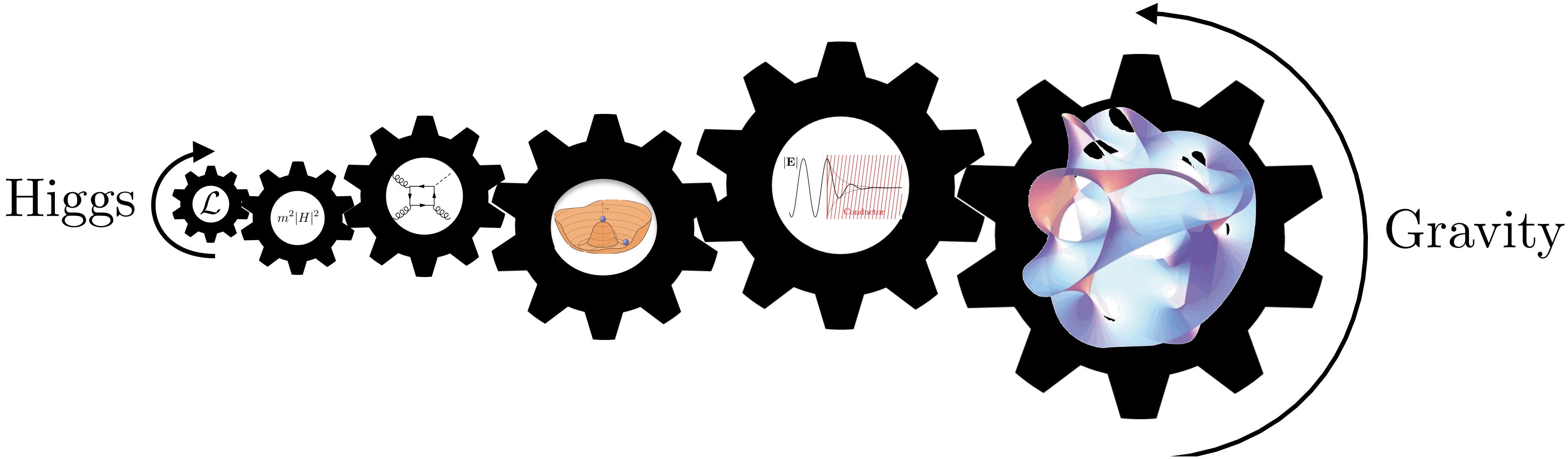
A Clockwork Theory



Moriond EW

La Thuile, Mar 20th 2017

Based on: Giudice, MM, 2016

And ongoing work: Giudice, Kats, MM, Torre, Urbano

The Problem of Hierarchies

Often stated that, without knowing e.g. the Planck-scale physics, we should naively expect:

But that is clearly not the case, by many orders of magnitude! Can be reconciled by a theory that permits scale separation…

- Supersymmetry
- Composite Higgs
- …

Which all predict new particles at $E \sim M_H$.

The Problem of Hierarchies

Thus, for example, without knowing the Planck-
scal… should expect:

Lets reexamine these statements
briefly for loopholes…

Maybe, e.g. the Planck scale isn't
quite what it seems…

B… ers
of magnitude…
theory that permits scale …

- Supersymmetry
- Composite Higgs
- …

Which all predict new particles at $E \sim M_H$.

On Masses and Scales

Masses and interaction scales are not physically equivalent. Seen by reinserting ħ into action.

$$\mathcal{L}_{\hbar \neq 1}$$

In terms of these dimensionful quantities:

$$[\hbar] = EL \ , \quad [\mathcal{L}] = EL^{-3} \ , \quad [\phi] = [A_\mu] = E^{1/2}L^{-1/2} \ , \quad [\psi] = E^{1/2}L^{-1}$$

$$[\partial] = [\tilde{m}] = L^{-1} \ , \quad [g] = [y] = E^{-1/2}L^{-1/2} \ , \quad [\lambda] = E^{-1}L^{-1}$$

we can quickly see the relationship between masses and interaction scales.

On Masses and Scales

Masses and interaction scales are <u>not physically equivalent</u>. Seen by reinserting ħ into action.

$$\mathcal{L}_{\hbar \neq 1}$$

In terms of dimensionful quantities

Masses

Couplings

<u>Planck Scale</u>

Interaction: $\mathcal{L} \sim \frac{h_{\mu\nu} T^{\mu\nu}}{M_P}$

UV-completion

Dimension: $[M_P] = \frac{[M_S]}{[\lambda_S]}$

Coupling

On Masses and Scales

Masse… …ction scales are not physically equiv… …ing of ħ into action.

In te…

Pl…

Interaction: $L \sim$ … M_P … ion

Dimension: $$[M_P] = \frac{[M_S]}{[\lambda_S]}$$ ← Coupling

What if the Planck scale is not where new physics arises?

What if quantum gravity is instead at the weak scale?

Then there must be an incredibly small coupling somewhere?

This talk...

The clockwork mechanism was first proposed by Choi & Im, Kaplan & Rattazzi, for scalar fields: Tiny coupling emerges from theory with no larg ll parameters.

See intro by Teresi yesterday.

Recently generalised to ferm osons, gravity in 1610.07962!

I will only sketch the gravity part, but other possibilities are equally interesting.

Then: Phenomenology for LHC...

Clockwork Graviton

A **wild speculation** that triggered this work…

- Take N+1 copies of gravity.
- This gives N+1 gravitons.
- Use them to construct clockwork gravity?

Clockwork Fierz-Pauli mass term for N gravitons:

$$\mathcal{L} = -\frac{m^2}{2}\sum_{j=0}^{N-1}\left(\left[h_j^{\mu\nu} - q h_{j+1}^{\mu\nu}\right]^2 - \left[\eta_{\mu\nu}(h_j^{\mu\nu} - q h_{j+1}^{\mu\nu})\right]^2\right)$$

Massless graviton present from shift symmetry:

$$h_j^{\mu\nu} \to h_j^{\mu\nu} + \frac{1}{q^j}\tilde{h}^{\mu\nu}$$

Clockwork Gravity

If such a theory exists then it would solve the hierarchy problem.

Imagine SM fields only "charged" under last diffeomorphism invariance, couple to last graviton.

$$-\frac{1}{M_N} h_N^{\mu\nu} T_{\mu\nu} \rightarrow -\frac{1}{M_P} \tilde{h}_0^{\mu\nu} T_{\mu\nu} \longrightarrow M_P = q^N M_N$$

- Cutoff of theory.
- Take $M_N \approx$ TeV.
- Should also take $M_H \approx M_N$.

- After clockworking, SM coupled to true massless graviton (and massive "graviton gears").

- Observed Planck scale clockworked!
- Exponentially greater than true cutoff of theory, and the weak scale.

Where could this theory come from?

The Clockwork Metric

This backwards "dimensional construction" process reveals the unique geometry

$$ds^2 = e^{\frac{4k|y|}{3}}\left(dx^2 + dy^2\right)$$

as a generator for clockwork theories.

Previously showed up in linear dilaton theory (Antoniadis, Dimopoulos, Giveon), as a dual to "Little String Theory" (Berkooz, Rozali, Seiberg).

Place a massless field in this geometry, make extra dimension a lattice, and you get the clockwork…

- Scalar
- Fermion
- Photon
- Graviton!

