How bright is the proton?

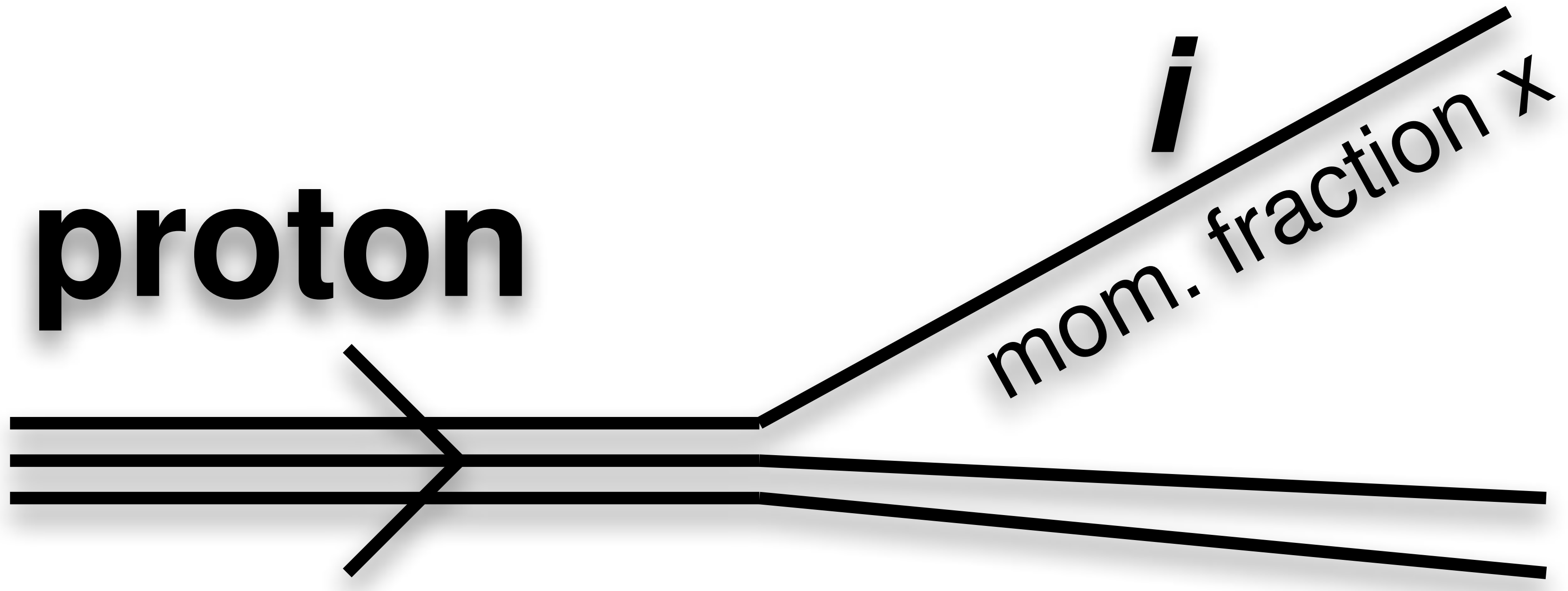
Determining the photon content of the proton

Gavin Salam (CERN)
with Manohar, Nason & Zanderighi

1607.04266 and work in progress

Moriond EW, March 2017, La Thuile

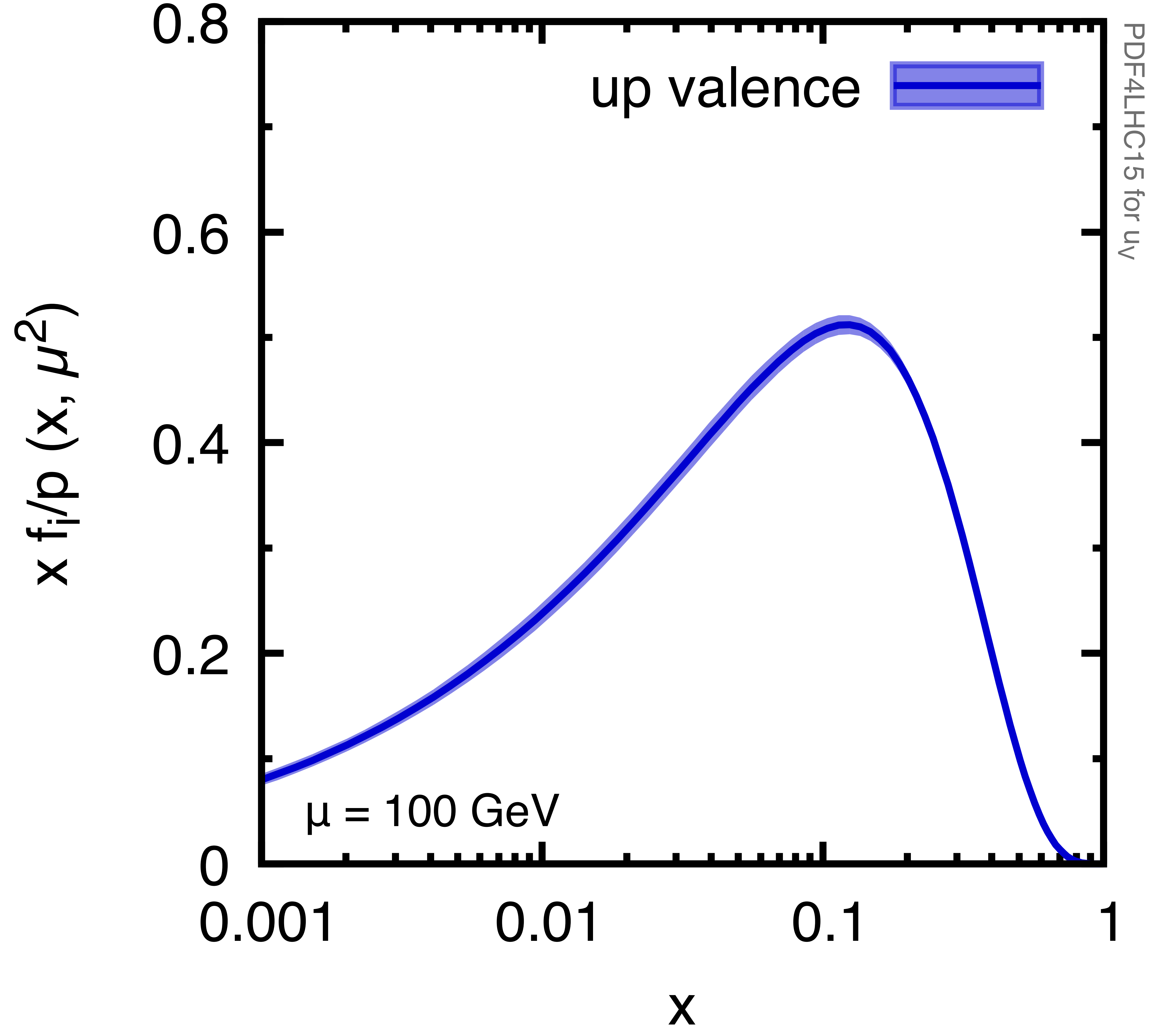
parton distribution functions (PDFs)



LHC physics
needs PDFs in region
$\sim 10^{-3} - 0.5$

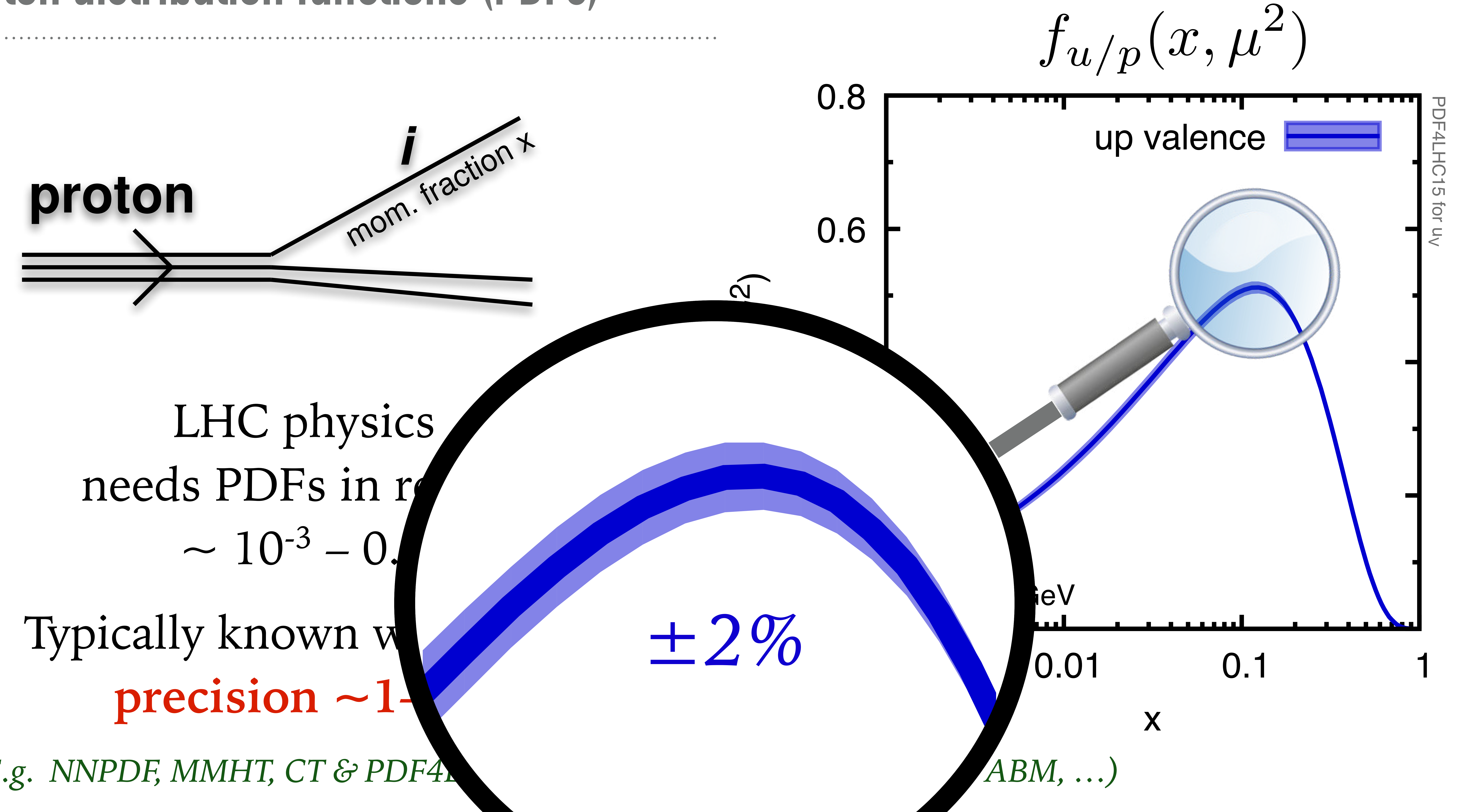
Typically known with good
precision ~1–3%

