Point Spread Function modelling and measurement

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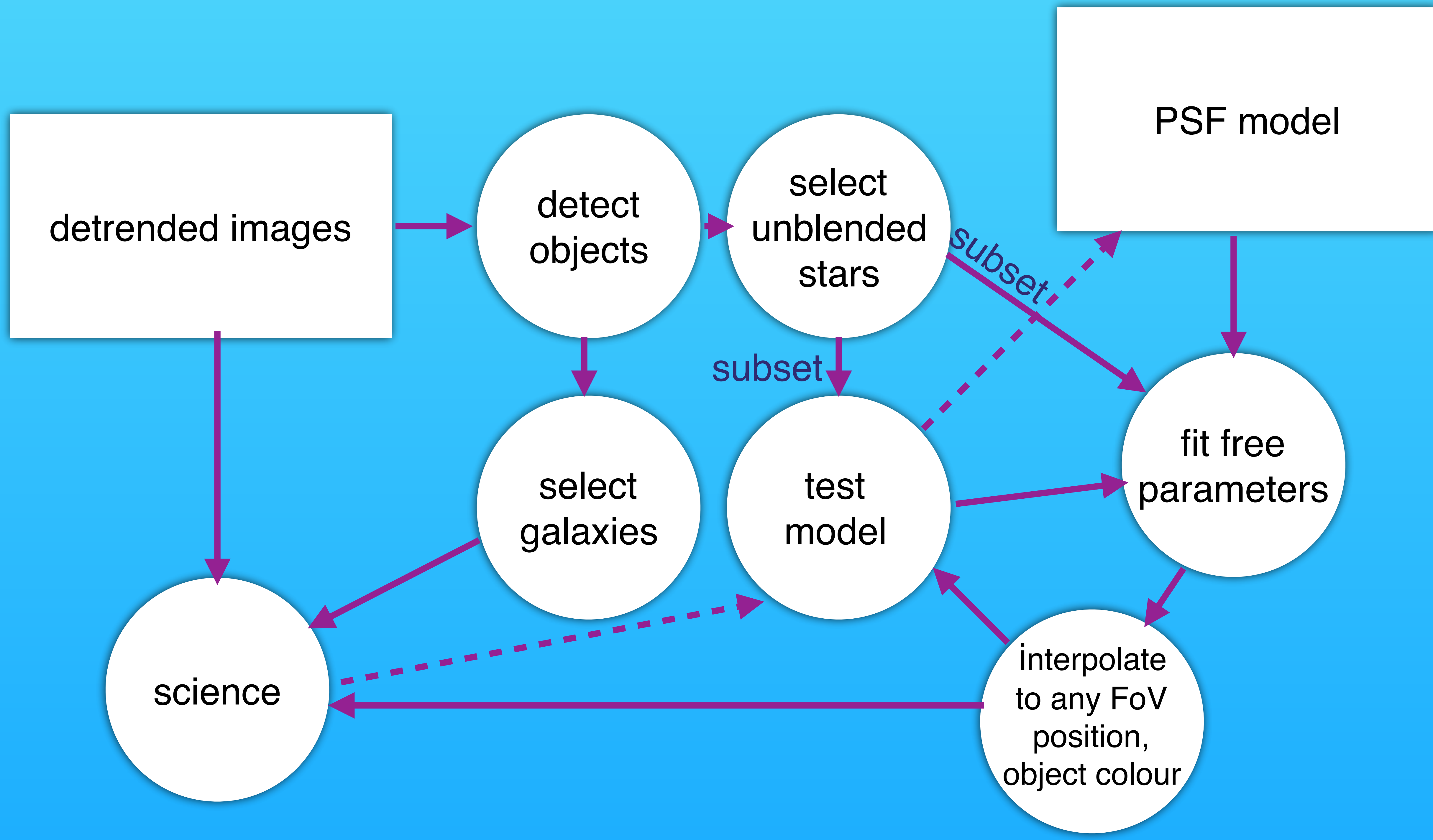
- knowledge of PSF on astronomical images essential for
  - object detection and deblending
  - photometry
  - profile fitting/component subtraction
  - galaxy morphology/shape
  - **weak lensing shear measurement**

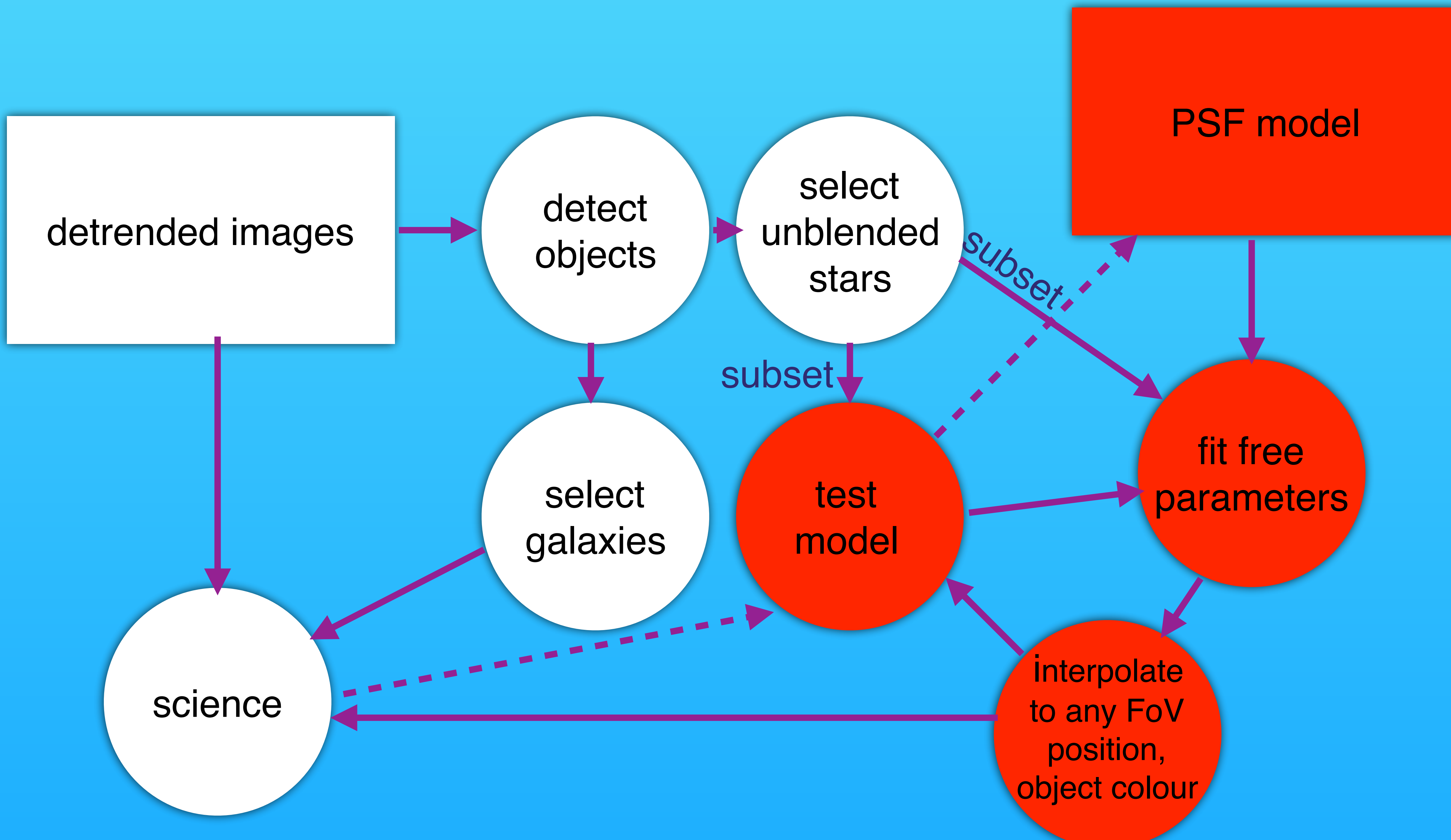
Point Spread Function modelling and measurement

- basics - understanding contributions to the PSF
  - atmosphere
  - optics
  - distortion
  - scattered light
  - focal plane and detector
- survey requirements - how well do we need to measure and model the PSF?
- measuring the PSF
  - atmosphere
  - wavefront domain
  - detector
- PSF quality assessment, current results & limitations
- summary

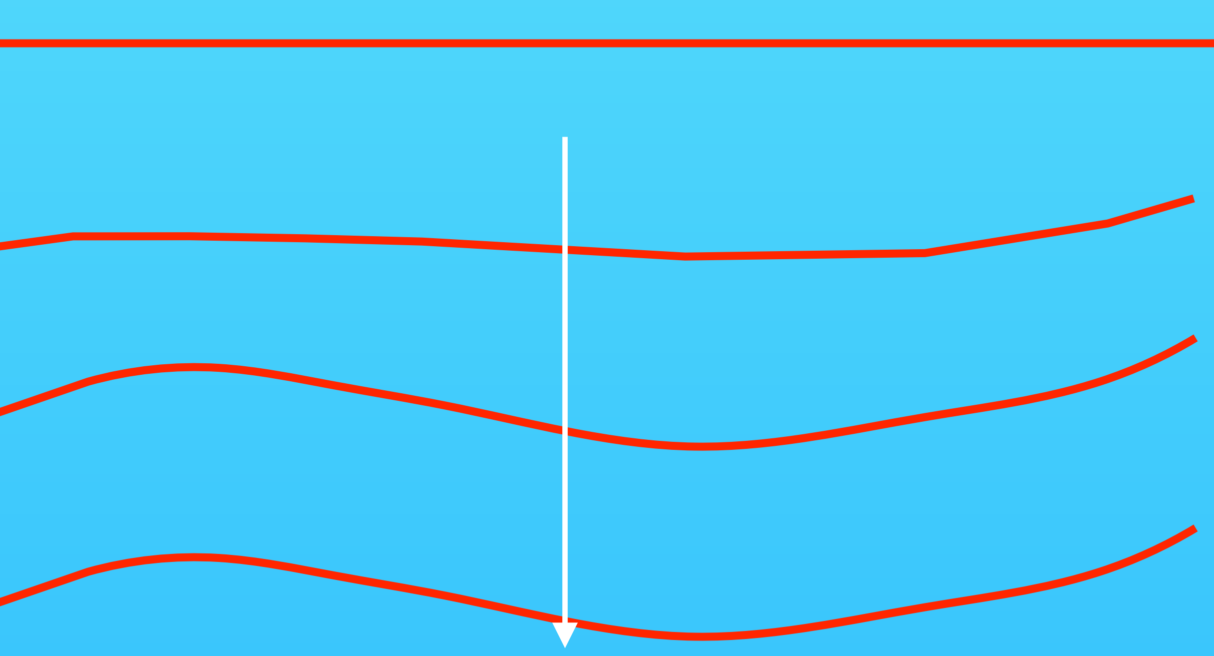
Point Spread Function modelling and measurement



Point Spread Function modelling and measurement



PSF basics: atmosphere



- optical path delays introduced by atmosphere (ignore wavefront amplitude variations in good seeing)
- refractive index fluctuations well described by Kolmogorov spectrum between “inner” and “outer” limits

