

$\varepsilon', \varepsilon, \dots$  et al & the K-UT  
*LD*

*[from the lattice]*

BNL-HET-032017

EW Moriond 2017

Based in part on Lehner, Lunghi + AS, PLB 2016

# Outline



- Reminiscences: Lattice & *the* UT = “SUT” for Standard UT
- Recent STRIDES on the lattice [**almost exclusively by our RBC-UKQCD Collab**] in tackling long-standing non-perturbative challenges in K-decays
- In concert with expts path towards [primarily] a K-UT
- summary

RBC means  
RIKEN-BNL-Research Center  
BNL  
Columbia

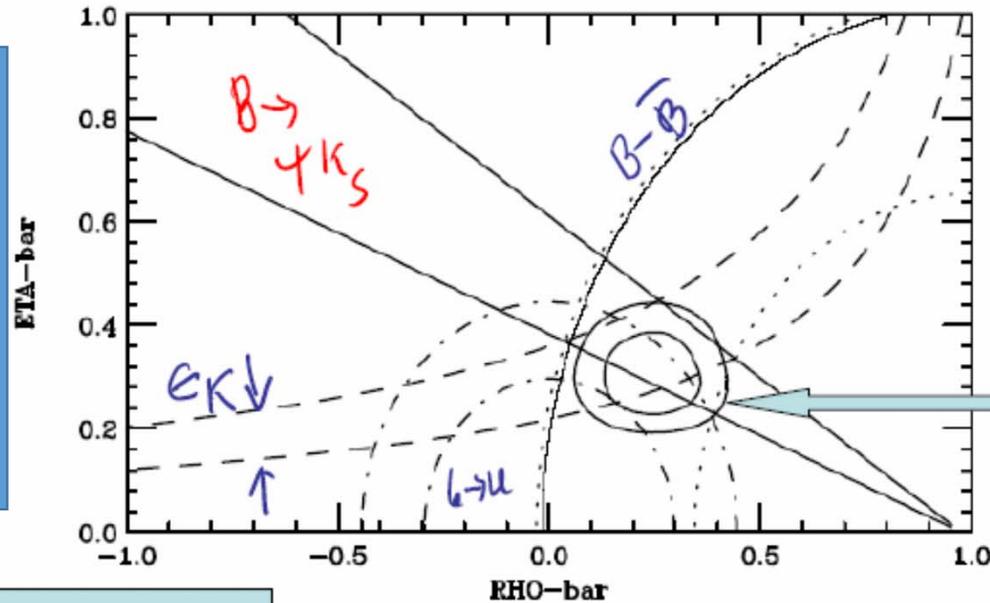
In the "beginning" "Dawn" of  
the asymmetric B-Fac era

Atwood & AS, hep-ph/0103197

B-CP Feb'01 Ise, Japan

1<sup>st</sup> Hint of  
confirmation of CKM  
CP description

Case-A1



REDUNDANT  
MEASUREMENTS  
CRUCIAL

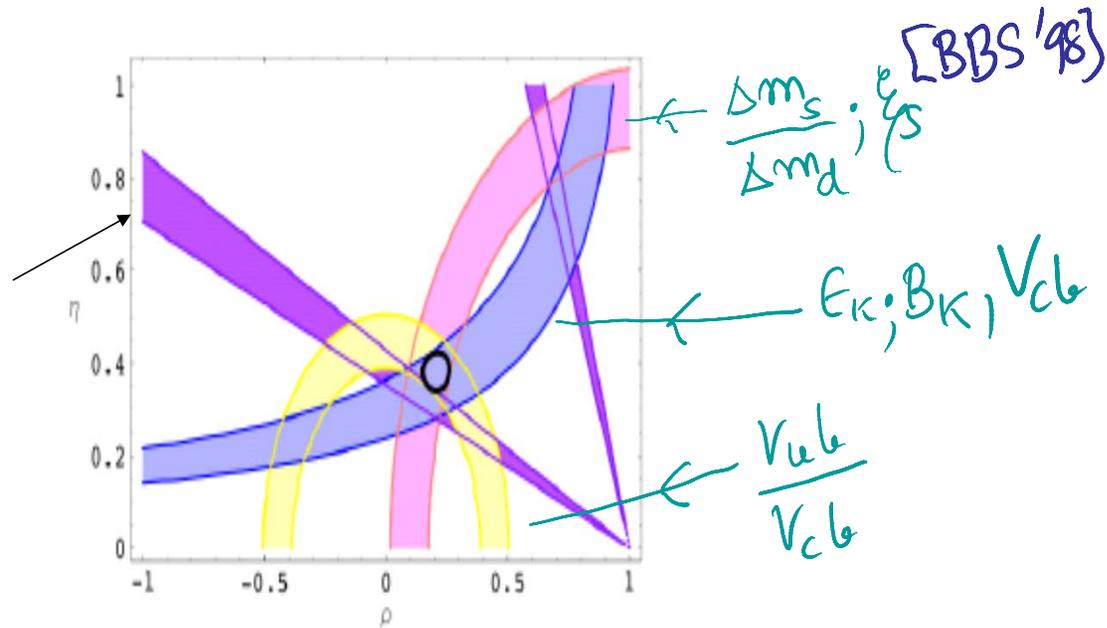
NOOSE

Most bands due  
To theory errors

New physics will be a perturbation, important  
to use clean theory and lots of statistics.

CIRCA ~ 2001

# Lunghi+AS, arXiv.0707.0212

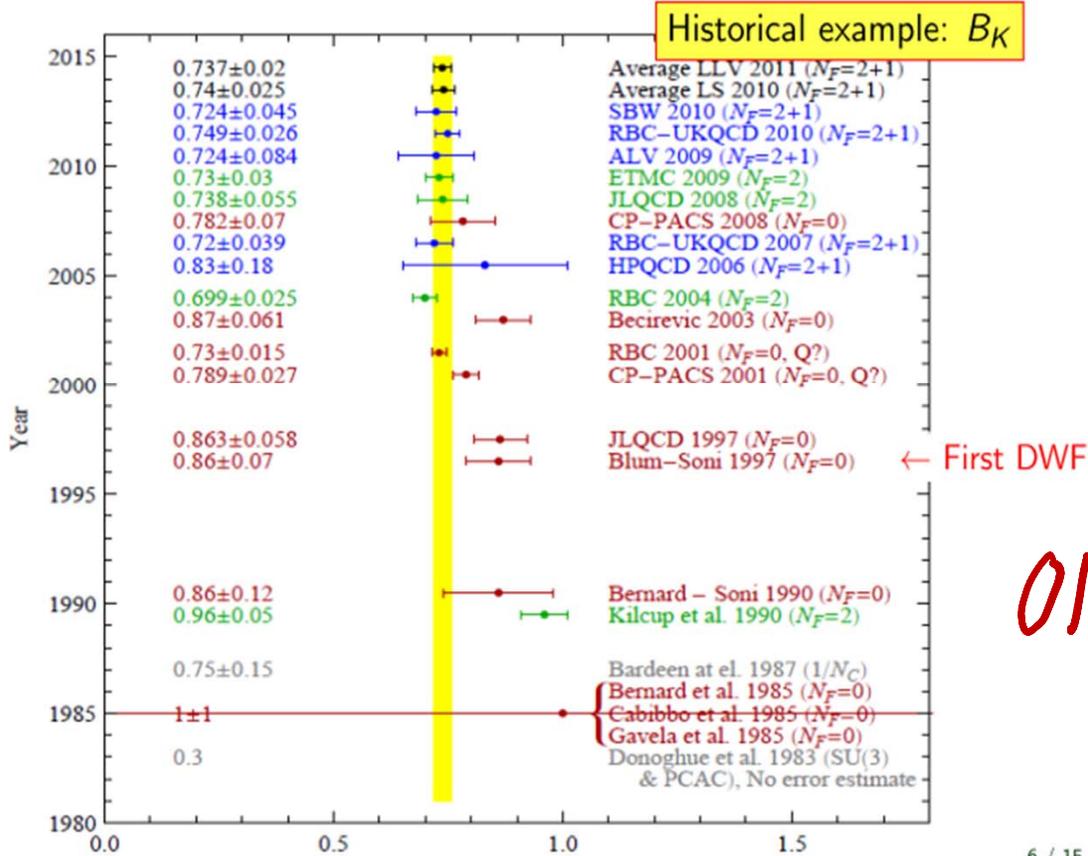


CKM CP  
WORKS  $\sim 15\%$

Figure 1: Unitarity triangle fit in the SM. The constraints from  $|V_{ub}/V_{cb}|$ ,  $\epsilon_K$ ,  $\Delta M_{B_s}/\Delta M_{B_d}$  are included in the fit; the region allowed by  $a_{\psi K}$  is superimposed.

C ALSO  
UTFITS '07

Power of the lattice: Only method to systematically reduce the NP error!



AB-initio Calculation

$$B_K = \frac{\langle \bar{s} \gamma_5 \psi \rangle^2}{\langle \bar{s} \psi \rangle^2 \langle \bar{u} u \rangle^2}$$

ONE ILLUSTRATION

=> Milestone: For the 1<sup>st</sup> time confirmation of the SM-CKM-paradigm of CPV

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Bファクトリー実験に参加している研究教育機関

ブドカー研究所 チェンナイ数理論科学 千葉大学  
 チョナム大学 シンガポール大学 イーファ女子大学  
 キーセン大学 キョウサン大学 ハワイ大学  
 広島工業大学 北京 清華研  
 ミスタウ 東京エネルギー 東京大学 理研新物理学部  
 カルメスルーニ大学 神奈川大学 コリア大学  
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 ローザメ大学 マックスプランク研究所  
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 ノバコリカ 科学技術学校 大阪大学 大阪府立大学  
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プリンストン大学 理化学研究所 佐賀大学  
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 トリア 物理学部 岡山県立大学  
 ウェイン大学 ウィーン工科大学  
 ハーツコブ工科大学 経世大学  
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BELLE コラボ <http://belle.kn.u-tokyo.ac.jp>  
 KEKB <http://www.kekb.jp>  
 BABAR <http://babar.jp>

Poster Designed by T. Iijima, Y. Iwasaki, S. Kataoka, N. Katayama, K. Miyabayashi

# 小林益川理論が正解だった！ Bファクトリーが放った決定打

Courtesy: Tom Browder

Critical Role of the B factories in the verification of the KM hypothesis was recognized and cited by the Nobel Foundation

A single irreducible phase in the weak interaction matrix accounts for most of the CPV observed in kaons and B's.