A Clockwork Dimension

Put gravity in this background and decompose to find 5D eigenstates (KK):

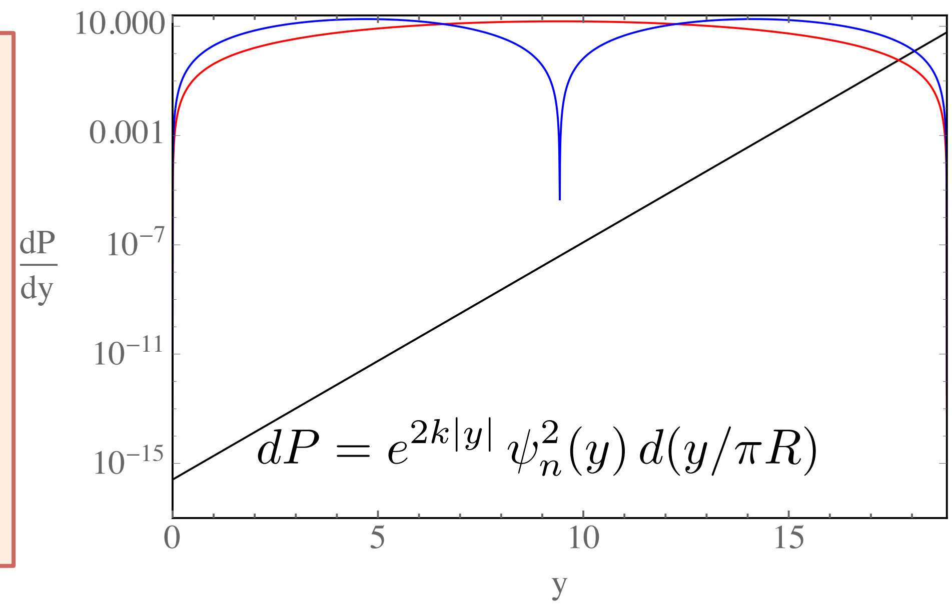
$$\phi(x,y) = \sum_{n=0}^{\infty} \frac{\tilde{\phi}_n(x)\,\psi_n(y)}{\sqrt{\pi R}}$$

Find a zero-mode:

Mass: $m_0^2 = 0$

Wavefunction:

$$\psi_0(y) = \sqrt{\frac{k\pi R}{e^{2k\pi R} - 1}}$$



A Clockwork Dimension

Put gravity in this background and decompose to find 5D eigenstates (KK):

$$\phi(x,y) = \sum_{n=0}^{\infty} \frac{\tilde{\phi}_n(x)\,\psi_n(y)}{\sqrt{\pi R}}$$

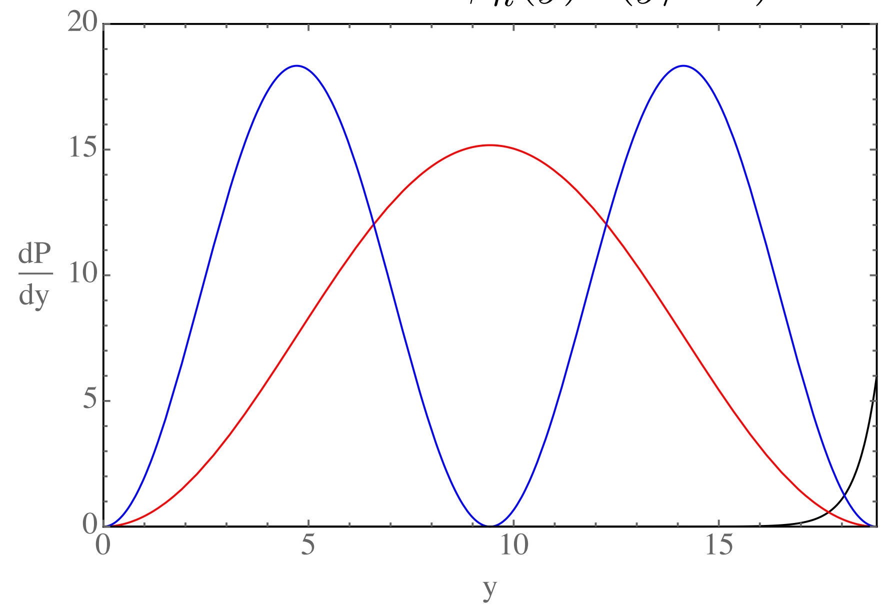
Find excited modes:

Mass: $m_n^2 = k^2 + \frac{n^2}{R^2}$

Wavefunction:

$$\psi_n(y) = \frac{n}{m_n R}\, e^{-k|y|} \left(\frac{kR}{n} \sin\frac{n|y|}{R} + \cos\frac{ny}{R} \right)$$

$$dP = e^{2k|y|}\,\psi_n^2(y)\, d(y/\pi R)$$



A Clockwork Dimension

Put gravity in this background and decompose to find 5D eigenstates (KK):

$$\sum^{\infty} \frac{\tilde{\phi}_n(x)\,\psi_n(y)}{\ldots}$$

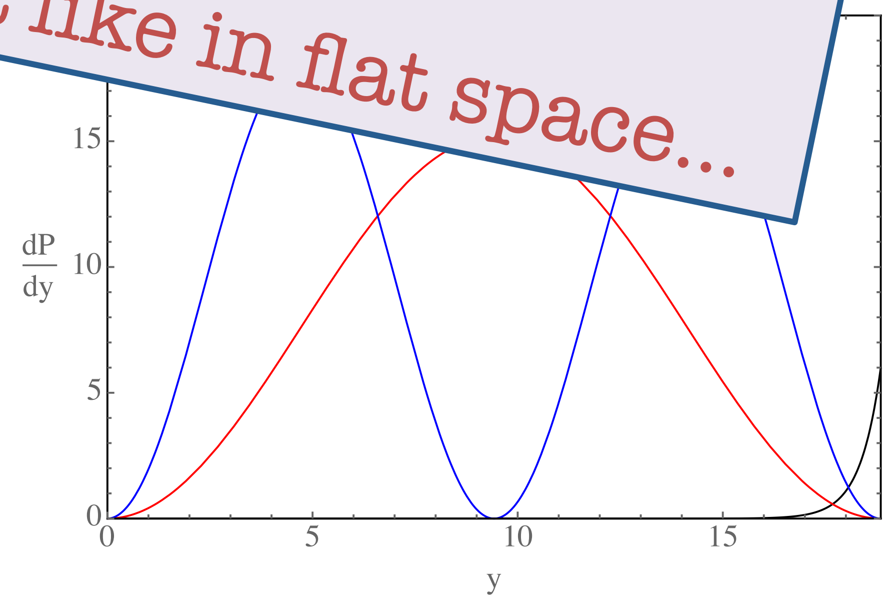
Zero mode density exponentially warped,
KK mode density just like in flat space…

Find excited m…

Mass: $m_n^2 = k^2 + \dfrac{n^2}{R^2}$

Wavefunction:

$$\psi_n(y) = \frac{n}{m_n R}\, e^{-k|y|} \left(\frac{kR}{n} \sin \frac{n|y|}{R} + \cos \frac{ny}{R} \right)$$



The Hierarchy Problem

Graviton 0-mode and KK states have same decomposition. If SM fields on brane at end:

$$\mathcal{L} = -\frac{h_{\mu\nu}(x,0)\, T^{SM}_{\mu\nu}(x)}{M_5^{3/2}} = -\sum_{n=0}^{\infty} \frac{\tilde{h}^{(n)}_{\mu\nu}(x)\, T^{SM}_{\mu\nu}(x)}{\Lambda_n}$$

Interaction scale

Excited graviton modes:

$$\Lambda_n = \sqrt{M_5^3\, \pi R \left(1 + \frac{k^2 R^2}{n^2}\right)}$$

True massless graviton:

Exponentially enhanced

$$\Lambda_0 = M_P = \sqrt{\frac{M_5^3}{k}}\sqrt{e^{2k\pi R} - 1}$$

Phenomenology

Things get really interesting when looking to the phenomenology…

This talk: Work in progress with Giudice, Kats, Torre, Urbano.