$f_{u/p}(x, \mu^2)$



E.g. NNPDF, MMHT, CT & PDF4LHC working group (+ also HERAPDF, ABM, …)

parton distribution functions (PDFs)

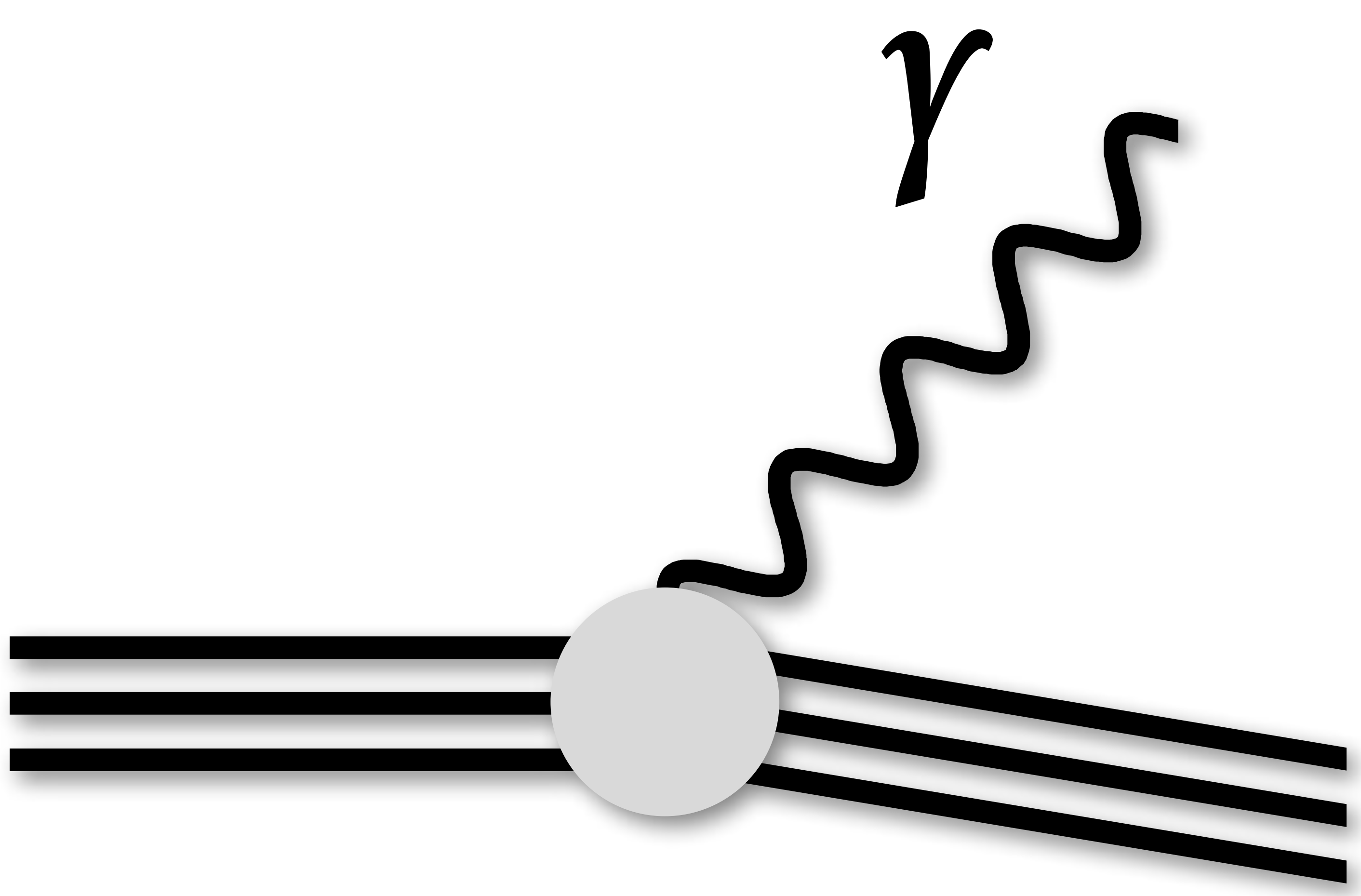


LHC physics
needs PDFs in r
~ 10^-3^ – 0.

Typically known w
precision ~1-

E.g. NNPDF, MMHT, CT & PDF4L ABM, …)

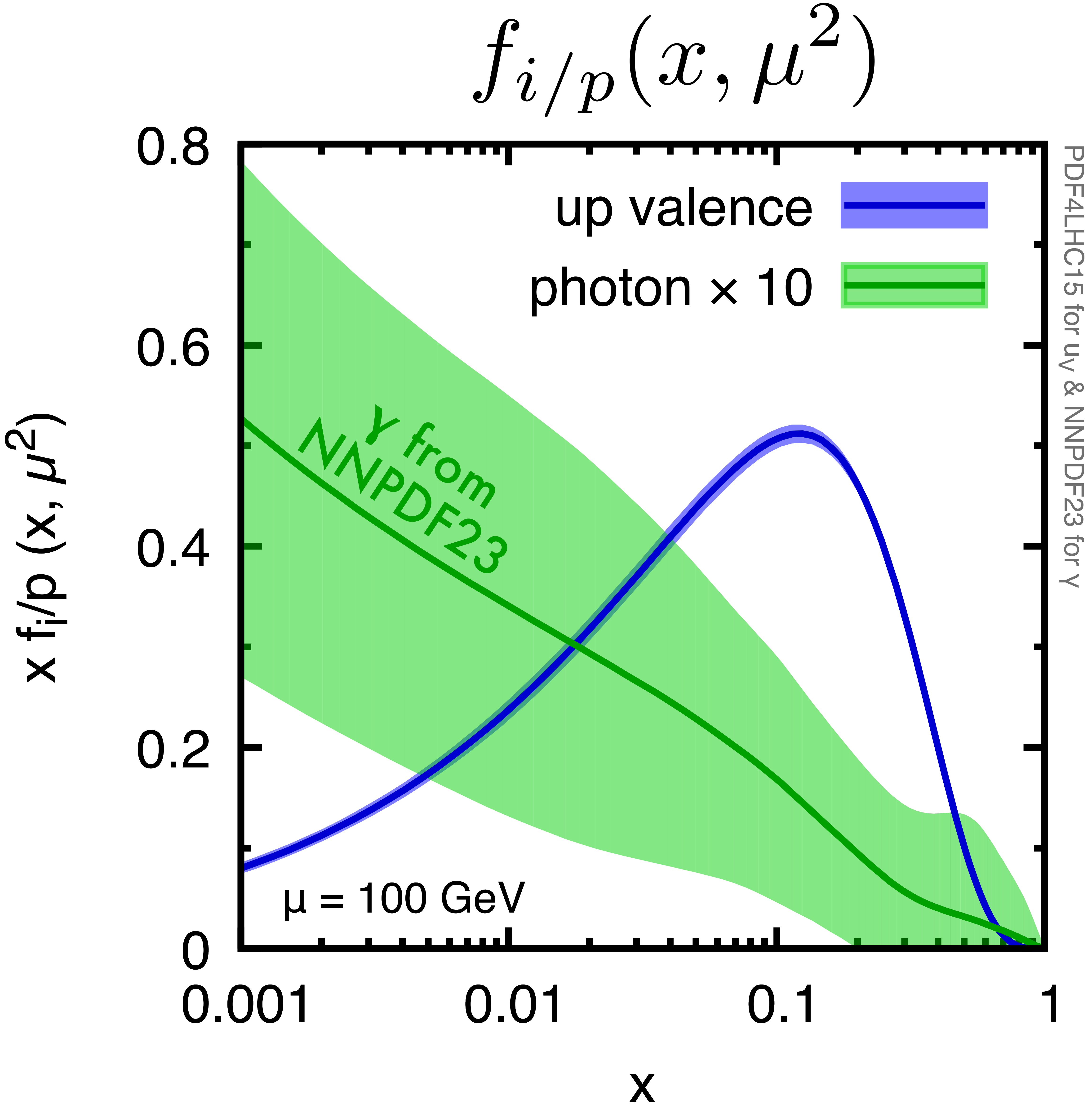
parton distribution functions (PDFs)