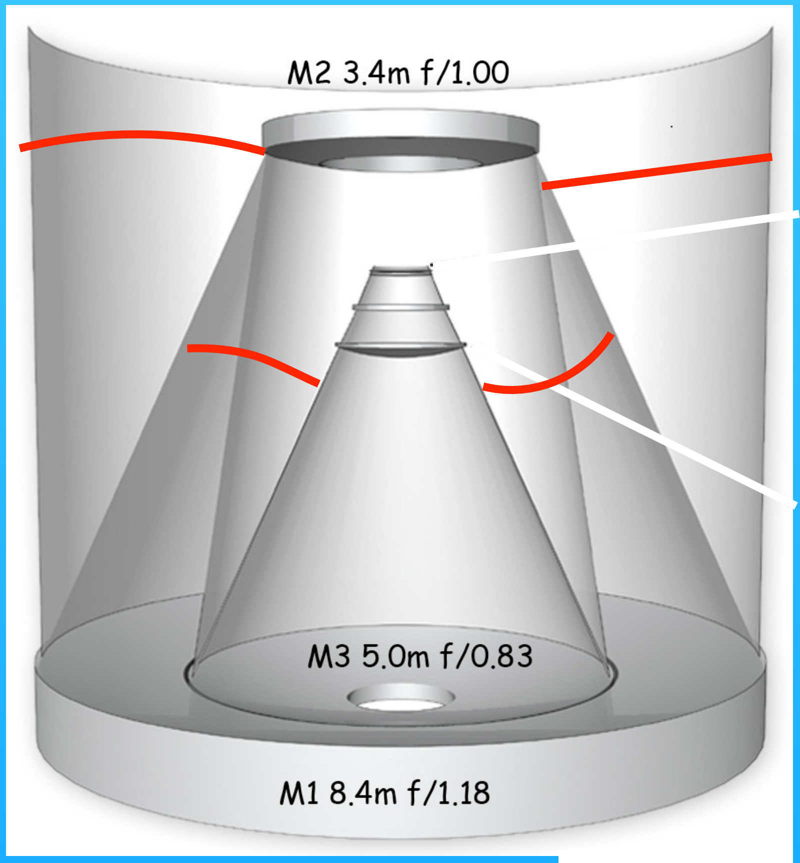
autocorrelation function

D varies as 2/3 power of separation r

$$D_n(r) = 2(A_0 - A(r)) = \underline{C_n^2(z)}\, r^{2/3}$$

structure function

refractive index structure constant varies with altitude z



LSST

$$\tilde{C}(\kappa) = 0.033 C_n^2(z) |\kappa|^{-11/3} \qquad \frac{1}{L_0} < |\kappa| < \frac{1}{l_0}$$

power spectrum

wavenumber

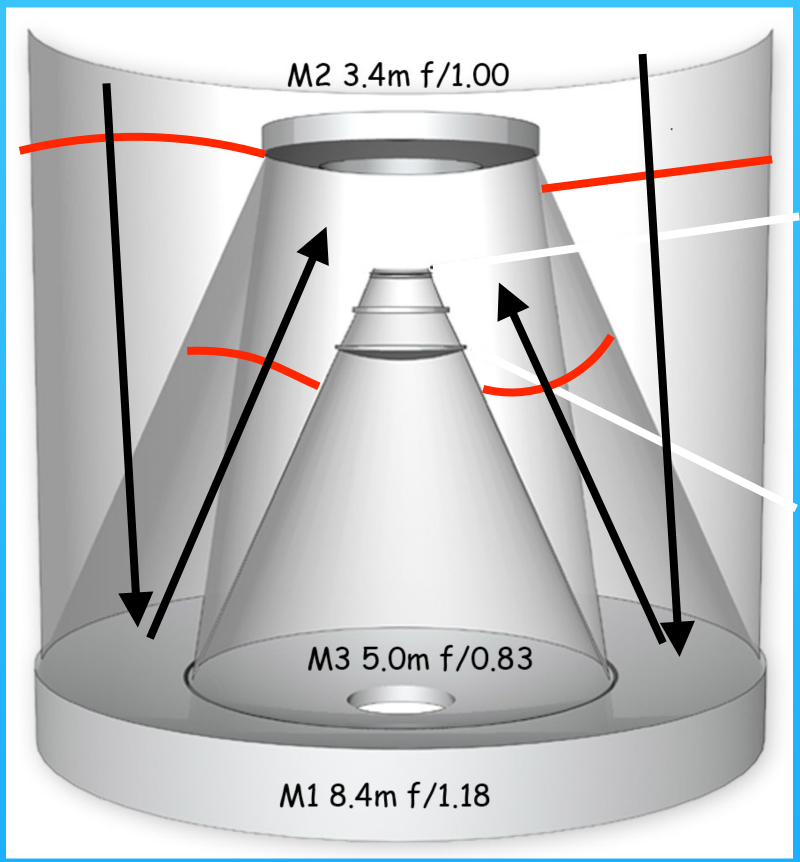
outer and inner scales

Dainty 2000

PSF basics: optics

- astronomical telescope (in Fraunhofer condition) forms image as Fourier transform of wavefront at (exit) pupil
- telescope aperture and optical elements modulate the wavefront amplitude
- idealised circular aperture -> Airy profile
- departures from ideal & alignment errors introduce optical path delays
- atmosphere and telescope optical path delays enter the Fourier transform as wavelength-dependent phases
- these effects are multiplicative in the (complex) Fourier domain -> convolutions in image domain
- the detector also modifies the image - dominant effects also convolutional, but not all

$$I(\theta) = I_0 \left[\frac{2 J_1(ka\sin\theta)}{ka\sin\theta} \right]^2$$



PSF basics: distortion

- as well as convolution, atmosphere and telescope cause distortion - locally affine transformation (shear, rotation, magnification)
- Fraunhofer condition relates complex wavefront angular to angular PSF

spectrum of detected photons

wavenumber

$$F_{\boldsymbol{\theta}} = \int \mathrm{d}\nu \, n(\nu) N(\nu)^{-1} \left| \sum_m a_m e^{\mathrm{i}\boldsymbol{k}_m^T \boldsymbol{\theta}} \right|^2$$

angular coordinate

normalisation

complex wavefront field

photon frequency

a,k

- note implicit wavelength dependence in a,k
- apply local affine transforma $\boldsymbol{\theta} = \mathsf{A}\boldsymbol{x}$, angular to detector coords

$$F_{\boldsymbol{x}} = \int \mathrm{d}\nu \, n(\nu) N(\nu)^{-1} \left| \sum_m a_m e^{\mathrm{i}\boldsymbol{k}_m^T \mathsf{A}\boldsymbol{x}} \right|^2$$

- atmospheric distortion is chromatic

photo R.Wayman

PSF basics: scattered light

- scattered light caused by telescope contamination
- reflections within optical system/camera
- wavelength-dependent wings on PSF

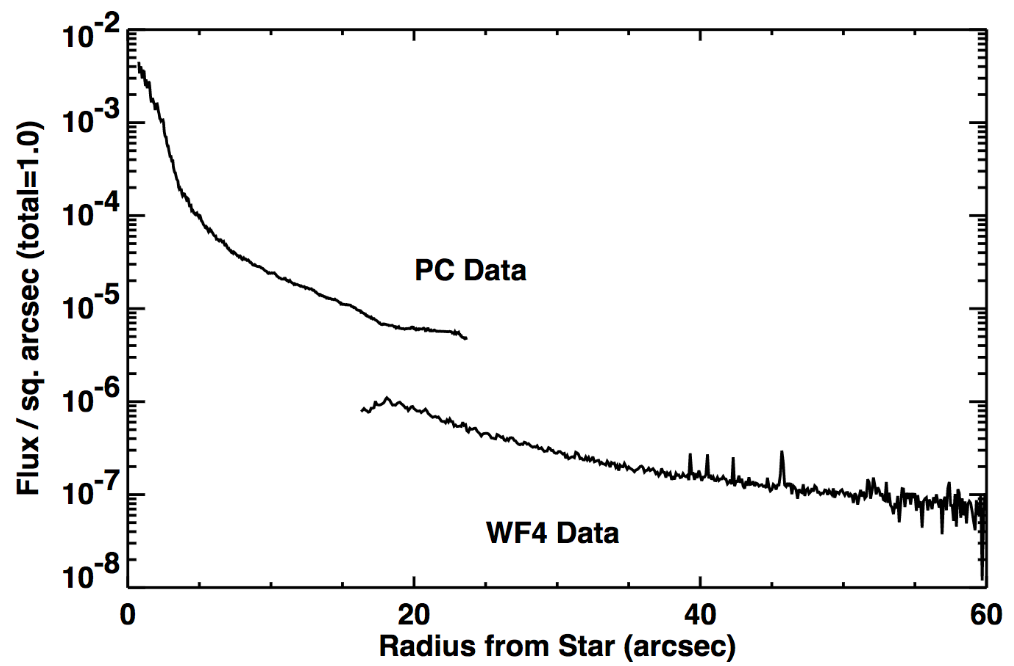
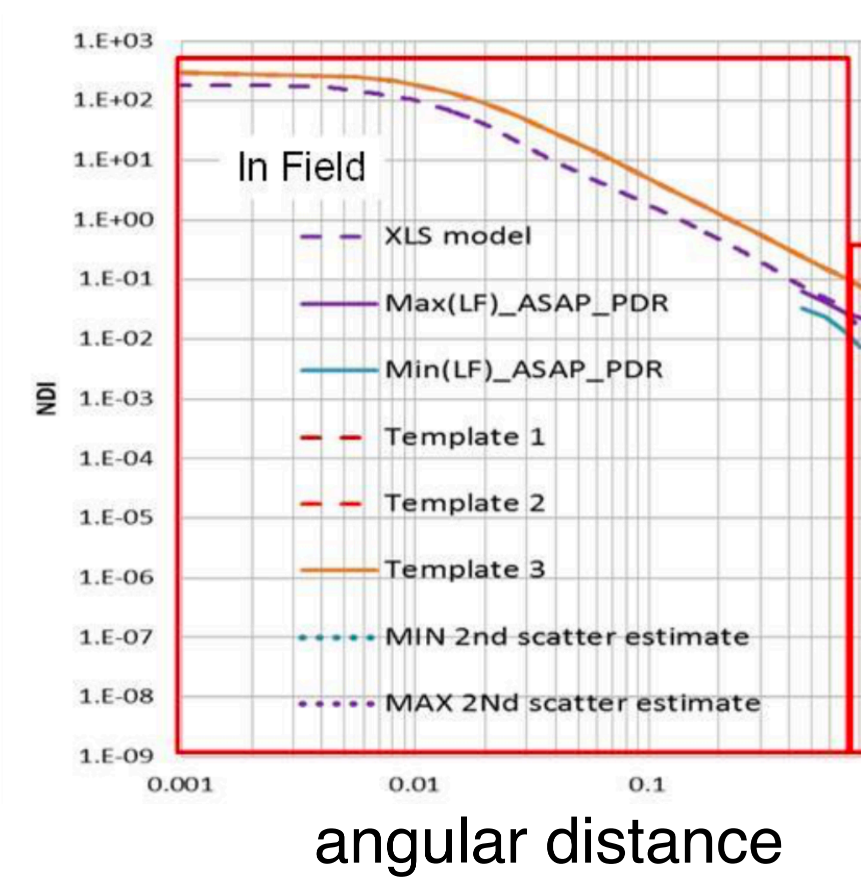


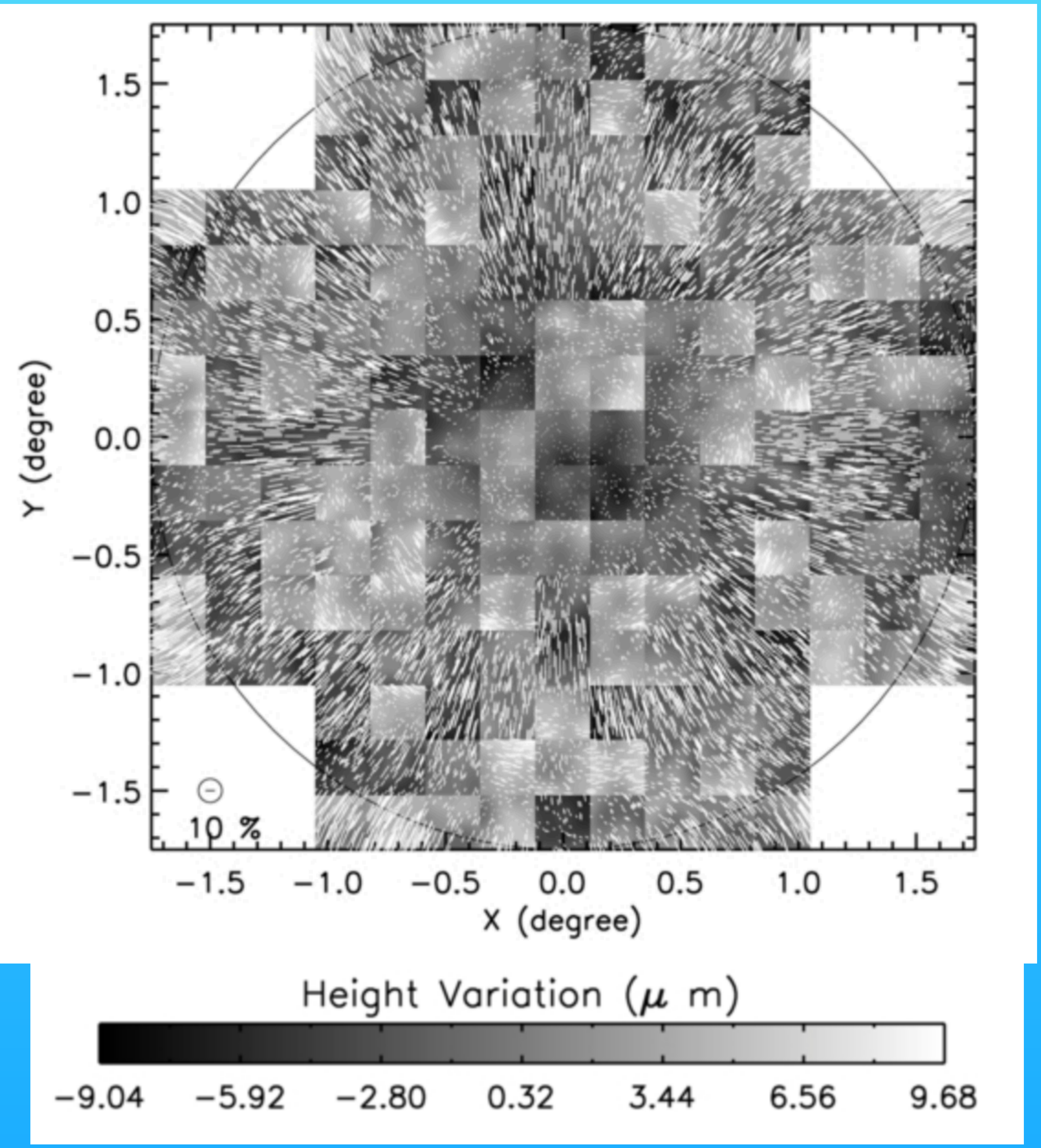
Fig. 3. Azimuthal average plot of a star centered in the PC. The background is higher in the camera containing the star due to CCD scatter. The total flux is normalized to one.