Previous related studies:

- Antoniadis, Arvanitaki, Dimopoulos, Giveon, 2011. (Large-k)
- Baryakhtar, 2012. (All-k)
- Cox, Gherghetta, 2012. (Dilatons)
- Giudice, Plehn, Strumia, 2004. Franceschini, Giardino, Giudice, Lodone, Strumia, 2011. (Large extra dimensions, pheno similar.)

Phenomenology

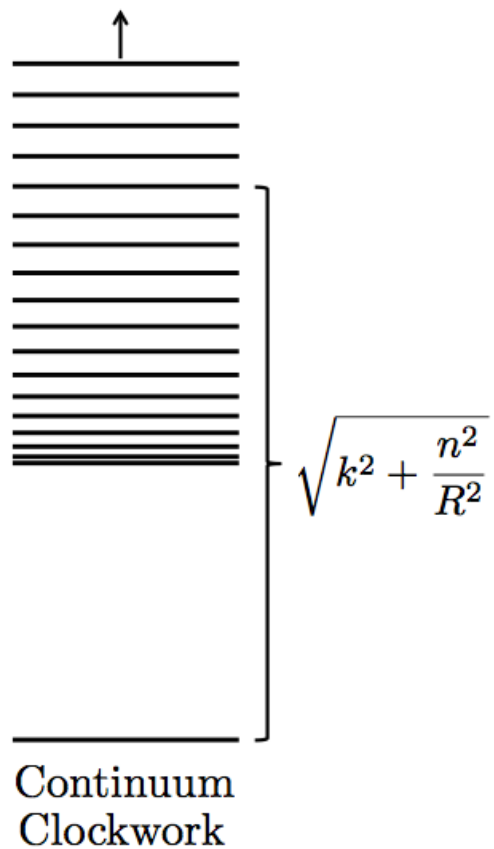
Irreducible prediction of clockwork gravity:

In this theory Planck scale is:

$$M_P \sim \sqrt{\frac{M_5^3}{k}} e^{k\pi R}$$

So if all other parameters at the weak scale, require:

$$kR \sim 11$$



But the mass spectrum is given by:

$$m_n \sim k\left(1 + \frac{n^2}{2(kR)^2}\right)$$

Thus the first few states will always be split by %'s, with the relative splitting decreasing for heavier modes.

This splitting is thus a key prediction of the theory.

Phenomenology

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n^2 / $kR)^2$

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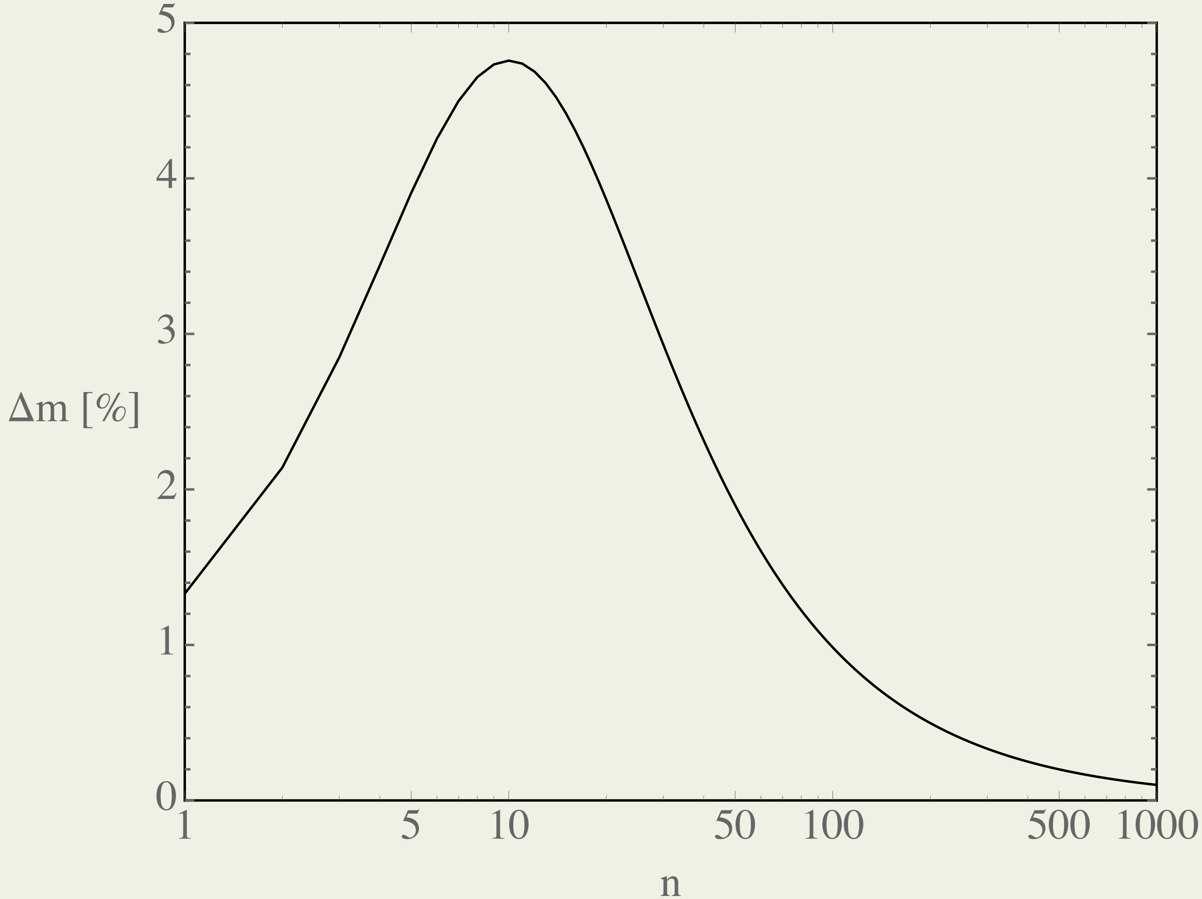
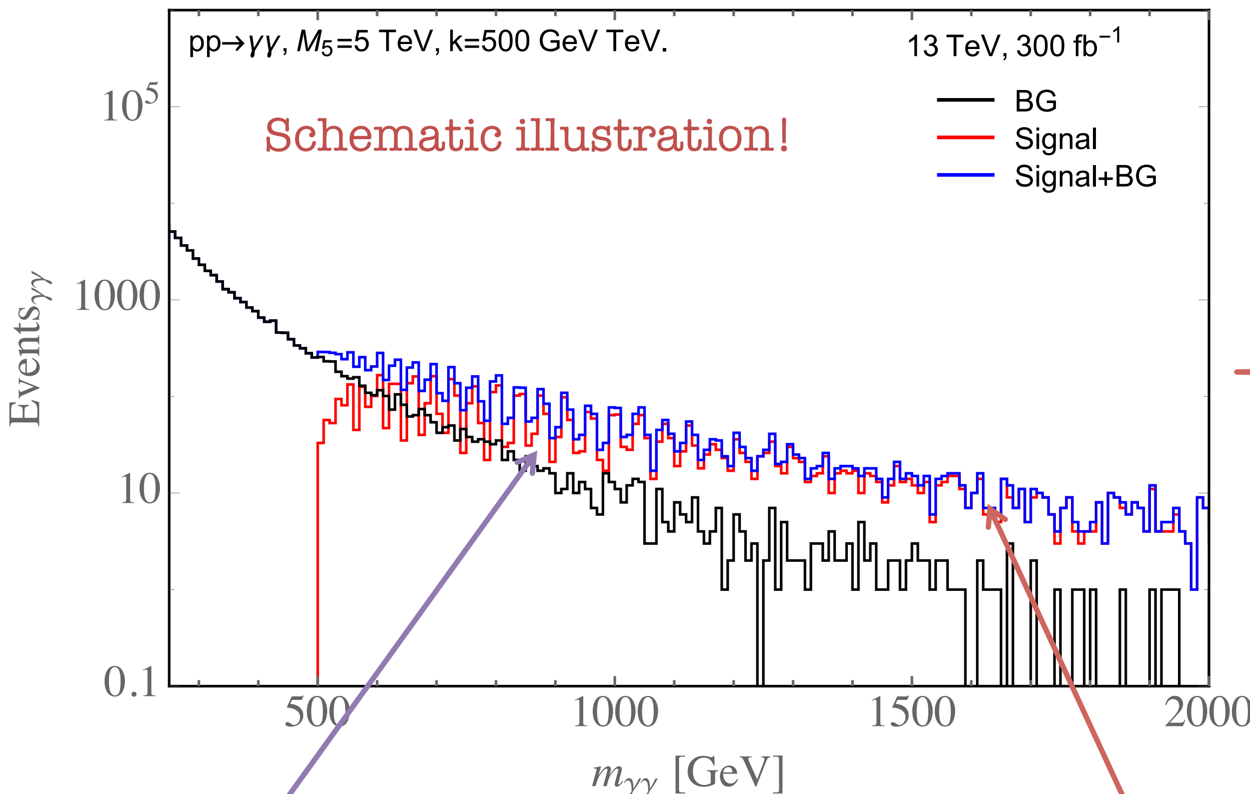
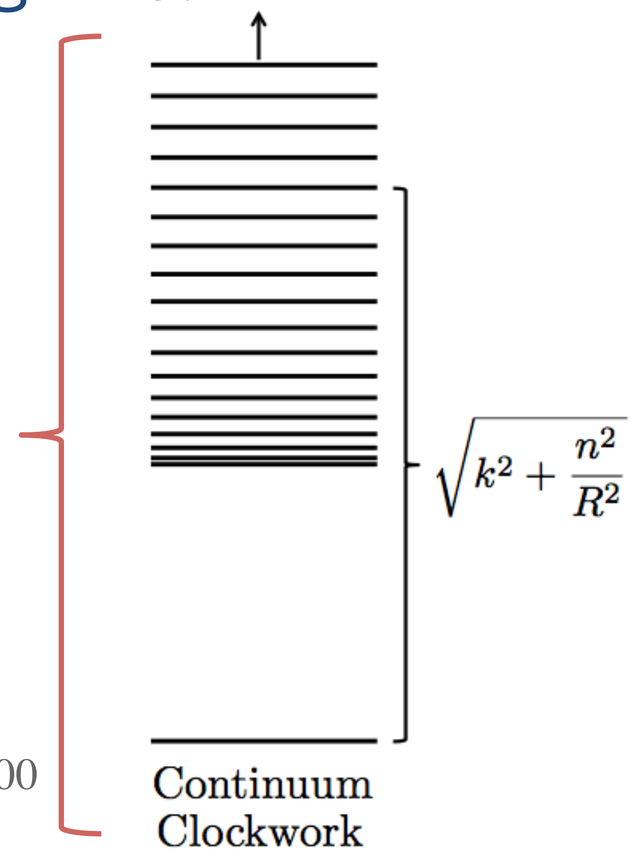
Clockwork mass splitting:



Phenomenology

At colliders would look something like:



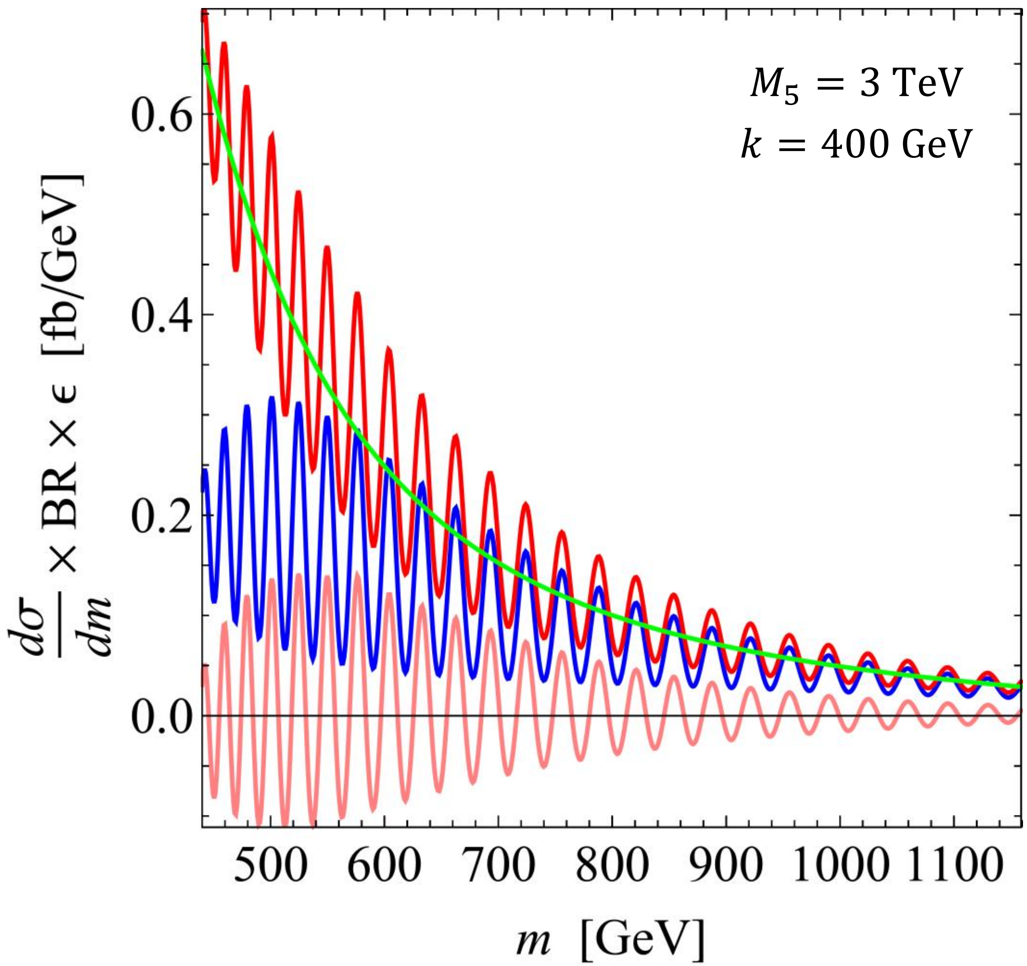


Most interestingly, due to splittings, signal appears to "oscillate". Thus get extra sensitivity by doing spectral analysis… The "power spectrum" of LHC data!