↪

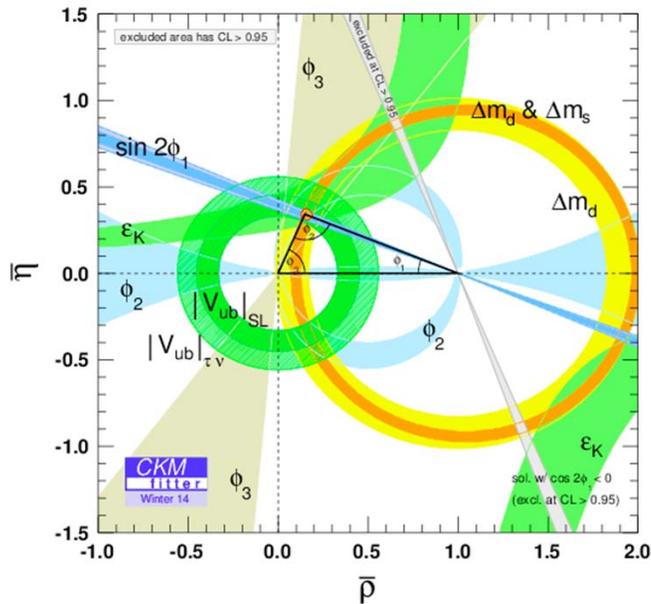
CP violating effects in the B sector are O(1) rather than O(10<sup>-3</sup>) as in the kaon system.

↪ Lattice

7

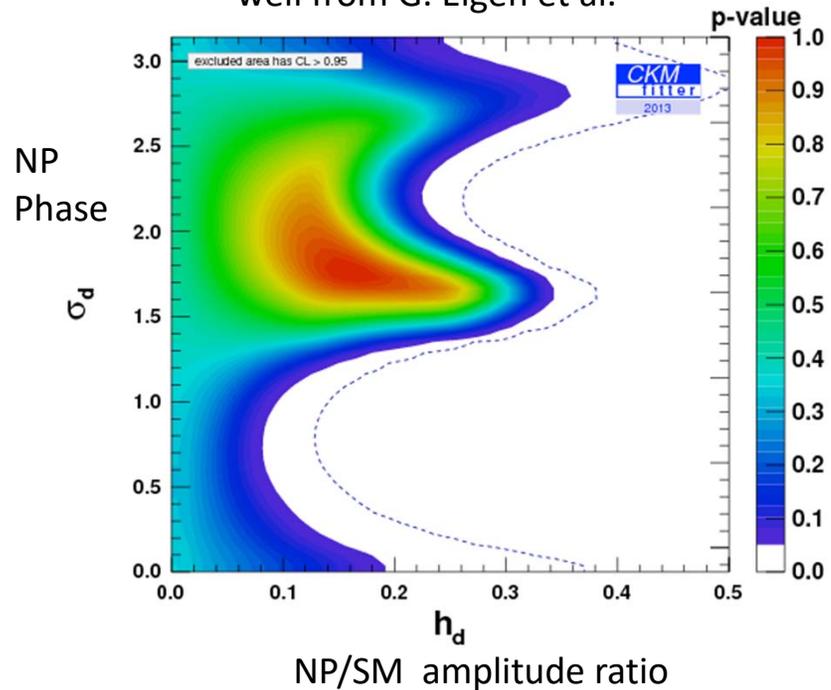
# Results from Global Fits to Data (CKMFitter Group)

Great progress on  $\phi_3$  or  $\gamma$  (first from B factories and now in the last two years from LHCb (several new results at ICHEP2014)). These measure the phase of  $V_{ub}$

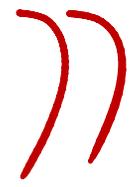


Looks good  
(except for an issue with  $|V_{ub}|$ )

ICHEP2014: Similar results from UTFIT (D. Derkach) as well from G. Eigen et al.



But a 10-20% NP amplitude in  $B_d$  mixing is perfectly compatible with all current data.



## A lesson from history (I)

---

"A special search at Dubna was carried out by E. Okonov and his group. They did not find a single  $K_L \rightarrow \pi^+ \pi^-$  event among **600 decays** into charged particles [12] (Anikira et al., JETP 1962). At that stage the search was terminated by the administration of the Lab. The group was unlucky."

-**Lev Okun**, "The Vacuum as Seen from Moscow"

---

1964:  $BF = 2 \times 10^{-3}$

A failure of imagination? Lack of patience?

CHRISTENSEN,  
CARMON, FITCH  
& TURLAY  
BNL 1964

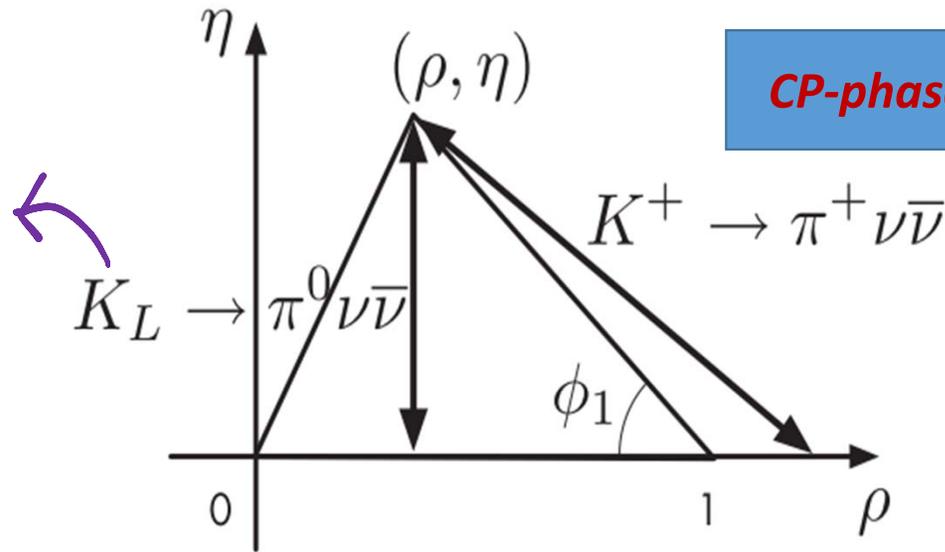
=> Precision! Precision! Precision!  
Need of the day essential for  
uncovering new phenomena  
=> Also, since we are searching for  
small effects, **using different probes  
may be more revealing**

- In B's, in conjunction with experiments, Lattice WME helped in attaining a milestone in our understanding of CP
- Analogously can lattice sharpen tests now via K's?
- Since  $m_K$  is  $\sim 10$  times lighter, the non-perturbative effects are much more difficult and quantitatively a lot bigger, can the lattice meet this long-standing challenge and render K-tests become precise?

A dream for some:  
 UT purely from  $K$ -decays

Blucher, Winstein and  
 Yamanaka '09 & many others

Nothing  
 to  
 Nothing



*CP-phase in special focus*

Fig. 14. Unitarity triangle.

A Faster way in the offing?

Promising developments on the lattice esp in  $\epsilon'/\epsilon$  and in other K-decays.....RBC-UKQCD

- Can the successful lattice treatment  $\epsilon'/\epsilon$  replace  $KL \Rightarrow \pi^0 \nu \nu$ ?
- Personal perspective: lattice calculation of  $\epsilon'/\epsilon$  an obsession for **over 1/3** of a century! & in fact reason for going into LGT ~'83
- O(10 grads!): Terry Draper (UCLA), George Hockney(UCLA), Cristian Calin (Columbia=CU), Jack Laiho(Princeton), Sam Li(CU), Matthew Lightman(CU), Elaine Goode(Southampton), Qi Liu(CU), Daiqian Zhang(CU) +
- obstacles & challenges (and mistakes!) ad infinitum.....
- Started with CWB (Wilson); for this physics Chiral symm on the lattice is a pre-requisite [off-shoot B-physics] => on to **DWF** (with Tom B)=> RBC with ChPT + quenched => huge quench pathologies=full QCD is mandatory for this physics; full QCD + ChPT=> large chiral corrections => RBC-UKQCD direct  $K \Rightarrow 2 \pi$  a la **Lellouch- Lüscher** @ threshold=> @physical kinematics.....

*WHY FOCUS with SUCH intense DETERMINATION*

Underlying Realization

*$\epsilon'$ : a possible gem in search of new phenomena*

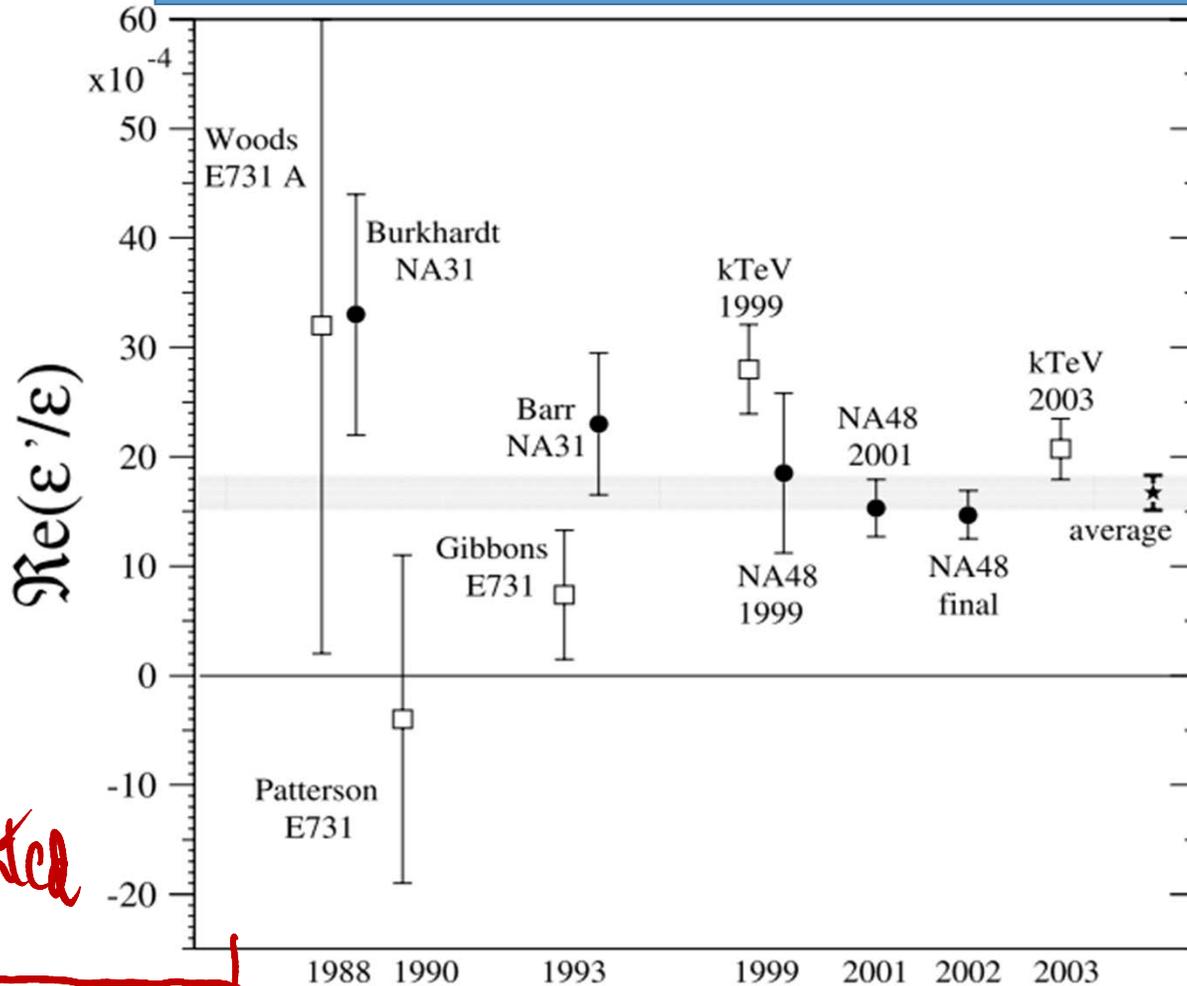
## Its presumed importance:

- lies in its very small size => Perhaps new phenomena has a better chance of showing up....smallness due in part to large [DOUBLE!!] cancellations among seemingly diff sectors seen from the perspective of the SM
- Exceedingly important monitor of flavor –alignment....new CP phase(s)
- **Simple naturalness arguments strongly suggest  $\epsilon_{ps}'$  very sensitive to**

### **BSM – CP odd phases**

- In many ways  $\epsilon_{ps}'$  is rather analogous to  $\text{nedm}$ .....both being very sensitive to BSM phases [however, experimental non-vanishing measurement of  $\text{nedm}$  would necessarily mean discovery of BSM-CP-odd phases...lattice or theory not needed.....This is in complete contrast to  $\epsilon_{ps}'$ ]
- Understanding  $\epsilon_{ps}'$  (and  $\text{nedm}$ ) is extremely important for learning how naturalness really works in nature

A monumental experimental achievement!