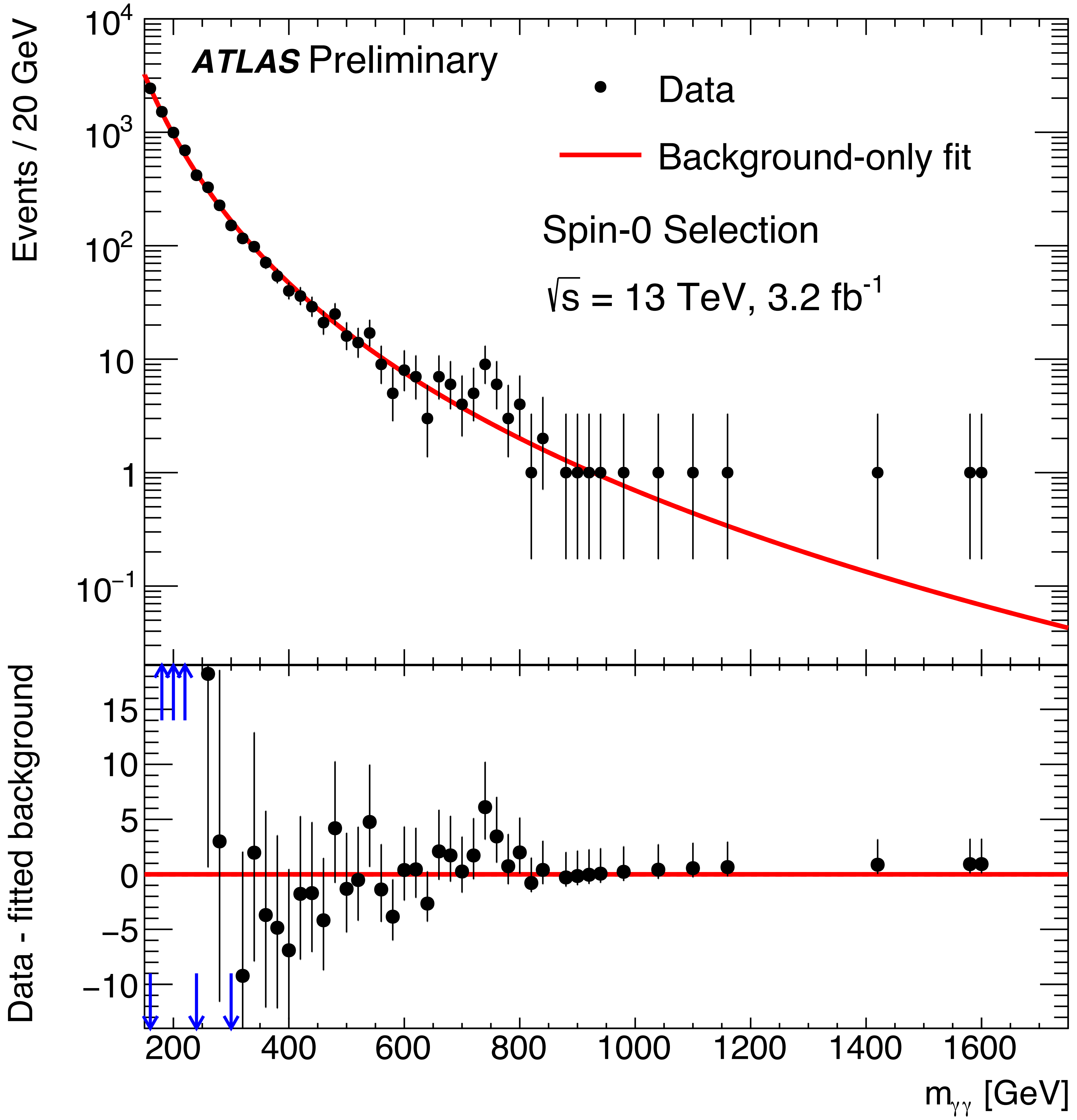
One exception:

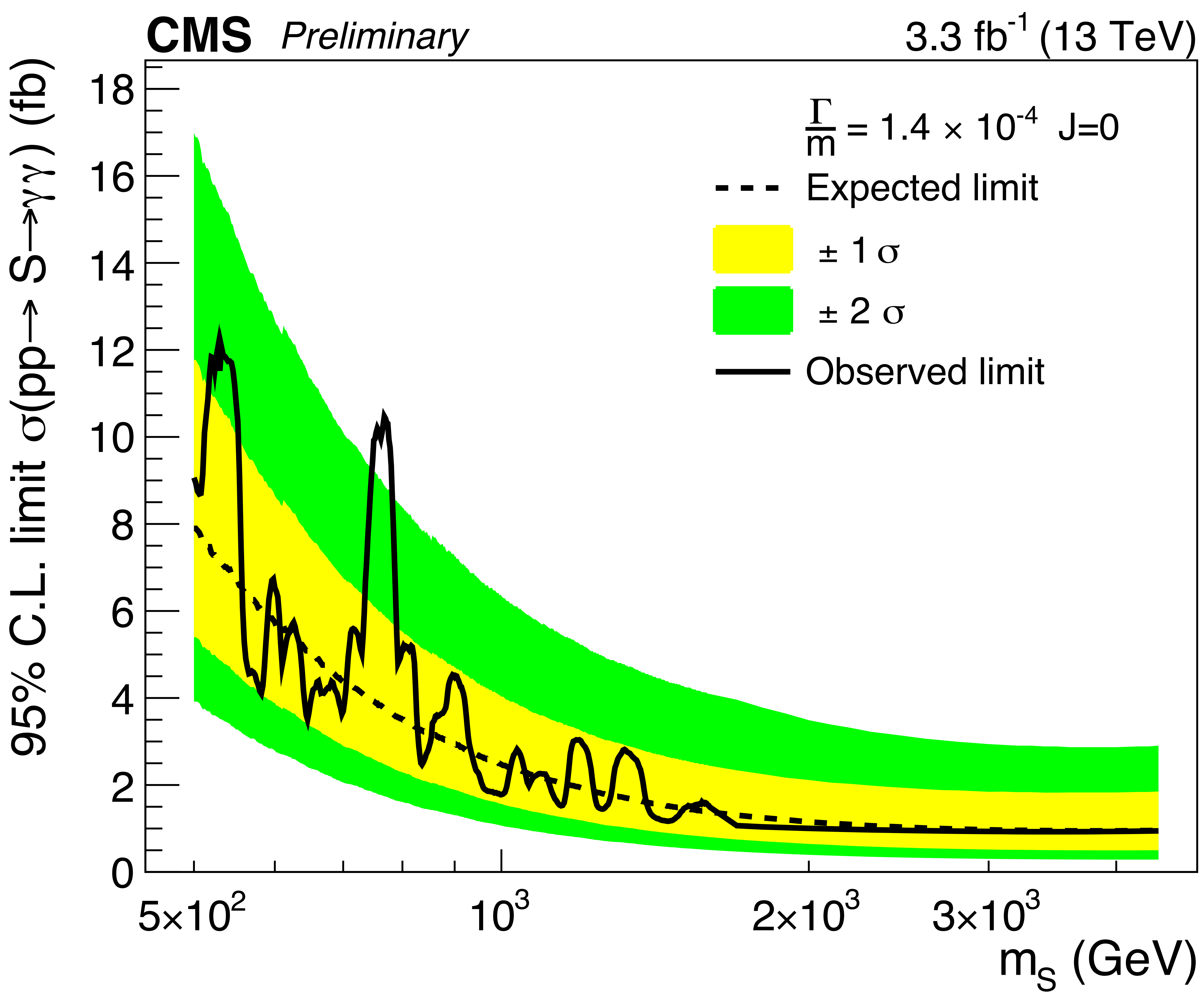
the photon distribution inside the proton

(had up to 100% uncertainty)



one year ago: A γγ resonance? From γγ → γγ ?





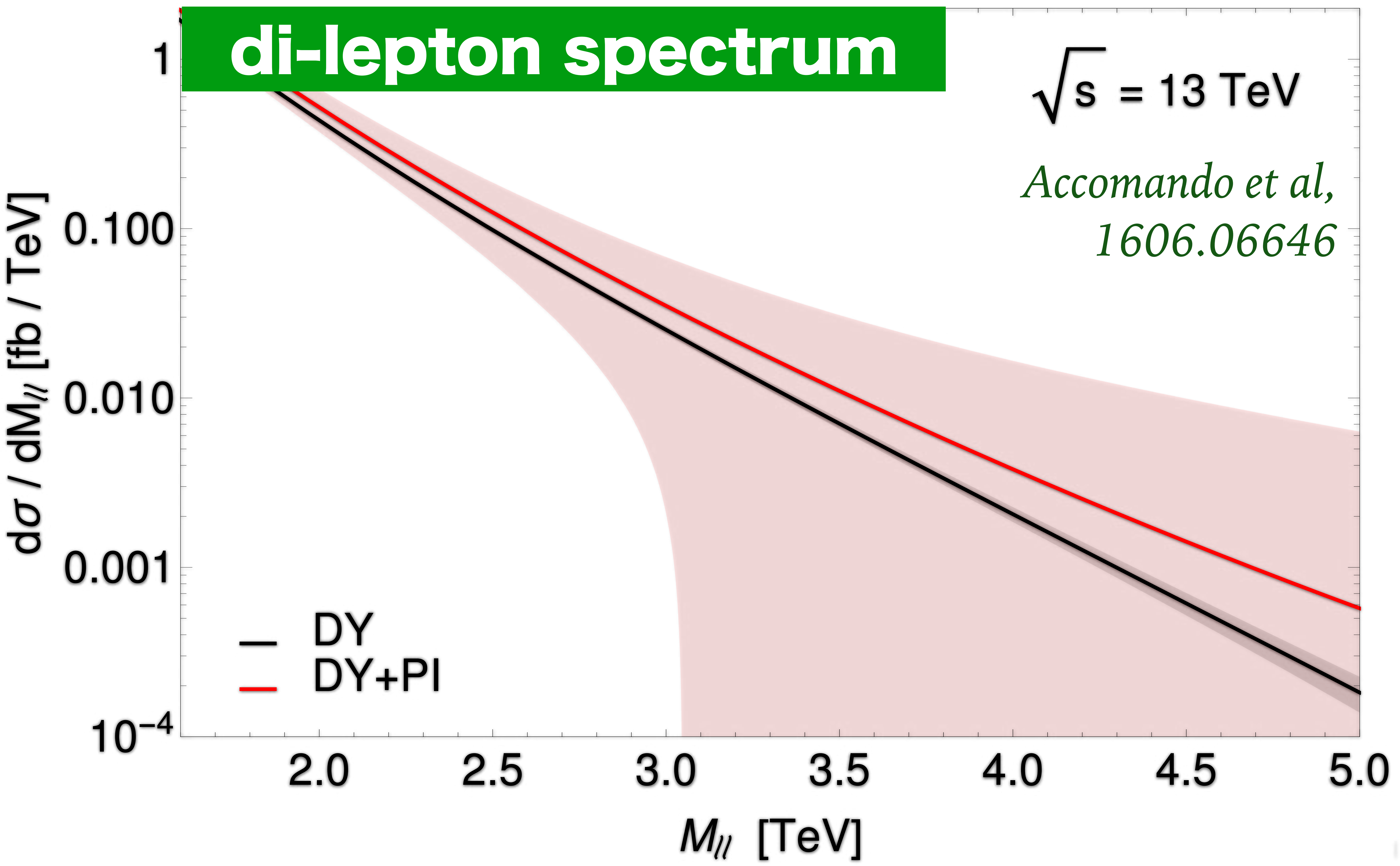
photon induced contribution to HW production

