(Joint) Astrometry

LSST in Lyon (June 2017)

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What is astrometry ?

- In principle, anything that has to do with measuring positions of astrophysical objects
- In practice, defining the reference frame is now provided by GAIA
- LSST will improve over GAIA only for faint (m>~20) objects.
- We are then concerned about relative astrometry
- It boils down to mapping position measurements in sensors coordinates on a global reference frame, possibly using common objects not in the reference catalog.

What for?

- In the context of "repeated imaging", relating positions measured in different images is mandatory:
  - Prior to co-adding (!)
  - Prior to subtracting
  - For all sorts of measurements carried out on individual images, e.g. lightcurve extraction, shape measurement, …

Why do we care about positions when measuring fluxes ?

If one shifts the position by δX (independent from the image) :

$$E[\hat{f}] = f\left(1 - \frac{(\delta X)^2}{R^2_{PSF}}\right)$$

$$R^{-2}_{PSF} \equiv 4\frac{\int (\partial_x P)^2 + (\partial_y P)^2 \ dx \ dy}{\int P^2 \ dx \ dy} = -4\frac{\int P \Delta P \ dx \ dy}{\int P^2 \ dx \ dy}$$

If the flux is variable and the position is not, then fitting all fluxes at the same position reduces the bias.

Why do we care about positions when measuring fluxes ?

- When measuring the light-curve of a point source there is a benefit at using the best possible (common) position estimator.
- This requires to map the coordinate systems of the involved images one on the other.

However....

- If δX is due to inaccuracies of image-to-image mappings (i.e. the floor of astrometric residuals)
- The flux bias vanishes in flux ratios
- …. which are actually used when considering the photometric calibration phase.
- So, the astrometric accuracy floor is not a first order issue when measuring lightcurves.

Why do we care about positions when measuring shapes ?

$$M = \frac{\int (X - X_0)(X - X_0)^T \; W(X - X_0) \; I(X) \; d^2X}{\int \; W(X - X_0) \; I(X) \; d^2X}$$