Konrad  
kleinknecht  
"Uncovering CPV"

16.6(2.3) x 10<sup>-4</sup>  
PDG 2014

LATTICE  
WORK STARTED

## Results for $\varepsilon'$

- Using  $\text{Re}(A_1)$  and  $\text{Re}(A_2)$  from experiment and  $\text{Im}(A_0)$  and  $\text{Im}(A_2)$  and the phase shifts,

and our lattice values for

→ EWP 185% /  
→ QCDP

$$\text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right) = \text{Re} \left\{ \frac{i\omega e^{i(\delta_2 - \delta_0)}}{\sqrt{2}\varepsilon} \left[ \frac{\text{Im}A_2}{\text{Re}A_2} - \frac{\text{Im}A_0}{\text{Re}A_0} \right] \right\}$$

LARGE CANCELLATION!!

RBC-UKQCD PRL'15  
EDITOR'S CHOICE

$$= \begin{array}{ll} 1.38(5.15)(4.43) \times 10^{-4}, & \text{(this work)} \\ 16.6(2.3) \times 10^{-4} & \text{(experiment)} \end{array}$$

Bearing in mind the largish errors in this first calculation, we interpret that our result are consistent with experiment at  $\sim 2\sigma$  level

$$\omega = \frac{\text{Re}A_2}{\text{Re}A_0} \sim 0.045$$

Computed  $\text{Re}A_2$  excellent agreement with expt  
Computed  $\text{Re}A_0$  good agreement with expt  
Offered an "explanation" of the Delta I=1/2 enhancement

# RBC-UKQCD progress in tackling LD (non-local) contributions

$$|\epsilon_K| = \frac{G_F^2 m_W^2 f_K^2 m_K}{12\sqrt{2}\pi^2 \Delta m_K^{\text{exp}}} \hat{B}_K \kappa_\epsilon \text{Im} \left( \eta_1 S_0(x_c) (V_{cs} V_{cd}^*)^2 + 2\eta_3 S_0(x_c, x_t) V_{cs} V_{cd}^* V_{ts} V_{td}^* \right) \epsilon'$$

$\epsilon'$  also provides a new horizontal band constraint on CKM matrix:

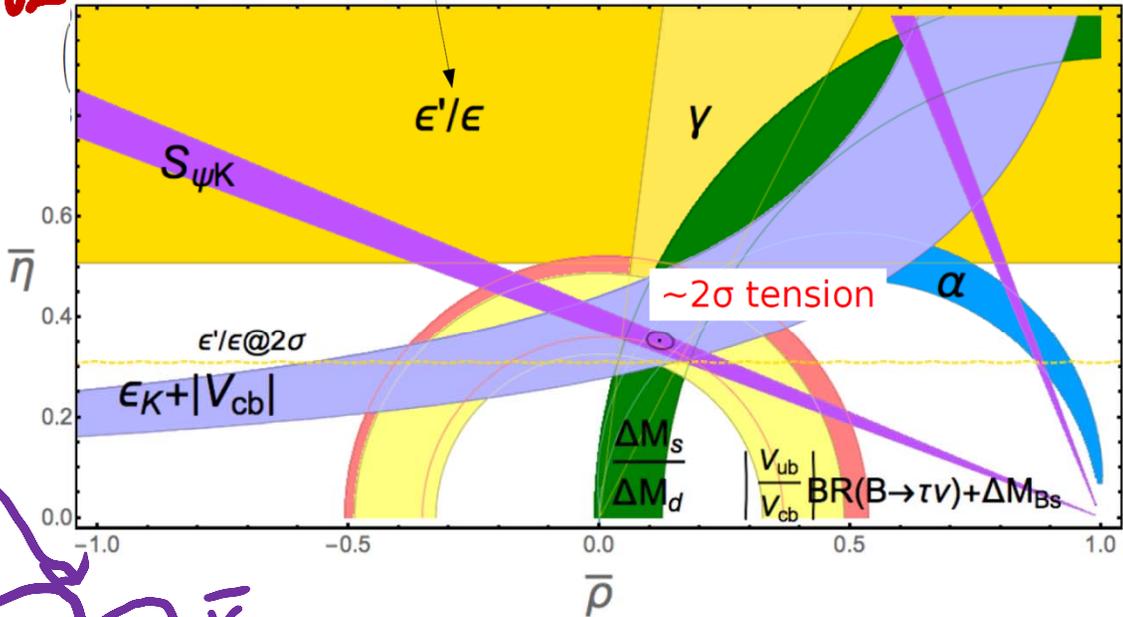
$\delta B_K \sim 2\%$  - Lattice since  $\sim 2013$

$\kappa_\epsilon = 0.96 \pm 0.02$  BGI 110

new constraint from our work!

[Lehner et al arXiv:1508.01801]

$\Delta m_K^{\text{Th}} \sim \Delta m_K^{\text{expt}} \pm 40\%$  (NNLO) B.G.'68



$$SD \Rightarrow \langle K | [\bar{s} \gamma_\mu d(x)]^2 | \bar{K}^0 \rangle$$



<

# LD/non-local effects

$s \text{---} d$   
 $d \text{---} s$   
 $u, c$

$\Rightarrow$

$s \text{---} d$   
 $d \text{---} s$   
 $u, c$

$+ \text{---} + \text{---}$

e.g.  $K \rightarrow \pi \pi \rightarrow \bar{K}$

Expect  $\Delta m_K^{LD} \sim 40\%$ ;  $E_K^{LD} \sim 5\%$ ;  $(K \rightarrow \pi^+ \nu \bar{\nu})^{LD} \sim \text{few}\%$

All 3 processes have been under study by RBC-UKQCD for past  $\sim 4$  years

$s \text{---} d$   
 $z \text{---} \nu$   
 $\nu$   
 $LD \Rightarrow C$

# Methodology [2<sup>nd</sup> order weak processes...]

Example  $K - \bar{K}$

J. Liu } CU  
Z. Bai }  
A. LAWSON }  
Southampton

$$T_{12} = -i \int d^4x \langle K^0 | T [\mathcal{H}_{|\Delta S|=1}^{(u,c)}(x) \mathcal{H}_{|\Delta S|=1}^{(u,c)}(0)] | \bar{K}^0 \rangle,$$

$$\Delta m_K^{\text{FV}} = 2 \sum_{n \neq n_0} \mathcal{A} = - \sum_{n \neq n_0} \frac{\langle \bar{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{m_K - E_n}$$

$$\mathcal{A} = \frac{1}{2} \langle \bar{K}^0(t_f) \int_{t_a}^{t_b} dt_2 \int_{t_a}^{t_b} dt_1 H_W(t_2) H_W(t_1) K^0(t_i) \rangle.$$

$$\mathcal{A} = - \sum_{n \neq n_0} \frac{\langle \bar{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{m_K - E_n} \left[ \underbrace{t_b - t_a}_{\text{red arrow}} - \frac{e^{-(E_n - m_K)(t_b - t_a)} - 1}{m_K - E_n} \right] e^{-(t_f - t_i)m_K} + \dots$$

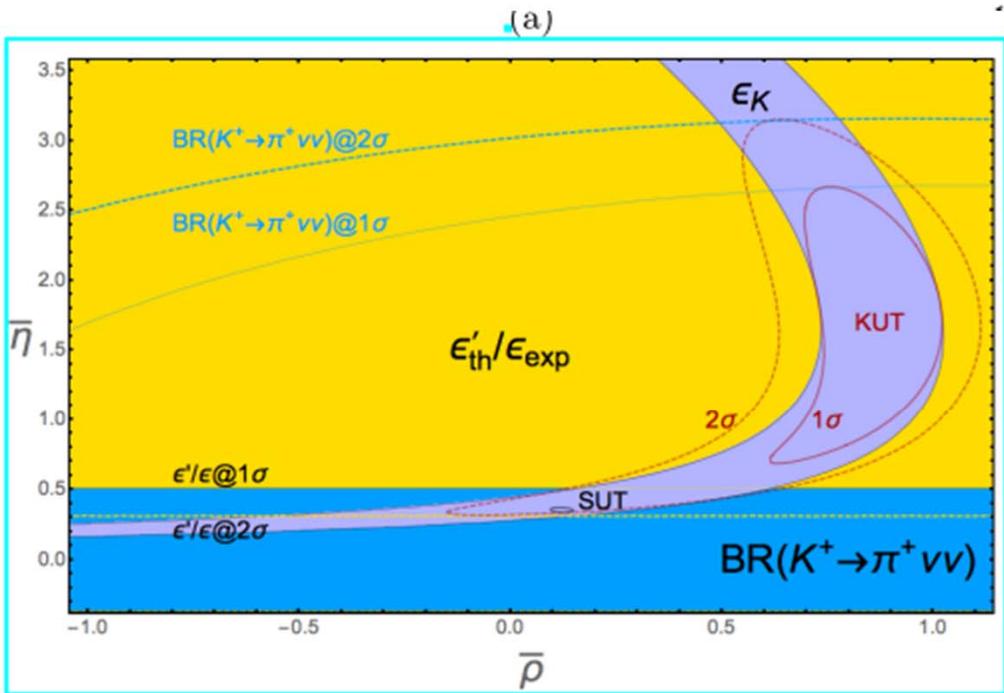
# More on K-decays=>rare K's

- KL => pi0 nu nu ... "Gold-plated", i.e Theory super-clean:  $A \propto m_t^2 \times \eta$   
**LITTENBERG PRD '89**  
**Nothing -> Nothing**
- Observe: The above expt is exceedingly challenging (esp for precision) and expensive.
  - use CP
- Bearing in mind positive developments on the lattice for tackling eps' and also bearing in mind the humungous challenge for precise experimental measurement of KL=>pi0 nu nu, it may be time now for kaon experimentalists to seriously think of improving precision on eps' from the current O(15%)

