

# $\epsilon'_K/\epsilon_K$ : Standard Model and Supersymmetry

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*Rencontres de Moriond:  
EW Interactions and Unified Theories  
La Thuile, 21 Mar 2017*

Flavour-changing neutral current (FCNC) transitions of Kaons probe virtual effects of very high mass scales.

Example:  $\epsilon_K$  quantifying CP violation in  $K-\bar{K}$  mixing is dominated by loops with top quarks.

⇒ experiment at 0.5 GeV probes physics at 173 GeV

- (i) theoretical control of  $\epsilon_K$  increases steadily (hadronic matrix elements and NNLO QCD under good control,  $\epsilon_K \propto |V_{cb}|^4$  issue improving),
- (ii)  $\epsilon'_K$  now tractable with lattice QCD,
- (iii) upcoming measurements of theoretically clean branching ratios  $B(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  (by NA62) and  $B(K_L \rightarrow \pi^0 \nu \bar{\nu})$  (by KØTØ),
- (iv) no new particles found at LHC:
  - ⇒ weaker rationale for Minimal Flavour Violation (MFV)

If the flavour structure of new physics is unrelated to the SM Yukawa sector, one expects the largest effects in Kaon (and  $\mu \rightarrow e$ ) FCNC processes.

Combine decay amplitudes  $A(K^0 \rightarrow \pi^+\pi^-)$  and  $A(K^0 \rightarrow \pi^0\pi^0)$  into

$$A_0 \equiv A(K^0 \rightarrow (\pi\pi)_{I=0}) \quad \text{and} \quad A_2 \equiv A(K^0 \rightarrow (\pi\pi)_{I=2}),$$

where  $I$  denotes the **strong isospin**.

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Indirect CP violation (from  $K-\bar{K}$  mixing):

$$\epsilon_K \equiv \frac{A(K_L \rightarrow (\pi\pi)_{I=0})}{A(K_S \rightarrow (\pi\pi)_{I=0})} = (2.228 \pm 0.011) \cdot 10^{-3} \cdot e^{i(0.97 \pm 0.02)\pi/4}$$

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Direct CP violation (from decay amplitude):

$$\epsilon'_K \simeq \frac{\epsilon_K}{\sqrt{2}} \left[ \frac{\langle (\pi\pi)_{I=2} | K_L \rangle}{\langle (\pi\pi)_{I=0} | K_L \rangle} - \frac{\langle (\pi\pi)_{I=2} | K_S \rangle}{\langle (\pi\pi)_{I=0} | K_S \rangle} \right] = (16.6 \pm 2.3) \cdot 10^{-4} \cdot \epsilon_K$$

discovered in **1999**

Experimentally well-known:

$$\begin{aligned}\operatorname{Re}A_0 &= (3.3201 \pm 0.0018) \times 10^{-7} \text{ GeV}, \\ \operatorname{Re}A_2 &= (1.4787 \pm 0.0031) \times 10^{-8} \text{ GeV}.\end{aligned}$$



assumes PDG convention for CKM elements

Master equation for  $\epsilon'_K$ :

$$\frac{\epsilon'_K}{\epsilon_K} = \frac{\omega_+}{\sqrt{2}|\epsilon_K^{\text{exp}}|\text{Re}A_0^{\text{exp}}} \left\{ \frac{\text{Im}A_2}{\omega_+} - \left(1 - \hat{\Omega}_{\text{eff}}\right) \text{Im}A_0 \right\}.$$

Here:

$$\omega_+ \simeq \frac{\text{Re}A_2}{\text{Re}A_0} = (4.53 \pm 0.02) \cdot 10^{-2}$$

$\hat{\Omega}_{\text{eff}} = (14.8 \pm 8.0) \cdot 10^{-2}$  quantifies isospin breaking.

Important theoretical ingredients:  $\text{Im}A_0$  and  $\text{Im}A_2$ , calculated from the effective  $|\Delta S| = 1$  hamiltonian describing  $s \rightarrow dq\bar{q}$  decays.