Second moments matrix

centroid

Some weight function

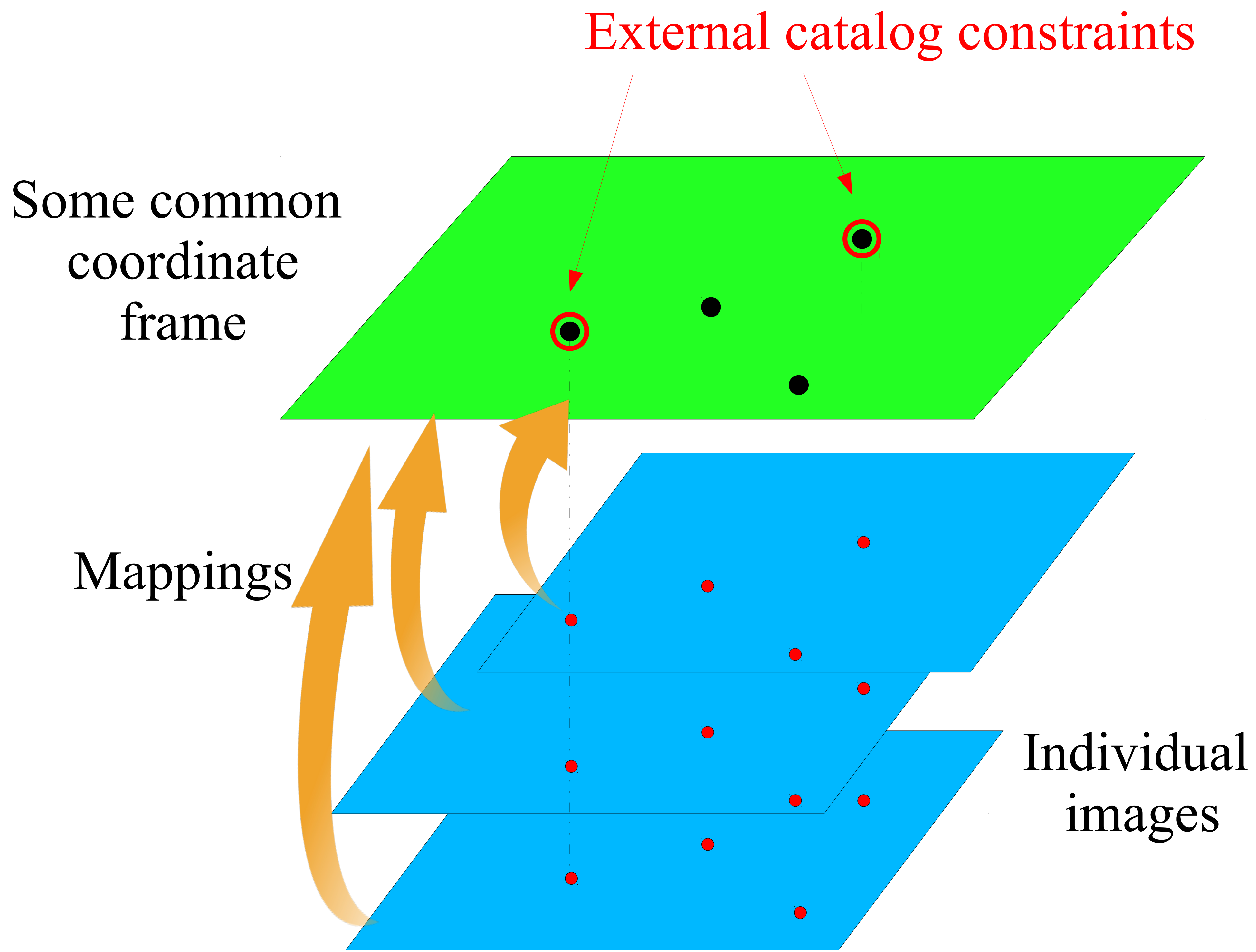
image

Again, a shift of X_0 will alter M, independently of the sign of the shift
→ the X_0 uncertainty causes a bias of M.

But this time, both the statistical (shot noise) and systematic (astrometric floor) contributions remain, because of the absence of a “calibration”.

Astrometric solution

- The goal is to map the pixel space of every image to some common frame (e.g. sidereal)
- Much lighter than determining all image-to-image mappings.
- Mappings to some undistorted space (e.g. tangent plane) allows one to remove the effects of optical distortions (important for shape measurements)


External catalog constraints
Some common coordinate frame
Mappings
Individual images

Various steps towards the astrometric solution

- Initial match (not part of the fitter but interesting to discuss anyway)
- Reading/filtering the catalogs
- Association (cross-id)
- Fit, iterations, outlier removal
    - Possibly re-associate
- Output : average catalog, WCS's, diagnostic ntuples, plots....

Initial combinatorial match

- Problem: matching a "reference catalog" to the one of an image.
- 4-parameter space: e.g. 2 offsets, rotation, scale.
- In practice, scale is often known to < 1% rotation angle to < 1°, location on the sky to < 1'. But not always.
- There is a handful of good algorithms:
  - See e.g. Scamp doc. and astro-ph/9907229, astrometry.net
- All work properly, provided the two catalogs overlap enough (!).
- The robustness of an algorithm primarily depends on how many times the right match could be found.

Fitting the (distorted) WCS

- Means fitting the mapping from pixel coordinates to e.g. tangent plane.
- It is less trivial than it seems, because we are fitting polynomials.
- One *has* to fit in transformed coordinates, and re-express the resulting polynomial.
- Best linear system solving methods :
  - SVD on the Jacobian (and check for degeneracies).
  - LDL^T on the Hessian (rather than LL^T, i.e. Cholesky)

Combinatorial matching: HSC

- HSC is challenging for combinatorial astrometric matching, because of huge optical distortions.
- We have to rely on an "instrument model", in order to project all catalogs from an exposure on some "undistorted" plane.
- A successful recipe to get this instrument model:
  - Find a set of exposures where each CCD of the mosaic was successfully matched (stand-alone) at least once.
  - Run the simultaneous astrometric fit on those matched images.
  - Use the output instrument model to combinatorial match whole exposures. This works(!)
  - Rerun the simultaneous astrometry on the whole sample

Three implementations of the simultaneous fitter

- SCAMP (Emmanuel Bertin 2008 ?)
  - The reference and the largest user base.
- WcsFit (Garry Bernstein, 2016)
  - Developed to fit a detailed instrument model for DECam.
- Jointcal (LSST-DM & co, ~2015-)
  - Just entered into the DM stack.

SCAMP (1)

$$\chi^2 = \sum_s \sum_a \sum_{b>a} w_{s,a,b} \|\boldsymbol{\xi}_a(\boldsymbol{x}_{s,a}) - \boldsymbol{\xi}_b(\boldsymbol{x}_{s,b})\|^2, \tag{13}$$

where $w_{s,a,b}$ is the non-zero weight for the pair of detections in fields a and b related to source s:

$$w_{s,a,b} = \frac{1}{\sigma_{s,a}^2 + \sigma_{s,b}^2}. \tag{14}$$

$\sigma_{s,f}$ is the positional uncertainty for source s in field f.

- Scamp minimizes the difference between mapped coordinates of measurement pairs.
- This is not exactly a maximum likelihood.

SCAMP (2)

- The default fitted model combines an instrument-specific mapping and an exposure anamorphism (atmosphere+...)
- Scamp incorporates the mechanics for combinatorial matching (possibly at the array level, using an embedded instrument layout).
- Can handle dozens of different reference catalogs.
- Parallaxes and proper motions (fitted separately...)
- Outputs the "average" catalog and WCS fits headers.
- Also outputs a lot of diagnostic plots.
- Any contender should provide at least these functionalities....