$K_L \rightarrow \pi^0 \nu \bar{\nu}$  : KOTO @ JPARC . T. YAMANAKA et al

# Sketch of an emerging K-UT

$$\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = \begin{cases} (8.64 \pm 0.60) \times 10^{-11} & \text{SM} \\ (17.3^{+11.5}_{-10.5}) \times 10^{-11} & \text{E949} \end{cases} \quad \text{BNL}$$



$$\text{Re}\left(\frac{\epsilon'}{\epsilon}\right)_K = \begin{cases} (16.7 \pm 1.6) \times 10^{-4} & \text{PDG 2015} \\ (1.36 \pm 5.21_{\text{stat}} \pm 4.49_{\text{syst}}) \times 10^{-4} & \text{ABC+UK@CTD '15} \end{cases}$$

LHS '15

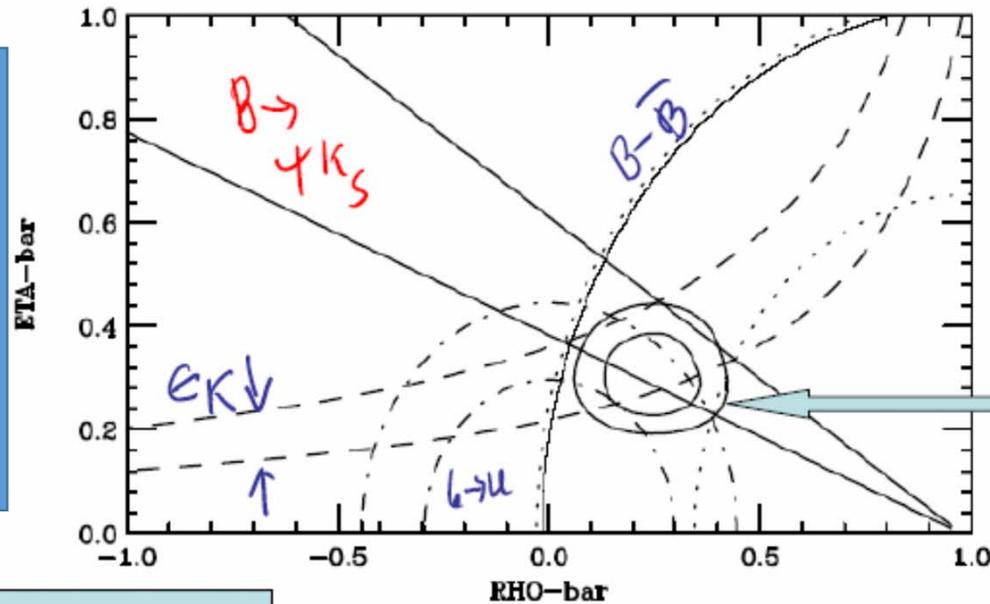
In the “beginning”

Atwood & AS, hep-ph/0103197

Case-A1

B-CP Feb'01 Ise, Japan

1<sup>st</sup> Hint of  
confirmation of CKM  
CP description



Most bands due  
To theory errors

New physics will be a perturbation, important  
to use clean theory and lots of statistics.

# Improvements in lattice eps' determination underway for past ~2 years

- Statistics X [ $> \sim 4$ ] now aiming for
- Systematics.....some already done..
- EM+ isospin....
- Completely diff method(s)
- A) excited  $\pi\pi$  state
- B) Revisit ChPT

[Previous result uses 215 configs]

$\delta[\Gamma_{\text{em}} A_0] \sim (15 \pm 8)\%$   
EMI

Cinigliano et al '04  
Buras et al 2015  
2.16  $\rightarrow$  2.56  
 $\downarrow$   
2.9  
6

TB student

$\rightarrow$  BDSPW '84; LAIHO + AS  
LOXPT  
RBCUKQCD, DMurphy et al  
[1511.01950] NLO

$\downarrow$   
D. HOYING

# SUPERCOMPUTERS OVER 3 CONTINENTS!

*Progress in the calculation of  $\epsilon'$  on the lattice*

C. Kelly

LAT/16

Resource	Million BG/Q equiv core-hours	Independent cfgs.
USQCD (BNL 512 BG/Q nodes)	50	220
RBRC/BNL (BNL 512 BG/Q nodes)	17	50
UKQCD (DiRAC 512 BG/Q nodes)	17	50
NCSA (Blue Waters)	108	380
KEK (KEKSC 512 BG/Q nodes)	74	296
Total	266	996

Table 1: A breakdown of the various resources we intend to utilize. Note that we require 4 molecular dynamics time units per independent configuration.

So for  $\approx 400$  new configs  
 $\approx 175$  new measurements

# Status of new measurements

Over past ~ few months New hardware [CORI-II]  
frantically tested [Chulwoo] + (Christ)  
data taking already begun .  
Expected to last  
many months

# Efforts to improve systematics

→ PRL '15

TABLE II. Representative, fractional systematic errors for the individual operator contributions to  $\text{Re}(A_0)$  and  $\text{Im}(A_0)$ .

Description	Error	Description	Error
Finite lattice spacing	12%	Finite volume	7%
Wilson coefficients	12%	Excited states	$\leq 5\%$
Parametric errors	5%	Operator renormalization	15%
Unphysical kinematics	$\leq 3\%$	Lellouch-Lüscher factor	11%
Total (added in quadrature)			27%

2nd lattice spacing under way  
 MATTIA BRUNO  
 ELAT '16

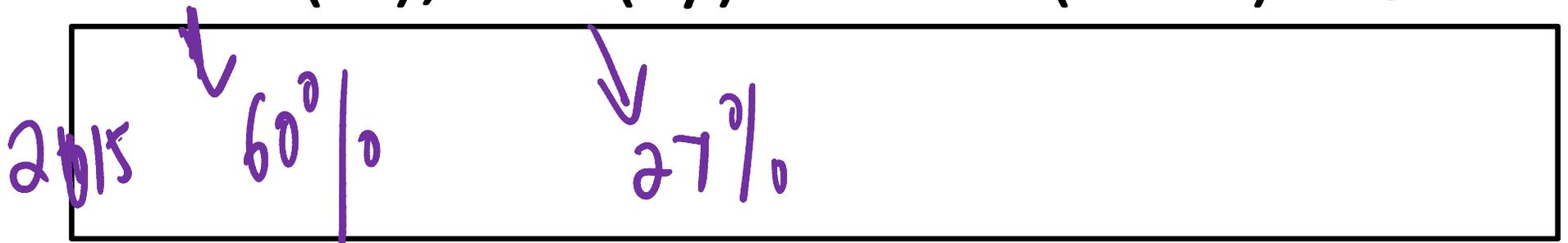
→ 8%

$[M = 1.53 \text{ GeV}]$

2014  $\rightarrow \delta(\text{Im } A_2) \sim 3\% \text{ [stat]} [12\% \text{ sys}]$   
 2020  $\Rightarrow \delta(\text{Im } A_2) \rightarrow 5\%$

Expectations for improved  
determination of  $\text{Im}A_0$  in  $\sim 5$   
years..... $\Delta[\text{Im}A_0]$   
 $\sim 10\%(\text{st}); 15\%(\text{sy}) \Rightarrow 18\%$  (total)

*Best  
Guesses*



# Prospects for improved determination of $B(K^+ \rightarrow \pi^+ \nu \bar{\nu})$

$$BR(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = \begin{cases} (8.64 \pm 0.60) \times 10^{-11} & \text{SM} \\ \left(17.3^{+11.5}_{-10.5}\right) \times 10^{-11} & \text{E949} \end{cases}$$

ARTAMONOV  
et al (BNL)  
PRL '08

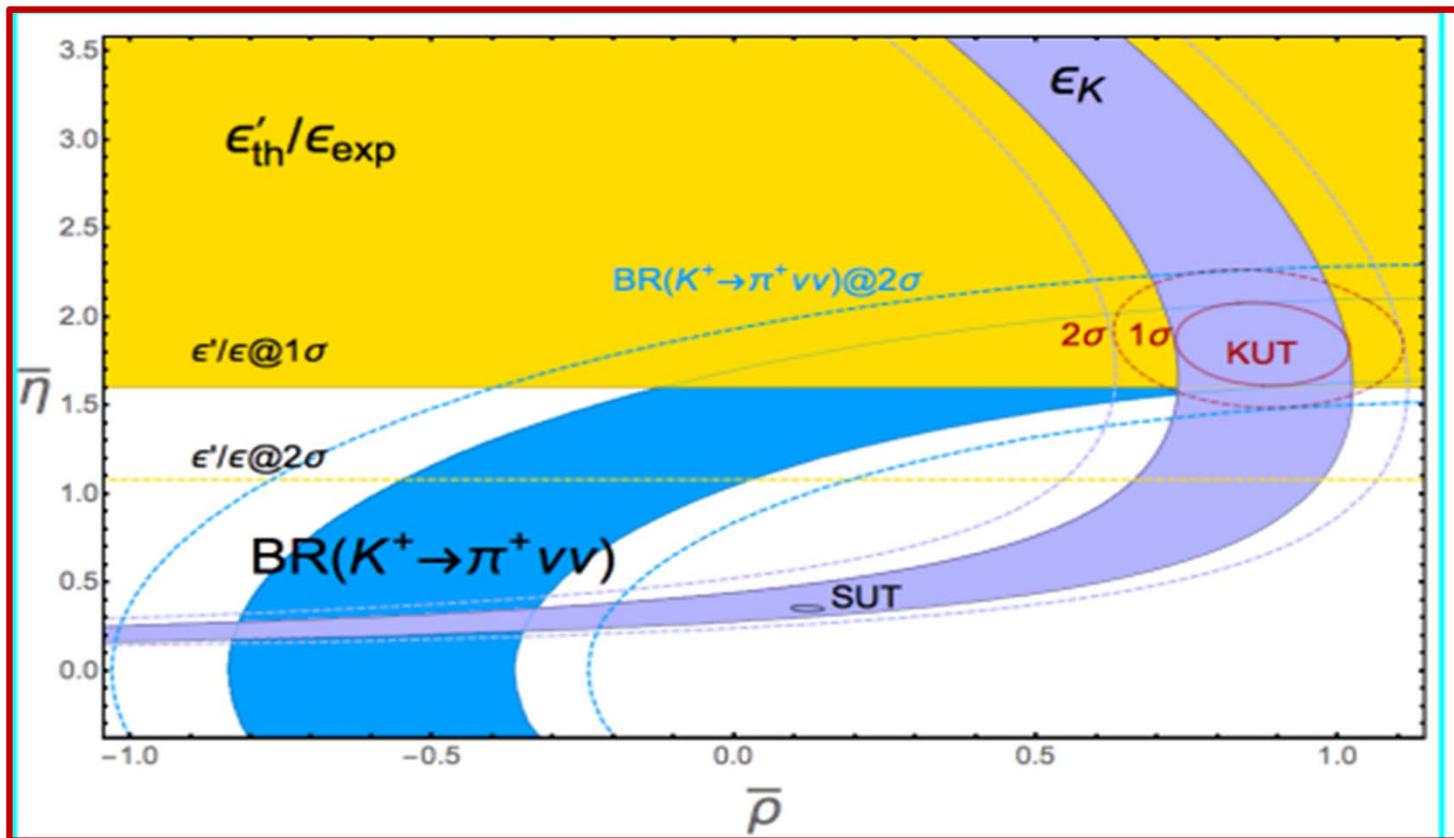
- **NA62 @CERN NOW in progress**....Final results expected < ~ 2 years

expect ~ 100 events  $\Rightarrow$   $\delta [7\%]$

Littenberg  
Panofsky  
Prize

C talk by Nicolas LURKIN [NA62]

# POSSIBLE KUT CIRCA 2020 / Speculation



NO  
unique  
 $\rho, \eta$  !!