$pp \to H W^+ (\to l^+ \nu) + X$ at 13 TeV	
non-photon induced contributions	91.2 ± 1.8 fb
photon-induced contribs (NNPDF23)	$6.0\,^{+4.4}_{-2.9}$ fb

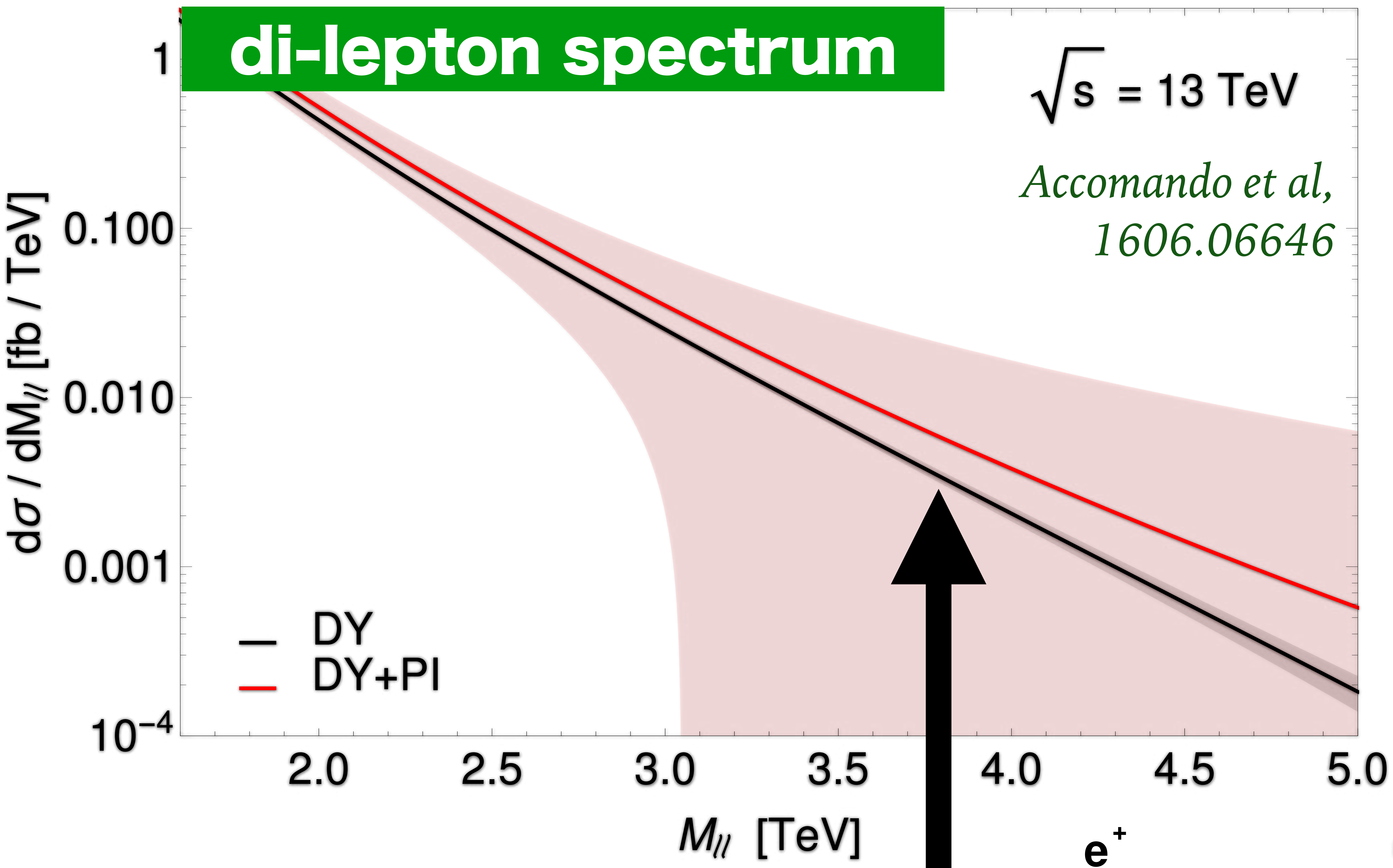
photon contribution brings the largest overall uncertainty