HST WFPC2 scattered and reflected light from surface of front-illuminated CCDs: Krist 1995



Euclid scattered light model

PSF basics: focal plane

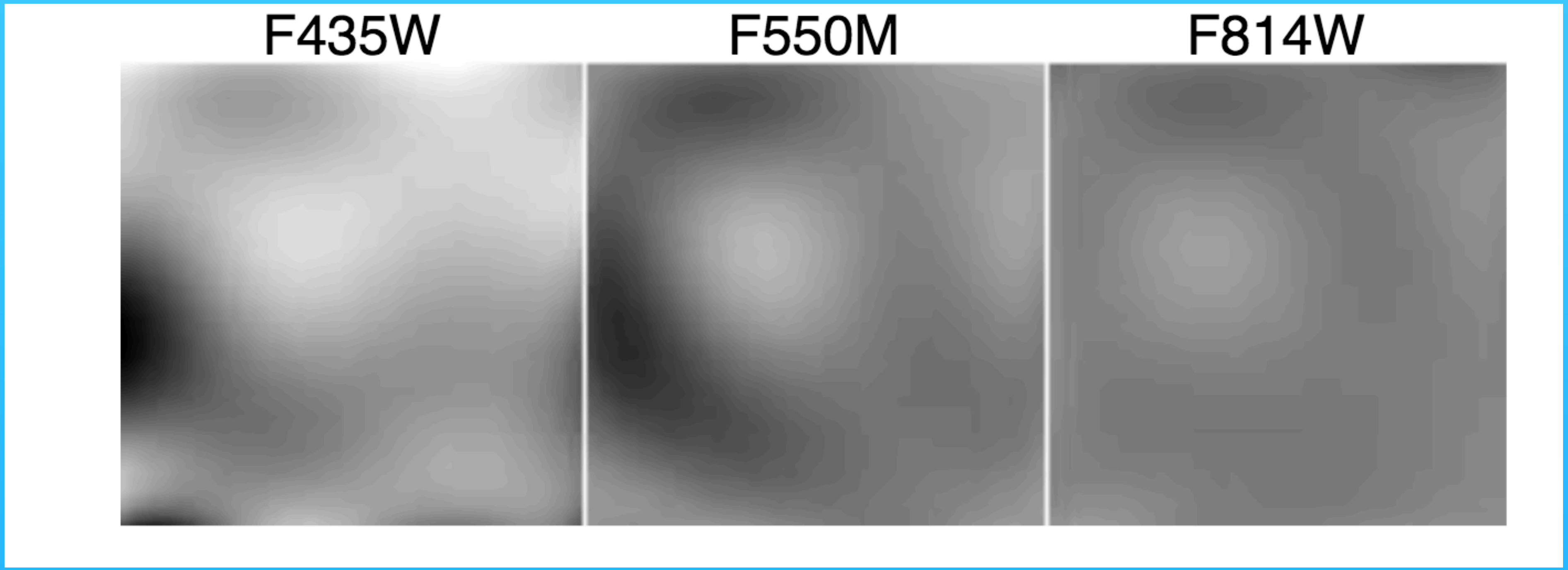


- CCD height/alignment variations
- important in fast telescopes e.g. LSST
  - wavelength dependent
- causes low order aberrations
- discontinuous at CCD boundaries

simulated effect of CCD height variations on PSF ellipticity: Jee & Tyson 2011

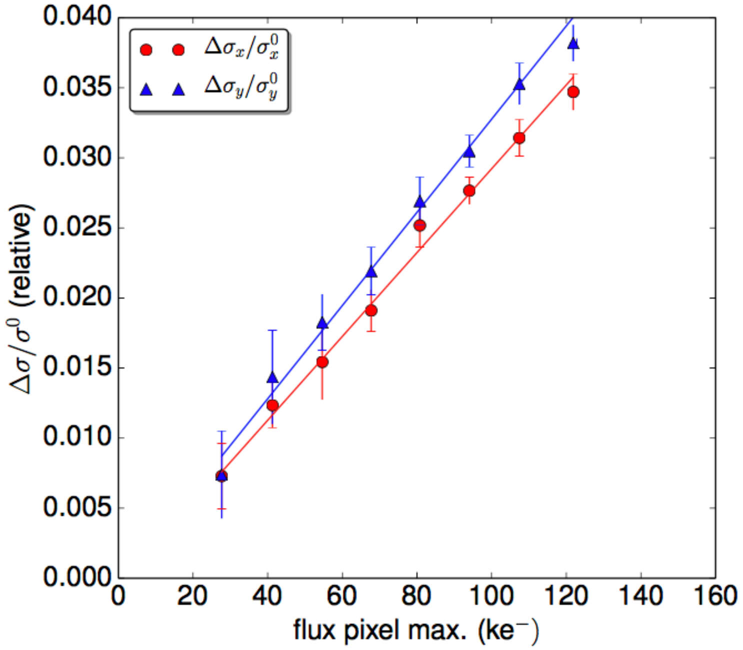
PSF basics: detector charge diffusion

- charge diffusion of electrons between pixels
- depends on depth in substrate (distance from electrodes) and hence wavelength
- varies with thickness and hence varies over field
- major contributor to LSST camera PSF

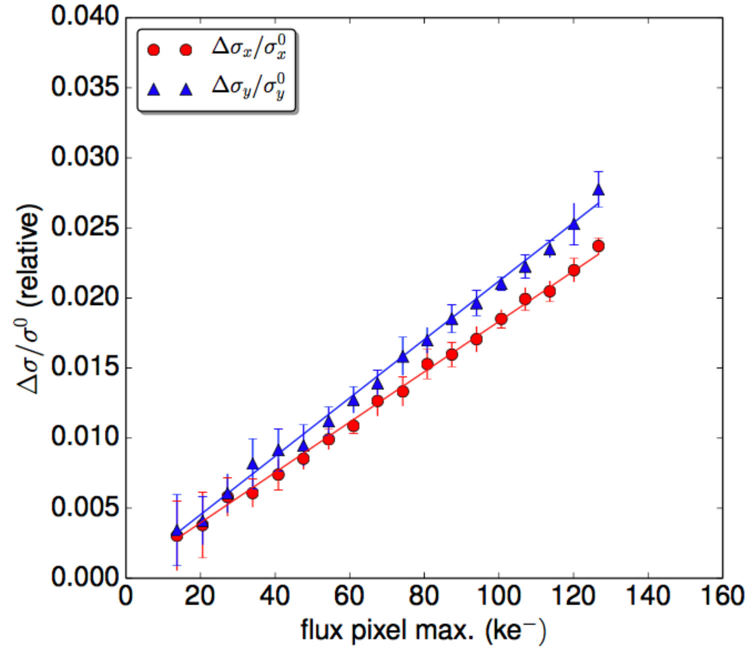


width of diffusion kernel at 3 wavelengths
HST ACS: Krist (2003)
correlates with CCD thickness

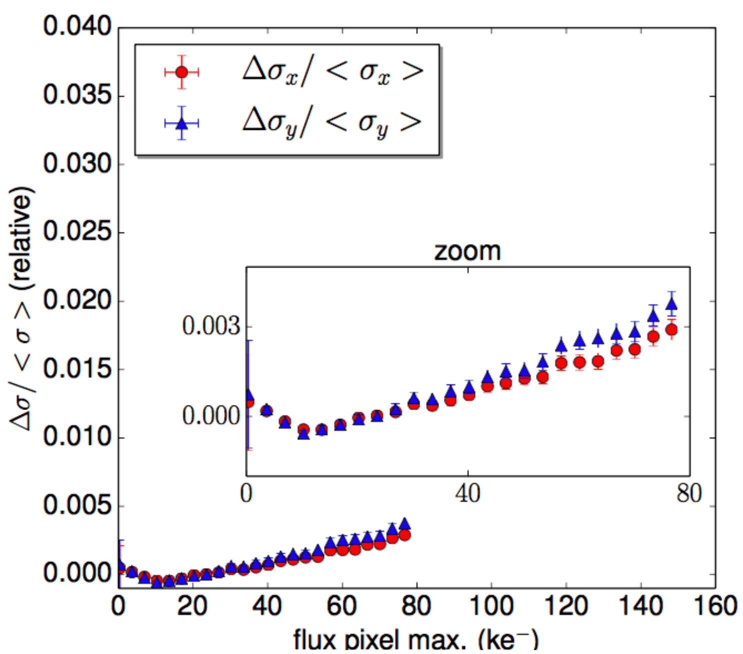
PSF basics: detector brighter-fatter effect



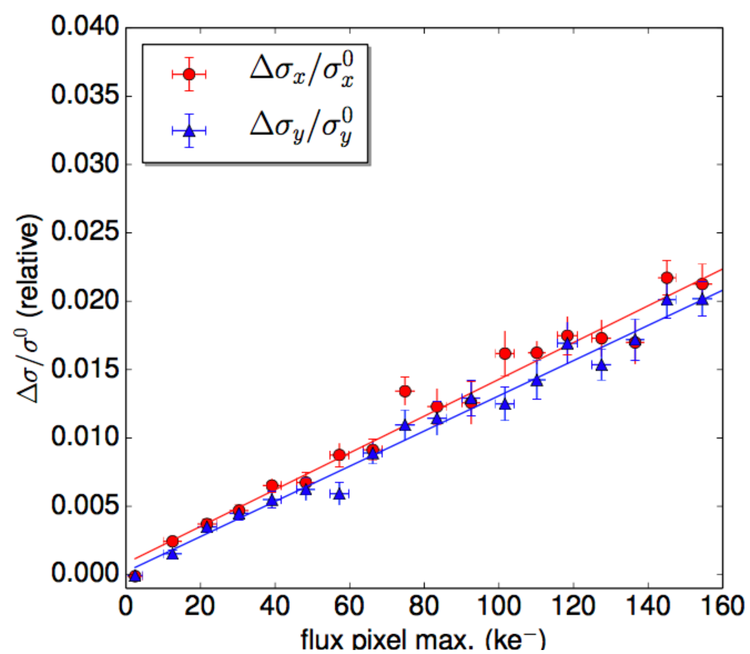
LSST 550nm



LSST 900nm



CFHT Megacam r band



DEcam r band

- as charge accumulates it modifies pixel boundary electric fields
- pixels appear smaller on sky with increasing flux
- worse for deep depletion devices (DECam, HSC, LSST)
- depends on wavelength

variation in effective PSF size with flux and wavelength: Guyonnet et al 2015

how well do we need to know the PSF?

- (weighted) centred second moments of image defined as

$$Q_{ij} = \frac{\int w(x) f(x - x_0) x_i x_j d^2 x}{\int w(x) f(x - x_0) d^2 x}$$

weight function needed to suppress noise in data and ensure finite behaviour for Airy profiles

centroid

- size-squared and ellipticity defined as

$$R^2 = Q_{11} + Q_{22}$$

size-squared

$$\chi = \left\{ \frac{Q_{11} - Q_{22}}{Q_{11} + Q_{22}}, \frac{2Q_{12}}{Q_{11} + Q_{22}} \right\}$$

ellipticity has 2 components

$$\epsilon = \left\{ \frac{Q_{11} - Q_{22}}{Q_{11} + Q_{22} + 2|Q|^{1/2}}, \frac{2Q_{12}}{Q_{11} + Q_{22} + 2|Q|^{1/2}} \right\}$$

alternative ellipticity definitions

write as complex ellipticity expressed in terms of axis ratio and position angle

$$\chi = \frac{(1 - q^2)}{(1 + q^2)} \exp[2i\theta]$$

Schneider 2005 arxiv:0509252

how well do we need to know the PSF?

- unweighted, centred second moments of convolving functions add

$$R^2 = R_G^2 + R_{\rm optics}^2 + R_{\rm atm}^2$$
$$R^2\chi = R_G^2\chi_G + R_{\rm optics}^2\chi_{\rm optics} + R_{\rm atm}^2\chi_{\rm atm}$$

observed Q | galaxy | optics | atmosphere

- infer moments of the galaxy from some observed moments

$$(R_G^2\chi_G)' = R^2\chi - R_{\rm optics}^2\chi_{\rm optics} - R_{\rm atm}^2\chi_{\rm atm}$$

- consider uncertainties in optics PSF, $\Delta R_{\rm optics}^2, \Delta\chi_{\rm optics}$

$$\Delta\chi_G{}' = \frac{\Delta R_{\rm optics}^2}{R_G^2}\chi_G - \left(\frac{R_{\rm optics}^2}{R_G^2}\Delta\chi_{\rm optics} + \frac{\Delta R_{\rm optics}^2}{R_G^2}\chi_{\rm optics}\right) = m\chi_g - c$$

multiplicative term arising from PSF size error

additive term arising from both size and ellipticity error

Hirata et al 2004
Paulin-Henrikssen et al 2009
Massey et al 2013

how well do we need to know the PSF?