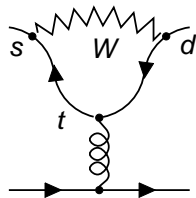
The enhanced sensitivity to  $\Delta I = 3/2$  transitions (such as electroweak penguins and boxes) is a **special feature** of  $\epsilon'_K$ .



$\text{Im}A_0$  is dominated by gluon penguins:

Operator:  $Q_6 = \bar{s}_L^j \gamma_\mu d_L^k \sum_q \bar{q}_R^k \gamma^\mu q_R^j$

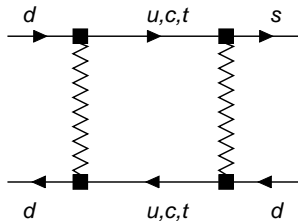
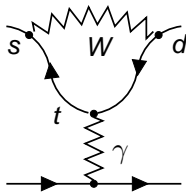
Matrix element:  $\langle (\pi\pi)_{I=0} | Q_6 | K^0 \rangle$



$\text{Im}A_2$  is dominated by photon penguin and box diagrams:

Operator:  $Q_8 = \frac{3}{2} \bar{s}_L^j \gamma_\mu d_L^k \sum_q e_q \bar{q}_R^k \gamma^\mu q_R^j$

Matrix element:  $\langle (\pi\pi)_{I=2} | Q_8 | K^0 \rangle$



$$\frac{\epsilon'_K}{\epsilon_K} = (16.6 \pm 2.3) \times 10^{-4} \quad (\text{experiments: NA62, KTeV})$$

$$\frac{\epsilon'_K}{\epsilon_K} = (1.1 \pm 4.7_{\text{lattice}} \pm 1.9_{\text{NNLO}} \pm 0.6_{\text{isosp. br.}} \pm 0.2_{m_t}) \times 10^{-4} \quad (\text{SM})$$

Kitahara, UN, Tremper, JHEP 1612 (2016) 078

The prediction uses the lattice-QCD results from **RBC-UKQCD**,  
Phys. Rev. Lett. **115** 212001 (2015).

Discrepancy with a significance of **2.8 $\sigma$** !

Our prediction uses the methodology of Buras et al. (JHEP 1511 (2015) 202) (taking **ReA<sub>0,2</sub>** from data), NLO formulae from Buras et al., and a new formula for the RG evolution.

## Standard Model:

Cabibbo-Kobayashi-Maskawa (CKM) factor:

$$\tau = -\frac{V_{td}V_{ts}^*}{V_{ud}V_{us}^*} \sim (1.5 - 0.6i) \cdot 10^{-3}$$

$$\epsilon_K^{\prime\text{SM}} \propto \text{Im } \tau \quad \text{and} \quad \epsilon_K^{\text{SM}} \propto \text{Im } \tau^2.$$

## Generic loop-induced new physics:

some flavour-violating parameter  $\delta$  with  $|\delta| \gg |\tau|$  to compensate for suppression from heavy new-physics mass

$$\epsilon_K^{\prime\text{NP}} \propto \text{Im } \delta \quad \text{and} \quad \epsilon_K^{\text{NP}} \propto \text{Im } \delta^2.$$

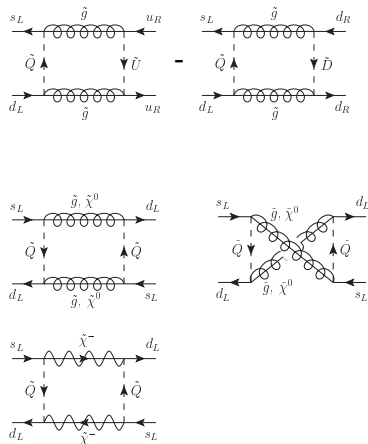
$\Rightarrow$  If  $\epsilon_K^{\prime\text{NP}} \sim \epsilon_K^{\prime\text{SM}}$ , expect  $\epsilon_K^{\text{NP}} \gg \epsilon_K^{\text{SM}}$ .