Can search for continuum spectrum at high energies. BG modelling essential…

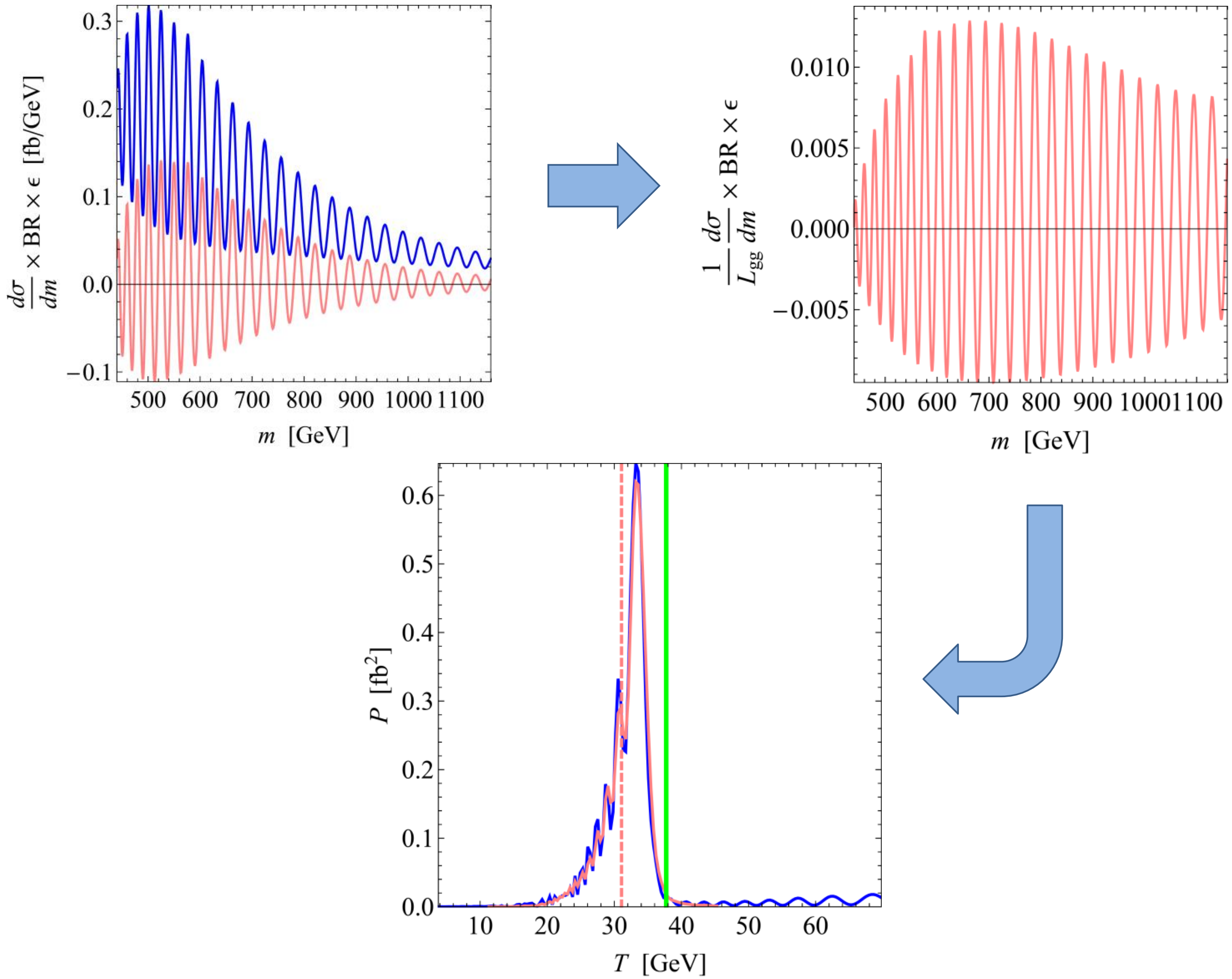
Phenomenology

In practice would want to perform a procedure to extract the oscillations, by subtracting off a smooth background:



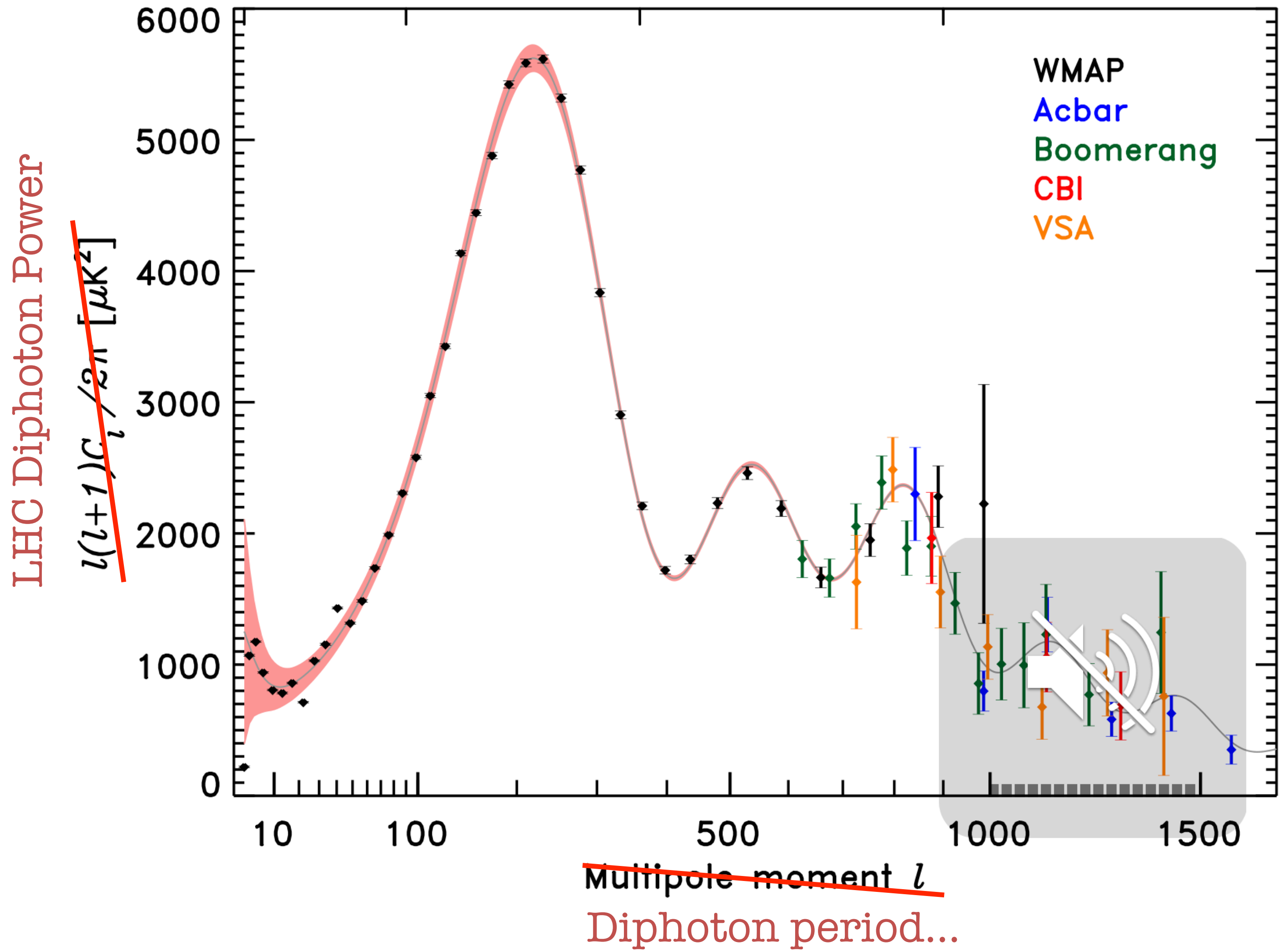
Phenomenology

The fourier transform would then exhibit a peak near the inverse radius:



Phenomenology

Irrespective of the clockwork, it would be a very cool thing to know the LHC power spectrum!!