non-photon numbers from LHCHXSWG (YR4)
including PDF uncertainties

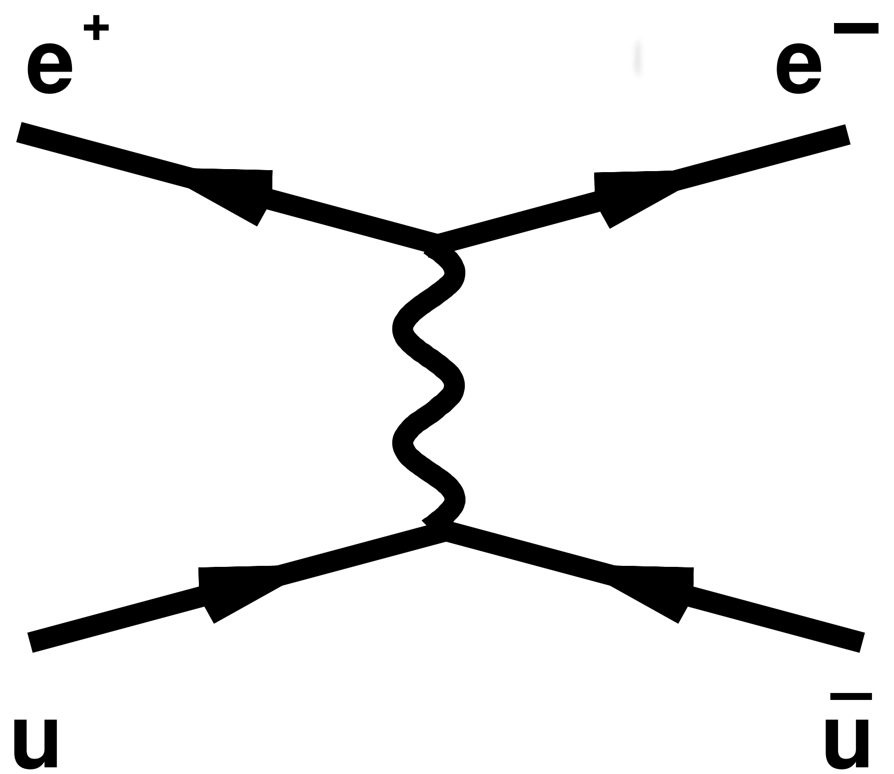
it matters in new-physics searches



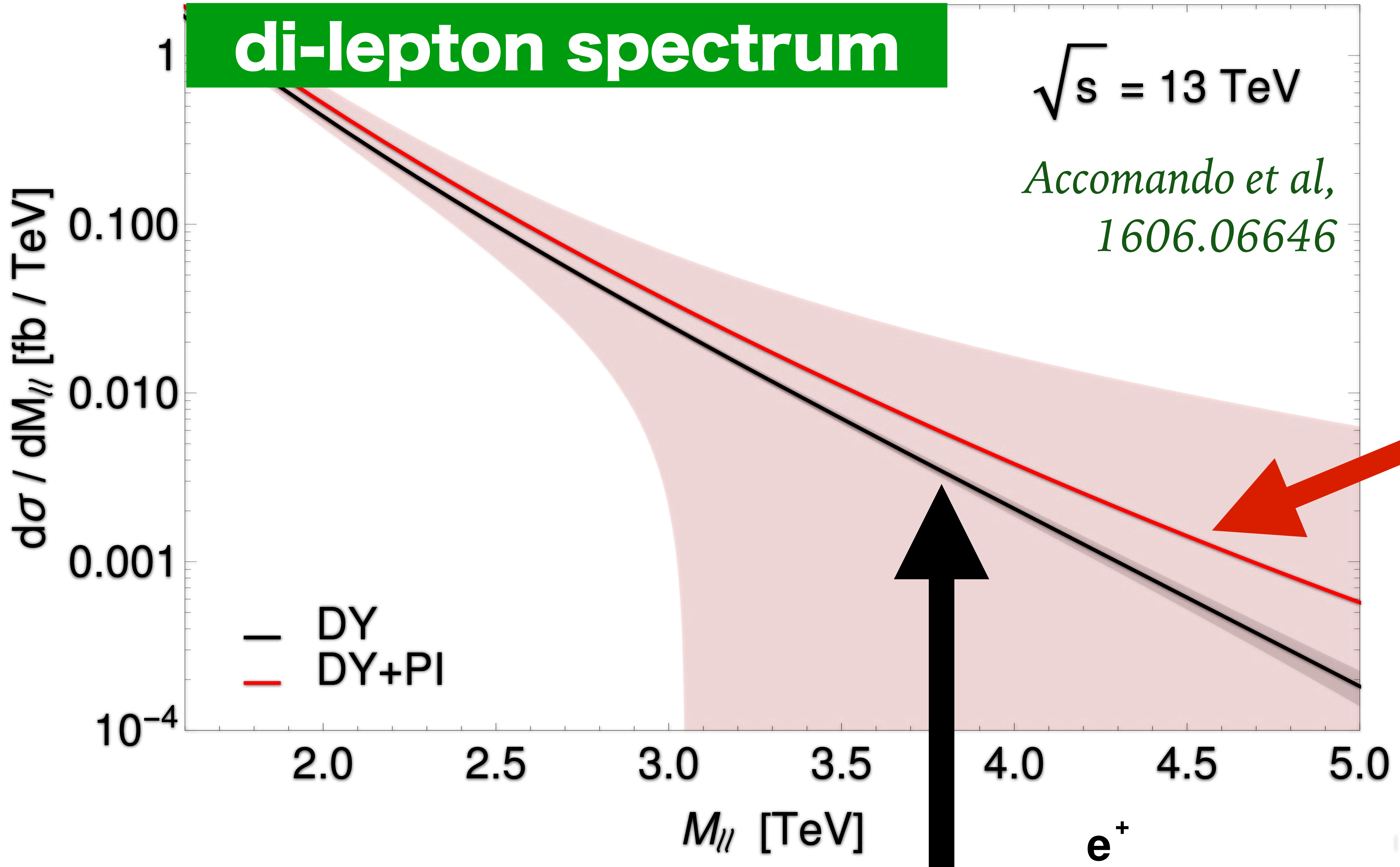
it matters in new-physics searches

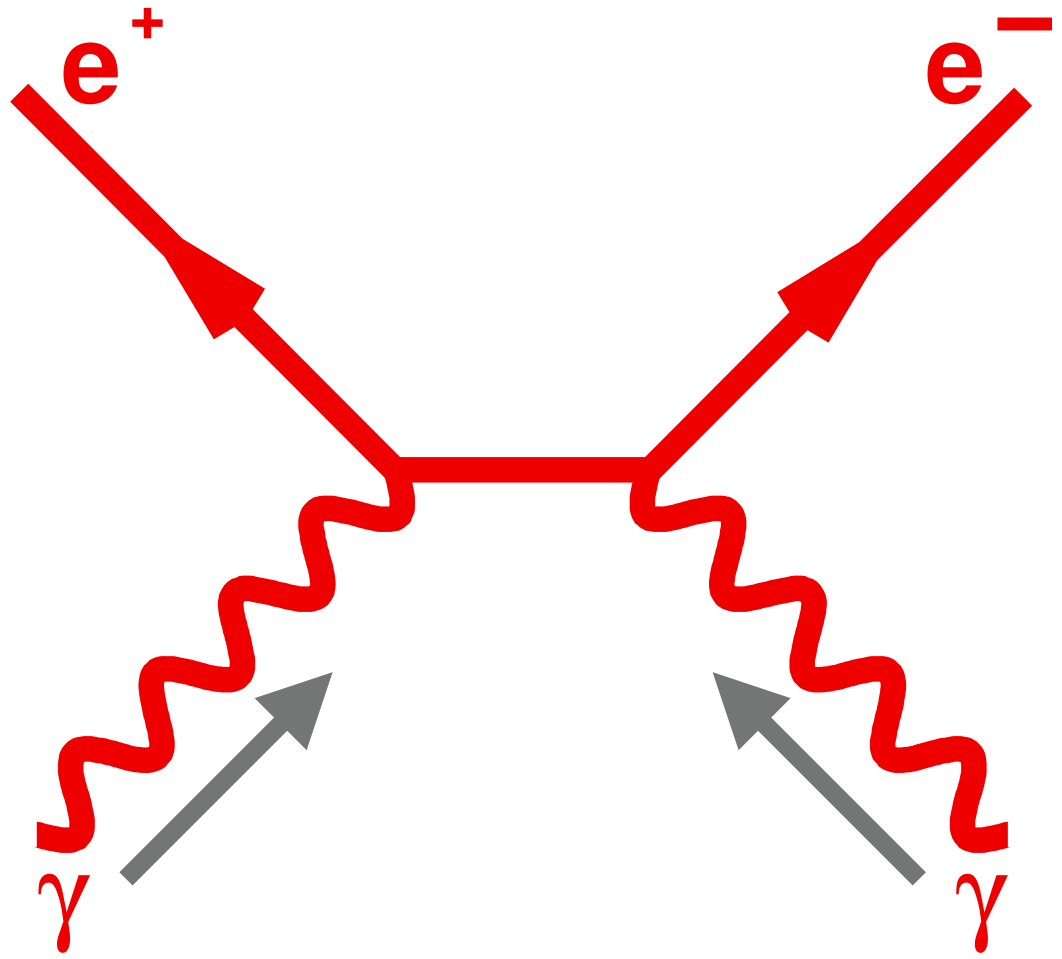


"normal" qqbar→ e^+e^- contribution



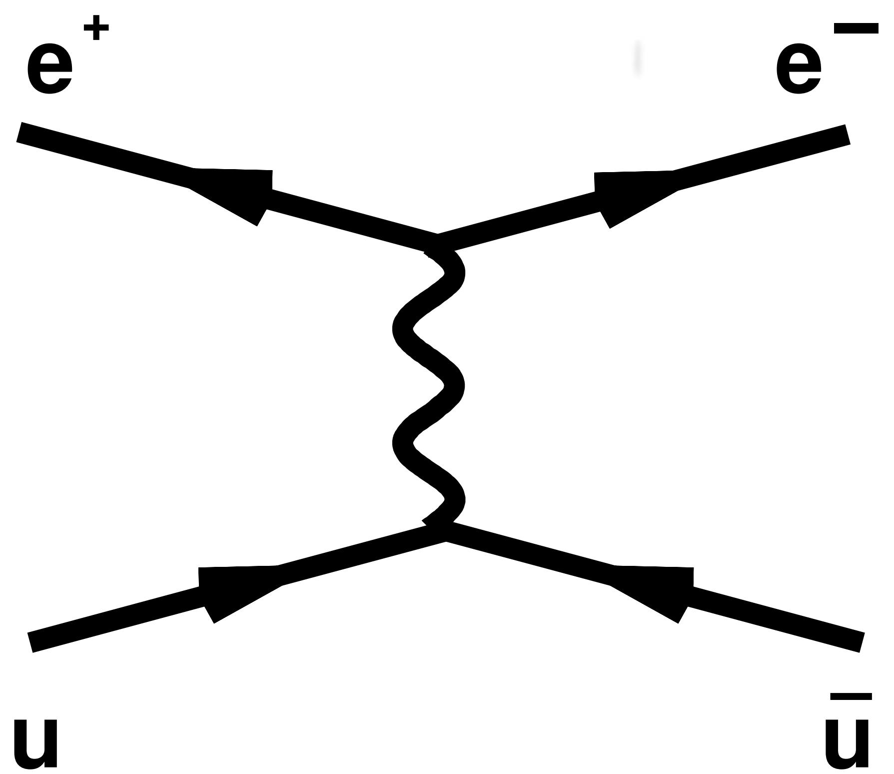
it matters in new-physics searches



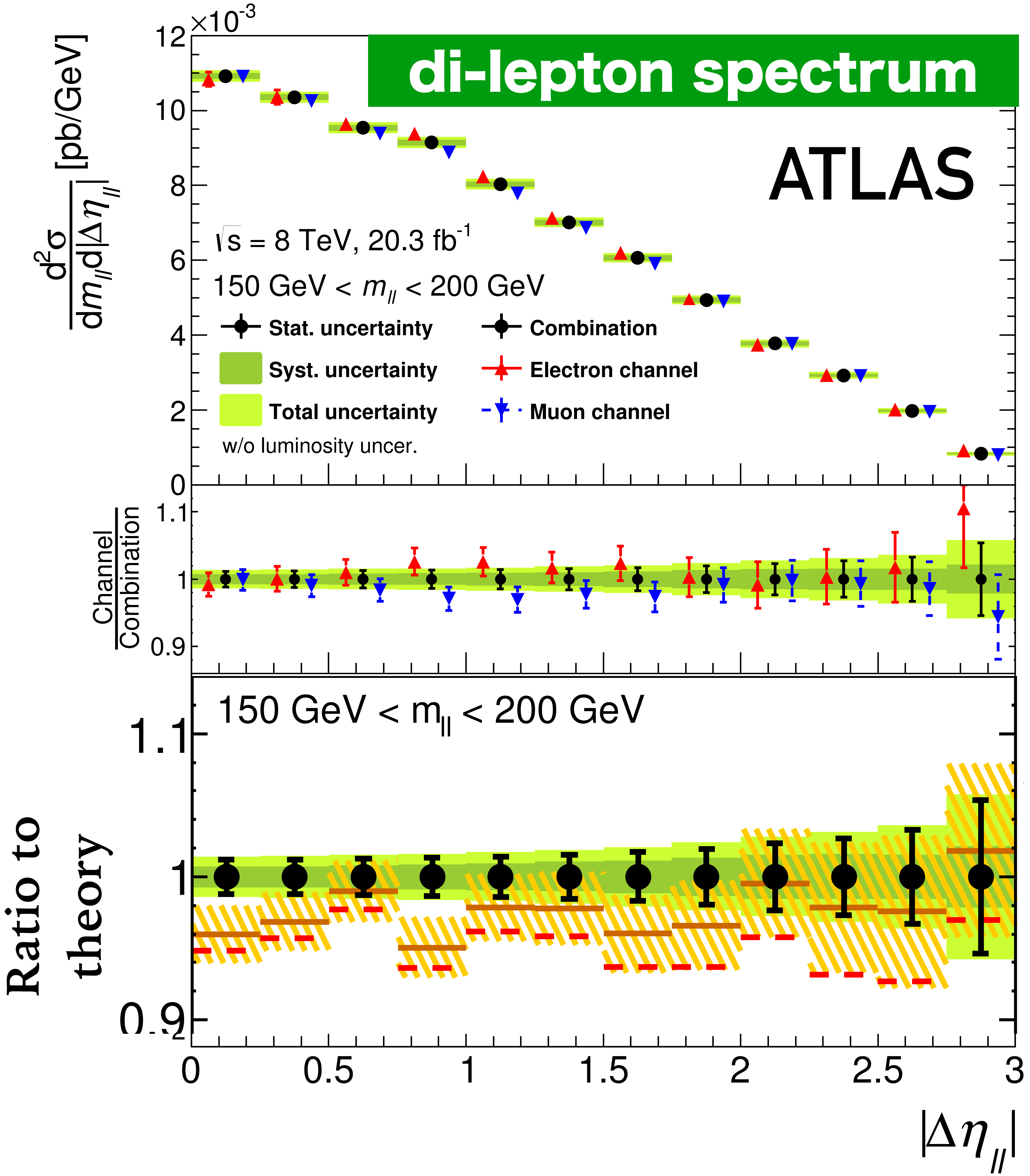


photon-induced contribution & uncertainty dominate [NNPDF23]