LLS PLB '16 EWN 2017; Soni HET

# Summary & Outlook

- In the past ~2 decades, B expts + lattice => SM-CKM paradigm of CP violation works to about an accuracy of ~ 15% through the SUT.
- In the past few years, **RBC-UKQCD collab has made significant progress in lattice methods enabling us to successfully tackle outstanding problems of  $\epsilon'/\epsilon$ , LD non-local contributions to  $\Delta m_K$ ,  $\epsilon_K$ ,  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ , ....**
- In conjunction with existing expt info and with anticipated improvements in key Kaon experiments, **a unitarity triangle based primarily on K-decays will become available in a few years.**
- Note also, the significant progress that has been made and is anticipated in lattice calculation of epsilon' in the next few years suggests that **the experimental community should re-examine the determination of epsilon' to better than the current accuracy of ~15%...Though admittedly very difficult, it may well be less so than precision measurement of  $KL \rightarrow \pi^0 \nu \bar{\nu}$ .**
- Upcoming BELLE-II (with 40-50 X more luminosity) and (of course) LHCb and upgrades ought to significantly improve precision due vast amount of relevant data
- ***All these efforts should lead to more stringent tests of the SM and much better clues to onset of new phenomena***



# XTRAS

# UKQCD Collaboration

- Edinburgh
  - Peter Boyle
  - Luigi Del Debbio
  - Julien Frison
  - Jamie Hudspith
  - Richard Kenway
  - Ava Khamseh
  - Brian Pendleton
  - Karthee Sivalingam
  - Oliver Witzel
  - Azusa Yamaguchi
- Plymouth
  - Nicolas Garron
- York (Toronto)
  - Renwick Hudspith
- Southampton
  - Jonathan Flynn
  - Tadeusz Janowski
  - Andreas Juttner
  - Andrew Lawson
  - Edwin Lizarazo
  - Antonin Portelli
  - Chris Sachrajda
  - Francesco Sanfilippo
  - Matthew Spraggs
  - Tobias Tsang
- CERN
  - Marina Marinkovic

m h c l KITP

# RBC Collaboration

- **BNL**
  - Chulwoo Jung
  - Taku Izubuchi (RBRC)
  - Christoph Lehner
  - Meifeng Lin
  - Amarjit Soni
- **RBRC**
  - **Chris Kelly**
  - Tomomi Ishikawa
  - Taichi Kawanai
  - Shigemi Ohta (KEK)
  - Sergey Syritsyn
- **Columbia**
  - Ziyuan Bai
  - Xu Feng
  - Norman Christ
  - Luchang Jin
  - Robert Mawhinney
  - Greg McGlynn
  - David Murphy
  - **Daiqian Zhang**
- **Connecticut**
  - **Tom Blum**

75% → 15%  
 41% ←

$$Q_2 = \frac{W \overline{W} \overline{d}}{s \quad u}$$

i	Re( $A_0$ )(GeV)	Im( $A_0$ )(GeV)
1	1.02(0.20)(0.07) $\times 10^{-7}$	0
2	3.63(0.91)(0.28) $\times 10^{-7}$	0
3	-1.19(1.58)(1.12) $\times 10^{-10}$	1.54(2.04)(1.45) $\times 10^{-12}$
4	-1.86(0.63)(0.33) $\times 10^{-9}$	1.82(0.62)(0.32) $\times 10^{-11}$
5	-8.72(2.17)(1.80) $\times 10^{-10}$	1.57(0.39)(0.32) $\times 10^{-12}$
6	3.33(0.85)(0.22) $\times 10^{-9}$	-3.57(0.91)(0.24) $\times 10^{-11}$
7	2.40(0.41)(0.00) $\times 10^{-11}$	8.55(1.45)(0.00) $\times 10^{-14}$
8	-1.33(0.04)(0.00) $\times 10^{-10}$	-1.71(0.05)(0.00) $\times 10^{-12}$
9	-7.12(1.90)(0.46) $\times 10^{-12}$	-2.43(0.65)(0.16) $\times 10^{-12}$
10	7.57(2.72)(0.71) $\times 10^{-12}$	-4.74(1.70)(0.44) $\times 10^{-13}$
Tot	4.66(0.96)(0.27) $\times 10^{-7}$	-1.90(1.19)(0.32) $\times 10^{-11}$

large  
 → cancel out  
 → dominant

TABLE I. Contributions to  $A_0$  from the ten continuum,  $\overline{\text{MS}}$  operators  $Q_i(\mu)$ , for  $\mu = 1.53$  GeV. Two statistical errors are shown: one from the lattice matrix element (left) and one from the lattice to  $\overline{\text{MS}}$  conversion (right).

## Application of chiral perturbation theory to $K \rightarrow 2\pi$ decays

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Chiral perturbation theory is applied to the decay  $K \rightarrow 2\pi$ . It is shown that, to quadratic order in meson masses, the amplitude for  $K \rightarrow 2\pi$  can be written in terms of the unphysical amplitudes  $K \rightarrow \pi$  and  $K \rightarrow 0$ , where 0 is the vacuum. One may then hope to calculate these two simpler amplitudes with lattice Monte Carlo techniques, and thereby gain understanding of the  $\Delta I = \frac{1}{2}$  rule in  $K$  decay. The reason for the presence of the  $K \rightarrow 0$  amplitude is explained: it serves to cancel off unwanted renormalization contributions to  $K \rightarrow \pi$ . We make a rough test of the practicability of these ideas in Monte Carlo studies. We also describe a method for evaluating meson decay constants which does not require a determination of the quark masses.

UNIVERSITY OF CALIFORNIA

Los Angeles

Lattice Evaluation of Strong Corrections  
to Weak Matrix Elements -  
The Delta-1 Equals One-Half Rule

A dissertation submitted in partial satisfaction of the  
requirements for the degree Doctor of Philosophy  
in Physics

by

Terrence Arthur James Draper

1984

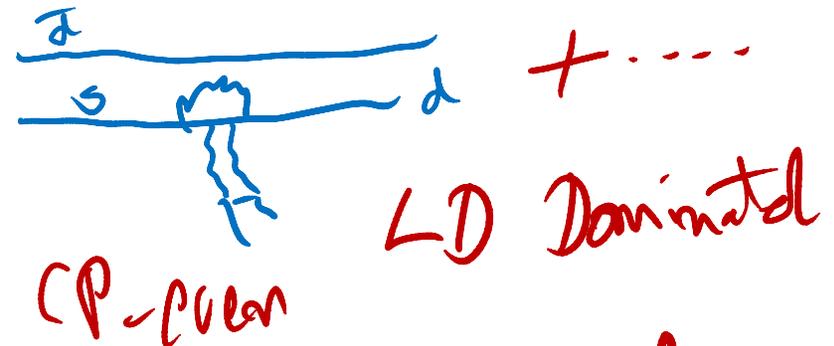
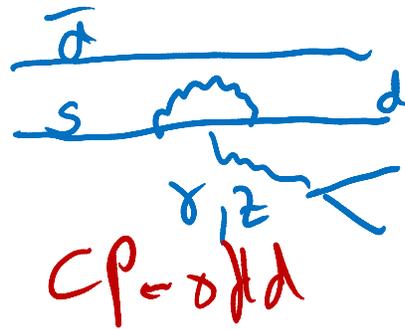
I. Wilson Fermions with Bernard ~'82 See also Martinelli et al [WF] Brower et al Sharpe et al [Stag F]	Lattice $\chi S$ is a pre-requisite for this physics Off-shoot B-physics important observables identified & studied=> evolved into UT	
II (a) DWF with Blum ~ '95 II(b) DWF with RBC[with Blum, Christ and Mawhinney became "flagship" project of RBC] ~'97.	LO $\chi$ PT; Quenched approx.[QA] Same QA is disastrous for this physics [Golterman-Pallante] pathologies; NPR of full $\Delta S=1$ accomplished for the 1 <sup>st</sup> time used since then.	CRAY @ NERSC  QCDSF ~ 1 TF
III. DWF with full QCD RBC, ~ '02	Used LO $\chi$ PT + full QCD Large chiral corrections	QCDSF ~ 1TF
IV. DWF with full QCD RBC + UKQCD, ~ '06	<b>Direct <math>K \Rightarrow \pi\pi</math>, [Lellouch-Luscher method] @ threshold</b>	QCDOC ~ 10 TF
V. DWF with full QCD, RBC + UKQCD ~ '11	Direct $K \Rightarrow \pi\pi$ , [Lellouch-Luscher method] ; physical kinematics	BG/Q ~ 100TF@BNL; RBRC;ANL; Edinburgh
Vi. Same ~now	Same	new hardware  ~1.5PF;NERSC;ANL;UK

HUGE # of OBSTACLES HAS to be overcome

~2006  
↓

$$K_L \Rightarrow \pi^0 e e$$

- Much more readily (seems) can do essentially the same physics as  $K_L \Rightarrow \pi \nu \nu$
- Very rich; e.g can study energy asymm....
- Long-standing challenge for theory: Reliably quantify CP-conserving contamination



- Challenge for expt: huge background from  $e e \gamma \gamma$  [
- $Br [K_L \Rightarrow \pi^0 e e]_{dir-CP} \sim 6 \times 10^{-12}$  ;  $CPC \sim 2 \times 10^{-12}$
- $Br [K_L \Rightarrow \gamma \gamma e e] \sim 5.95 \times 10^{-7}$  !! PDG Imposes severe demands on energy resolution

H. Greenlee  
PRD '90

→ AJB '93

# A possible difficulty: strong phases

- The continuum and our lattice determinations of strong phase difference differs at the  $\sim 2\sigma$  level:

$$\phi_{\epsilon'} = \delta_2 - \delta_0 + \frac{\pi}{2} = \begin{cases} (42.3 \pm 1.5)^\circ & \text{PDG [2]} \\ (54.6 \pm 5.8)^\circ & \text{RBC [47, 48]} \end{cases}$$