Phenomenology

Other searches include:

High mass diphoton spectrum. ATLAS and CMS both have 7 TeV limits, we reinterpret 13 TeV resonance searches.

High mass dilepton spectrum, electrons and muons work. ATLAS and CMS have 13 TeV results.

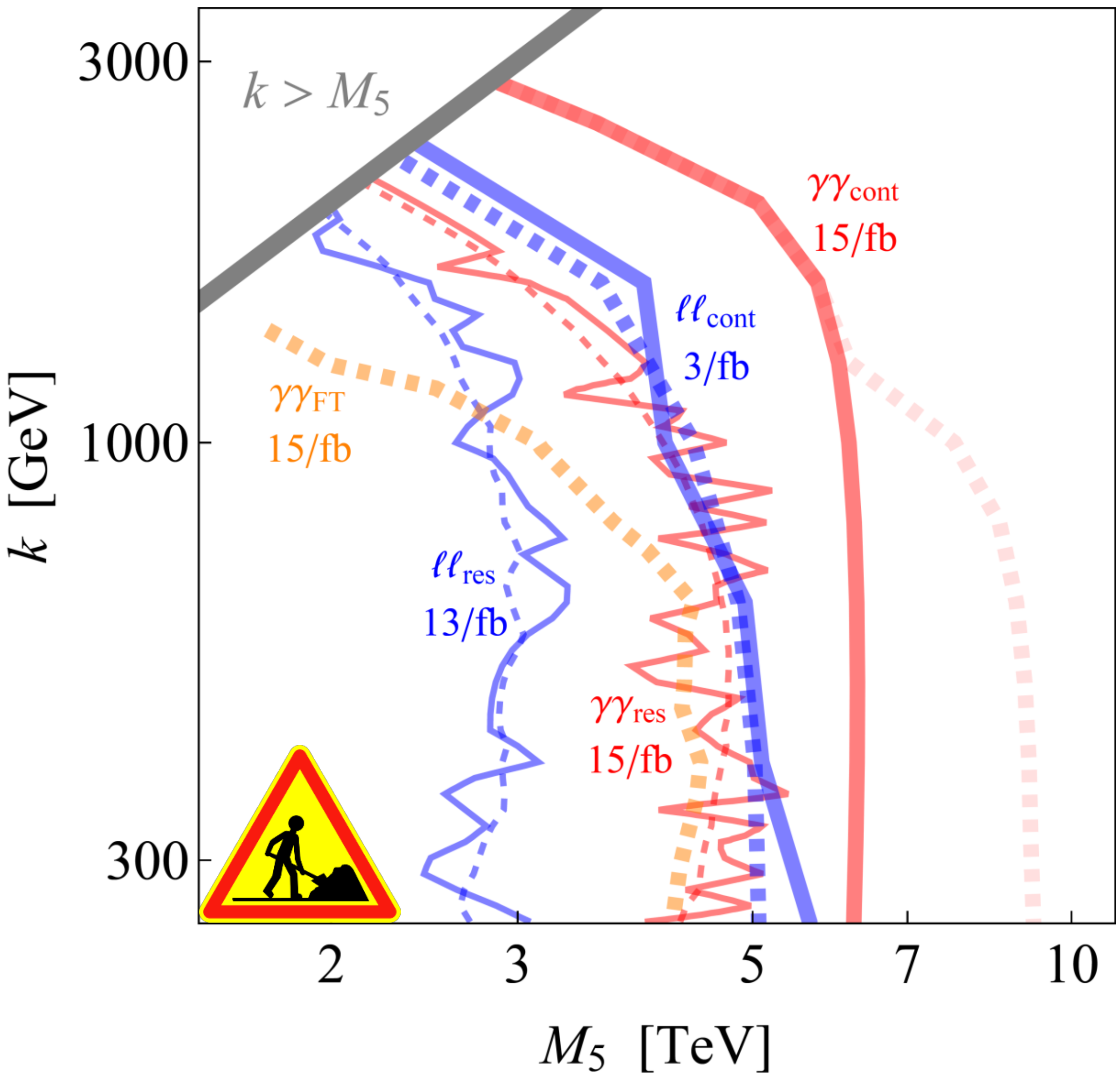
Angular distributions in dijet spectrum.

ATLAS has great analyis at 13 TeV, with 15.7fb^{-1}, but we cannot recast as error bars cannot be read from plot, and are not publicly available,

Standard searches for diphoton and dilepton searches should also give constraints, however it is not clear how the neighboring close by resonances will impact sensitivity in resonance fits.

Phenomenology

Very preliminary summary of constraints:



Work in progress. Note that although the fourier-transform search has not been optimised:

It is clearly a worthwhile analysis to perform!

More phenomenology...

The extra-dimensional scenario contains other interesting signatures

Beam Dump

Displaced vertices

Astrophysics

I did not discuss it, but the clockwork mechanism is more general than extra dimensional scenario, with applications to

Comp Higgs?

Ahmed, Dillon

Inflation?

Kehagias, Riotto

Flavour?

Dark Matter?

Hambye, Teresi, Tytgat

Axions?

Farina, Pappadopulo, Rompineve, Tesi...

Outlook

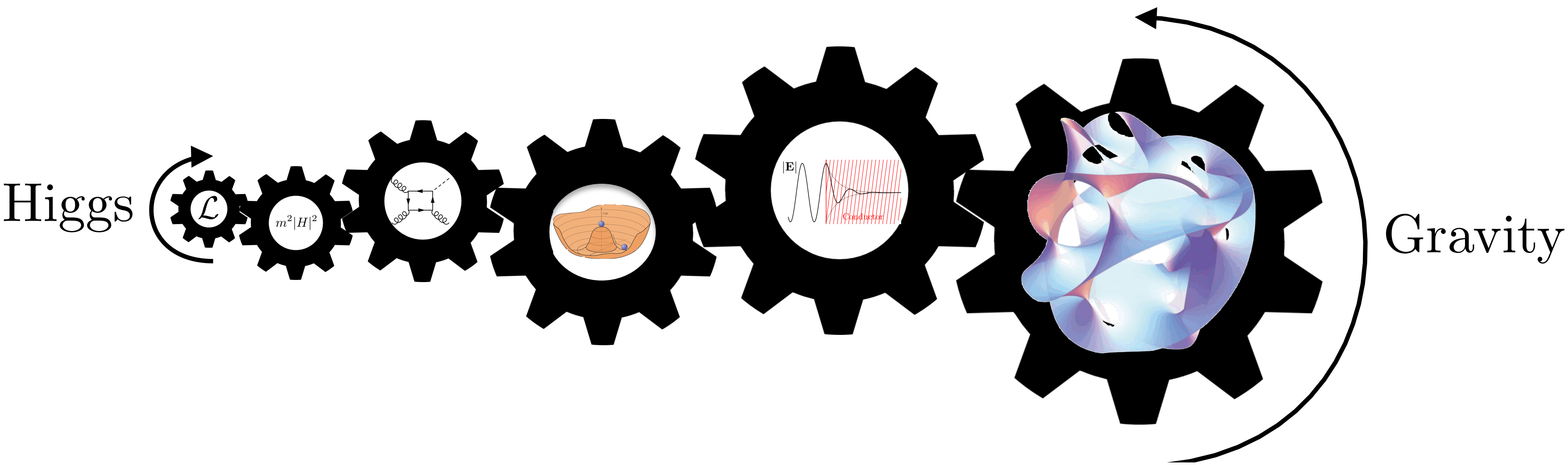
The time is ripe to reexamine hidden assumptions regarding new physics at high energies.

