k \times \text{Morphology}_k + \text{noise}$$

$$Y = A \cdot S + \text{noise}$$

$$(Y \in \mathbb{R}^{B\times N}, A \in \mathbb{R}^{B\times K},\ S \in \mathbb{R}^{K\times N})$$

$$||Y - A \cdot S||_2^2 + g(A, S)$$

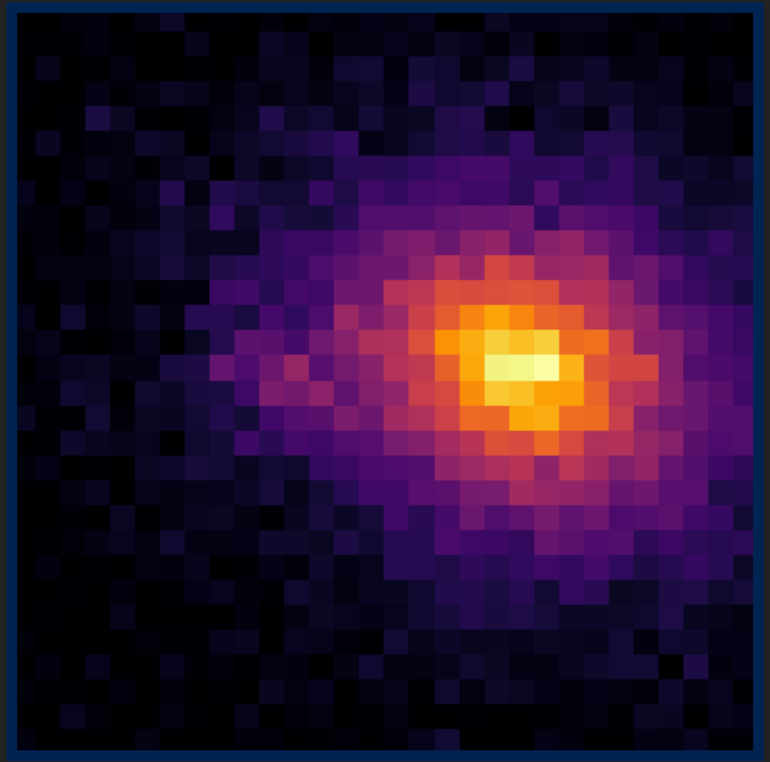
BSS VIA NON-NEGATIVE MATRIX FACTORIZATION

- per-object constraints with linear operators: gradients, symmetry, FFT …

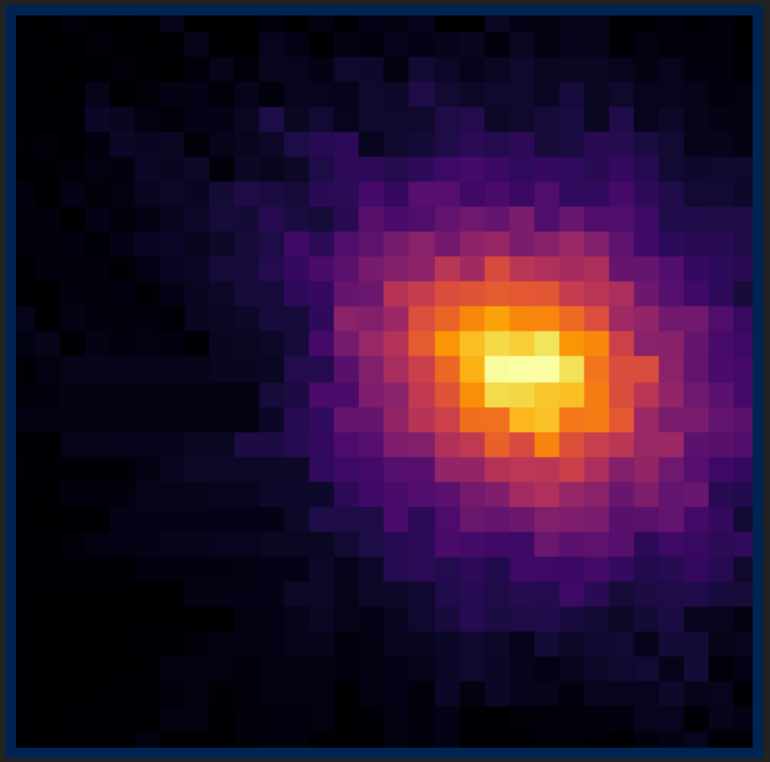
$$g(A,S) \to \sum_k g_k(A_k) + h_k(\mathsf{L}S_k)$$

- SED: sum=1, particular colors, distribution of observed colors

3-band RGB



NMF: no constraint



NMF: with monotonicity

THE STANDARD NMF SOLVER

Objective function f(A,S) is quadratic in A and S

1. solve for A under constraints (at least non-negative)
2. solve for S under constraints
3. repeat until convergence

Alternating Least-Squares (ALS)