- write shear estimate as

$$\hat{g} = \frac{\langle \chi_G \rangle}{1-\langle \chi_G^2 \rangle} \simeq \frac{1}{1-\langle \chi_G^2 \rangle} \frac{\langle \chi_G{}' + c \rangle}{1+m}$$

- combine biases from all sources

$$m = \frac{\Delta R_{\text{optics}}^2 + \Delta R_{\text{atm}}^2 + \Delta R_{\text{det}}^2}{R_G^2}$$

$$c = \frac{R_{\text{optics}}^2 \Delta\chi_{\text{optics}} + R_{\text{atm}}^2 \Delta\chi_{\text{atm}} + R_{\text{det}}^2 \Delta\chi_{\text{det}}}{R_G^2} + \frac{\Delta R_{\text{optics}}^2 \chi_{\text{optics}} + \Delta R_{\text{atm}}^2 \chi_{\text{atm}} + \Delta R_{\text{det}}^2 \chi_{\text{det}}}{R_G^2}$$

- relative contributions for LSST:

	FWHM	FWHM^2
atmosphere	0.5 arcsec	0.25 arcsec^2
telescope	0.2	0.04
camera	0.3	0.09

how well do we need to know the PSF?

- link to cosmology - e.g. Amara & Refregier 2008, Massey et al 2013
- in principle m could be calibrated

  - require uncertainty in m for LSST/Euclid $\sigma(m) \leq 0.002$

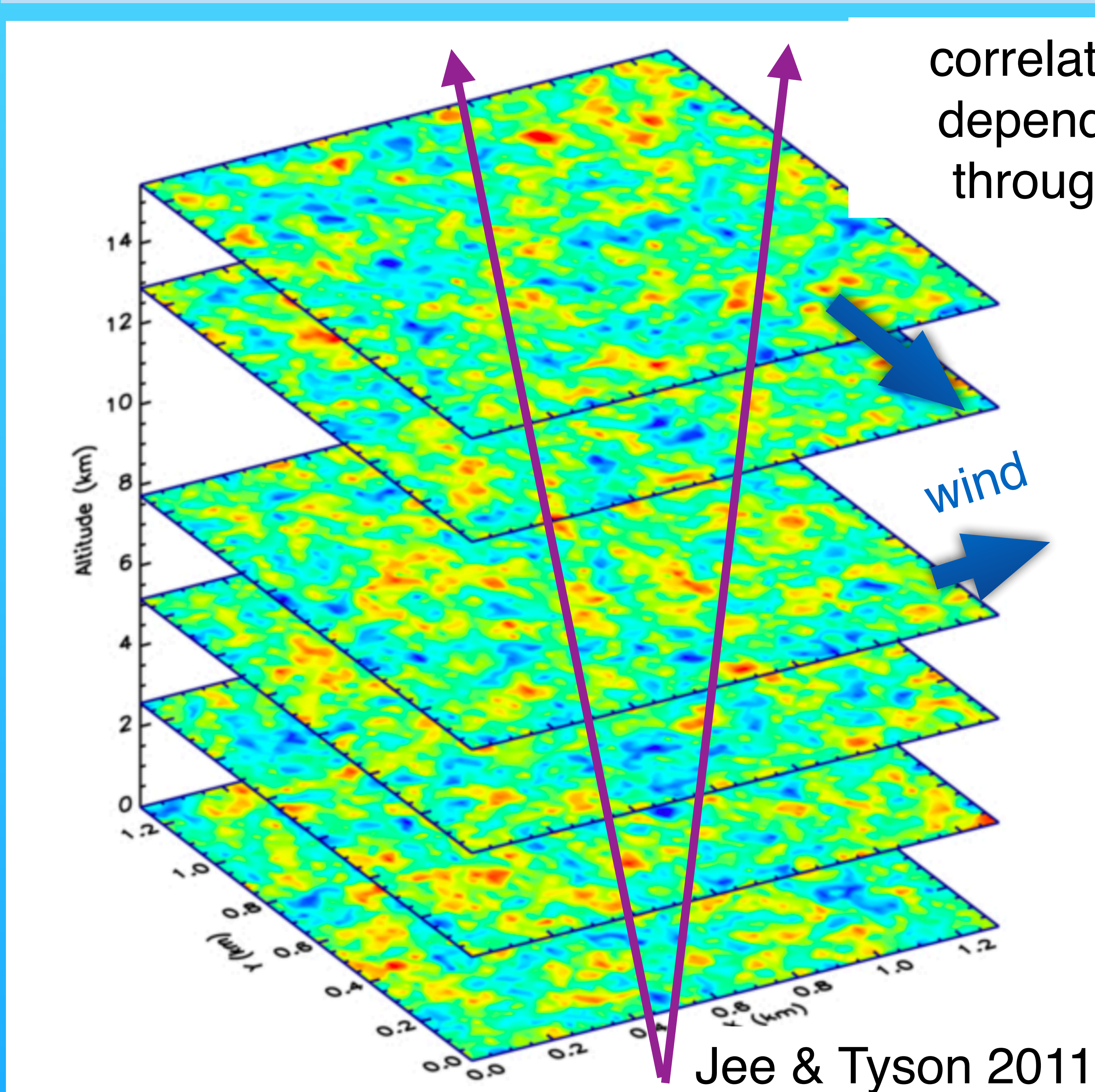
- m depends on $\dfrac{\Delta R^2_{PSF}}{R^2_G}$

- hence require knowledge of PSF size to 0.1 percent
- very challenging!
- much harder target to achieve from the ground

Measuring the PSF: atmosphere

- Rapidly varying wavefront phases, strongly dependent on direction
- very short integrations show “speckle pattern”
- In long integrations average out to produce PSF with broad wings

Dainty & Fienup 1987



Jee & Tyson 2011

correlation across FoV depends on sightlines through 3D structure

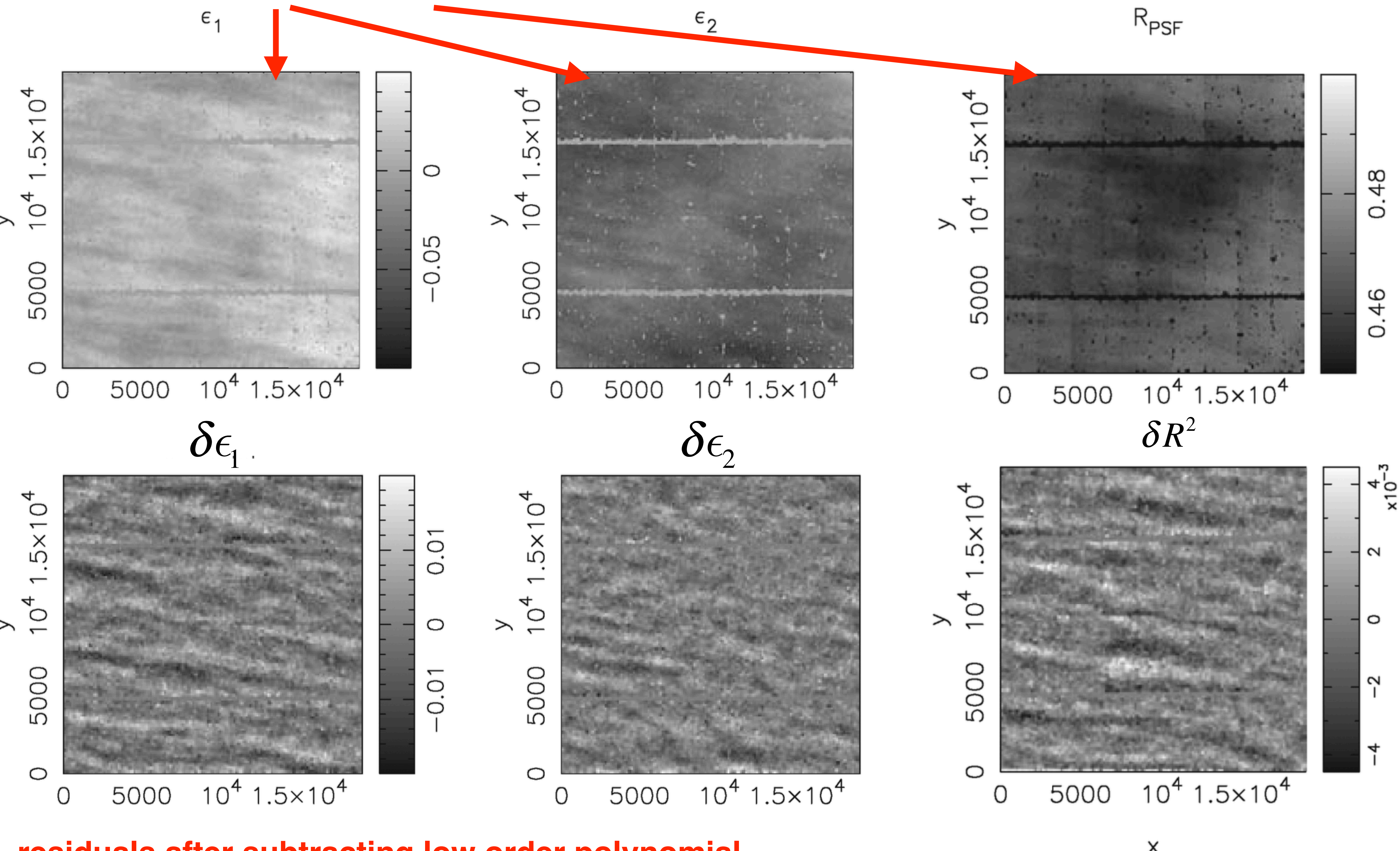
time evolution depends on multilayer wind and turbulence evolution

“ground layer” and “dome seeing” turbulence can be significant contributors at conventional observatories