Outlook

The time is ripe to search for new theories that may have unconventional signatures.

Outlook

The clockwork provides a new general approach for addressing a number of BSM puzzles, generating hierarchies without a parametric hierarchy,



and offers a new source of exotic and unexplored collider signatures and cosmology.

Anticipating questions...

An Analogy

Is there a physical picture for what is going on?

When modes are decomposed as KK states:

$$h_{\mu\nu}(x,y) = \sum_{n=0}^{\infty} \frac{\tilde{h}^{(n)}_{\mu\nu}(x)\,\psi_n(y)}{\sqrt{\pi R}}$$

they must satisfy the following equation of motion:

$$\left(\partial_y^2 + 2k\partial_y + \partial_x^2\right)\tilde{h}^{(n)}_{\mu\nu}(x)\,\psi_n(y) = 0$$

Remind you of anything?

An Analogy

When modes are decomposed as KK states:

$$h_{\mu\nu}(x,y) = \sum_{n=0}^{\infty} \frac{\tilde{h}^{(n)}_{\mu\nu}(x)\,\psi_n(y)}{\sqrt{\pi R}}$$

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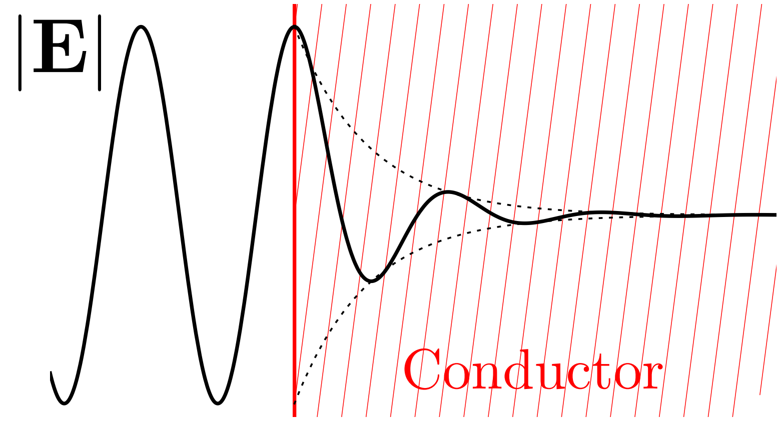
$$\left(\partial_y^2 + 2k\partial_y + \partial_x^2\right)\tilde{h}^{(n)}_{\mu\nu}(x)\,\psi_n(y) = 0$$

Maxwell's equations for EM wave in a conductor:

$$\left(\nabla^2 - \mu\sigma\partial_t - \mu\epsilon\partial_t^2\right)\mathbf{E} = 0$$



An Analogy

W... decomposed as KK states:

$$\ldots (y)$$

they must satis...

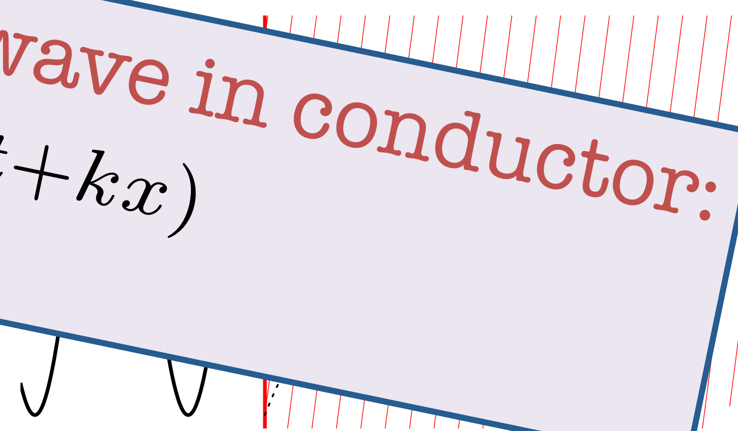
$$\left(\partial_y^2 + 2k\partial_y + \partial_x^2\right) h_{\mu\nu}^{(n)}(x) \ldots$$

General solution for stationary 4D particle:

$$\sim e^{-ky} e^{i\left(m_n t + \sqrt{m_n^2 - k^2}\, y\right)}$$

M... EM wave in a conductor:

$$\left(\nabla^2 - \mu\sigma\partial_t - \mu\epsilon\partial_t^2\right) \ldots$$



General solution for EM wave in conductor:

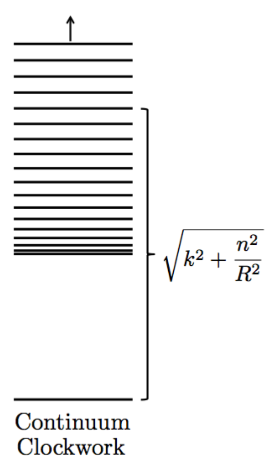
$$\sim e^{-\delta x} e^{i(\omega t + kx)}$$

Isn't this just RS??

It is useful to compare with other theories. For the continuum (5D) story

	m_n^2	Λ_n^2	M_P^2
LED	$\frac{n^2}{R^2}$	$\frac{M_P^2}{2}$	$M_5^3\, 2\pi R$
RS	$\approx [(n+\frac{1}{4})\pi\hat{k}]^2$	$\approx \frac{M_5^3}{\hat{k}}$	$\frac{M_5^3}{\hat{k}}(e^{2\hat{k}\pi R}-1)$
CK	$k^2 + \frac{n^2}{R^2}$	$M_5^3 \pi R\left(1+\frac{k^2R^2}{n^2}\right)$	$\frac{M_5^3}{k}(e^{2k\pi R}-1)$

Mass spectrum distinctive. Band gap, maybe followed by near continuum



Warping of Planck scale very reminiscent of Randall-Sundrum.

Isn't this just RS??

It is useful to compare with other theories. For the discrete story

	m_j^2	q_j
LED	$\frac{N^2}{\pi^2 R^2}$	1
RS	$\frac{N^2}{\pi^2 R^2} e^{-\frac{2\hat{k}\pi R j}{N}}$	$e^{\frac{\hat{k}\pi R}{N}}$
CW	$\frac{N^2}{\pi^2 R^2}$	$e^{\frac{k\pi R}{N}}$

From this perspective the clockwork emerges as a special theory. No hierarchy of mass scales or parameters, but generates an exponential hierarchy of couplings.

Thus we see that while the clockwork dimension clearly shares similarities with RS, it is distinct in a number of respects.

Grand Scheme of Things

This metric has previously arisen in a very different context.