THE STANDARD NMF SOLVER

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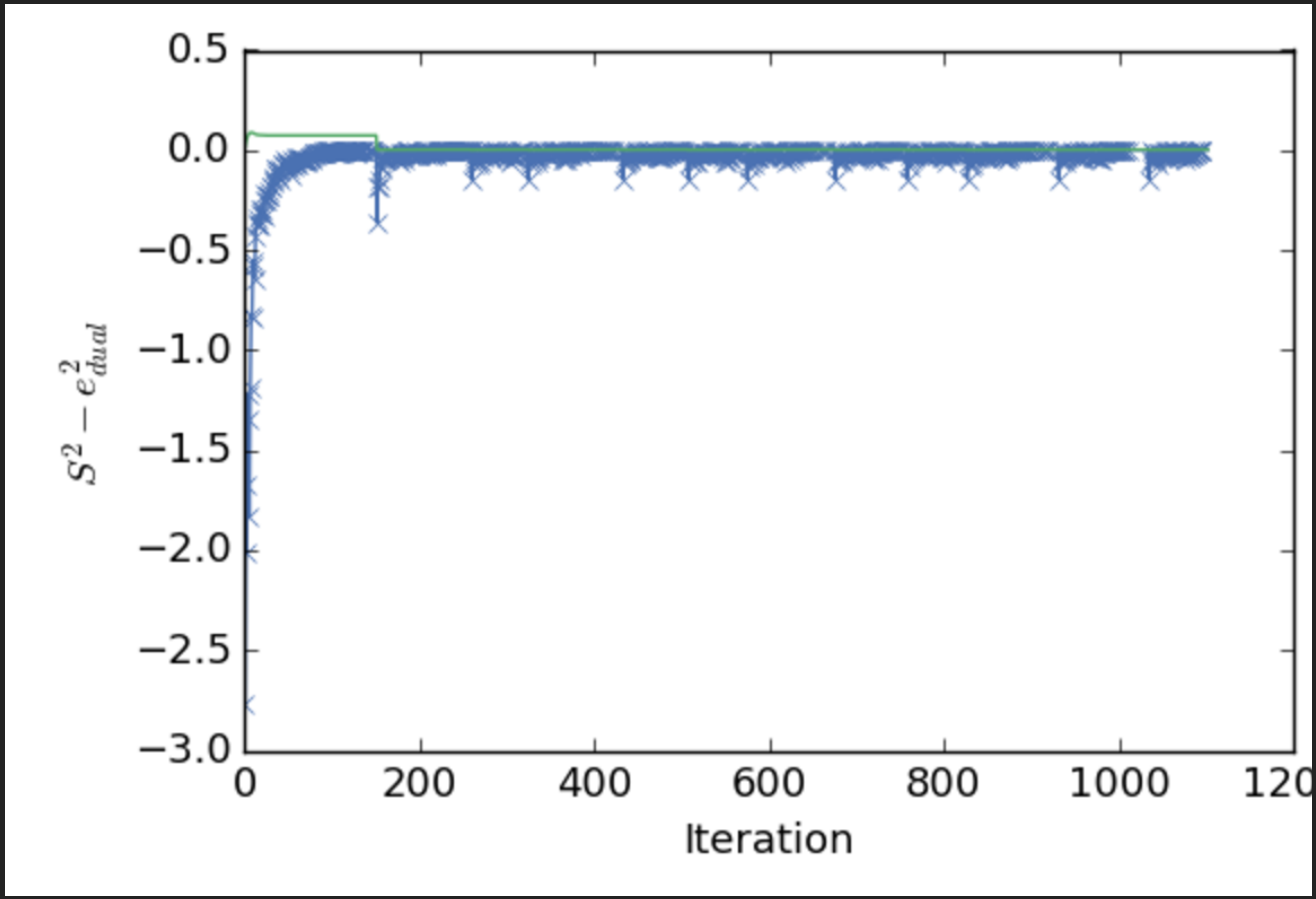

EFFICIENT GENERAL SOLVER: ADMM

- enforce constraints with dual variable $f\left(x_1\right)+g_1\left(z_{11}\right): \mathsf{L}_{11} x_1-z_{11}=0$

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- Alternative Direction of Method of Multiplier (ADMM):

$$
\begin{aligned}
x_1^{k+1} & := \underset{x_1}{\operatorname{argmin}}\left\{f(x_1)+\lambda_{11}^{k\top} \mathsf{L}_{11} x_1+\frac{1}{2\rho_{11}}\|\mathsf{L}_{11} x_1-z_{11}^k\|_2^2\right\} \\
z_{11}^{k+1} & := \underset{z_{11}}{\operatorname{argmin}}\left\{g_1(z_{11})-\lambda_{11}^{k\top} z_{11}+\frac{1}{2\rho_{11}}\|\mathsf{L}_{11} x_1^{k+1}-z_{11}\|_2^2\right\} \\
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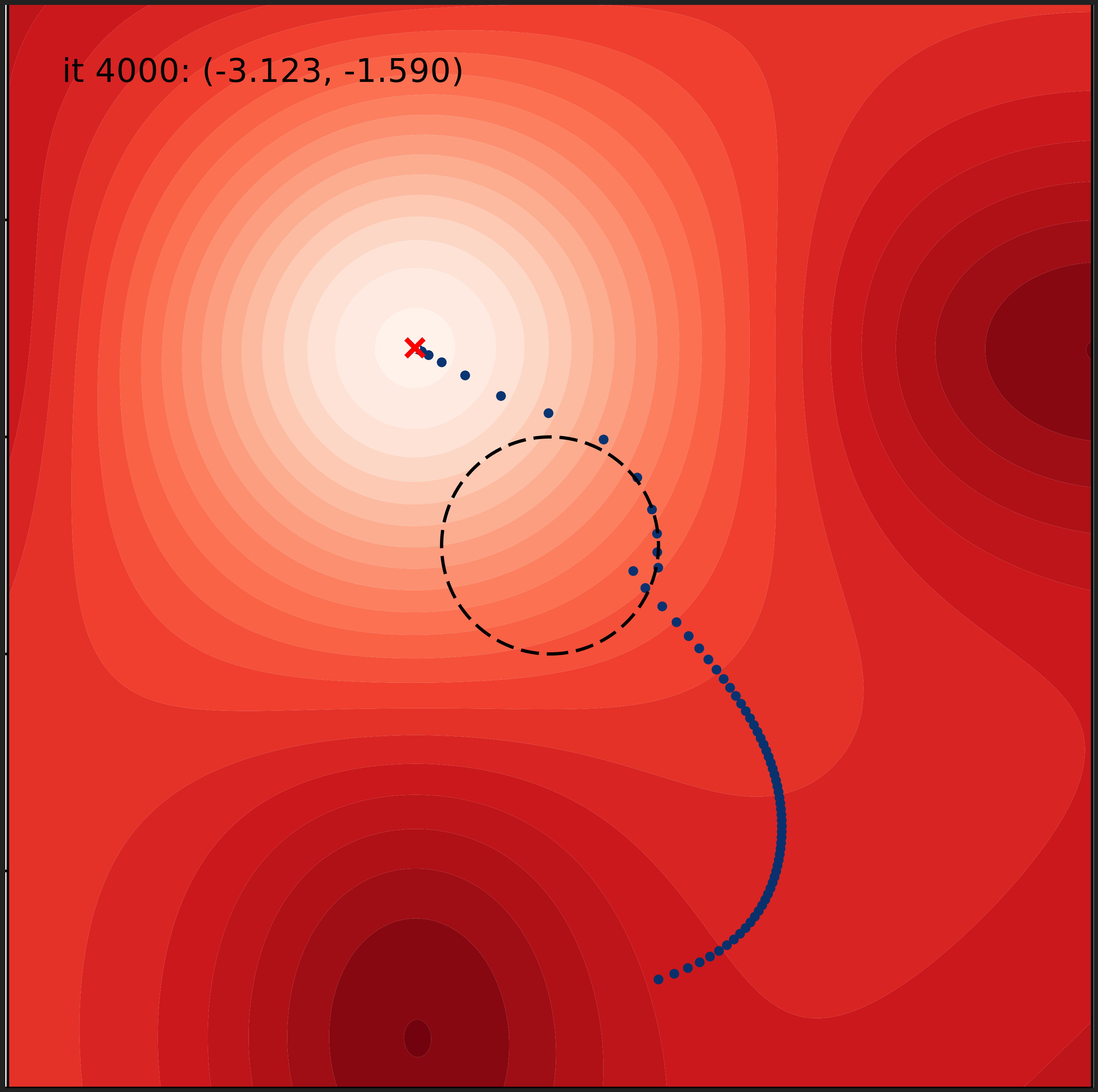
\end{aligned}
$$