WcsFit (1703.01679)

- Written by G. Bernstein to finely map the instrumental distortions of Decam, from dithered exposures of dense stellar fields.
- Actually fits positions of common objects.
- Does not rely on sparse linear algebra, thanks to a trick:

$$\chi^2 \approx \chi^2(\boldsymbol{\pi}_0) + 2\mathbf{b}\cdot\Delta\boldsymbol{\pi} + \Delta\boldsymbol{\pi}\cdot\mathbf{A}\cdot\Delta\boldsymbol{\pi},$$

$$b_\mu \equiv \frac{1}{2}\frac{\partial\chi^2}{\partial\pi_\mu} = \sum_i w_i \left(\mathbf{x}^w(\mathbf{x}_i^p, \boldsymbol{\pi}_0) - \bar{\mathbf{x}}_{\alpha_i}\right)\cdot\left(\frac{\partial\mathbf{x}^w(\mathbf{x}_i^p, \boldsymbol{\pi})}{\partial\pi_\mu} - \frac{\partial\bar{\mathbf{x}}_{\alpha_i}}{\partial\pi_\mu}\right)$$

$$A_{\mu\nu} \equiv \left(\frac{\partial\mathbf{x}^w(\mathbf{x}_i^p, \boldsymbol{\pi})}{\partial\pi_\mu} - \frac{\partial\bar{\mathbf{x}}_{\alpha_i}}{\partial\pi_\mu}\right)\cdot\left(\frac{\partial\mathbf{x}^w(\mathbf{x}_i^p, \boldsymbol{\pi})}{\partial\pi_\nu} - \frac{\partial\bar{\mathbf{x}}_{\alpha_i}}{\partial\pi_\nu}\right).$$

Position of sources treated as the average of transformed measurements

WcsFit (2)

- The user provides the fitted model at run time, by specifying a combination of transformations.
- The code does its best to eliminate degeneracies, but there is no failsafe algorithm.
- An example of the fitted components:

Table 2. Components of the DECam astrometric model

Description	Name	Type	Max. Size
Tree ring distortion	⟨*band*⟩/⟨*device*⟩/rings	Template (radial)	$\approx 0''.05$
Serial edge distortion	⟨*band*⟩/⟨*device*⟩/lowedge	Template (X)	$0''.03$
Serial edge distortion	⟨*band*⟩/⟨*device*⟩/highedge	Template (X)	$0''.03$
Optics	⟨*band*⟩/⟨*device*⟩/poly	Polynomial (order= 4)	$\gg 1''$
Lateral color[a]	⟨*band*⟩/⟨*device*⟩/color	Color×Linear	$\approx 0''.04$
CCD shift	⟨*epoch*⟩/⟨*device*⟩/ccdshift	Linear	$\approx 0''.1$
Exposure	⟨*exposure*⟩	Linear	$\gg 1''$
Differential chromatic refraction	⟨*exposure*⟩/dcr	Color×Constant	$\approx 0''.05$

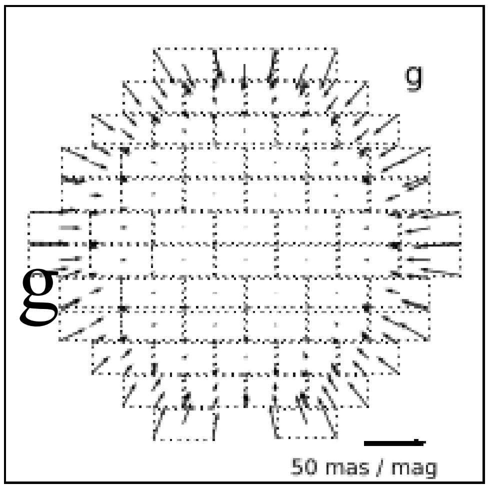
Degeneracy ? (Optics, CCD shift)

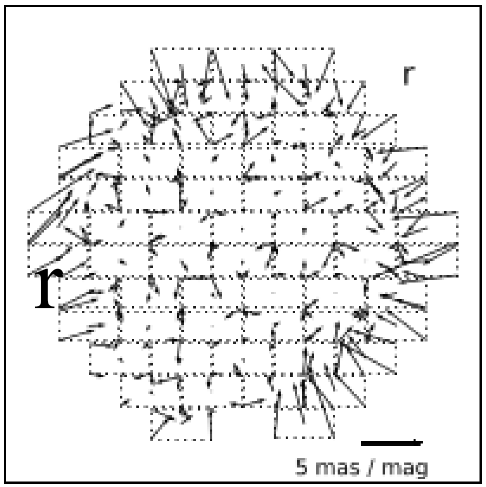
Next slide (Lateral color)

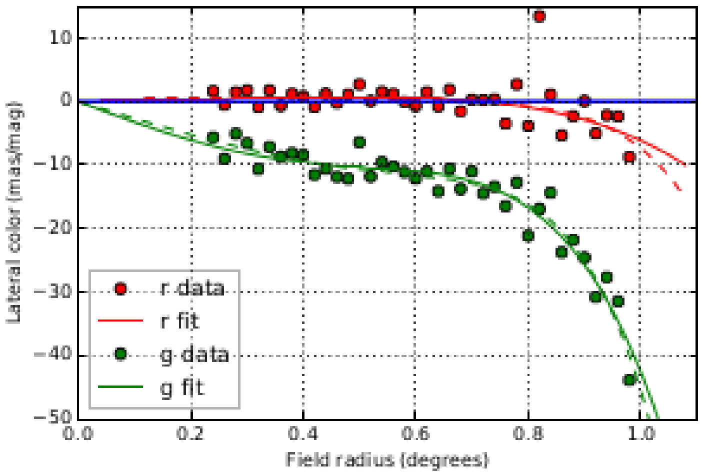
Not sufficient for refraction (Exposure)