In string theory we could make the choice

$$M_P^2 = \frac{M_s^8\, V_6}{g_s^2}$$

$M_s \sim V_6^{-1/6} \sim \text{ TeV}$

$g_s \sim 10^{-15}(M_s/\text{TeV})(M_s^6 V_6)^{1/2}$

This limit of tiny string coupling is known as "Little String Theory". Studied for many interesting properties.

Grand Scheme of Things

The holographic dual of Little String Theory was proposed by Aharony, Berkooz, Kutasov, Seiberg.

This dual is an extra-dim theory with metric:

$$ds^2 = e^{\frac{4k|y|}{3}}(dx^2 + dy^2)$$

Thus, from a very different starting point, we have arrived at the same continuum theory.

In fact, already studied as a solution to hierarchy problem! (Antoniadis, Dimopoulos, Giveon)

A Clockwork Scalar

Choi & Im, Kaplan & Rattazzi

Take N+1 copies of original story, assume $\lambda \approx 1$, such that at low energies only have Goldstones:

$$\phi_j \sim \frac{f}{\sqrt{2}} e^{i\pi_j/f} \quad , \quad j = 0, .., N$$

Now explicitly break N of the U(1) symmetries explicitly with spurions,

$$\mathcal{L} = \mathcal{L}(\phi_j) - \sum_{j=0}^{N-1} \epsilon \phi_j^* \phi_{j+1}^3 + h.c.$$

This action is justified by symmetry assignments for spurions.

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$$\mathcal{L} = \mathcal{L}(\phi_j) - \sum_{j=0}^{N-1} \epsilon \phi_j^* \phi_{j+1}^3 + h.c.$$

Can take other "q"

This action is justified by symmetry assignments for spurions.

A Clockwork Scalar

Action given by

"Interaction basis π"

$$\mathcal{L} = \frac{1}{2}\sum_{j=0}^{N}(\partial_\mu \pi_j)^2 - \frac{m^2 f^2}{2}\sum_{j=0}^{N-1}\left(e^{\frac{i}{f}(q\pi_{j+1}-\pi_j)} + h.c.\right)$$

Spontaneous symmetry breaking pattern:

$$\mathrm{U}(1)^{N+1} \to \emptyset$$

So expect $N+1$ Goldstones.

Explicit symmetry breaking:

$$\mathrm{U}(1)^{N+1} \to \mathrm{U}(1)$$

So expect N pseudo-Goldstones and one true Goldstone.

Can identify true Goldstone direction from remaining shift symmetry

$$\pi_j \to \pi_j + \kappa/q^j$$

A Clockwork Scalar

Identify Goldstone couplings by promoting shift parameter to a field:

$$\pi_j \to \pi_j + a(x)/q^j$$

Now, imagine we had some fields charged under last $U(1)_N$, thus coupled to π_N. Coupling to true massless Goldstone becomes:

$$\frac{\pi_N}{f} \to \frac{a_0}{q^N f}$$

Exponentially small coupling has been generated from a theory with no exponential parameters!

A Clockwork Scalar

Peculiar spectrum, reminiscent of some Condensed Matter systems...

$$\mathcal{L} = -\frac{1}{2}\sum_{j=0}^{N} \partial_\mu \pi_j \partial^\mu \pi_j - \frac{m^2}{2}\sum_{j=0}^{N-1} (\pi_j - q\,\pi_{j+1})^2 + \mathcal{O}(\pi^4)$$

Mass matrix

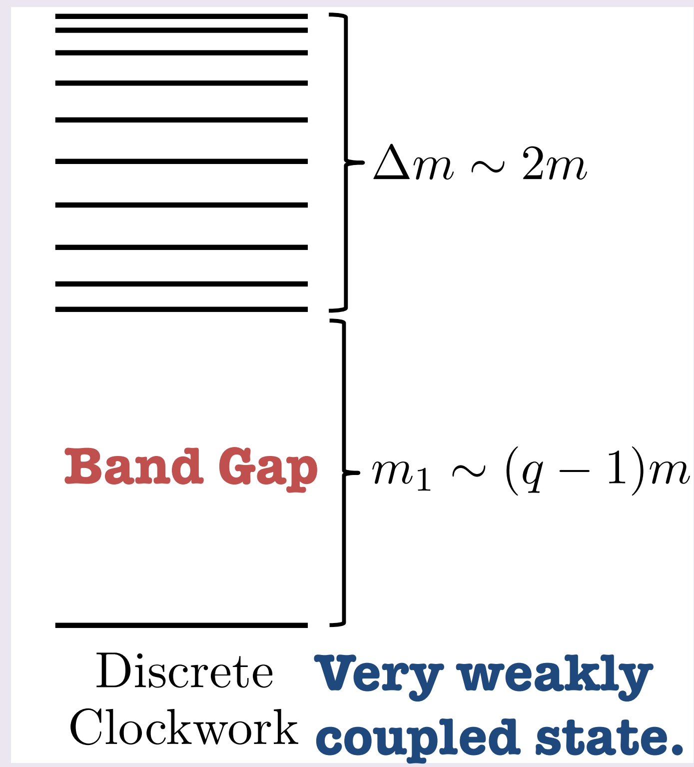
$$M_\pi^2 = m^2 \begin{pmatrix} 1 & -q & 0 & \cdots & & 0 \\ -q & 1+q^2 & -q & \cdots & & 0 \\ 0 & -q & 1+q^2 & \cdots & & 0 \\ \vdots & \vdots & \vdots & \ddots & & \vdots \\ & & & & 1+q^2 & -q \\ 0 & 0 & 0 & \cdots & -q & q^2 \end{pmatrix}.$$

Eigenvalues for "Clockwork Gears"

$$m_{a_k}^2 = \left(q^2 + 1 - 2q\cos\frac{k\pi}{N+1}\right) m^2$$

$k = 1, .., N$

Mass spectrum



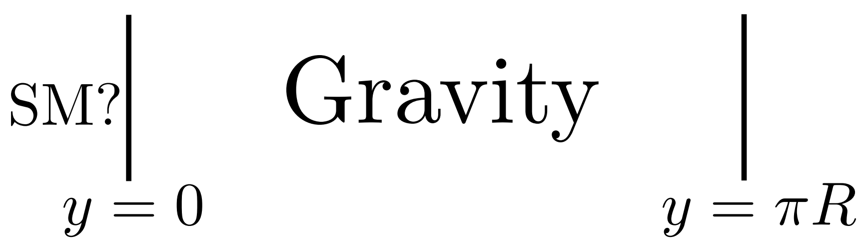
Continue to Continuum

Could clockwork gravity make sense as a lattice version of an extra-dimensional theory?

Imagine a general background geometry

$$ds^2 = X(|y|)dx^2 + Y(|y|)dy^2 \quad , \quad dx^2 = -dt^2 + d\vec{x}^2 \quad .$$

in a 5D interval of length πR:



Continue to Continuum

Reduce dimension to a lattice, like a crystal:

$$y_j = ja \quad , \quad Na = \pi R \quad , \quad \int dy \to \sum_j$$

$$\partial_y \phi(y) \to \frac{1}{a}(\phi_{j+1} - \phi_j) \quad , \quad F(ja) \to F_j$$

The action now in "clockwork form". For example, for scalars

$$\mathcal{S} = -\frac{1}{2}\int d^4x \left[\sum_{j=0}^{N}(\partial_\mu \phi_j)^2 + \sum_{j=0}^{N-1} m_j^2 \left(\phi_j - q_j \phi_{j+1}\right)^2\right]$$