- x-update can be made with single gradient step, z often by projection

$$
x_1^{k+1}=\nabla f\left(x_1^k-\frac{\mu_1}{\rho_1 1}\mathsf{L}_{11}^{\top}\left(\mathsf{L}_{11}x_1^k-z_{11}^k+u_1^k\right)\right)
$$

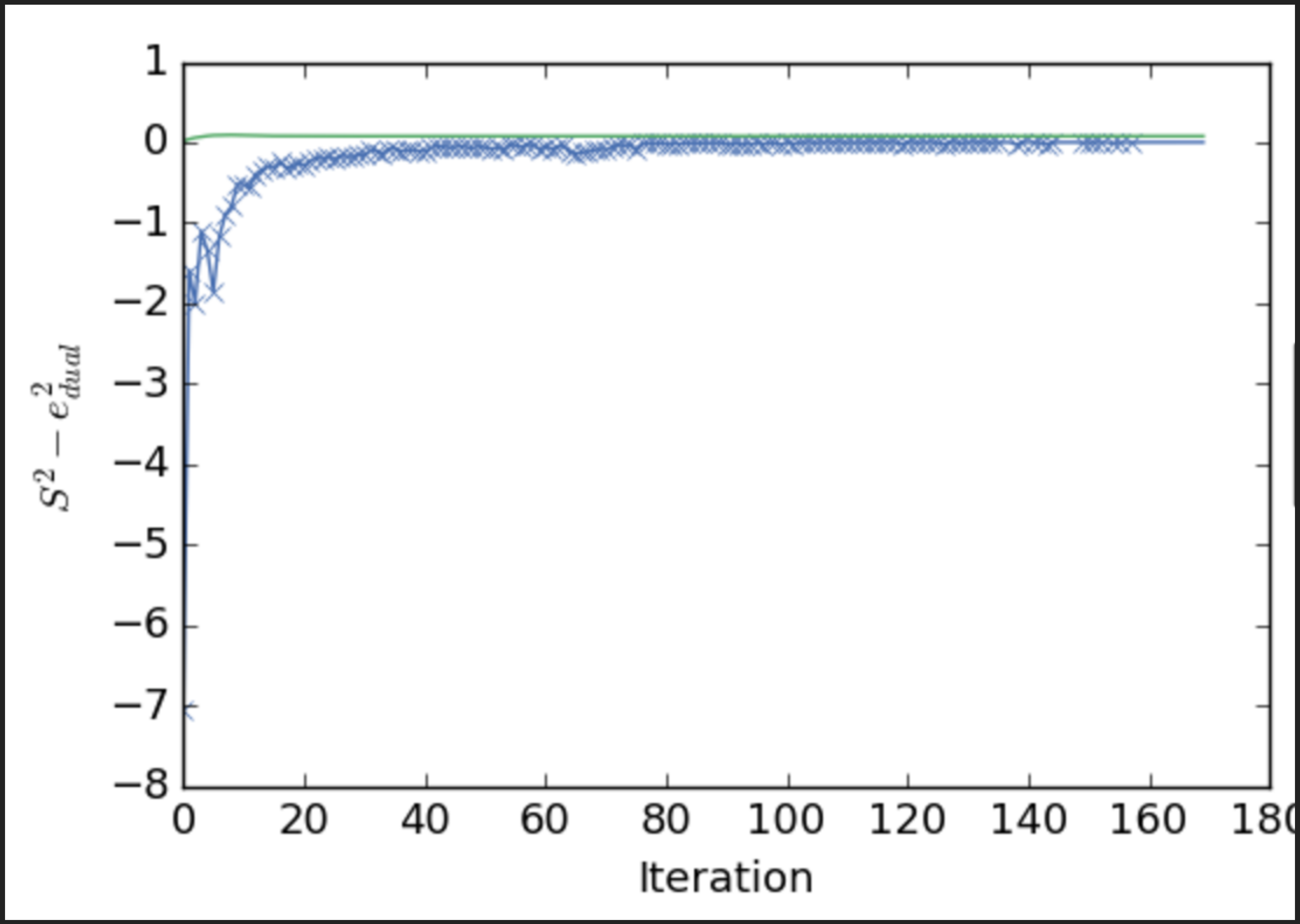
EFFICIENT GENERAL SOLVER: ADMM

- enforce constraints with dual variable $f\left(x_1\right)+g_1\left(z_{11}\right): \mathsf{L}_{11} x_1 - z_{11} = 0$