$\Rightarrow$  Need clever ideas to suppress  $\epsilon_K^{\text{NP}}$ .

# Supersymmetry

The **MSSM** has a mechanism

- to enhance  $\text{Re}A_2$ , because it permits strong-isospin violation through splittings between right-handed up-squark and down-squark masses (**Trojan penguins**),  
Grossman, Kagan, Neubert 1999.
- to suppress the  $\mathbf{K} - \bar{\mathbf{K}}$  mixing amplitude thanks to the Majorana nature of the gluinos, with negative interference of two box diagrams. Crivellin, Davidkov 2010

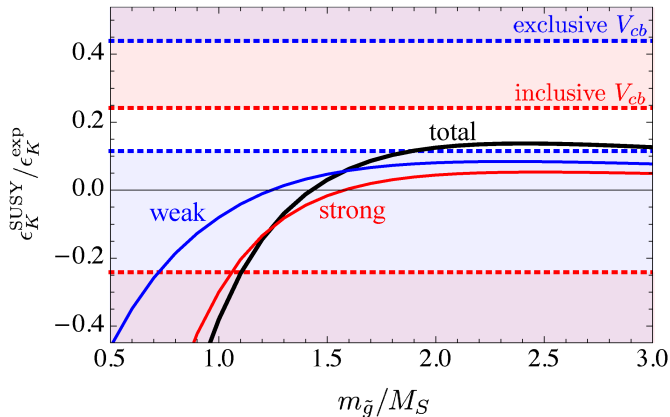


The second feature makes the **MSSM** contribution to  $\mathbf{K} - \bar{\mathbf{K}}$  mixing vanish for  $M_{\tilde{g}} \sim 1.5M_{\tilde{q}}$ , it stays small for  $M_{\tilde{g}} > 1.5M_{\tilde{q}}$ .

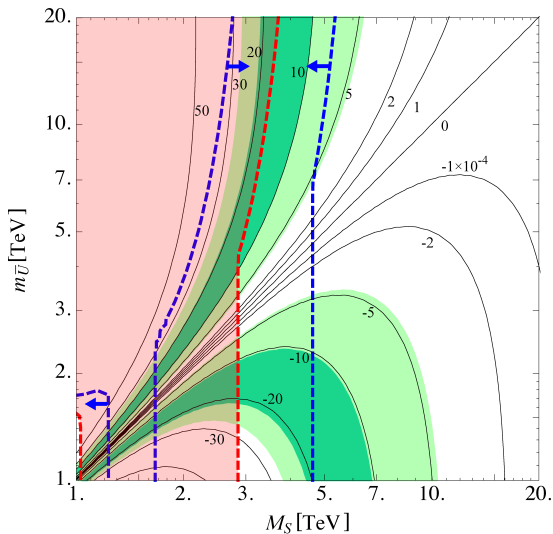
Choose:

Sparticle masses  $M_S \sim 10$  TeV,  $M_{\tilde{g}} > 1.5M_S$ , flavour mixing in down-squark mass matrix only with  $\arg \Delta_{sd}^{LL} = \pi/4$ .

$M_S = 10$  TeV



# Explain $\epsilon'_K$



x-axis: generic sparticle mass,  $M_{\tilde{g}} = 1.5M_S$

y-axis: right-handed up-squark mass

red region: excluded by  $\epsilon_K$  if  $|V_{cb}|$  from inclusive decays is correct

blue dashes: delimit allowed region, if  $|V_{cb}|$  from exclusive decays is correct

The (near) future of Kaon physics:

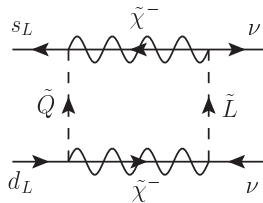
$$B(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \stackrel{\text{SM}}{=} (8.3 \pm 0.3) \cdot 10^{-11} \quad \text{NA62 (CERN)}$$

$$B(K_L \rightarrow \pi^0 \nu \bar{\nu}) \stackrel{\text{SM}}{=} (2.9 \pm 0.2) \cdot 10^{-11} \quad \text{KØTØ (J-PARC)}$$

These branching ratios are theoretically extremely clean.