PSF variations across 75s exposures at CFHT

variation in ellipticity and size across 1 deg field

Heymans et al 2012



residuals after subtracting low order polynomial

PSF variations across 75s exposures at CFHT

variation in ellipticity and size across 1 deg field

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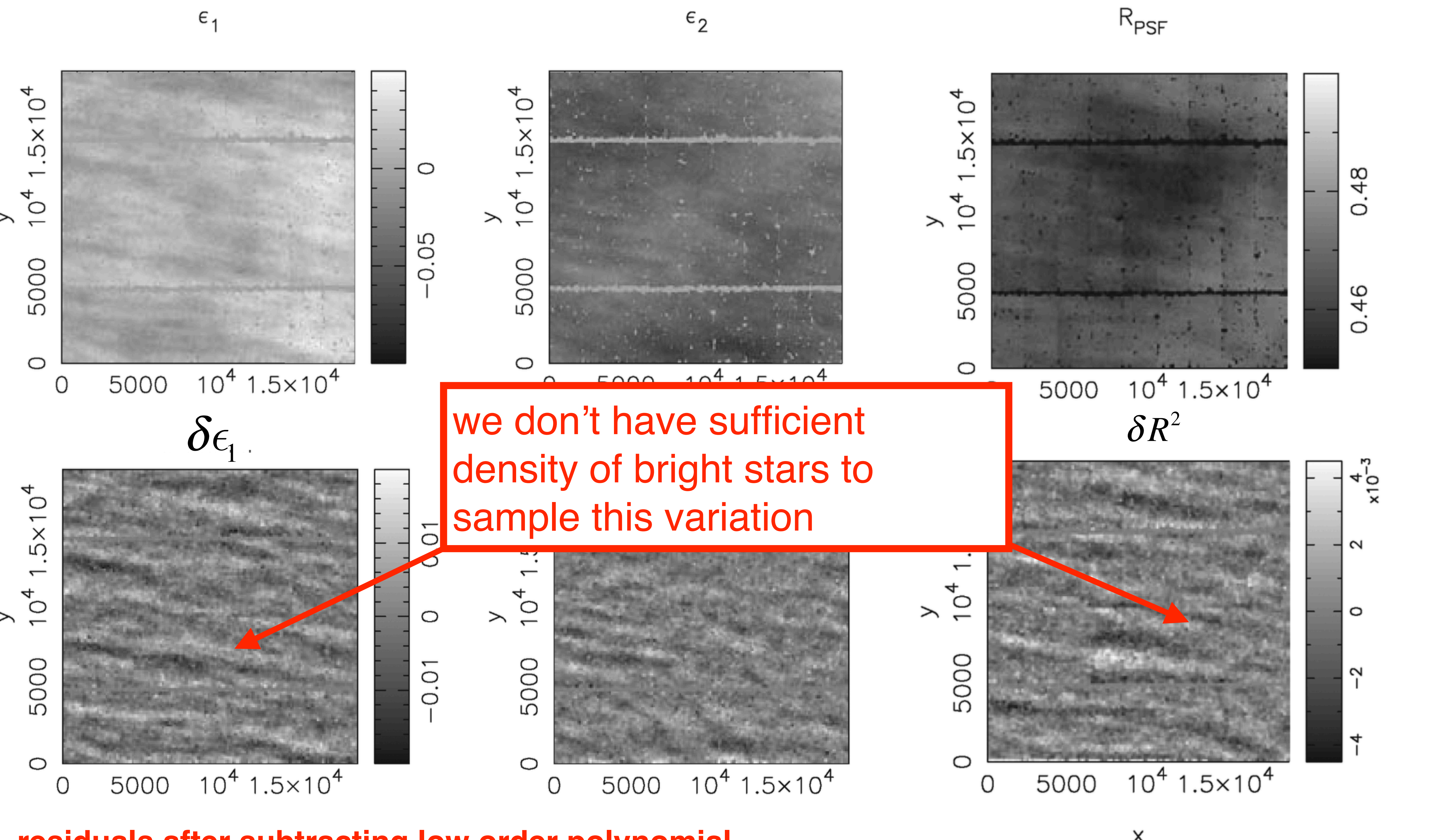
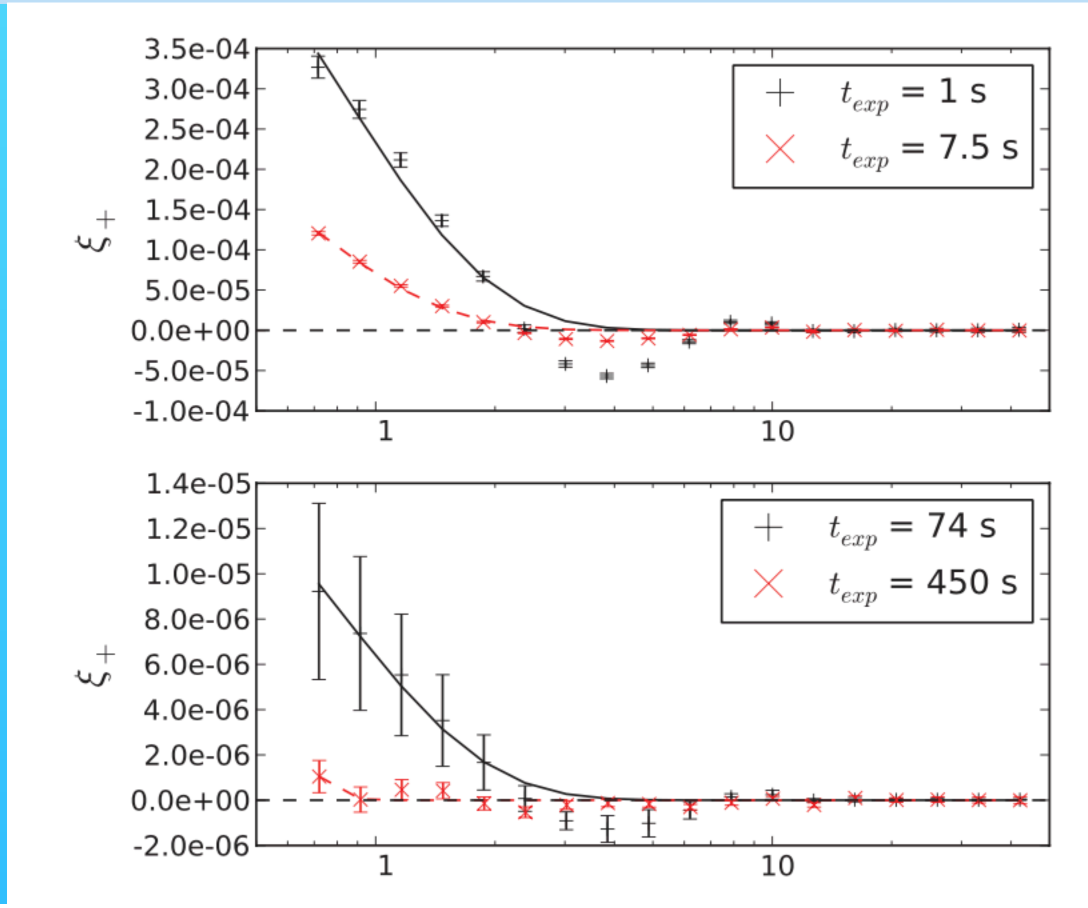


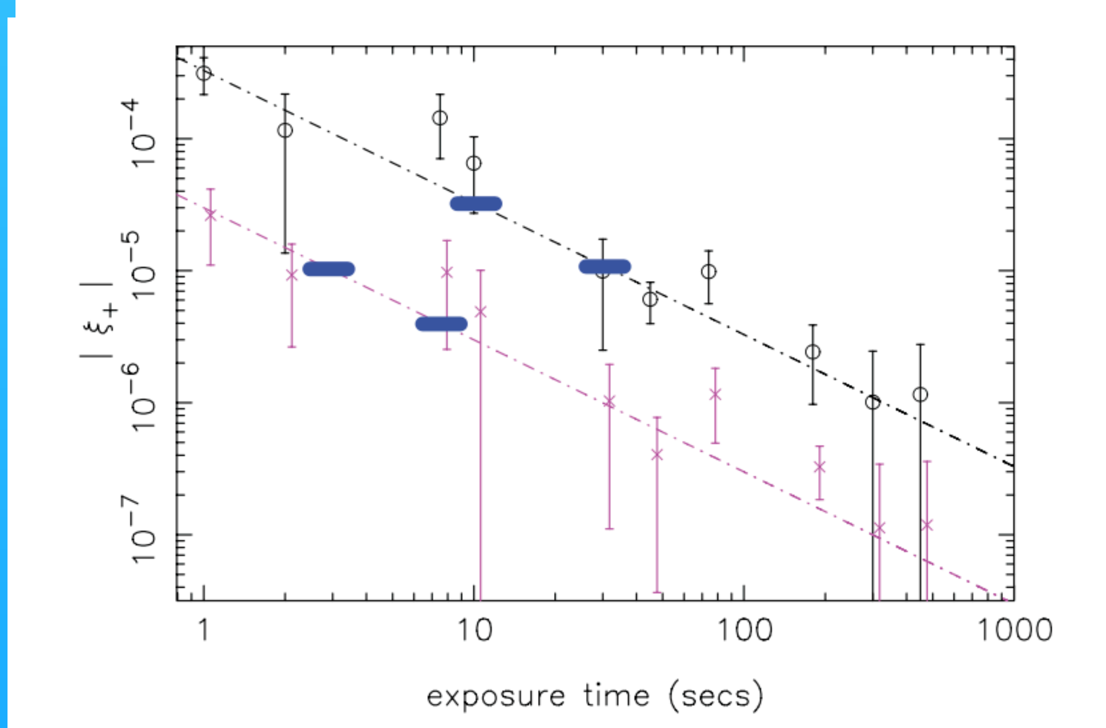
we don't have sufficient density of bright stars to sample this variation

residuals after subtracting low order polynomial

Measuring the PSF: atmosphere

- In weak lensing, care about angular autocorrelation function of residuals



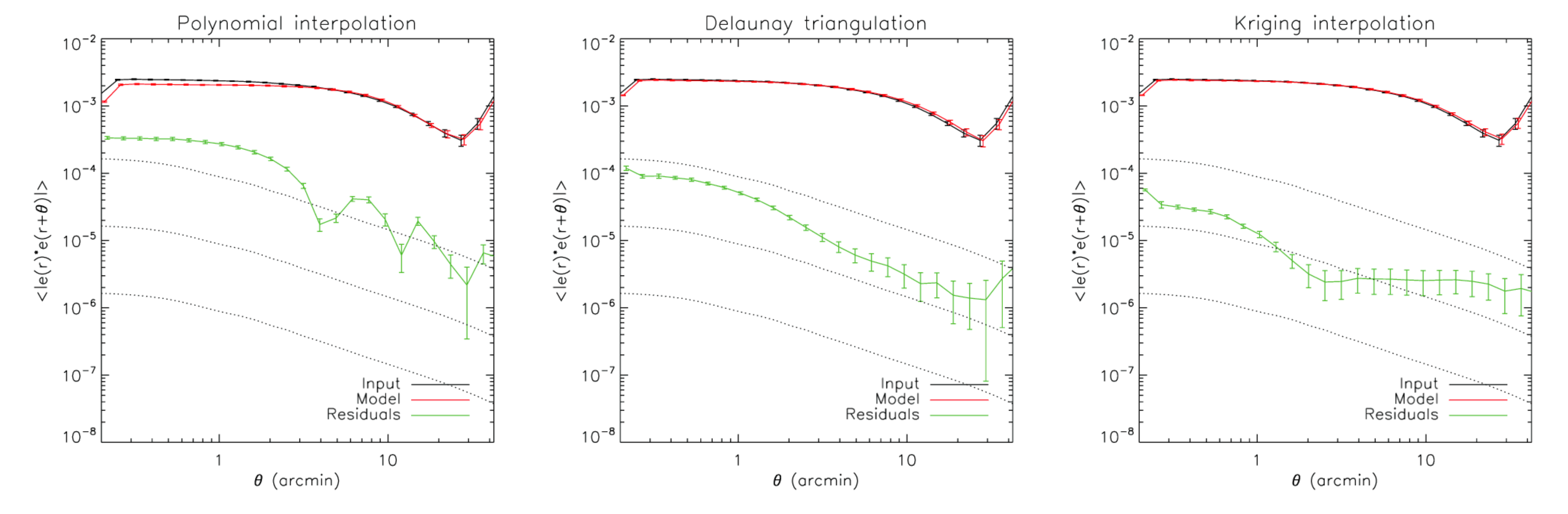


- Produces correlated signal on same scale as the cosmic signal
- amplitude decreases as integration time $t^{-1/2}$ (de Vries et al 2007, Jee & Tyson 2011, Cook et al 2013, Heymans et al 2012)
- expect uncertainty in ellipticity variation to average away as N^{-1} for N exposures (Heymans et al 2012)
- but size-squared R^2 must be measured, has non-central distribution, need to estimate mean effect
- **size variations/uncertainty are probably the dominant worry**

Measuring the PSF: ground based surveys

- Fit functions in some choice of basis set: examples -
  - pixels (lensfit, Miller et al 2007-13; PSFex, Bertin & Moneti 2010-17)
  - Gauss-Laguerre “shapelets” (PSFex; GaaP, Kuijken 2008)
  - Principal Components (Jee et al 2007, Jee & Tyson 2011)

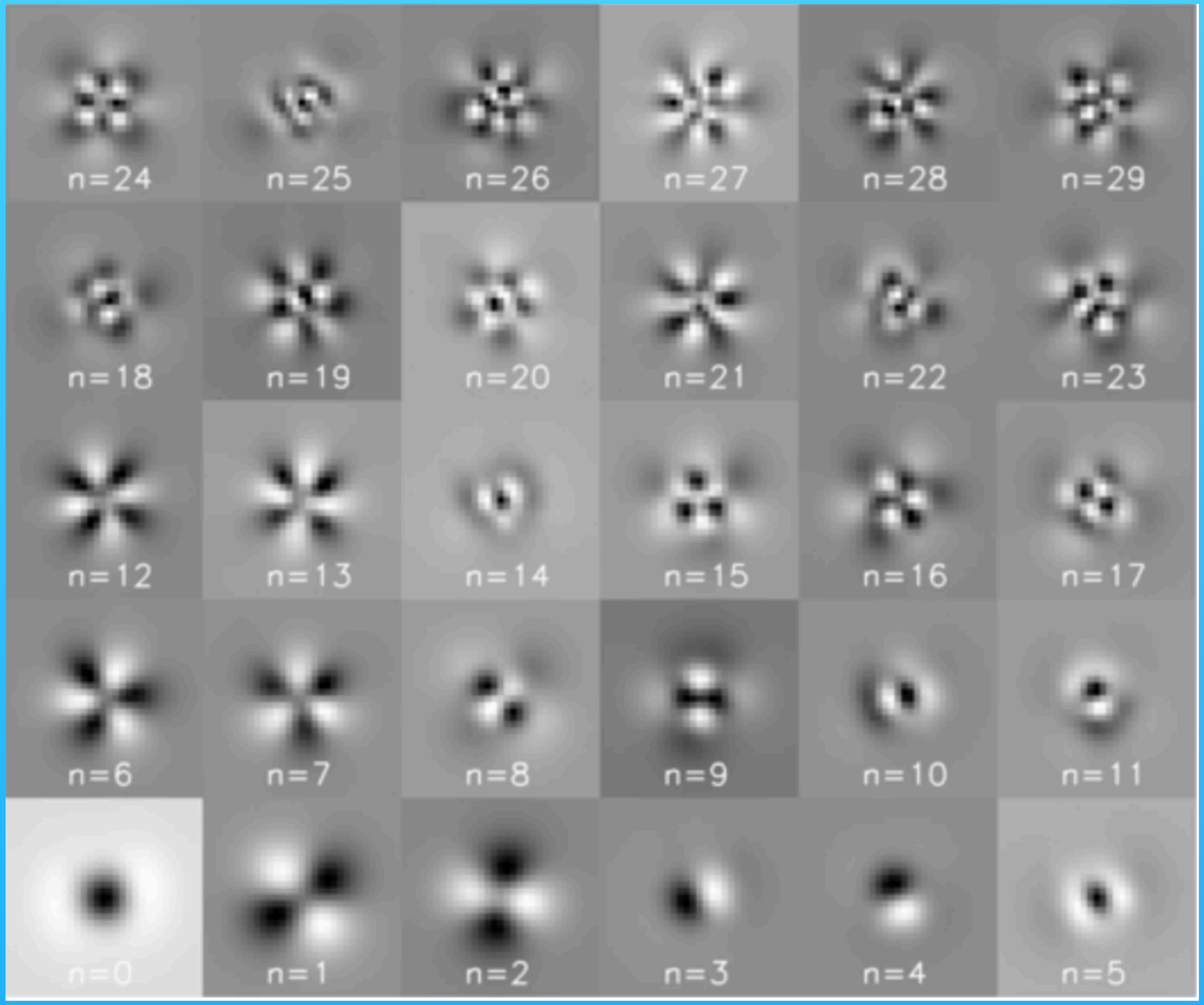
- Choose interpolating function
  - see Bergé et al 2012
  - multiscale analysis (e.g. Chang et al 2012)