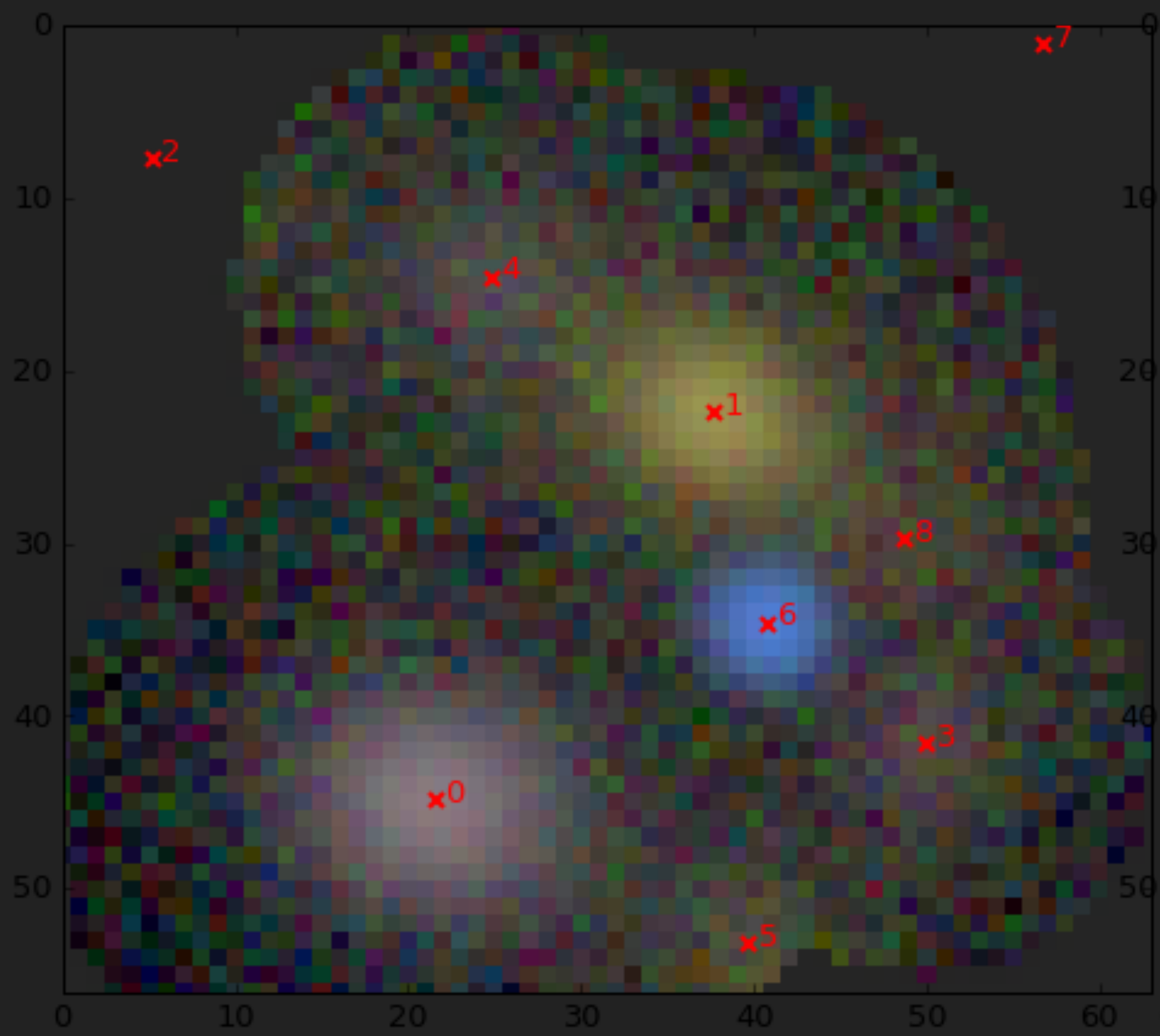


GLMM: MOOLEKAMP & MELCHIOR (IN PREP)