WcsFit for Decam

Chromatic terms (per chip/band) for g-i color







Large chromaticity of the Decam corrector.
It can (will?) eventually become a static part of the instrument model

Jointcal (1)

- Developed for DM, from a precursor written for SNLS.
- Fits both mappings and common objects positions, possibly using reference objects:

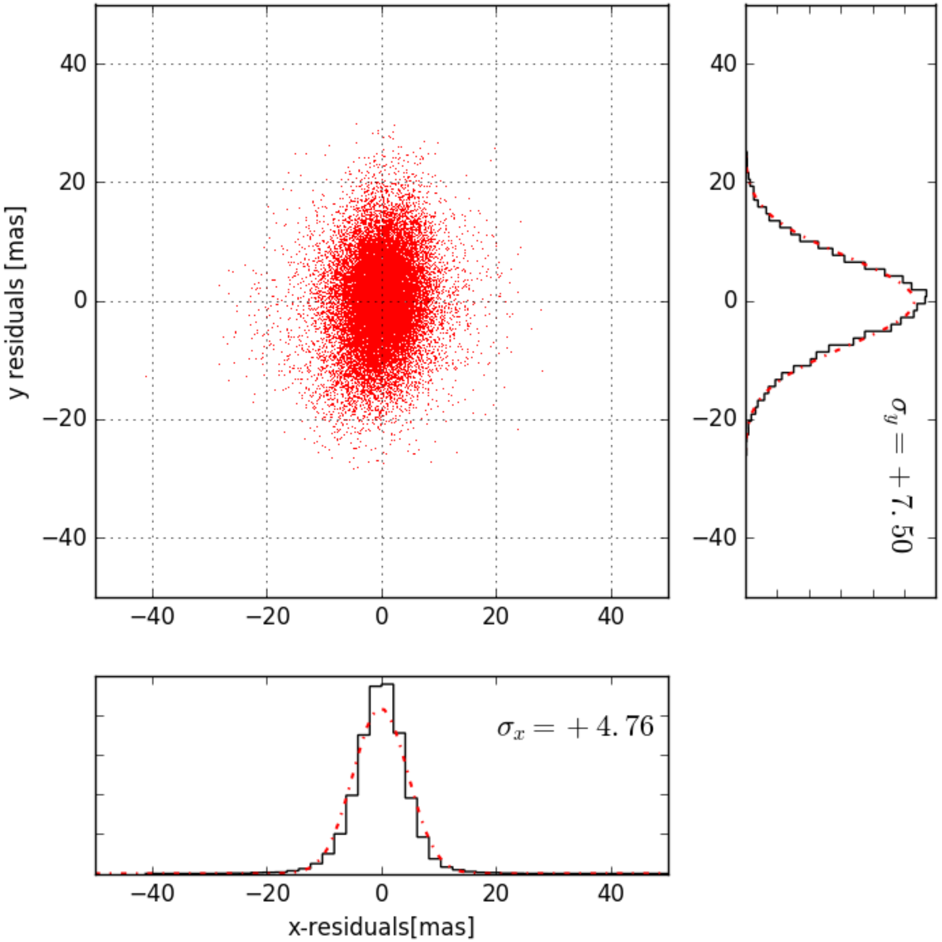
$$\chi^2 = \sum_{\gamma,i} [M_\gamma(X_{\gamma,i}) - P_\gamma(F_k)]^T W_{\gamma,i} [M_\gamma(X_{\gamma,i}) - P_\gamma(F_k)] \qquad \text{(meas. terms)}$$
$$+ \sum_j [P(F_j) - P(R_j)]^T W_j [P(F_j) - P(R_j)] \qquad \text{(ref. terms)}$$

- Relies on sparse linear algebra for expressing and solving the system, using the LDL^T factorization of *cholmod*, using its "factorization update" capability (for outlier removal).
- The fitted model is abstract for the fitter.

Jointcal (2)