Measuring the PSF: Principal Components

- principal components in principle powerful way to capture chief variations in a dataset
- additive variations may not be well-matched to telescope PSF variation

  - e.g. size variations are not additive & could generate many principal components



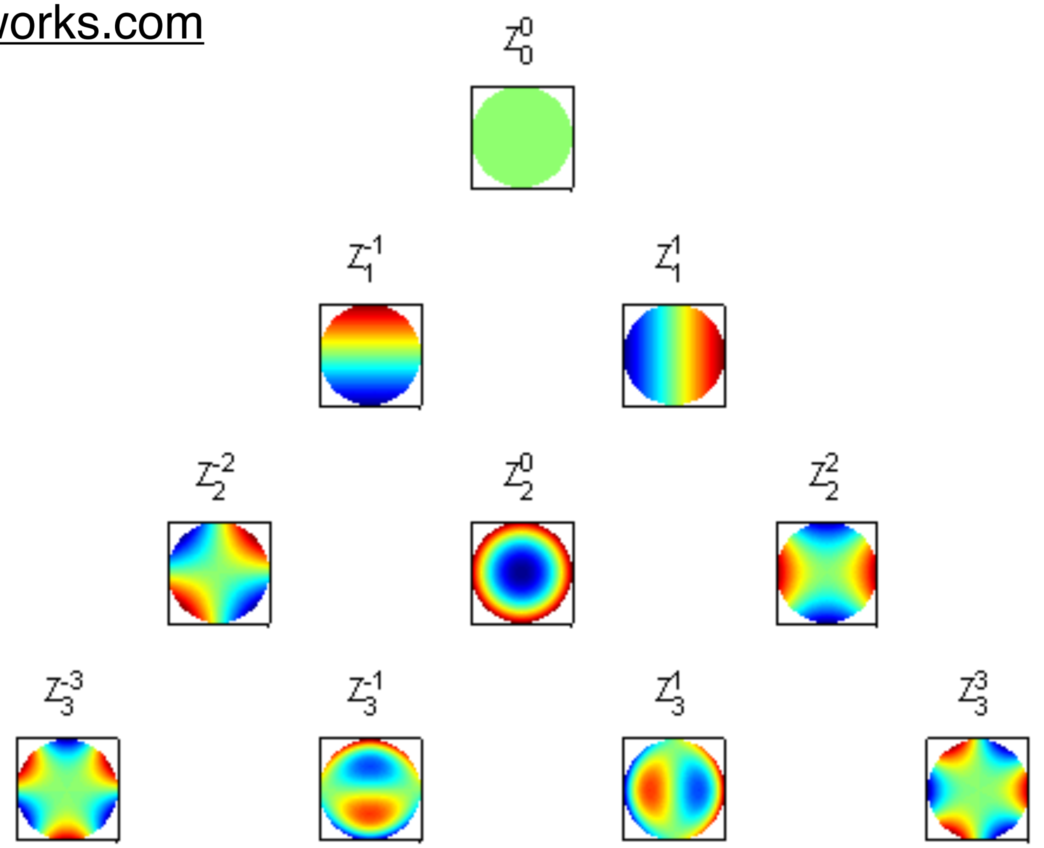
- any truncation of basis sets is liable to cause bias

Jee & Tyson 2011

Measuring the PSF: wavefront domain

- Fourier-domain basis set
  - dominant causes of variation in optical PSF (telescope & atmosphere) are phases in the wavefront domain
  - the chromatic optical PSF may be calculated by applying linear wavelength dependences to wavenumbers and wavefront amplitudes
  - use interpolating function in wavefront domain
  - additional convolution effects (detector and guiding) easily included
  - telescope (& atmosphere distortion) may be applied to wavefront (& autocorrelation) domain

mathworks.com



$$F_{\boldsymbol{x}} = \int \mathrm{d}\nu\, n(\nu) N(\nu)^{-1} \left| \sum_m a_m e^{\mathrm{i}\boldsymbol{k}_m^T \mathsf{A} \boldsymbol{x}} \right|^2$$

- low order wavefront phase modes well described by Zernike polynomials
- interpolate Zernike amplitudes across focal plane
- higher orders may as well use grid basis set
- mode amplitudes can be free parameters

e.g. Euclid PSF modelling

Measuring the PSF: phase retrieval

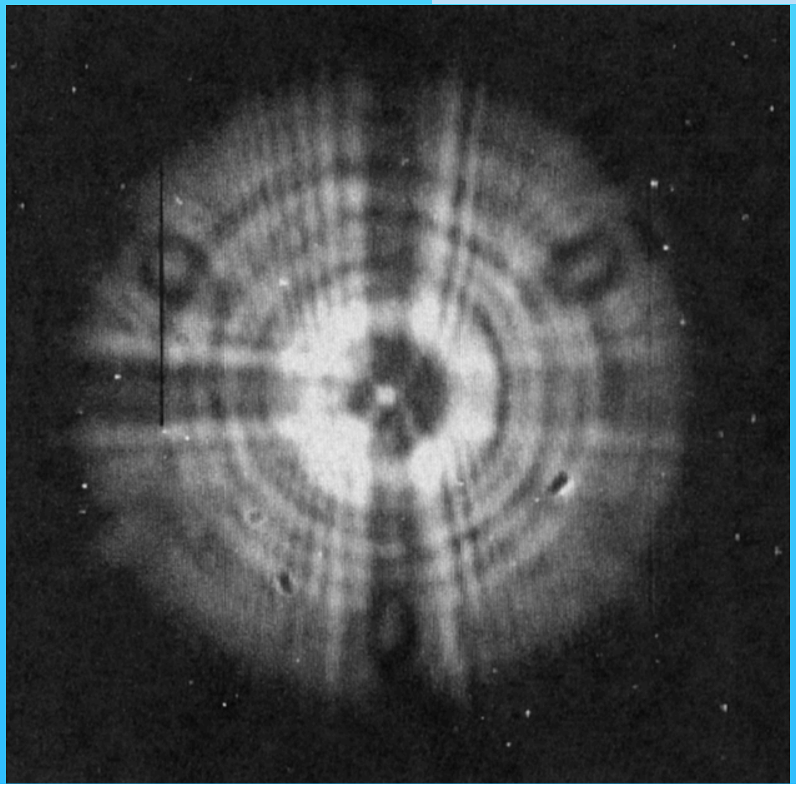
- PSF intensity is modulus-squared of FT of exit pupil wavefront, integrated over wavelength
- wavefront amplitude distribution usually known and invariant
- wavefront phases are usually not known and variable

$$F_{\boldsymbol{x}} = \int \mathrm{d}\nu\, n(\nu) N(\nu)^{-1} \left| \sum_m a_m e^{\mathrm{i}\boldsymbol{k}_m^T \mathsf{A}\boldsymbol{x}} \right|^2$$

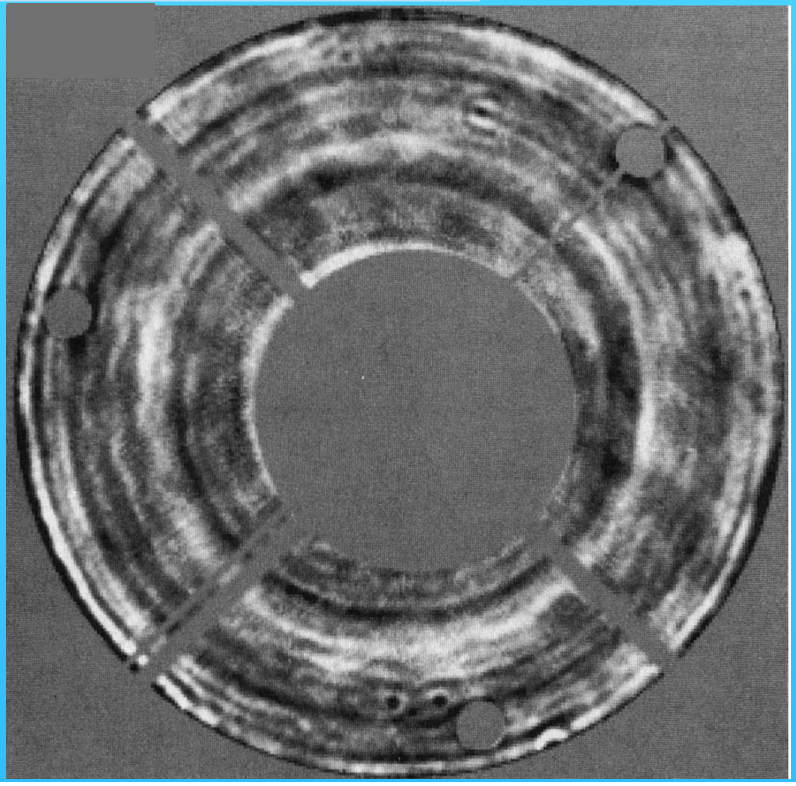
- Intensity is a real quantity, phase information of electric field at image plane has been lost
  - PSF intensity is FT of wavefront autocorrelation function
- Can we recover the wavefront phases?
  - not in 1D case, no unique solutions
  - in 2D lots of redundancy between wavevectors
  - wavefront phases may be found by iterative methods
- Degeneracies still exist
  - may be broken using out-of-focus images

Measuring the PSF: phase retrieval

- Mature field in microscopy
- Used to diagnose faulty HST optics
- Measure HST mirror errors purely from in-orbit data!