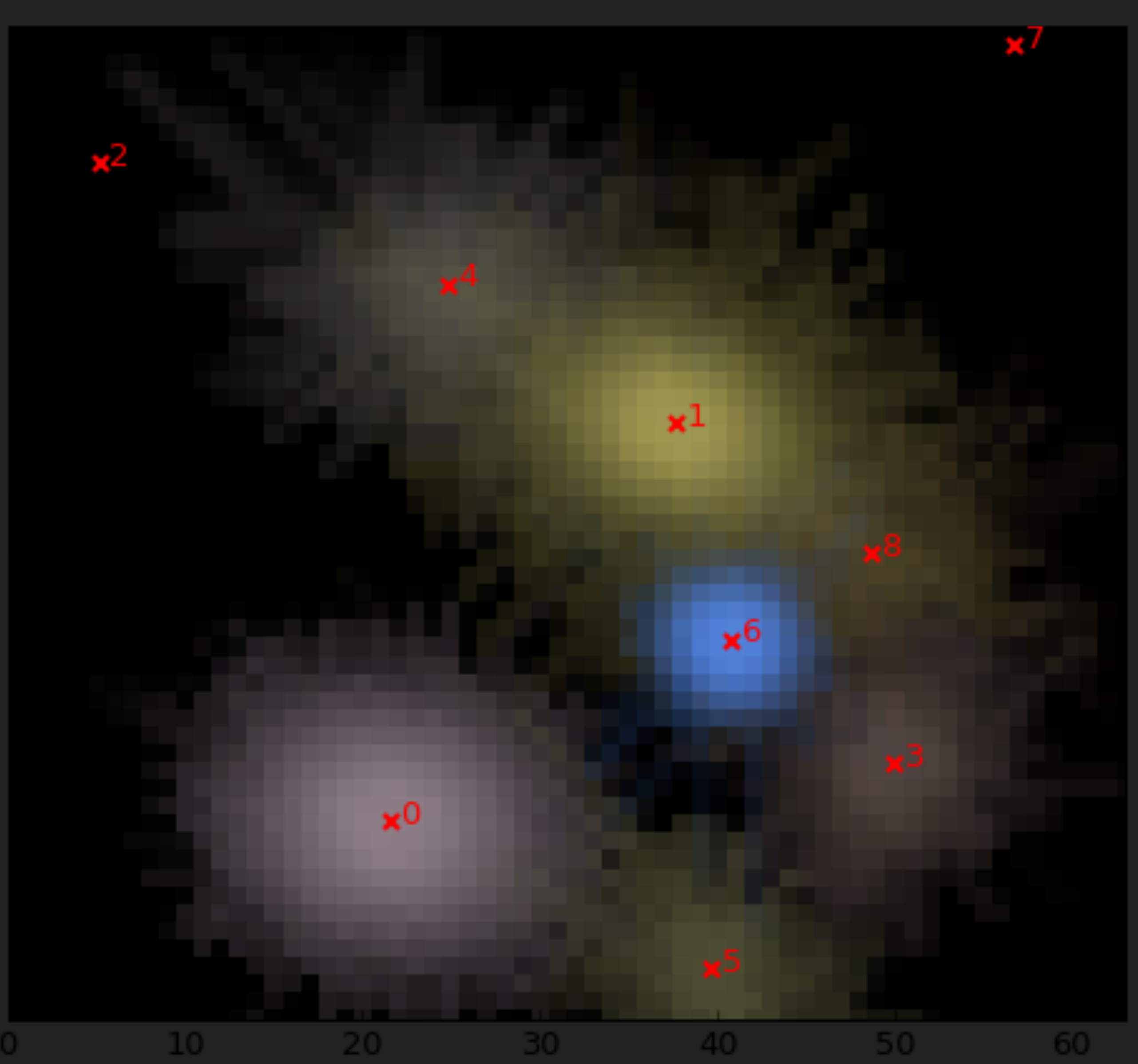
- Generalization to multiple variables (for NMF: A and S)