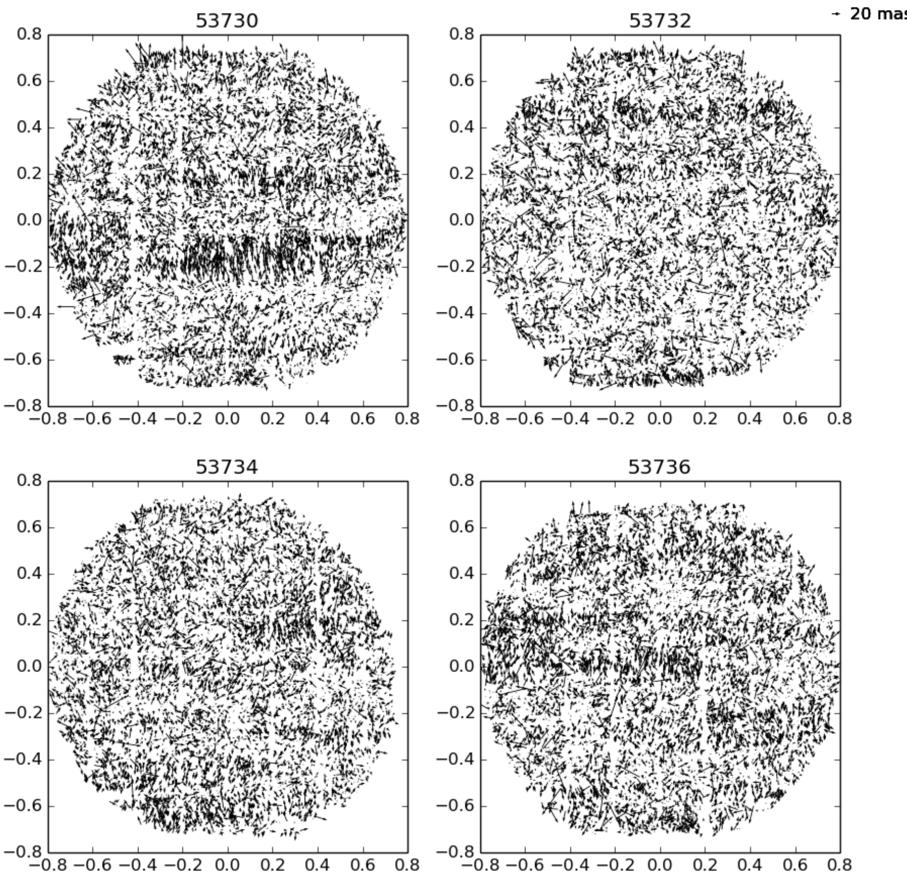
- So far, the code only contains two models:
  - Images are mapped independently $T_{expo,CCD}(X)$.
  - Images are mapped as $T_{expo}(T_{CCD})(X)$ (ConstrainedModel)
    - T_{expo} = Identity for one exposure.
  - In both instances mappings are polynomials.
- Results that follow come from reductions of HSC data on Cosmos. We(*) have been only using the ConstrainedModel (very similar to what SCAMP does). Uses Gaussian-Weighted positions.

(*) LPNHE LSST team



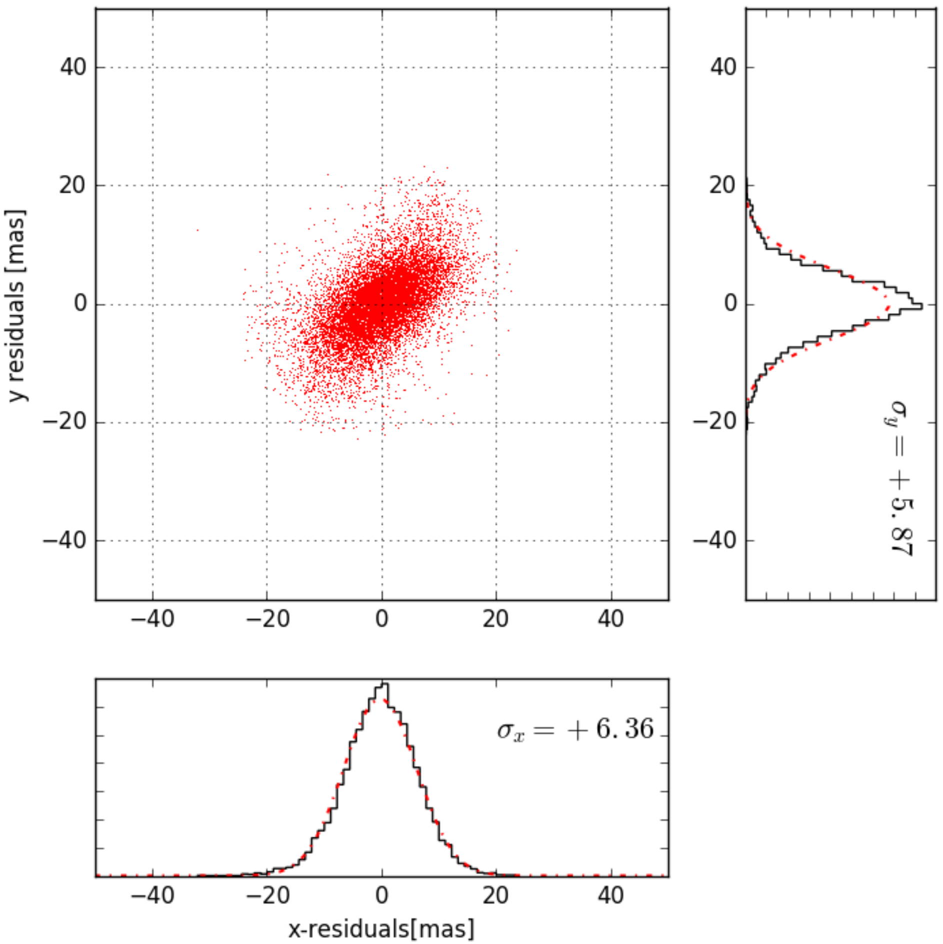
All residuals (m<~20)