defocussed HST data
Krist & Burrows 1995

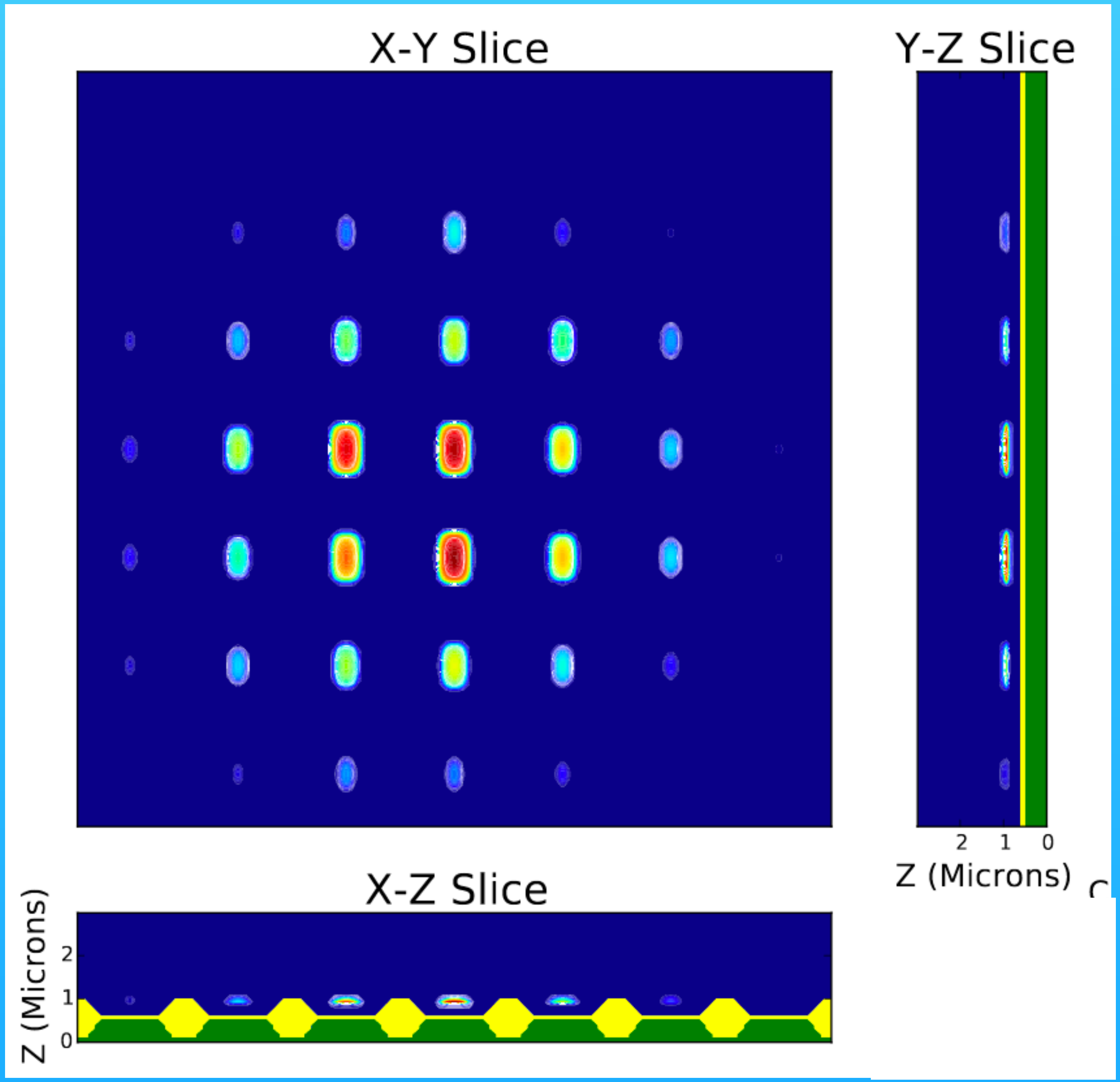


inferred HST primary mirror
surface errors, 30nm amplitude

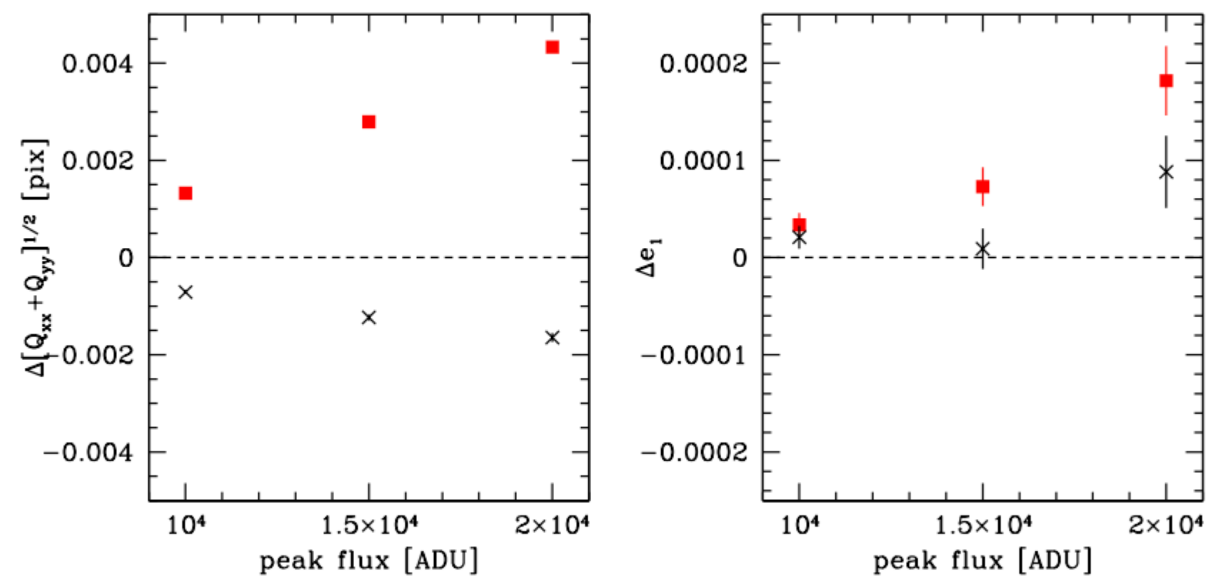
- Used in ground based active optics (slow telescope correction) and adaptive optics (fast seeing correction)
- Large de-focus - “curvature sensing” (Roddier & Roddier 1993) - LSST & DECam wavefront sensors (Xin et al 2013) - move the detector, telescope beam unchanged
- Small defocus - phase diversity (Fienup 1998) - fixed detector, defocus telescope, changes beam

Measuring the PSF: detector brighter-fatter effect

- two methods to measure B-F effect in lab
  1. project spots onto detector, vary intensity - difficult to achieve sufficiently small, accurate spots (e.g. Tyson et al 2014, Niemi et al 2015) and confused with non-linear QE (Gruen et al 2015)
  2. measure noise covariance in flat-field images - B-F introduces pixel covariance (Antilogus et al 2014)
- basic model based on (2) only corrects ~90% of effect (Gruen et al 2015)
- effect depends on silicon resistivity, hence varies between chips



residual size & ellipticity variation with flux after correction - Gruen et al 2015



- improved model informed from 3D modelling of distribution of charge and fields in substrate - see Lage et al 2017

Measuring the PSF: multiple exposures

- Usually observe multiple, dithered exposures to improve sampling, artefact rejection (e.g. cosmic rays) and fill-in of gaps/bad pixels
- Coaddition requires interpolation of data

  - distorts PSF
  - introduces pixel covariance
  - CCD gaps and edges introduce discontinuous changes in PSF - hard to model from coadd

1. Measure PSFs on individual exposures, but at each position calculate coadded PSF taking into account interpolation, which exposures used, infer covariance (e.g. Bosch et al 2017)
2. Make measurements only from individual exposures, joint model fitting (e.g. lensfit, Miller et al 2013)

- both methods require very careful handling of residual astrometric registration errors

Measuring the PSF quality

- measure residuals between stars and PSF model
- compute rho/Rowe statistics

$$\rho_1(\theta) = \left\langle \delta\epsilon_i \delta\epsilon_j^\star \right\rangle \text{for pairs i, j separated by } \theta$$

$$\rho_2(\theta) = \left\langle \delta\epsilon_i \delta\epsilon_j^\star \right\rangle$$

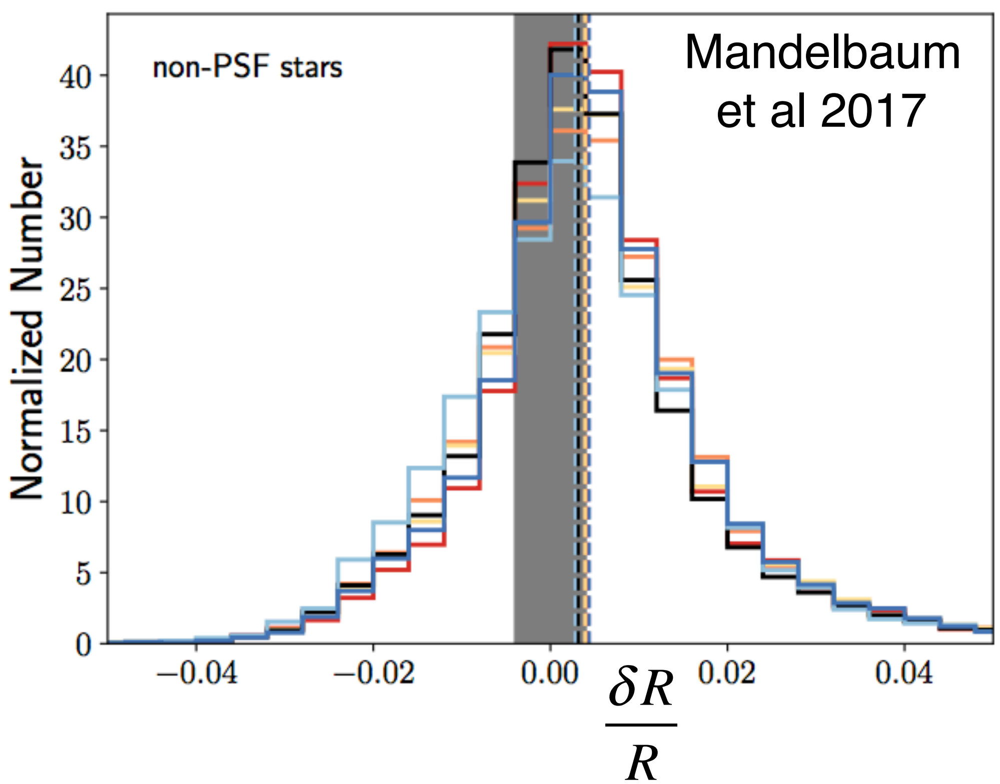
Rowe et al 2010

Jarvis et al 2016

$$\rho_3(\theta) = \left\langle \epsilon_i \frac{\delta R_i^2}{R_i^2} \epsilon_j^\star \frac{\delta R_j^2}{R_j^2} \right\rangle$$

$$\rho_4(\theta) = \left\langle \epsilon_i \frac{\delta R_i^2}{R_i^2} \delta\epsilon_j^\star \right\rangle$$

$$\rho_5(\theta) = \left\langle \epsilon_i \frac{\delta R_i^2}{R_i^2} \epsilon_j^\star \right\rangle$$

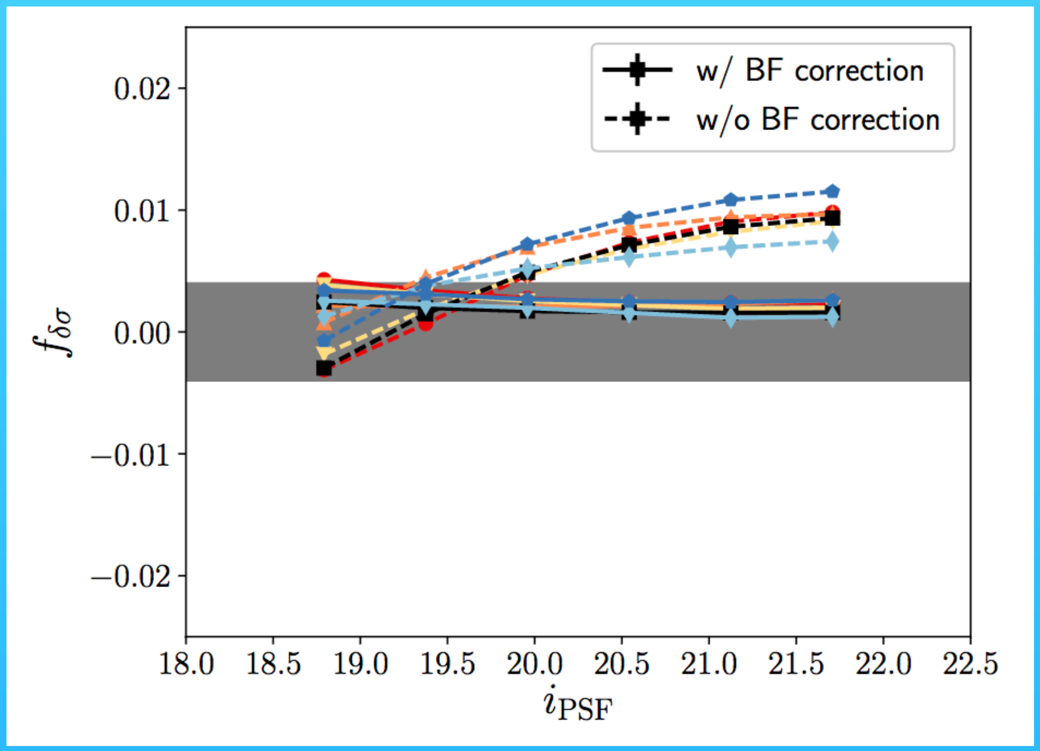


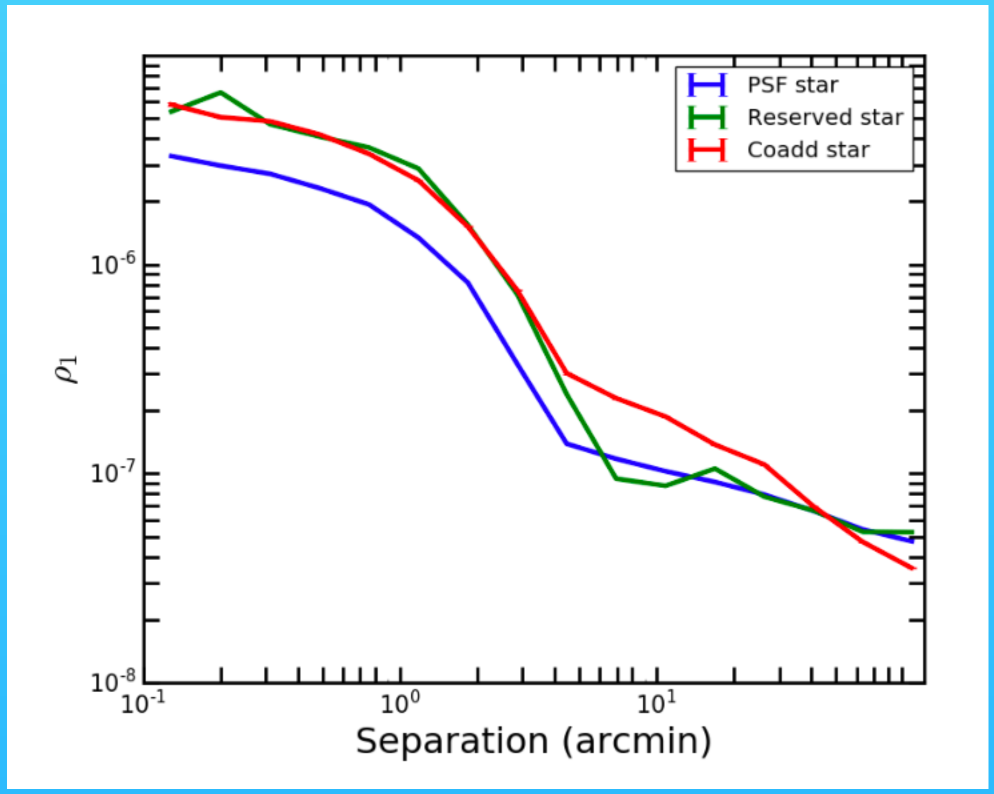
$\frac{\delta R}{R}$

- distribution of size residuals

Measuring the PSF: results and limitations

- HyperSuprimeCam survey as a test-bed for LSST
  - amazing data quality, seeing > 0.4 arcsec
  - using PSFex - Bosch et al 2017, Mandelbaum et al 2017





- correction for brighter-fatter effect
- uses 2D model - Coulton et al in prep
- relationship to Gruen et al model?