RESULTS ON 5-BAND HSC-LIKE DATA

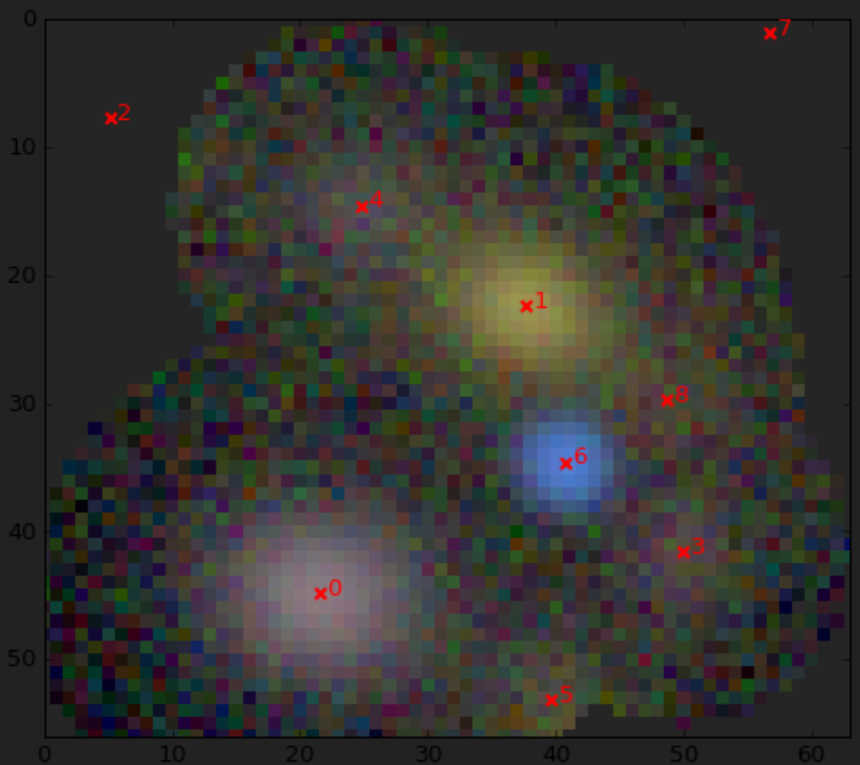


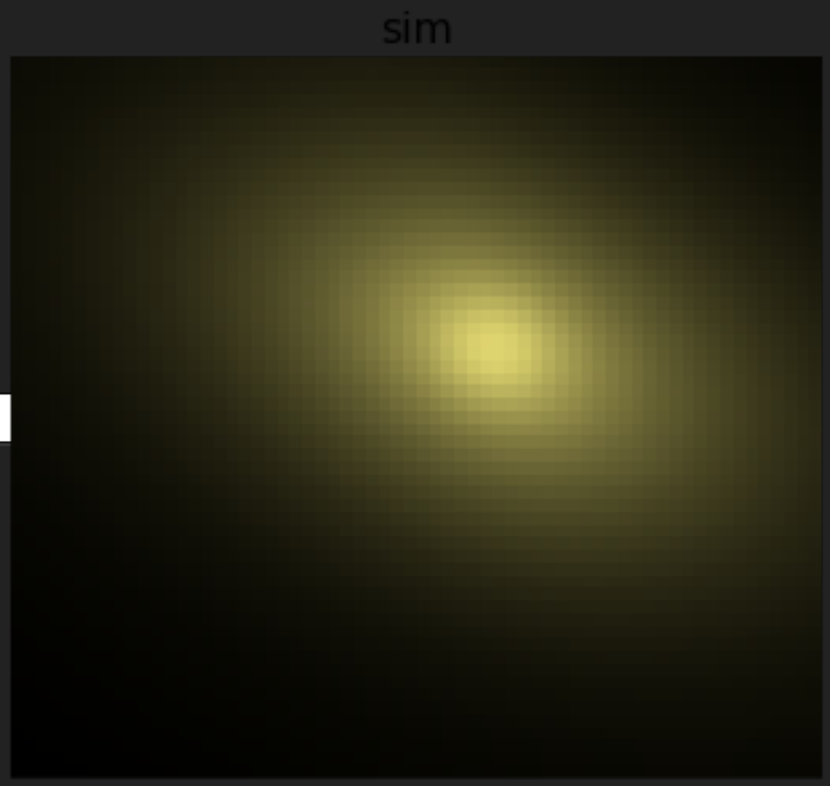
simulated HSC-like 5 band data



NMF deblender model

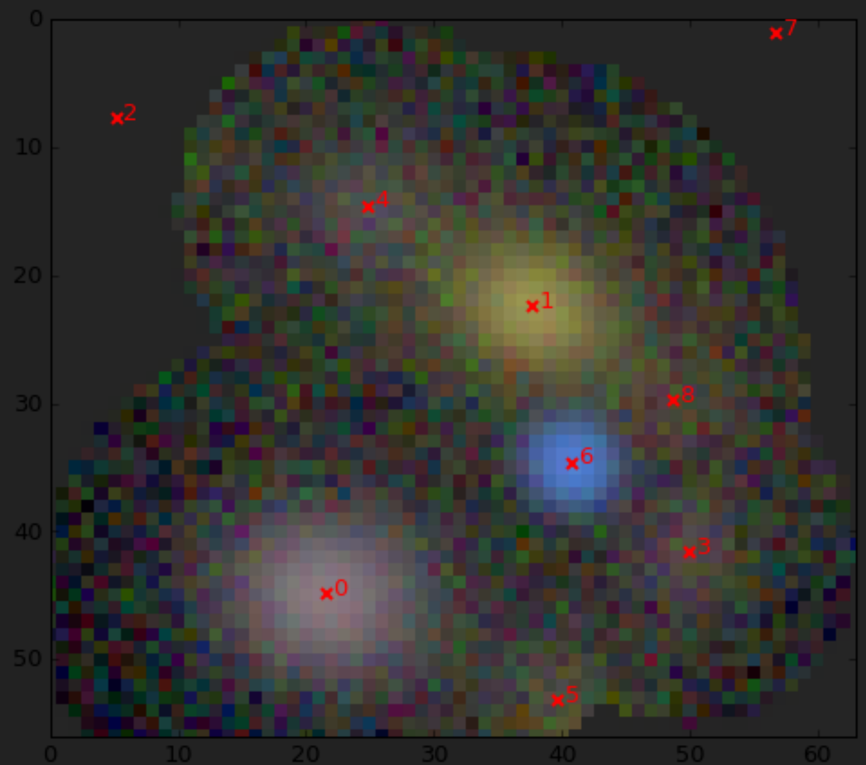
RESULTS: SDSS-STYLE VS NMF