Residuals per exposure
As a function of position
In the focal plane



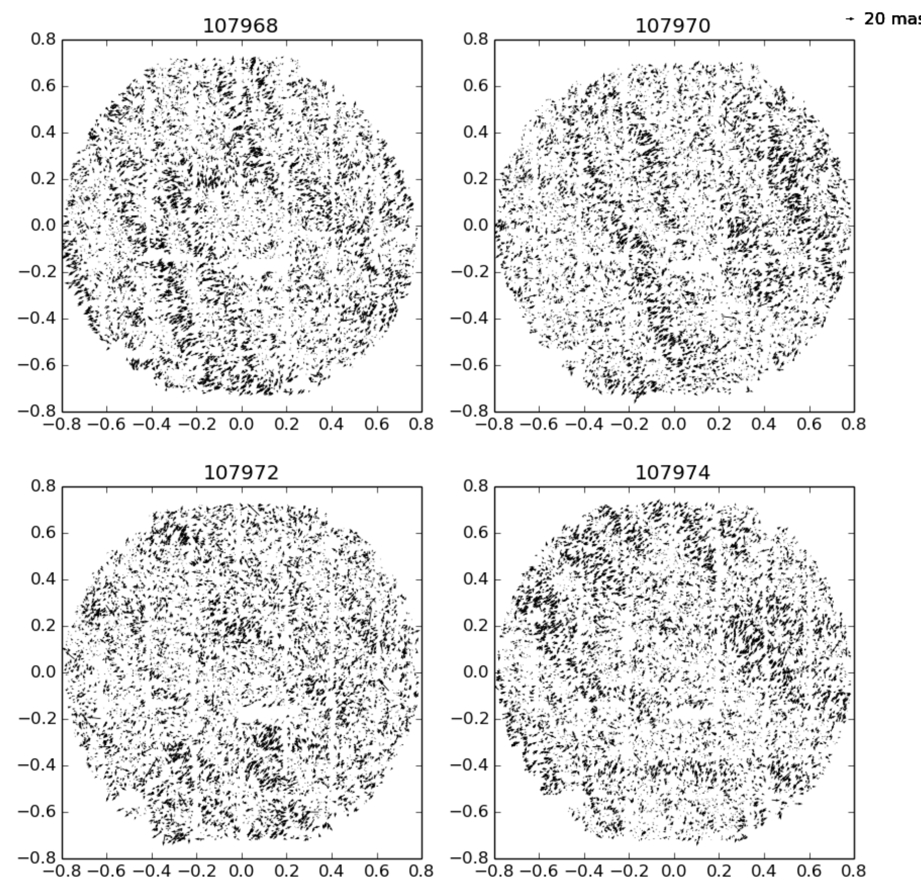
Night 57402, z band.
17 exposures on Cosmos

- 1280 s of wall time (1 core)
- 509 k 2d-measurements, 138 k parameters
- Computing derivatives: 20s
- “squaring”: 80s
- Factorize-solve : 20 s



All residuals (m<~20)

Residuals per exposure



Night 57841, z band.
11 exposures.

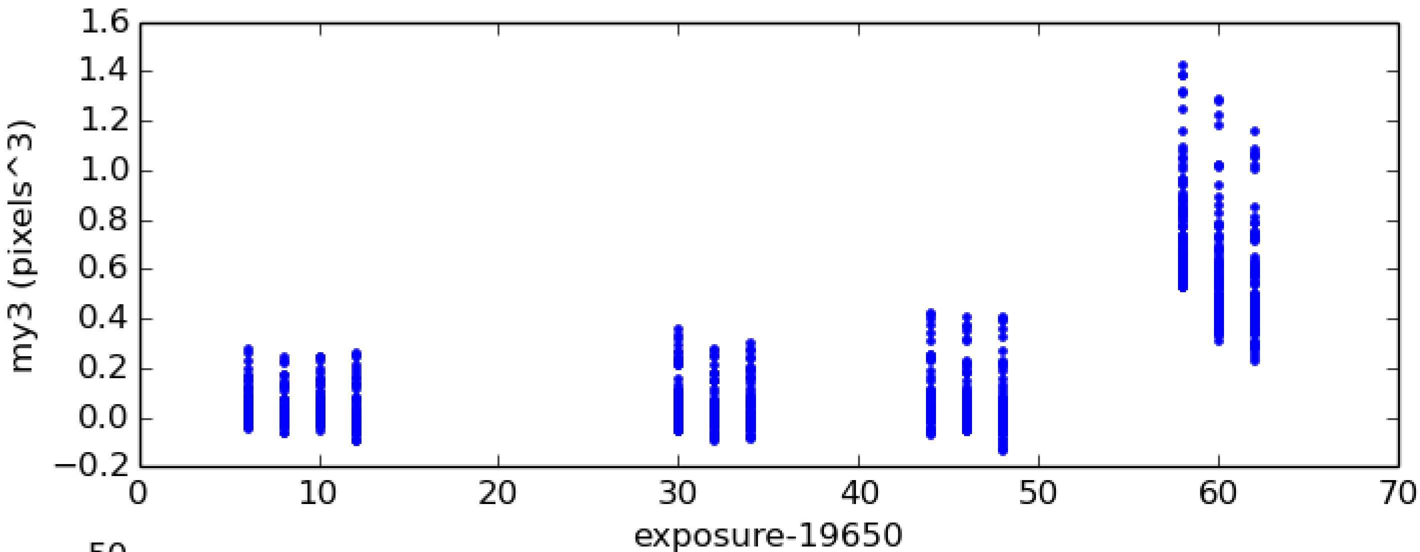
Source of these residual patterns

- Their variability from exposure to exposure points towards the atmosphere
- This kind of pattern is expected from high altitude refraction index variations.
- Then, the displacements are the gradient of a scalar field. G. Bernstein checks that.
- Getting rid of those residuals at scales > a few arcmin, means several hundred parameters per exposure. This is a lot.