"normal" qqbar→ e^+e^- contribution

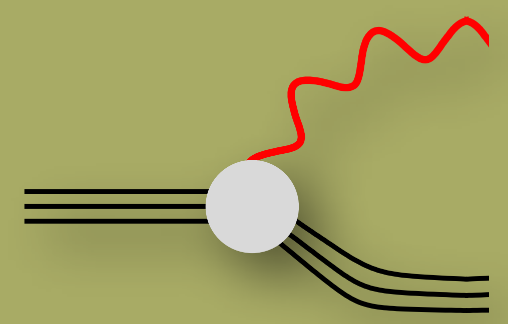
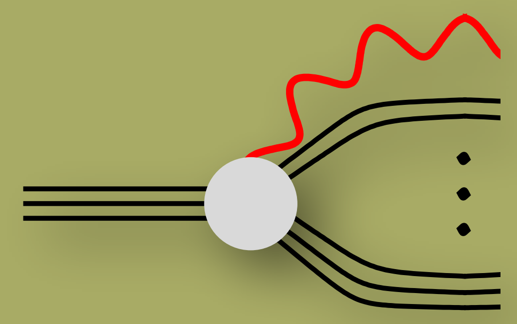


model-independent γ PDF fit (c. 2013)



model-independent γ PDF fit (c. 2013)

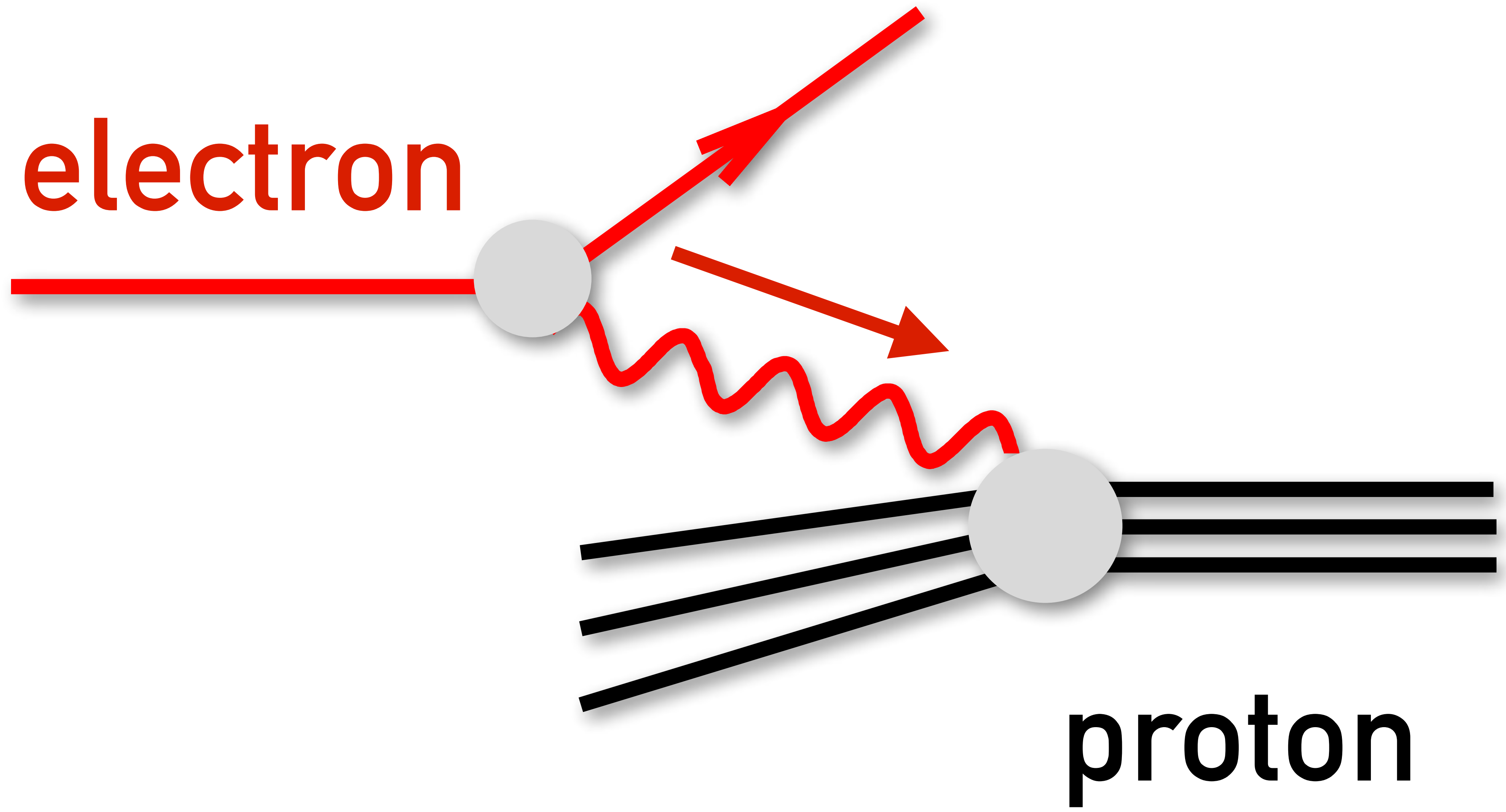
Widely discussed photon-PDF estimates

	 elastic	 inelastic	LHAPDF public computer-readable form?
Gluck Pisano Reya 2002	dipole	model	✗
MRST2004qed	✗	model	✓
CT14qed_inc	dipole	model (data-constrained)	✓
Martin Ryskin 2014	dipole (only electric part)	model	✗
Harland-Lang, Khoze Ryskin 2016	dipole	model	✗
NNPDF23qed (& NNPDF30qed)	no separation; fit to data		

elastic part long known: Budnev, Ginzburg, Meledin & Serbo, Phys.Rept. 1974

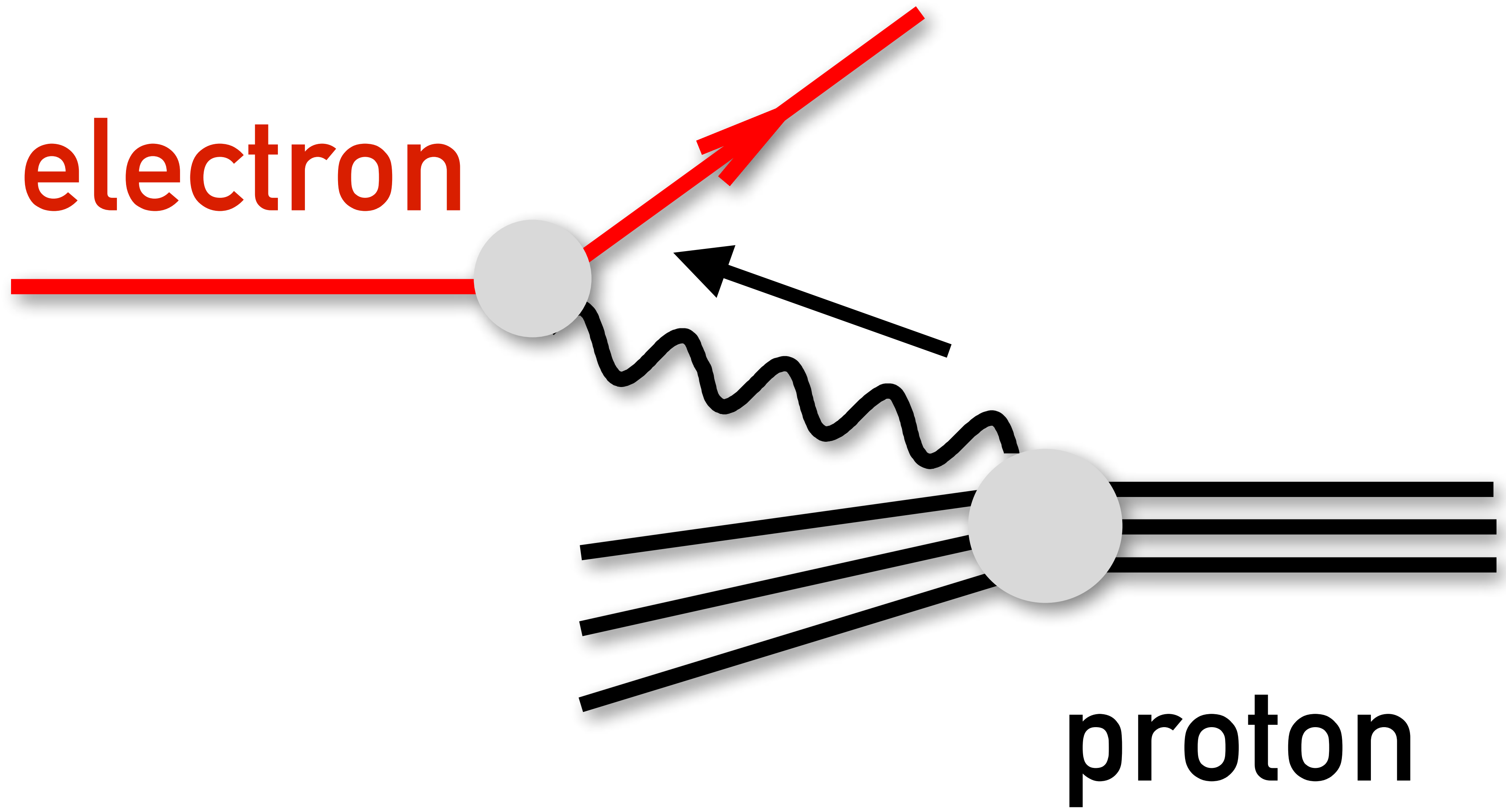
How do you do better? → Use electron–proton scattering

- ➤ Experiments have been going on for decades
- ➤ Usually seen as photons from electron probing proton structure



How do you do better? → Use electron–proton scattering

- Experiments have been going on for decades
- Usually seen as photons from electron probing proton structure
- But **can be viewed as electron probing proton's photonic field**



- Everything about unpolarized EM electron–proton interaction encoded in two "structure functions" $F_2(x,Q^2)$ & $F_L(x,Q^2)$

$$\frac{d\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{xQ^4}\left(\left(1 - y + \frac{y^2}{2}\left(1 + 2x^2\frac{m_p^2}{Q^2}\right)\right)F_2(x,Q^2) - \frac{y^2}{2}F_L(x,Q^2)\right)$$

Photon PDF in terms of F_2 and F_L — the LUXqed approach

$$x f_{\gamma/p}(x,\mu^2) = \frac{1}{2\pi\alpha(\mu^2)} \int_x^1 \frac{dz}{z} \left\{ \int_{\frac{x^2 m_p^2}{1-z}}^{\frac{\mu^2}{1-z}} \frac{dQ^2}{Q^2} \alpha^2(Q^2) \left[\left(z p_{\gamma q}(z) + \frac{2x^2 m_p^2}{Q^2} \right) F_2(x/z, Q^2) - z^2 F_L\left(\frac{x}{z}, Q^2\right) \right] - \alpha^2(\mu^2) z^2 F_2\left(\frac{x}{z}, \mu^2\right) \right\}$$