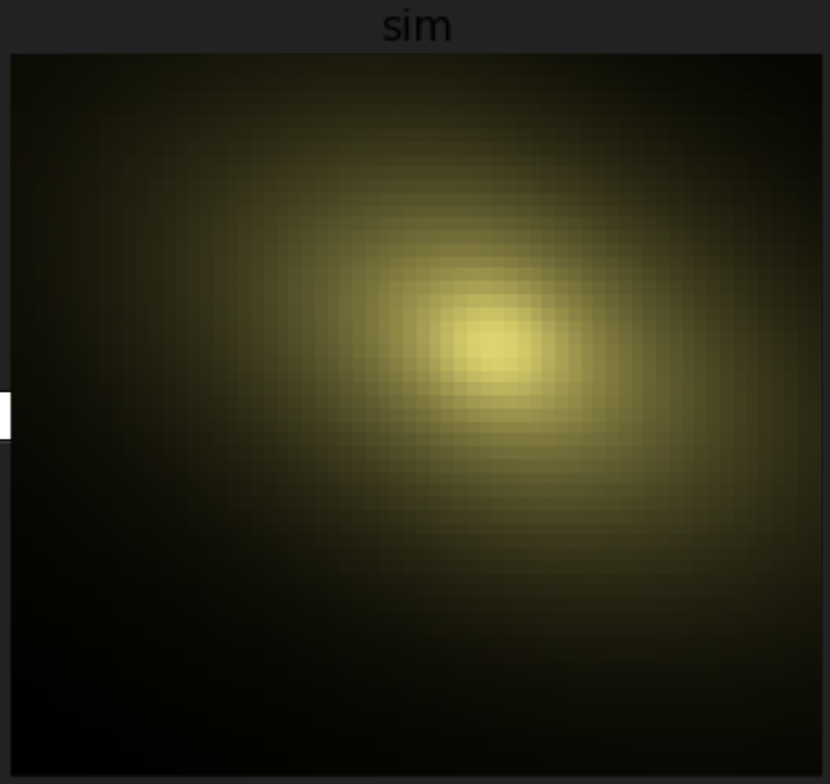




Peak 1: Truth

RESULTS: SDSS-STYLE VS NMF





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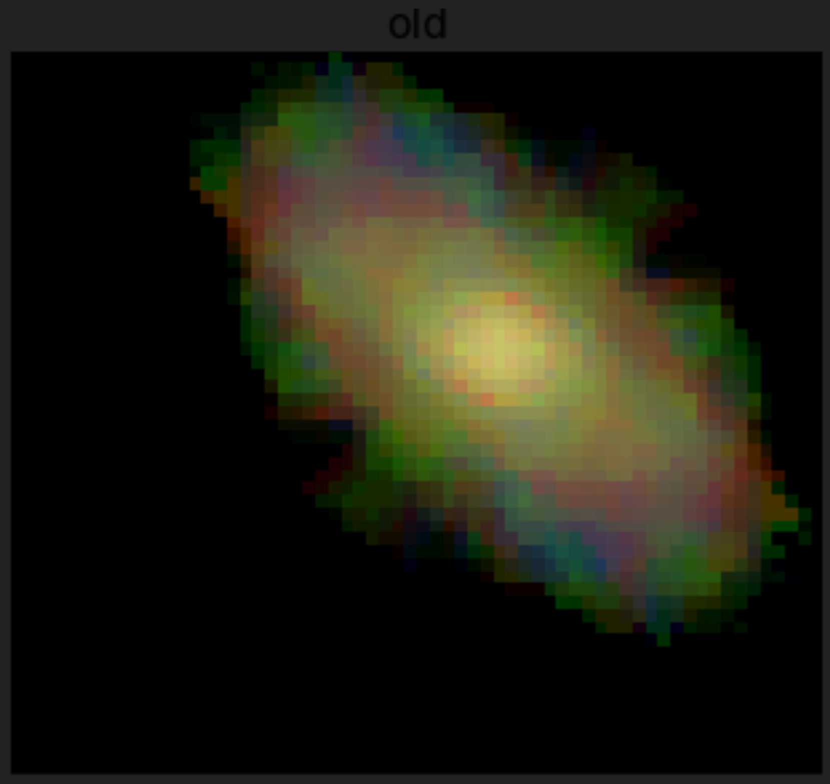
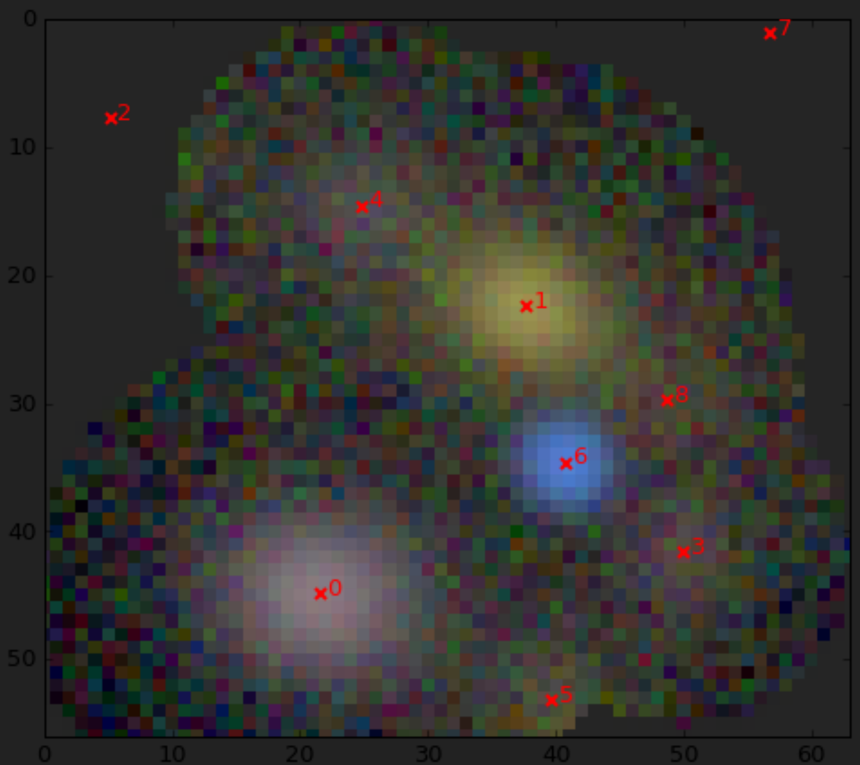
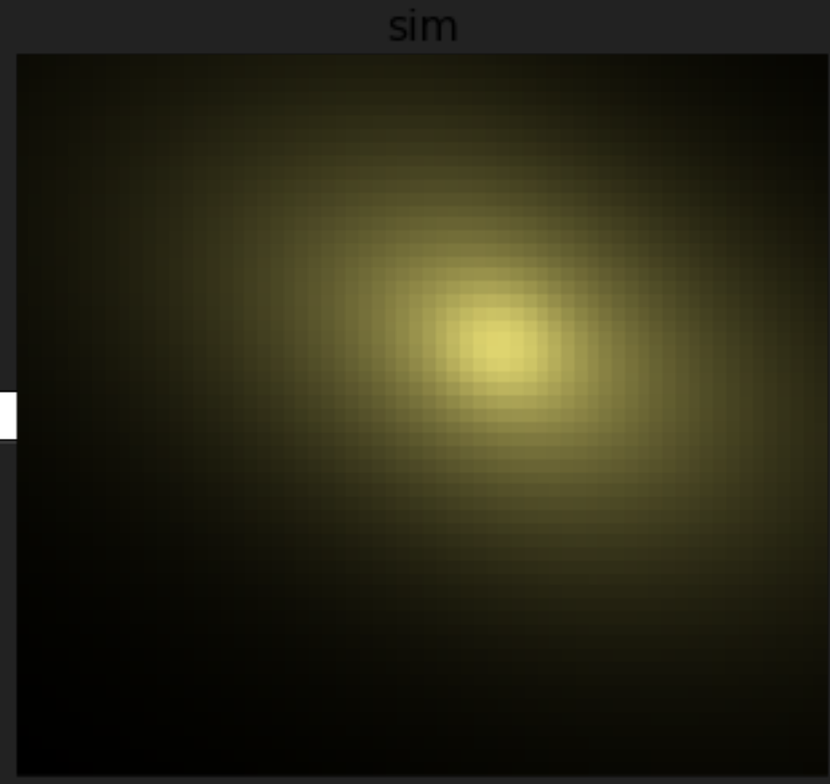