→ Not directly accessible expt  
 RBC-UKQCD

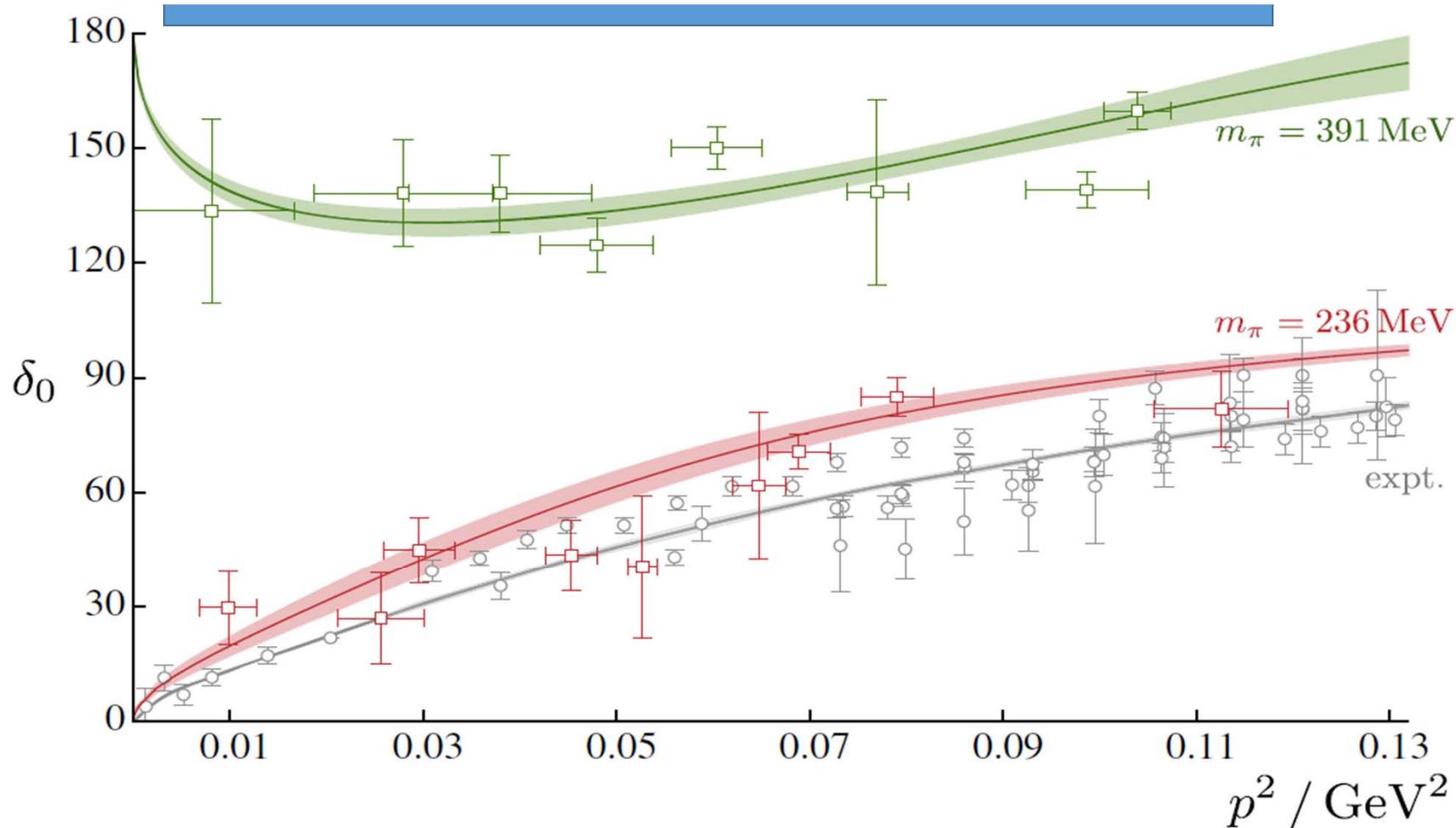
$\phi_\epsilon \sim 43.5 \pm 0.5^\circ$

Fortunately, due to the central value of the combination  $\delta_2 - \delta_0 + \pi/2 - \phi_\epsilon$  and to the large uncertainties in the determination of the various matrix elements, these two choices yield almost identical results; ~~for definiteness, we~~

→ Lehner, Langanke + AS, 1508.01801

**Isoscalar  $\pi\pi$  Scattering and the  $\sigma$  Meson Resonance from QCD**Raul A. Briceño,<sup>1,\*</sup> Jozef J. Dudek,<sup>1,2,†</sup> Robert G. Edwards,<sup>1,\*</sup> and David J. Wilson<sup>3,§</sup>

(for the Hadron Spectrum Collaboration)



Tree

$$Q_1 = (\bar{s}_\alpha d_\alpha)_L (\bar{u}_\beta u_\beta)_L,$$

$$Q_2 = (\bar{s}_\alpha d_\beta)_L (\bar{u}_\beta u_\alpha)_L,$$

$$Q_3 = (\bar{s}_\alpha d_\alpha)_L \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_L,$$

$$Q_4 = (\bar{s}_\alpha d_\beta)_L \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_L,$$

$$Q_5 = (\bar{s}_\alpha d_\alpha)_L \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_R,$$

$$Q_6 = (\bar{s}_\alpha d_\beta)_L \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_R,$$

$$Q_7 = \frac{3}{2} (\bar{s}_\alpha d_\alpha)_L \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\beta)_R,$$

$$Q_8 = \frac{3}{2} (\bar{s}_\alpha d_\beta)_L \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\alpha)_R,$$

$$Q_9 = \frac{3}{2} (\bar{s}_\alpha d_\alpha)_L \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\beta)_L,$$

$$Q_{10} = \frac{3}{2} (\bar{s}_\alpha d_\beta)_L \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\alpha)_L,$$

SM

EWP

~~T=2~~

(8,8)

QCD  
(8,1)

$T=0$

$m_q \rightarrow 0$

$\rightarrow$  const

$m \rightarrow 0$

SM  
eg  
QCD

SM  
 $\frac{1}{2}, 2$

8, 1, 8

EWP

## More demands on the calculation

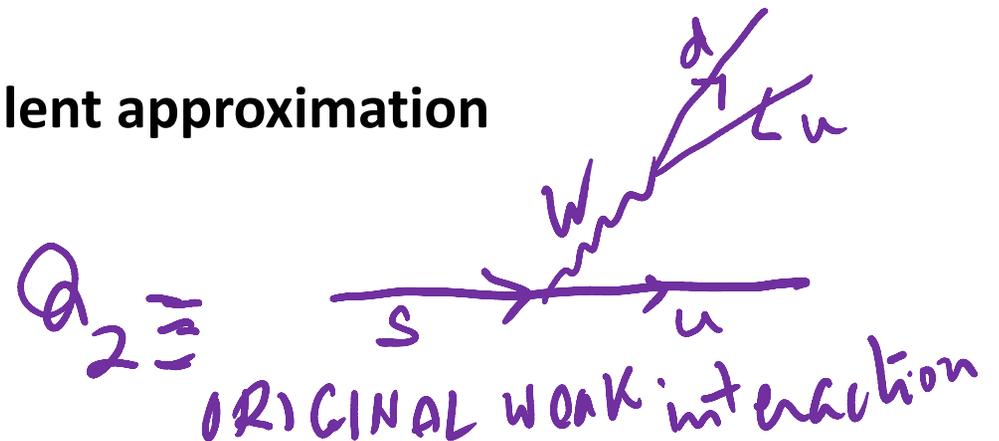
- ~ The 1995 discovery of the huge top mass has accentuated the cancellation of  $l=0$  and  $l=2$  contributions to  $\epsilon_s$  significantly, putting additional demands on the calculation but also enhancing the potential for discovery of new physics

$$c_8 \propto m_t^2 / M_W^2$$

# Regarding ReA0

- Lattice calculation [over and over again over the past ~16 years] show that at a scale greater than about 1.5 GeV, **contribution of penguin operators, to Re A0 is completely negligible.... $<O(\sim 1\%)$ ...only tree operators matter**

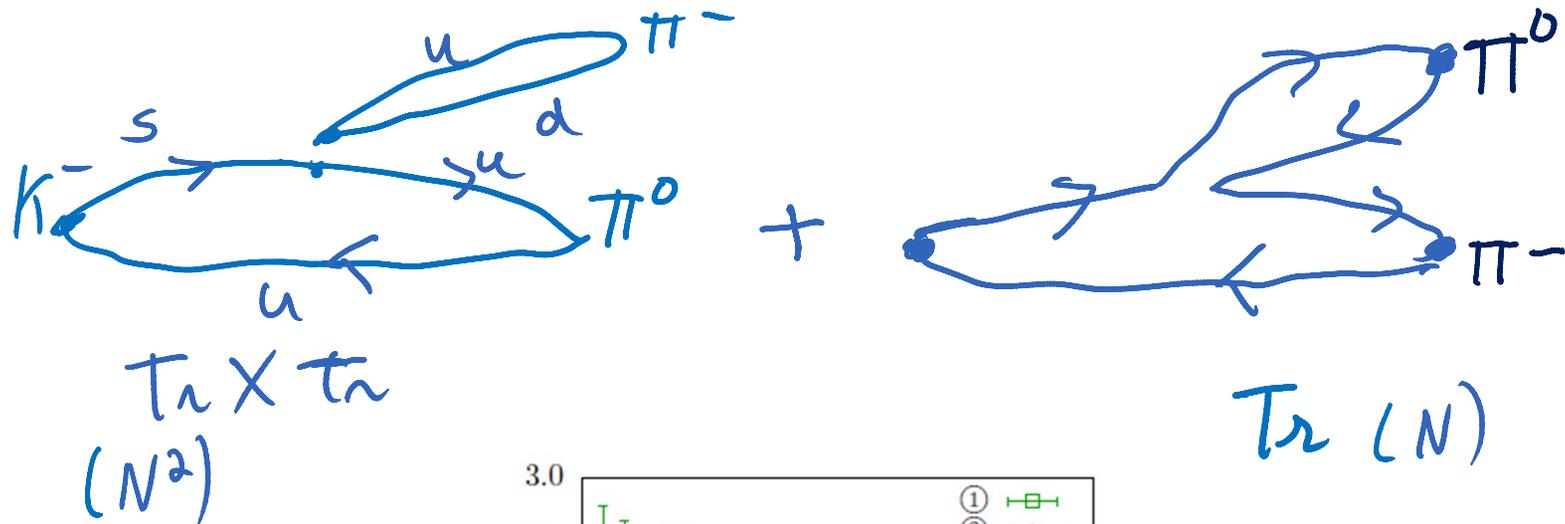
Re A0  $\sim$  c1 Q1 + c2 Q2 to an excellent approximation