Measuring the PSF: results and limitations

- HyperSuprimeCam survey as a test-bed for LSST
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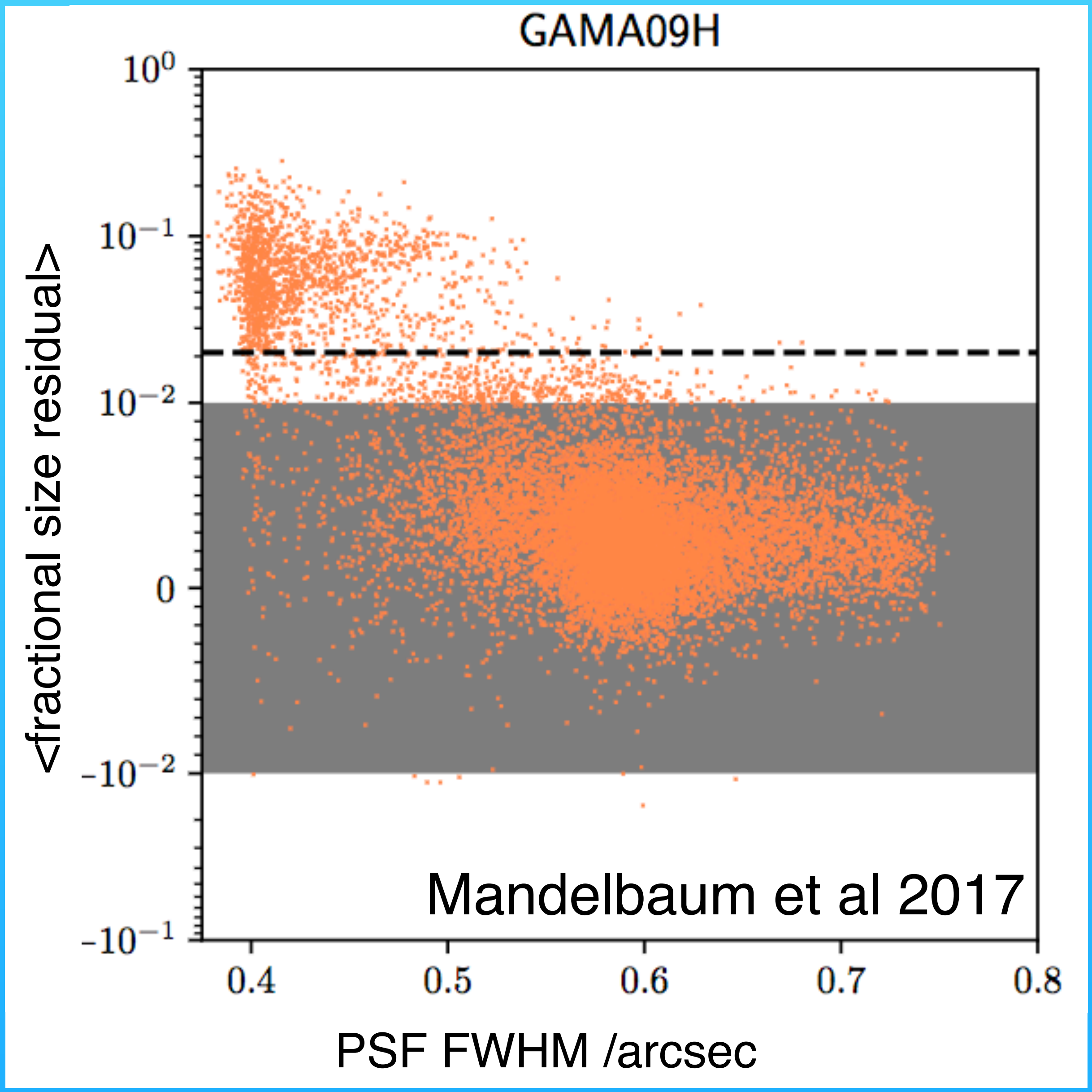
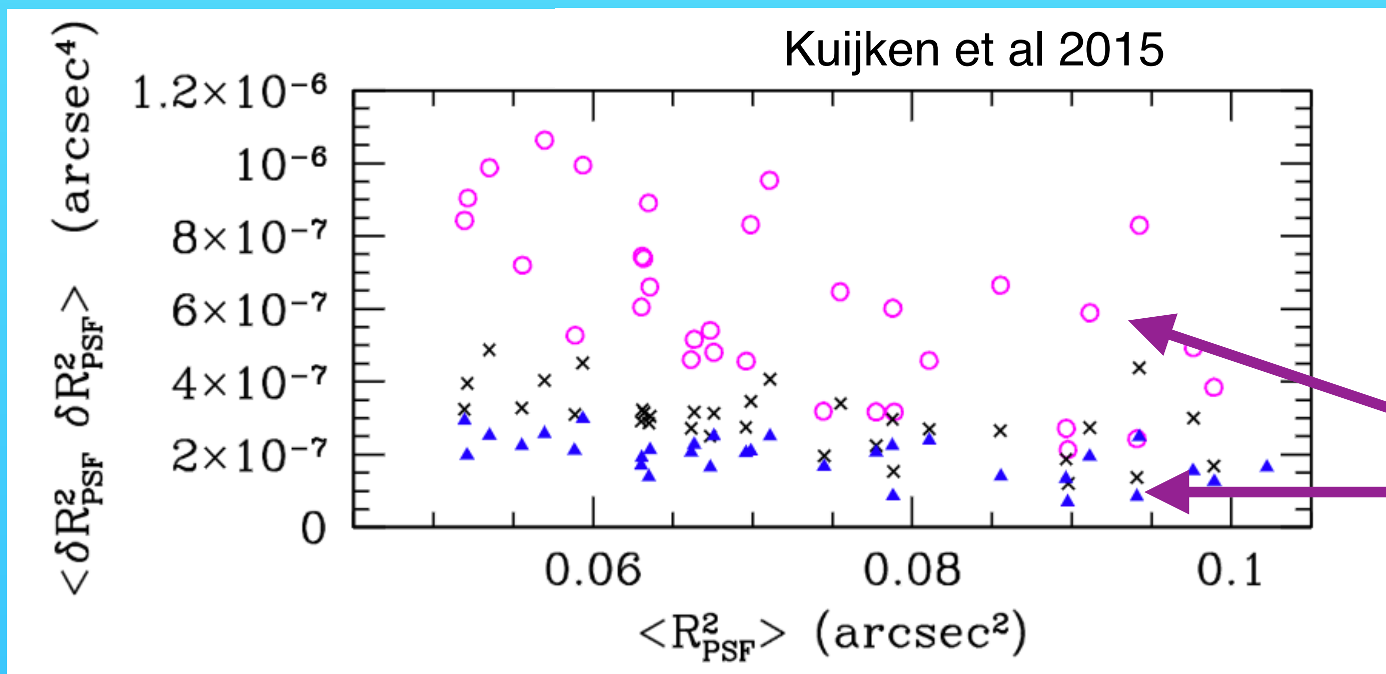


- Best seeing data is the hardest to model!
- difficult to model PSF with FWHM < 0.5 arcsec with 0.17 arcsec pixels
- discontinuity probably caused by switching basis functions at 0.5 arcsec: native pixel sampling >0.5 arcsec, oversampled pixels < 0.5 arcsec (Bosch et al 2017)

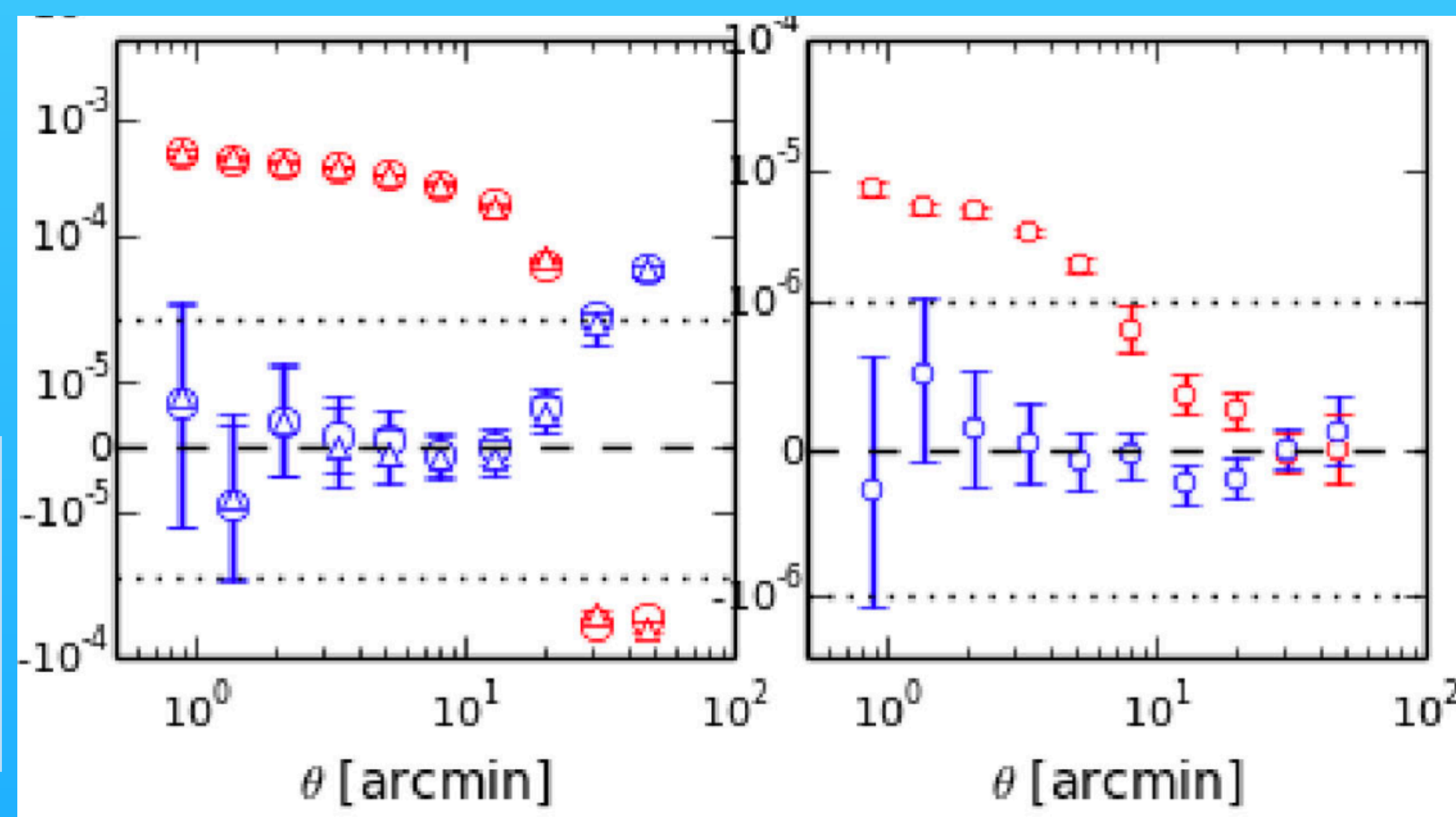
Measuring the PSF: results and limitations



- Interpolating function should be optimised for camera (e.g. KiDS survey)

effect of introducing CCD-dependent coefficients

KiDS rho statistics, individual exposure



Kuijken et al 2015

- KiDS residuals acceptable for 1500 deg² survey
- not for LSST

Modelling and measuring the PSF: summary

- Accuracy of PSF modelling in current ground-based surveys just about OK
- Need factor 10 improvement for LSST & Euclid
  - need to measure size of PSF to 0.1 percent
- Atmosphere limits ground-based accuracy
  - sparse density of bright stars limits accuracy on arcmin scales
  - combining measurements averages out fluctuations
  - not clear if this will be adequate for LSST weak lensing survey
- Algorithm improvements needed for good seeing data
- Telescope can be modelled in wavefront domain but requires calibration observations
- Detector effects are also important

There is plenty to do - interesting combination of optics, modelling and statistical analysis - please join us!