Peak 1: SDSS deblender

Peak 1: NMF deblender

RESULTS: SDSS-STYLE VS NMF





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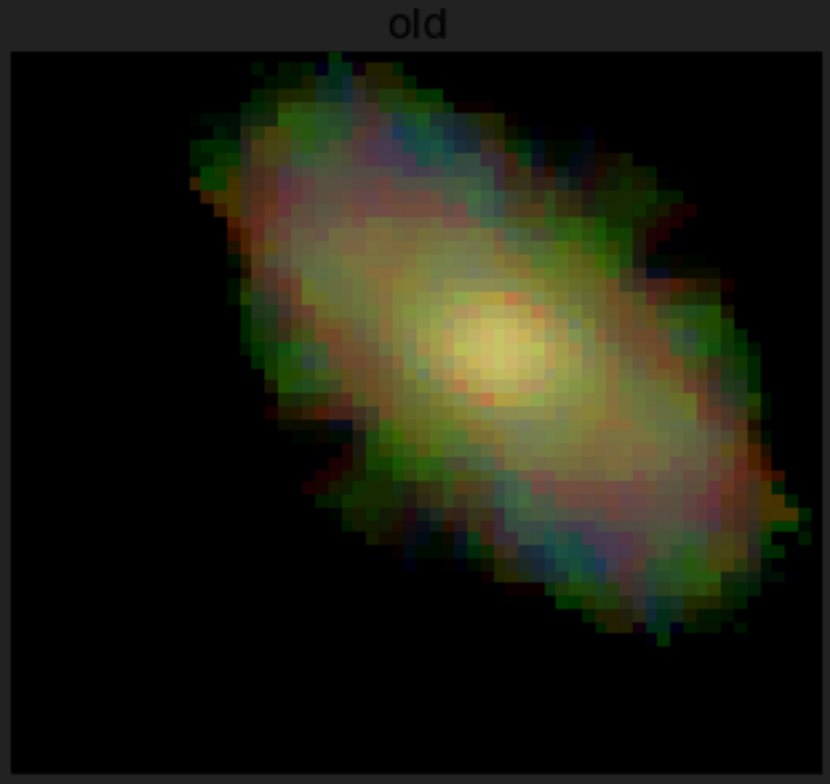
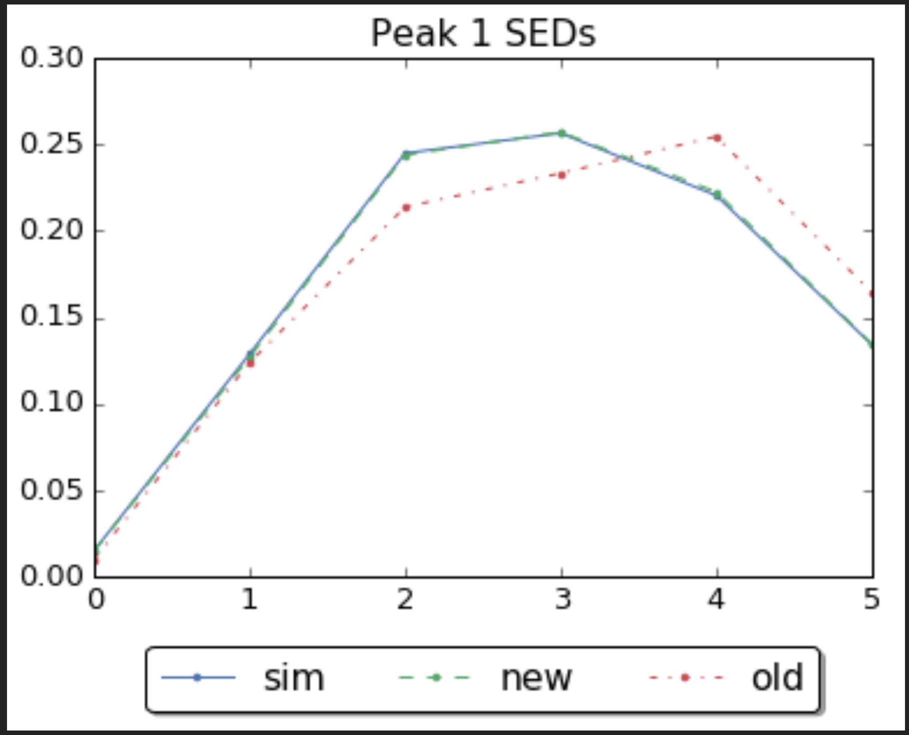


Peak 1: SDSS deblender

Peak 1: NMF deblender



Peak 1: SEDs

A FEW MORE TRICKS

- Sparsity vs PSF convolution // on-the-fly PSF matching

$$||Y - A \cdot P \cdot S||_2^2$$

- Shift operators for centroiding

$$||Y - A \cdot T \cdot P \cdot S||_2^2$$

- Constraints operators are identical, only likelihood term get adjusted

NEXT STEPS

- Testing in various situations (sims and HSC data)
- Adding spin-decomposition constraints
- High-performance implementation as part of LSST stack
- Fast centroid updates if M or S operators are used
- Error estimation with MCMC
- Faint-source detection: Garbage collector, and repeat
- Re-assembling of components to best-guess galaxies

CONCLUSIONS

- whenever you have an additive mixture situation: think of NMF
- soft priors and hard constraints can be implemented
- [side project: unmixing photo-z's in clusters]
- Code will be public within ~1 month
- Current studies of color priors

THE CASE FOR GROUND & SPACE

DES data from Melchior et al. (2015)

THE CASE FOR GROUND & SPACE

CLASH WFC3/IR data, image by Dan Coe