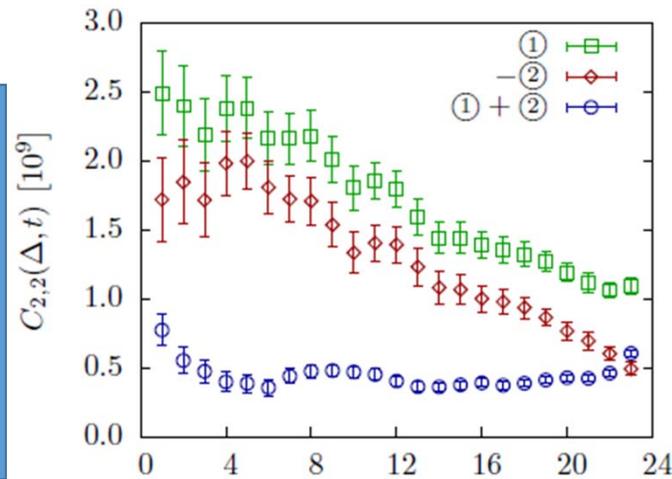
# Useful op identity and its (ab)uses

- $Q4 = Q3 + Q2 - Q1$
- Current lattice cal show rather largish central value for  $[- Q1/Q2]$  with rather errors compared to expectation from large N
- Also  $Q3/Q4$  is small but with largish errors

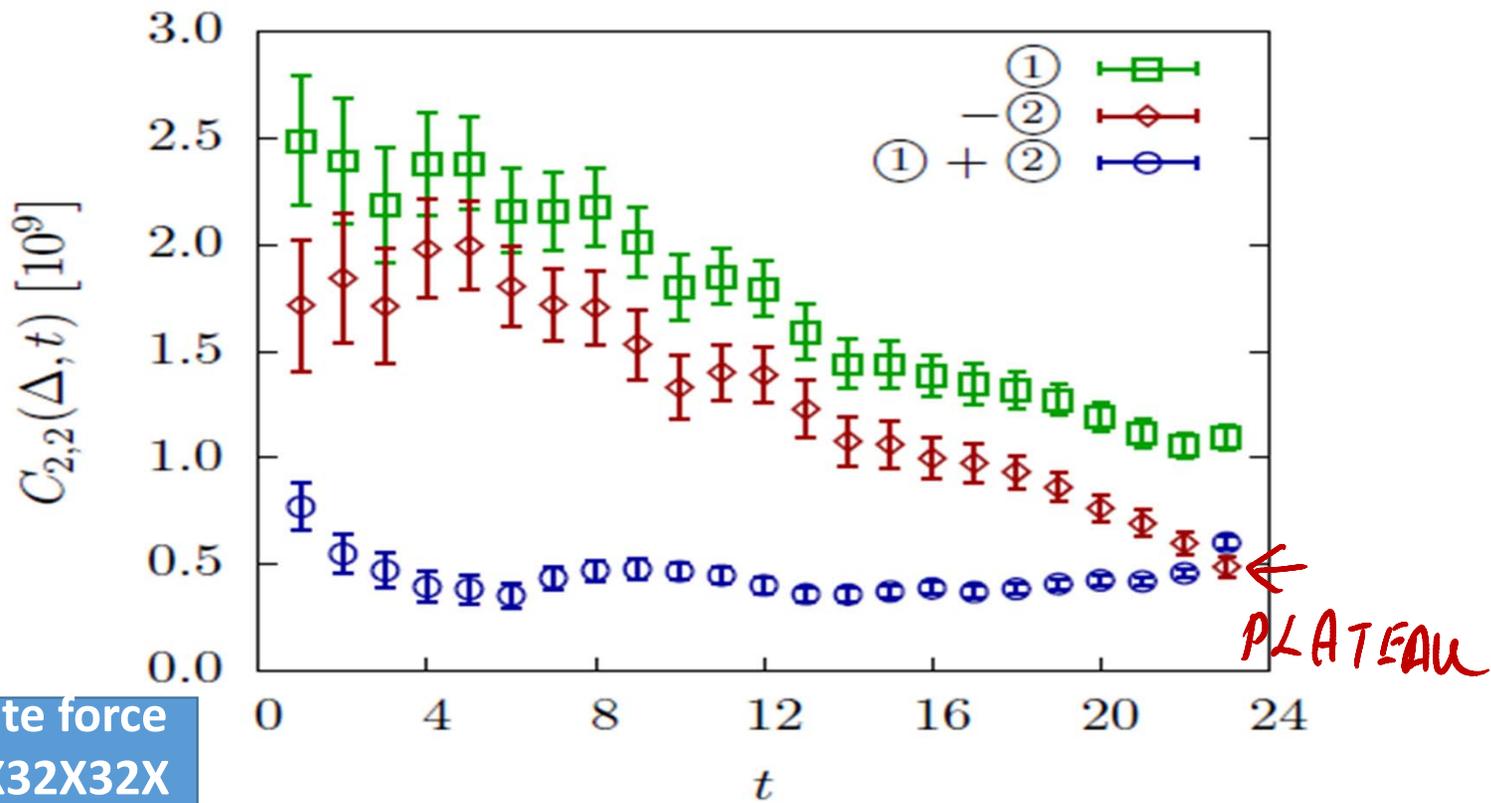
# Dissecting 3/2 Amp on the lattice



Simplest basic step is significantly different from phenomenological expectations



DRAMATIC CANCELLATION!



Brute force  
32X32X32X  
64X16

FIG. 2: Contractions (1), -(2) and (1) + (2) as functions of  $t$  from the simulation at physical kinematics and with  $\Delta = 24$ .

QCDOC 10 Tf

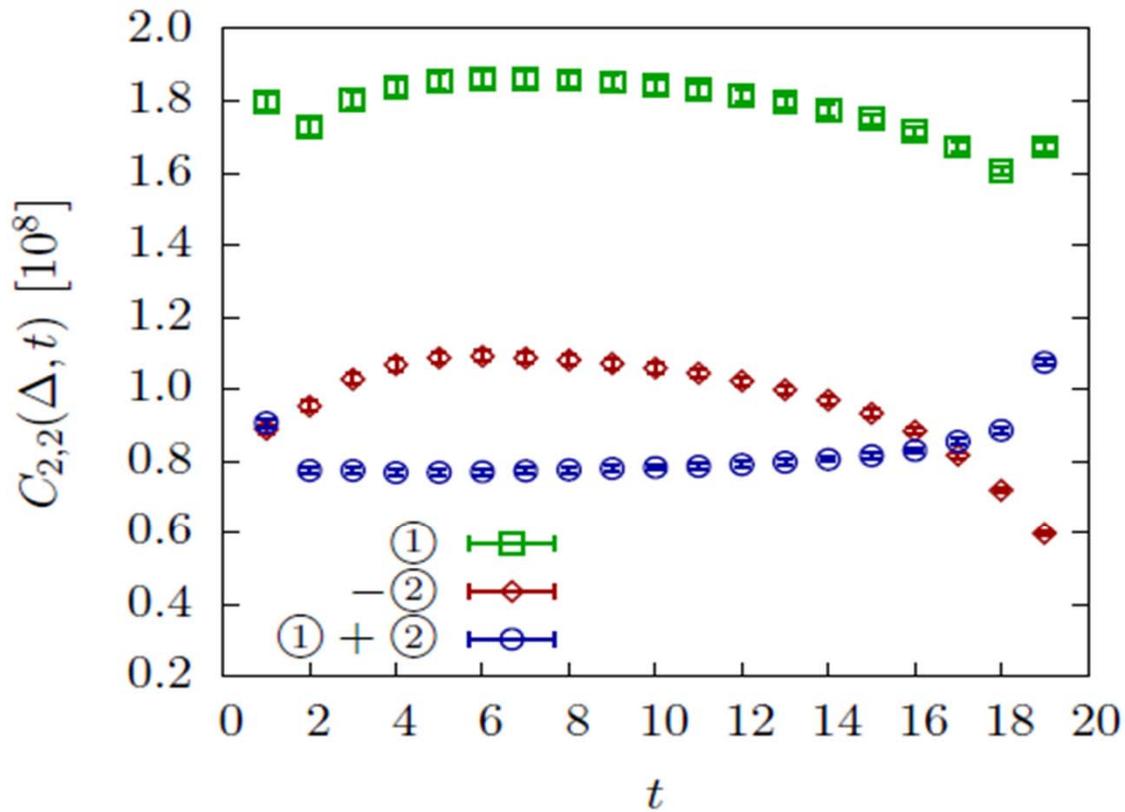


FIG. 3: Contractions ①, -② and ① + ② as functions of  $t$  from the simulation at threshold with  $m_\pi \simeq 330$  MeV and  $\Delta = 20$ .

## Mass depends of ReA2, A0

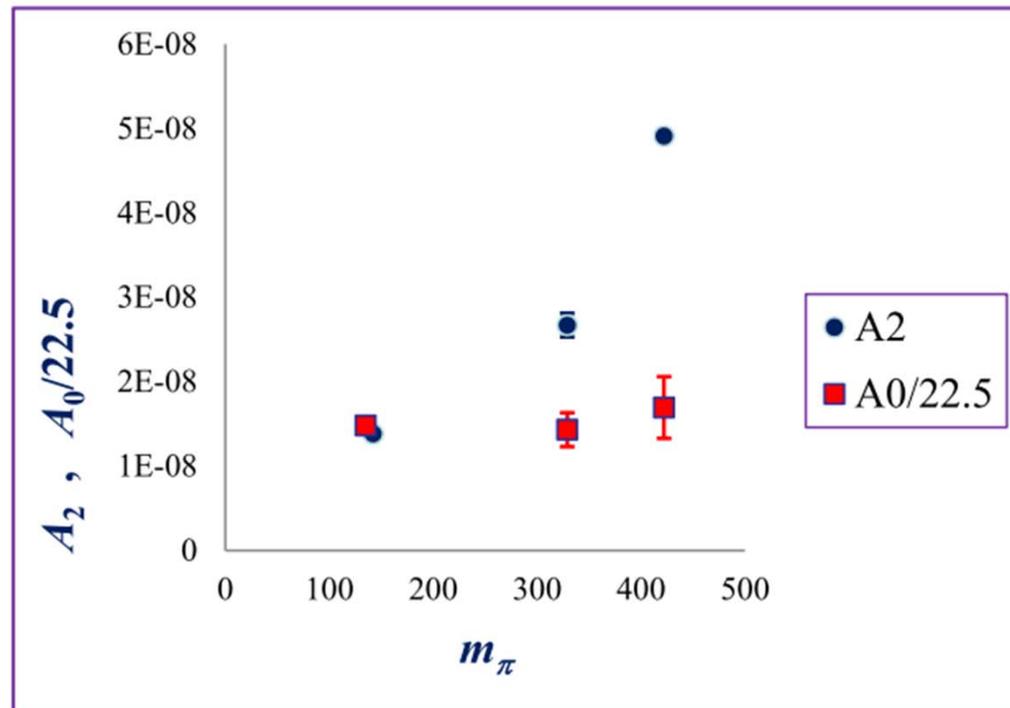
PRL  
2013

	$a^{-1}$ [GeV]	$m_\pi$ [MeV]	$m_K$ [MeV]	$\text{Re}A_2$ [ $10^{-8}$ GeV]	$\text{Re}A_0$ [ $10^{-8}$ GeV]	$\frac{\text{Re}A_0}{\text{Re}A_2}$	notes
$16^3$ Iwasaki	1.73(3)	422(7)	878(15)	4.911(31)	45(10)	9.1(2.1)	threshold calculation
$24^3$ Iwasaki	1.73(3)	329(6)	662(11)	2.668(14)	32.1(4.6)	12.0(1.7)	threshold calculation
IDSDR	1.36(1)	142.9(1.1)	511.3(3.9)	1.38(5)(26)	-	-	physical kinematics
Experiment	-	135-140	494-498	1.479(4)	33.2(2)	22.45(6)	

TABLE I: Summary of simulation parameters and results obtained on three DWF ensembles.