This includes terms

$\alpha L (\alpha_s L)^n$

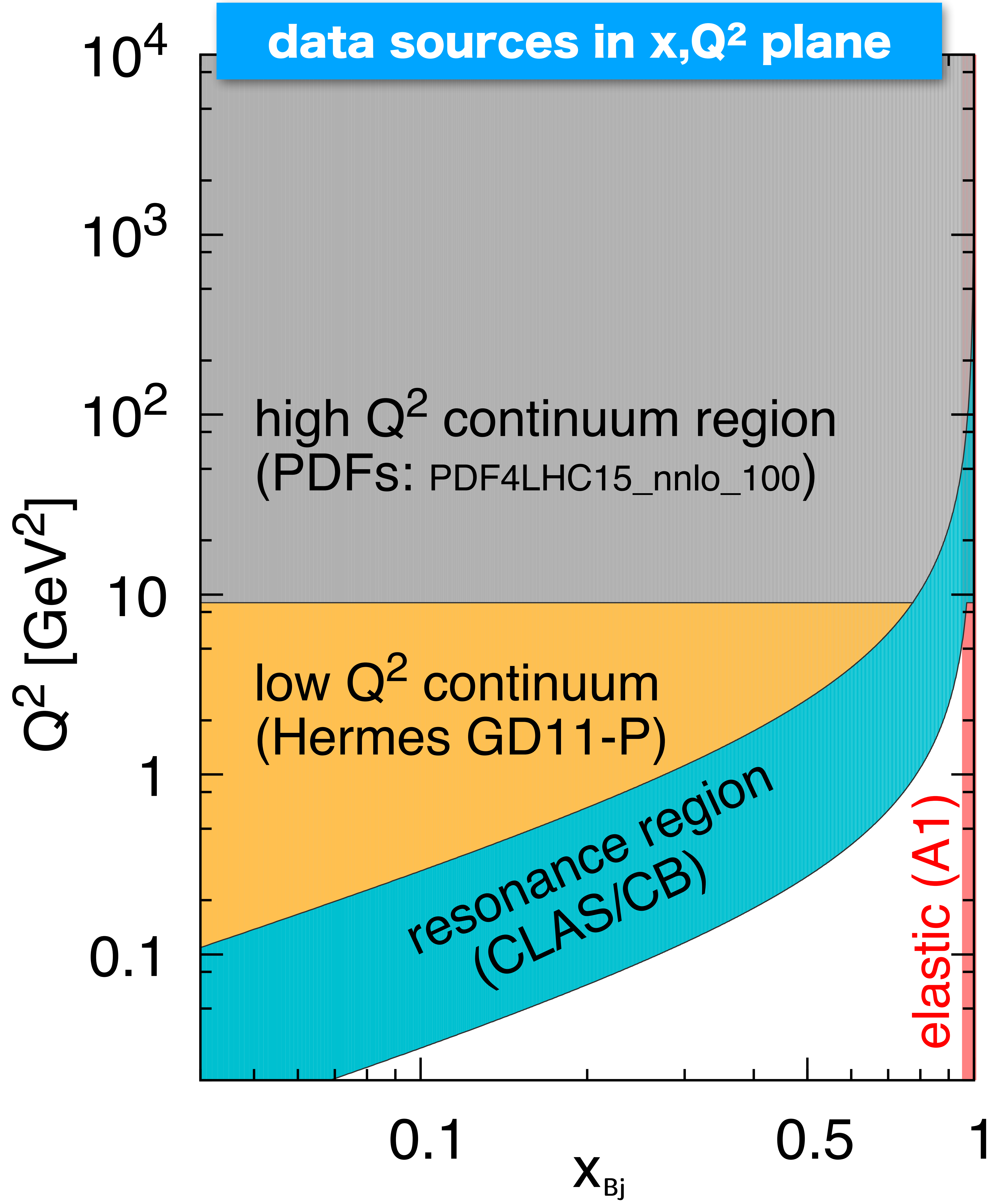
$\alpha (\alpha_s L)^n$

$\alpha^2 L^2 (\alpha_s L)^n$

$(L = \ln \mu^2/\Lambda^2)$

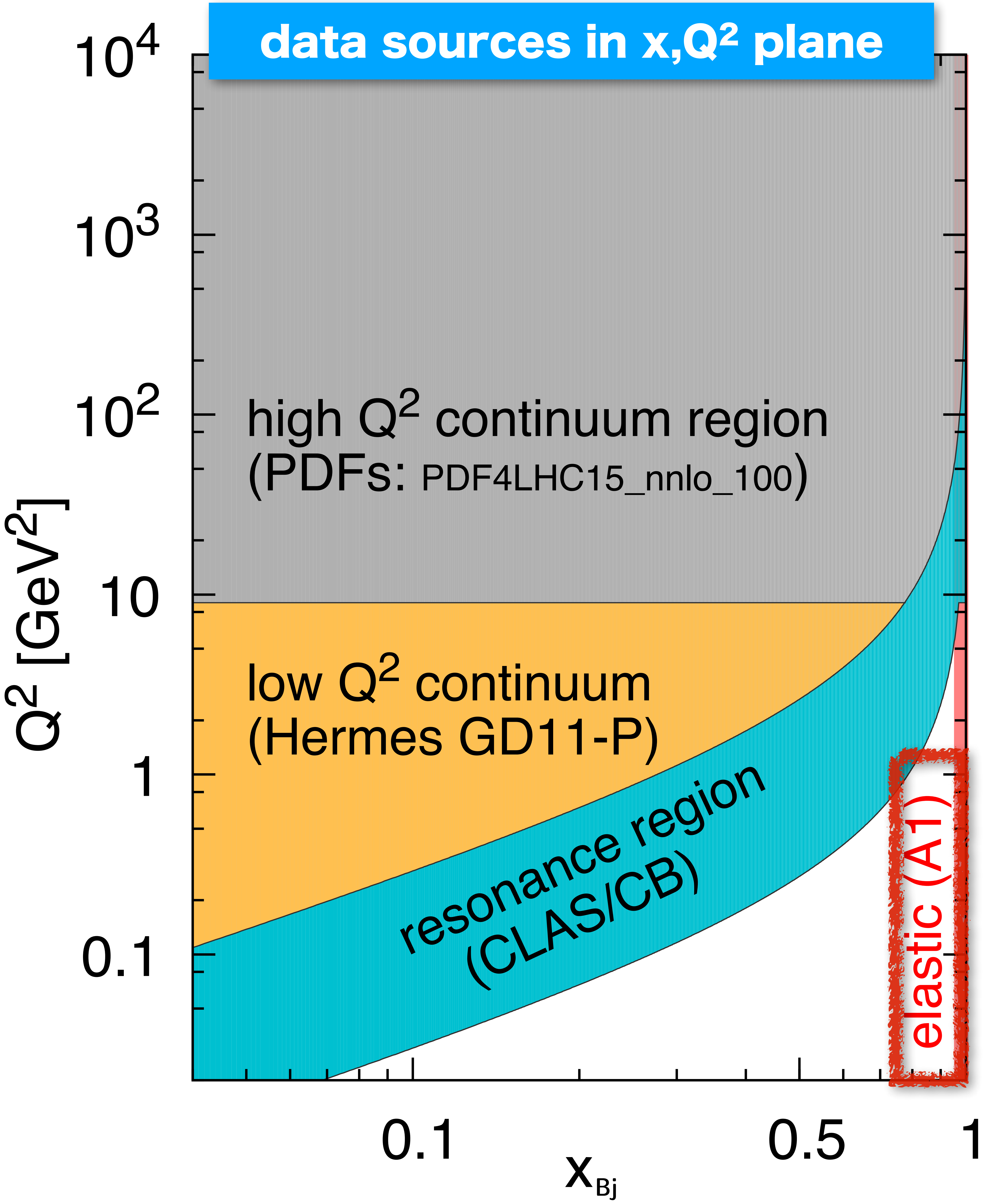
Work in progress goes one order higher (e.g. extra power of α_s)

It subsequently emerged that two "forgotten" papers, Anlauf et. al, CPC70(1992)97 Mukherjee & Pisano, hep-ph/0306275, had the correct integrand (but not the limits)