In our **MSSM** scenario:

Contributions from wino-like chargino box:



Giancarlo D'Ambrosio, Andreas Crivellin, Teppei Kitahara, UN, 1703.05786

Our **MSSM** scenario makes falsifiable predictions for  $B(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  and  $B(K_L \rightarrow \pi^0 \nu \bar{\nu})$ .

Furthermore: if the new-physics contribution to  $\epsilon'_K$  is positive (as indicated by present data), find

$$\text{sgn} [B(K_L \rightarrow \pi^0 \nu \bar{\nu}) - B^{\text{SM}}(K_L \rightarrow \pi^0 \nu \bar{\nu})] = \text{sgn} (m_{\bar{U}} - m_{\bar{D}})$$

Here  $\bar{U}$  and  $\bar{D}$  denote the right-handed **up** and **down squarks**, respectively.

⇒ more in Teppei Kitahara's talk in today's Young Scientists' Forum ⇐



- The new lattice results for the matrix element  $\langle (\pi\pi)_{I=0} | Q_6 | K^0 \rangle$  from **RBC-UKQCD** points to a tension between the experimental value of  $\epsilon'_K$  and the Standard-Model prediction.
- If **new physics** enters through loops, a sizable effect in  $\epsilon'_K$  requires a new source of flavour violation which is much larger than the **CKM factor**  $\text{Im} \frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*} \sim 6 \cdot 10^{-4}$ . But then the effect on  $\epsilon_K$  will typically be too big.
- In the **MSSM** one can simultaneously enhance  $\epsilon'_K$  and suppress the new-physics contributions to  $\epsilon_K$ . This requires flavour mixing among **left-handed squarks**, masses of right-handed **up-type squarks** different from those of the **down-type squarks**, and a **gluino mass** above 1.5 times the mass of the left-handed squarks.
- $B(K \rightarrow \pi\nu\bar{\nu})$  data will test our scenario.  $B(K_L \rightarrow \pi^0\nu\bar{\nu})$  can determine, whether the **right-handed up squark** is heavier or lighter than the **right-handed down squark**.

# Backup

Standard Model:

Cabibbo-Kobayashi-Maskawa (CKM) factor:

$$\tau = -\frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*} \sim (1.5 - 0.6i) \cdot 10^{-3}$$

$$\epsilon_K^{\prime\text{SM}} \propto \text{Im} \frac{\tau}{M_W^2} \quad \text{and} \quad \epsilon_K^{\text{SM}} \propto \text{Im} \frac{\tau^2}{M_W^2}.$$

Generic new physics:

Some flavour-violating parameter:  $\delta$

$$\epsilon_K^{\prime\text{NP}} \propto \text{Im} \frac{\delta}{M^2} \quad \text{and} \quad \epsilon_K^{\text{NP}} \propto \text{Im} \frac{\delta^2}{M^2}.$$

with new heavy particle mass  $M \gg M_W$ .

But data require  $|\epsilon_K^{\text{NP}}| \leq |\epsilon_K^{\text{SM}}|$ , so that

$$\left| \frac{\epsilon_K^{\text{NP}}}{\epsilon_K^{\text{SM}}} \right| \leq \frac{|\epsilon_K^{\text{NP}} / \epsilon_K^{\text{SM}}|}{|\epsilon_K^{\text{NP}} / \epsilon_K^{\text{SM}}|} = \mathcal{O} \left( \frac{\text{Re } \tau}{\text{Re } \delta} \right).$$

If new physics enters through loops, we need  $|\delta| \gg |\tau|$  to compensate for  $M \gg M_W$ , but then  $\epsilon_K$  prohibits large effects in  $\epsilon_K'$ .

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Solutions in the literature:

- tree-level new physics (e.g.  $Z'$ ) with  $|\delta| \sim |\tau|$
- fine-tuning of the CP phase to get  $\text{Re } \delta \sim 0$
- exploit special features of supersymmetry (this talk).