**Due to the cancellation, 3/2 amplitude decreases significantly as the pion mass is lowered towards its physical value**

## Compare $A_2$ and $A_0/22.5$



NHCE  
KITP,  
Aug 15

# Proof of the pudding: underlying method is systematically improvable

- BK in full QCD with DWF '07 error  $O(7\%)$
  - ~2012 many discretizations , WA error  $O(1-2\%)$
  - K13, A2, fB's , BB's.....
  - Mq's....
- No doubt that A0, A2 for eps' will not go that way for quite sometime to come.....to ~10% total
- After that EM& isospin effects need to be ascertained quantitatively.

One measure

C.k e Lat 16

	1.32 GeV 24I	1.33 GeV 32ID	1.53 GeV 32ID	2.29 GeV 24I	2.29 GeV stepscaled, no $G_1$	2.29 GeV stepscaled inc. $G_1$
(1,1)	0.06076(15)	0.063454(63)	0.05978(13)	0.036954(41)	0.03948(16)	0.03948(16)
(2,2)	0.203(19)	0.204(15)	0.300(68)	0.0795(70)	0.080(33)	0.092(35)
(2,3)	0.310(21)	0.317(16)	0.363(76)	0.1486(59)	0.153(36)	0.161(42)
(2,4)	0.0120(48)	0.0076(42)	0.030(22)	0.0033(23)	0.0083(89)	0.0086(93)
(2,5)	0.0120(42)	0.0005(31)	0.015(20)	0.0039(15)	0.0074(53)	0.0081(77)
(3,2)	0.283(22)	0.268(15)	0.264(87)	0.1547(42)	0.143(23)	0.174(26)
(3,3)	0.391(25)	0.414(17)	0.44(11)	0.2207(39)	0.238(25)	0.297(32)
(3,4)	0.0012(59)	0.0002(34)	0.019(27)	0.0077(13)	0.0057(59)	0.0017(66)
(3,5)	0.0128(62)	0.0264(43)	0.008(27)	0.0190(11)	0.0247(46)	0.0090(68)
(4,2)	0.118(70)	0.094(53)	0.24(25)	0.037(24)	0.07(10)	0.06(11)
(4,3)	0.113(76)	0.073(62)	0.26(30)	0.026(20)	0.08(11)	0.09(13)
(4,4)	0.006(20)	0.006(16)	0.076(88)	0.0318(80)	0.024(30)	0.023(31)
(4,5)	0.023(18)	0.019(12)	0.046(79)	0.0014(50)	0.033(19)	0.027(26)
(5,2)	0.239(28)	0.205(28)	0.19(17)	0.0957(84)	0.048(63)	0.033(74)
(5,3)	0.404(34)	0.347(37)	0.25(20)	0.1885(90)	0.101(77)	0.075(99)
(5,4)	0.0106(80)	0.0174(98)	0.039(62)	0.0028(26)	0.044(20)	0.031(23)
(5,5)	0.0810(100)	0.0740(93)	0.016(56)	0.0303(23)	0.012(15)	0.062(21)
(6,6)	0.00461(21)	0.005040(92)	0.006154(96)	0.001631(63)	0.00185(24)	0.00185(24)
(6,7)	0.002132(46)	0.003396(25)	0.002073(59)	0.001567(12)	0.002002(43)	0.002002(43)
(7,6)	0.02221(12)	0.024916(60)	0.024131(91)	0.018826(21)	0.02260(20)	0.02260(20)
(7,7)	0.127052(87)	0.133903(52)	0.12284(25)	0.080743(12)	0.08705(11)	0.08705(11)

Table 2: Preliminary values of  $\Xi$  obtained using the renormalization matrices obtained at various scales on the 32ID and 24I lattices, as well as using the step-scaled matrices with and without  $G_1$ .

# MOTHER of all (lattice) calculations to date: A Personal Perspective

- Calculation  $K \Rightarrow \pi$  &  $\epsilon$ ' were the reasons I went into lattice over 1/3 of a century ago!
- 9 + PhD thesis: Terry Draper (UCLA), George Hockney(UCLA), Cristian Calin (Columbia=CU), Jack Laiho(Princeton), Sam Li(CU), Matthew Lightman(CU), Elaine Goode(Southampton), Qi Liu(CU), Daiqian Zhang(CU)

*many PhD's et al also will*

EWM 2017; Soni HET

$\epsilon' / \epsilon$ : Direct CPV **EXPERIMENTAL ROUTE**

$$\eta_{+-} = |\eta_{+-}| e^{i\phi_{+-}} = \frac{A(K_L \rightarrow \pi^+\pi^-)}{A(K_S \rightarrow \pi^+\pi^-)}$$

$$\eta_{00} = |\eta_{00}| e^{i\phi_{00}} = \frac{A(K_L \rightarrow \pi^0\pi^0)}{A(K_S \rightarrow \pi^0\pi^0)}$$

$$\eta_{+-} = \epsilon + \epsilon', \quad \eta_{00} = \epsilon - 2\epsilon'$$

$$\epsilon' = \frac{1}{3} [\eta_{+-} - \eta_{00}] \Rightarrow 0(10^{-3}) - 0(10^{-3}) \Rightarrow 10^{-6}$$

$$\epsilon = \frac{1}{3} [2\eta_{+-} + \eta_{00}]$$

$K \rightarrow 2\pi$

**FROM THEORY**

$$\text{Re}\left(\frac{\epsilon'}{\epsilon}\right) = \frac{\omega}{\sqrt{2}|\epsilon|} \left[ \frac{\text{Im}(A_2)}{\text{Re}(A_2)} - \frac{\text{Im}(A_0)}{\text{Re}(A_0)} \right]; \quad \omega \approx \frac{\text{Re}A_2}{\text{Re}A_0}$$

$I = 2 \text{ amp}$   
 $I = 0 \text{ amp}$

BNL '64  
CRONIN +  
FITCH N.P.

DIRECT ~~CP~~

Indirect CP

$$|\epsilon| = 2.228(11) \times 10^{-3},$$

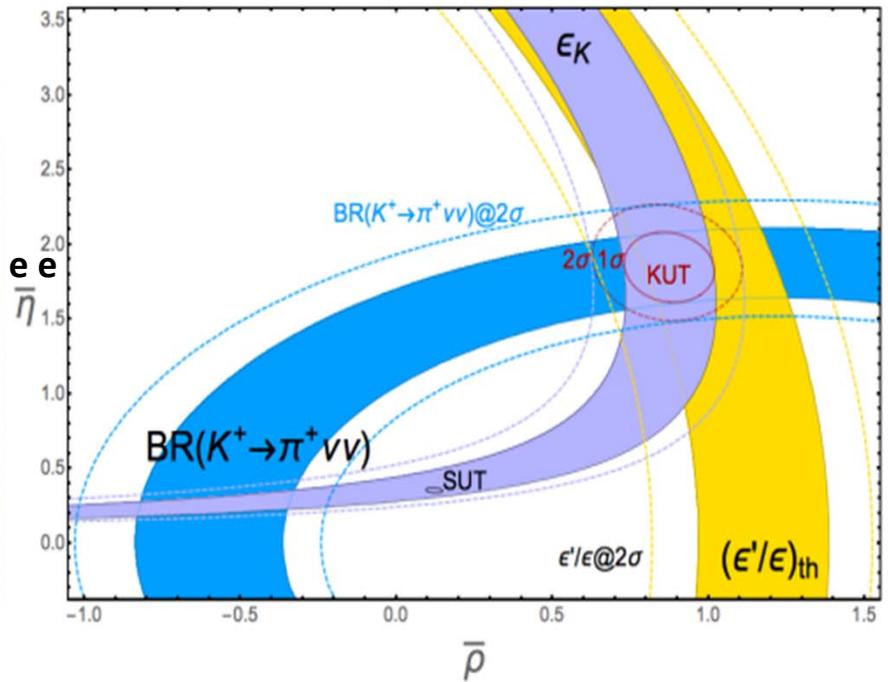
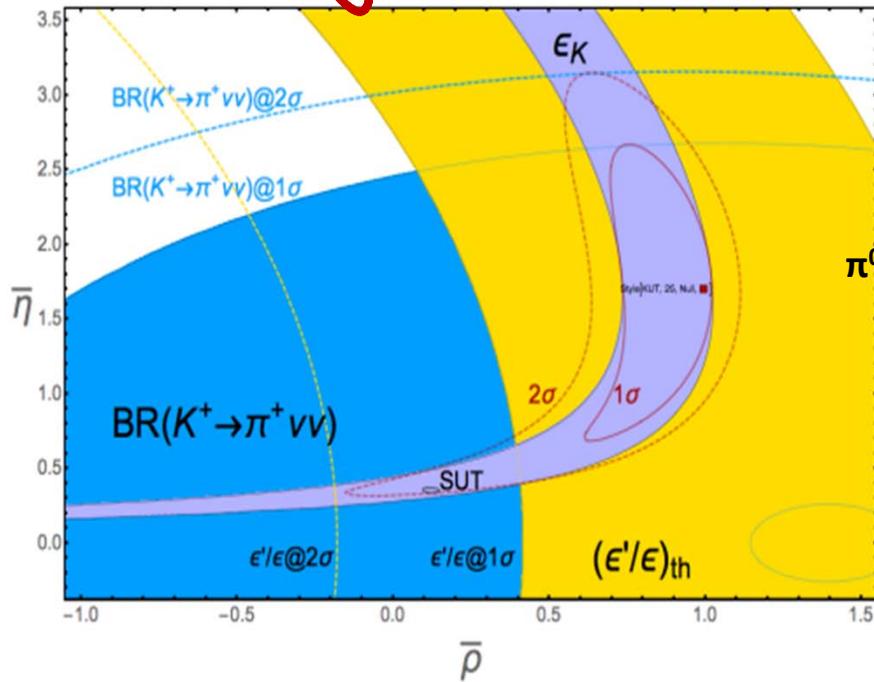
$$\text{Re}(\epsilon'/\epsilon) = 1.65(26) \times 10^{-3}.$$

$\epsilon' \approx 0 (10^{-6})!$

# A new observable on the horizon

*CP conserving observable*

$\epsilon'_{th}/\epsilon_K$



# Lattice $\epsilon'/\epsilon$ & SUT: CIRCA $\sim 2015$