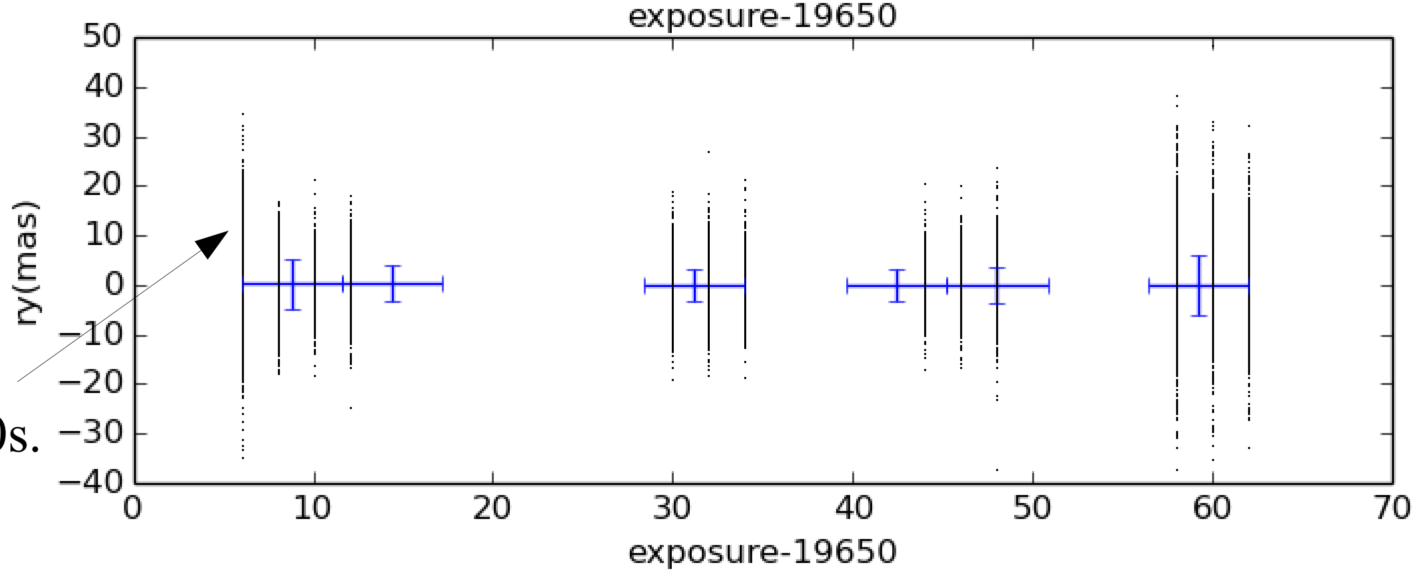
Odd PSF terms

Skewed PSFs

3rd Gaussian moments (y^3) of stars



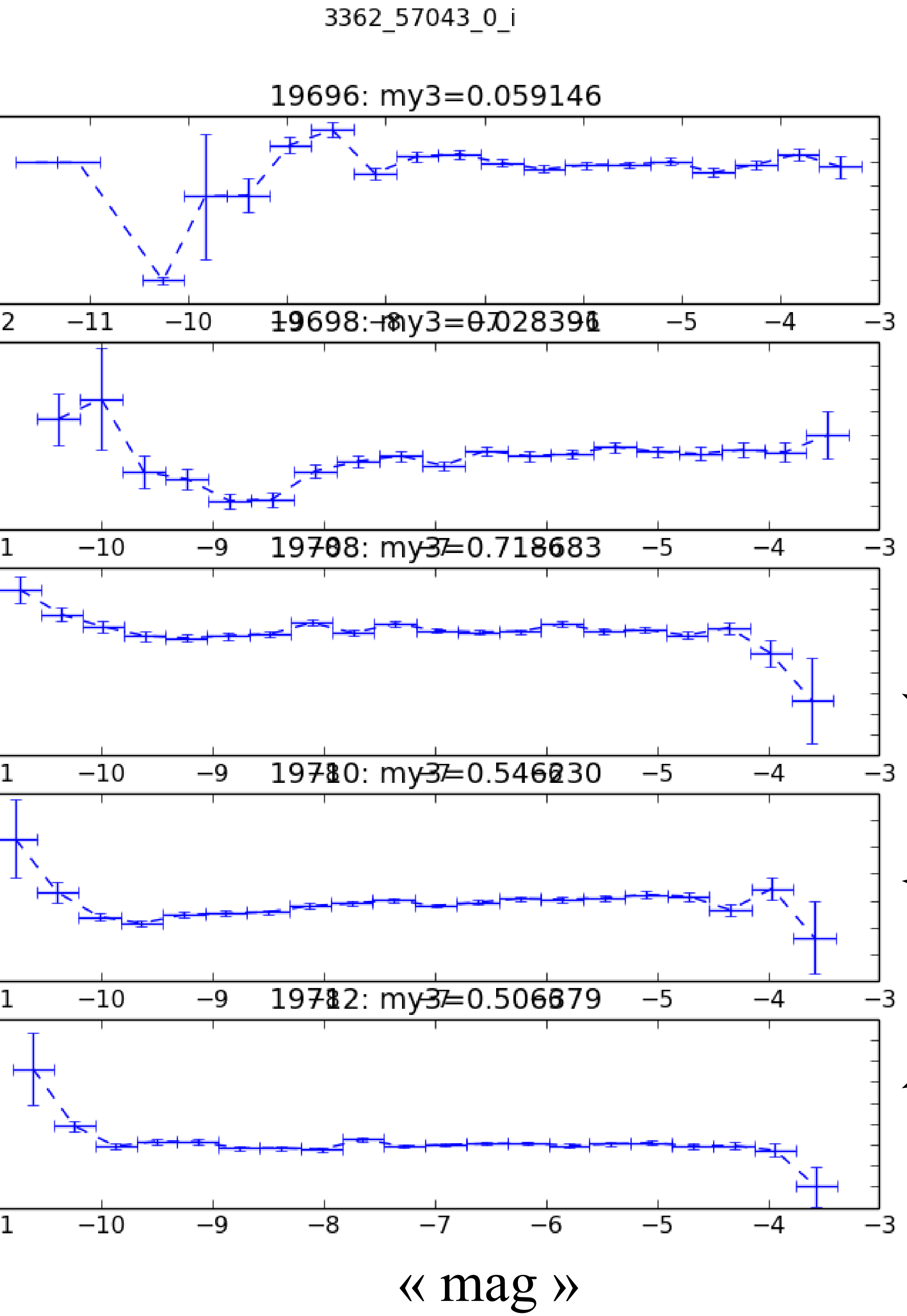
Residuals along y (m>~20)



Night 57043, i band, 300s exposures, first is 30s.

P. Astier, LSST in Lyon (2017)

Odd PSF terms



Astrometric residuals
To the night average

Exposures
with poor
residuals
(and large 3rd moments)

Jointcal status (1)

Diff. Atm. Refraction → One parameter per Band. What about HSC ???

Atm. Refraction →
Flexure of the corrector →
Per exposure Mosaic-wide anamorphism

Atmospheric turbulence → ?? per exposure

Optical distortions → Per “run”(TBD)/band
Mechanics of the mosaic → CCD → tang. plane mapping

Tree rings →
Side shifts →
Chromatic aberrations →
Fixed after determination from a specific Fitter (to be done)

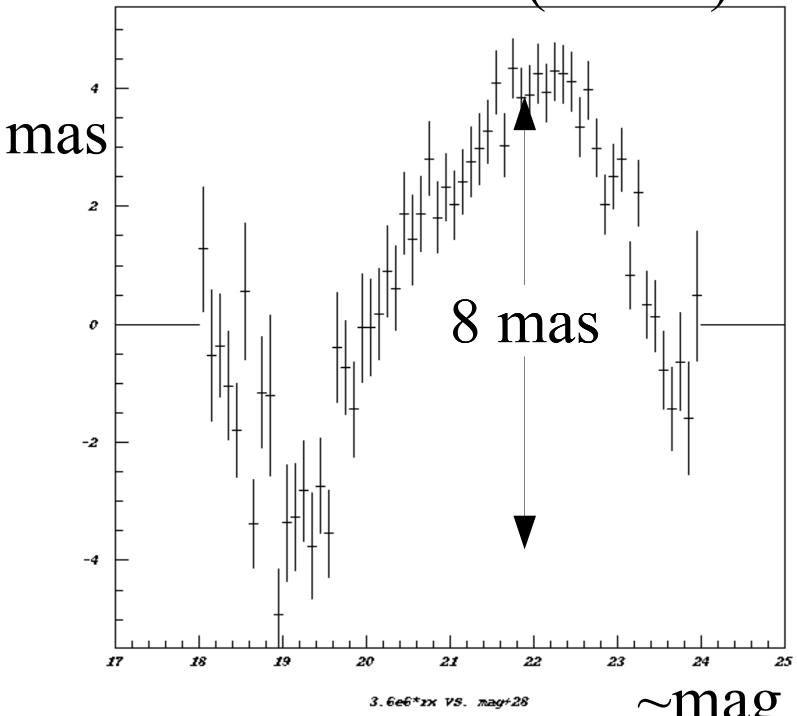
Jointcal status/future (2)

- Any layer added to the model should come with a scheme to lift the added degeneracies.
- For some reason (guiding?), odd PSF components probably compromise the astrometric solution.
- Atmospheric turbulence requires a lot of parameters per exposure to be modeled. Some sort of post-processing would be welcome.
- Depending on the fit size, some parallelism could be needed.
- …. Proper motions, …..

HSC: effect of PSF skewness

- Position estimation: SDSS-like coordinates, i.e Gaussian fit.

Residual(rx) vs mag, visit 19712 (i-band)



- The average residual depends on how extended the object is, and hence on magnitude.

- The skewness of stars is consistent across the mosaic.

- Current fix: exclude skewed-PSF exposures from stacking.

- Is there a general way to measure positions, accounting for PSF skewness?