DATA

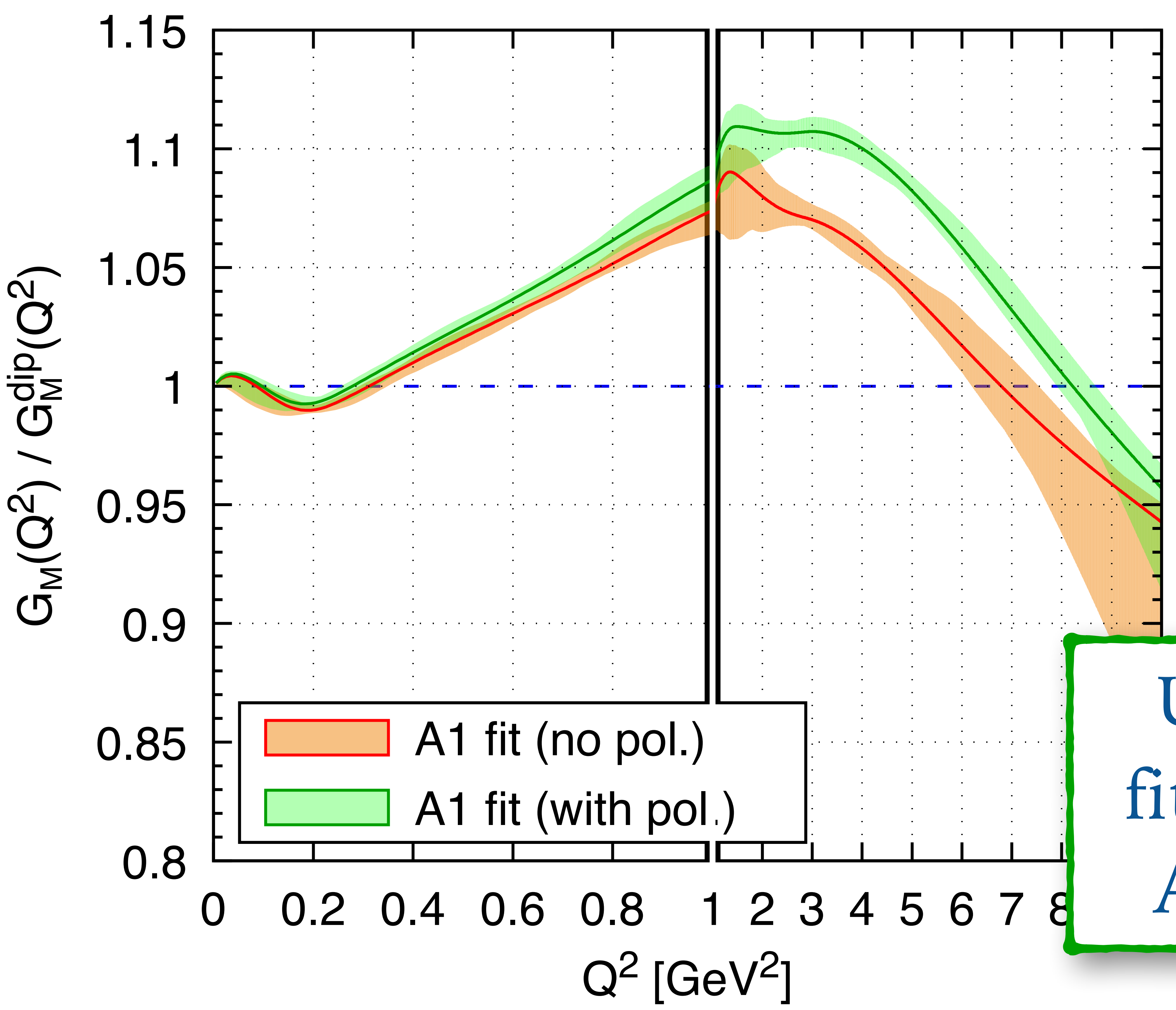
- x, Q^2 plane naturally breaks up into regions with different physical behaviours and data sources
- We don't use F_2 and F_L data directly, but rather various fits to data



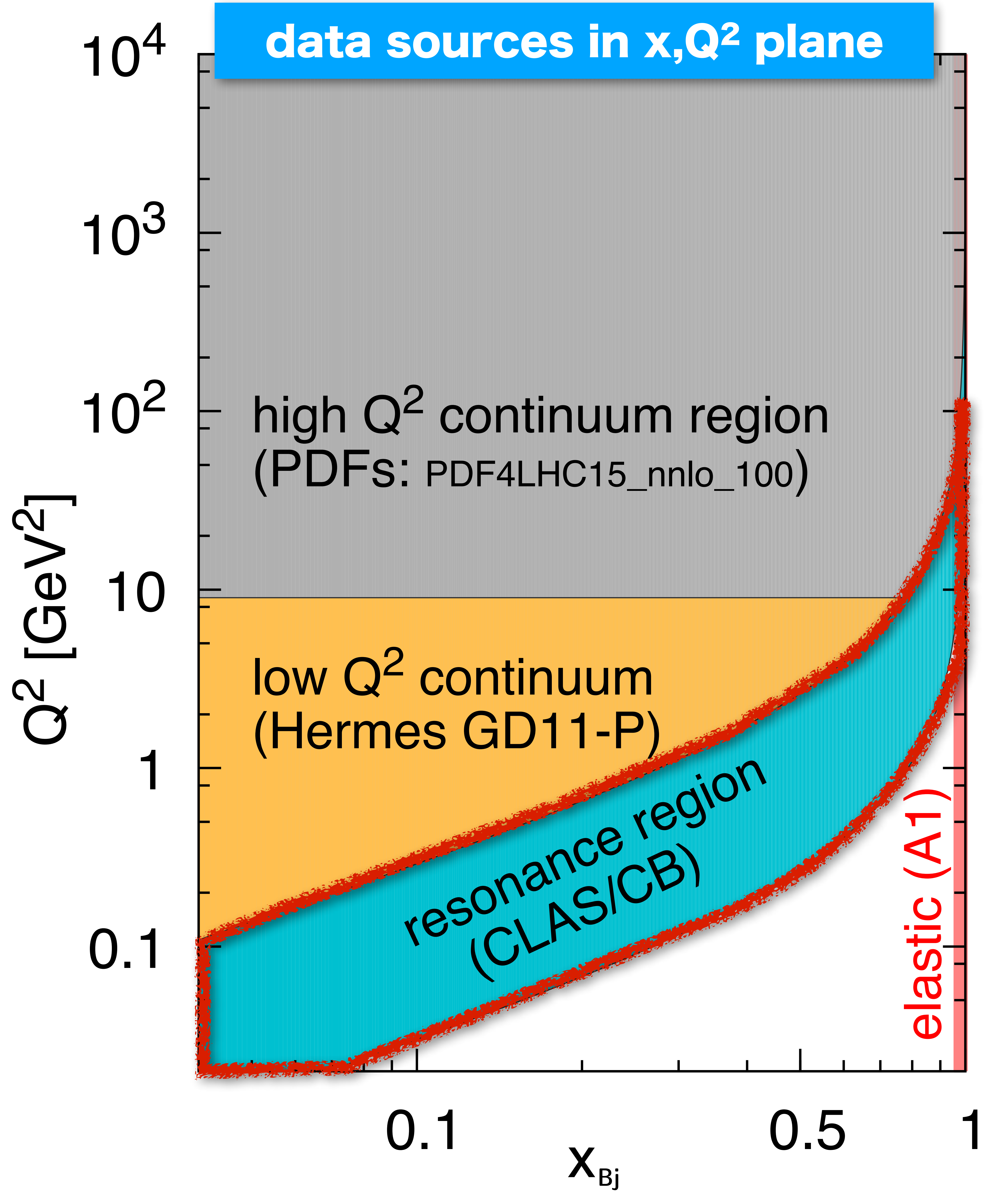
ELASTIC COMPONENT

- Elastic component of $F_{2/L}$ lives at $x=1$
- Get from Sachs Form factors, G_E, G_M

$$F_2^{\rm el}(x_{\rm bj},Q^2) = \frac{[G_E(Q^2)]^2+[G_M(Q^2)]^2\tau}{1+\tau}\delta(1-x_{\rm bj}), \qquad \tau = Q^2/(4m_p^2)$$

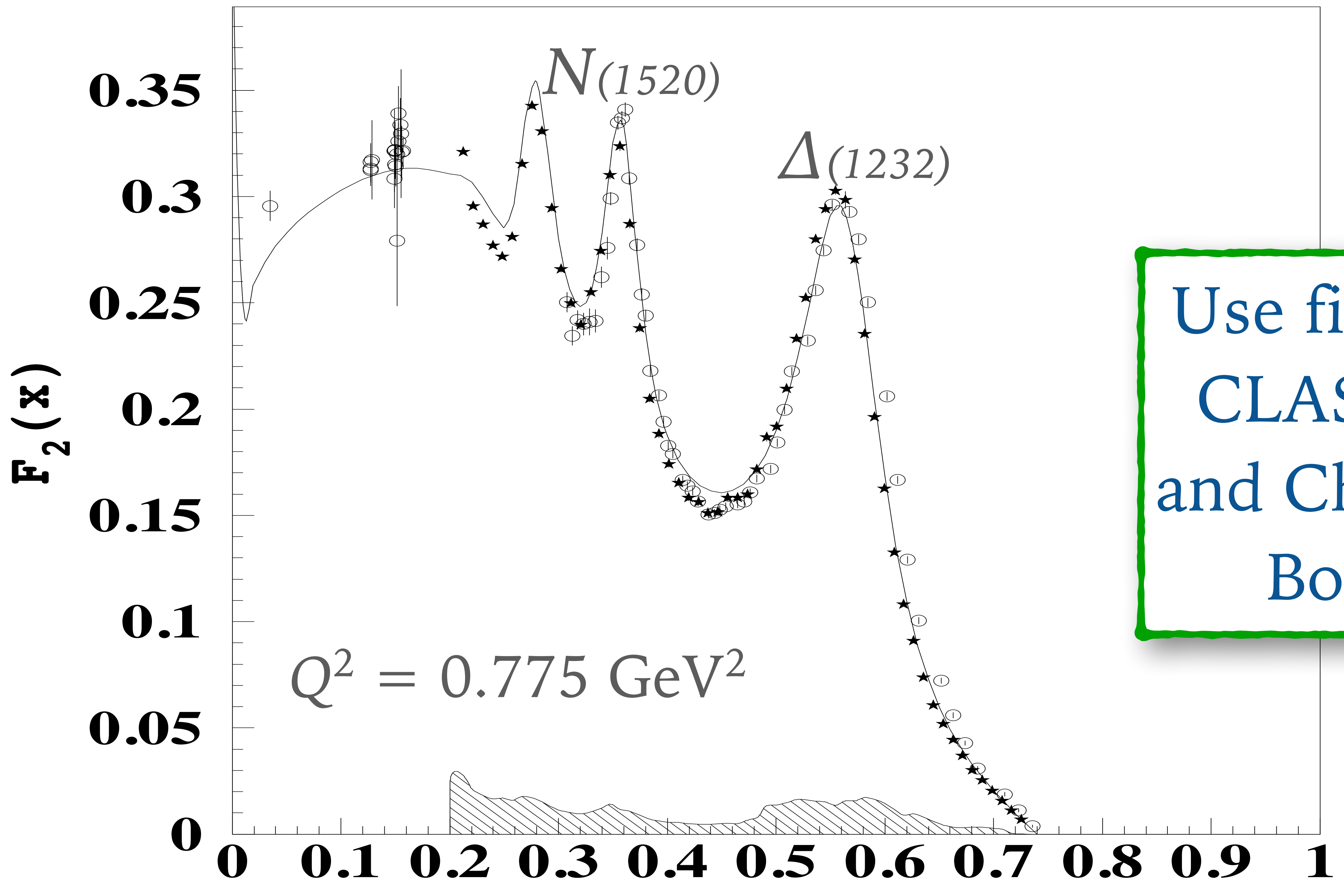


Use global fits from the A1 Collab.

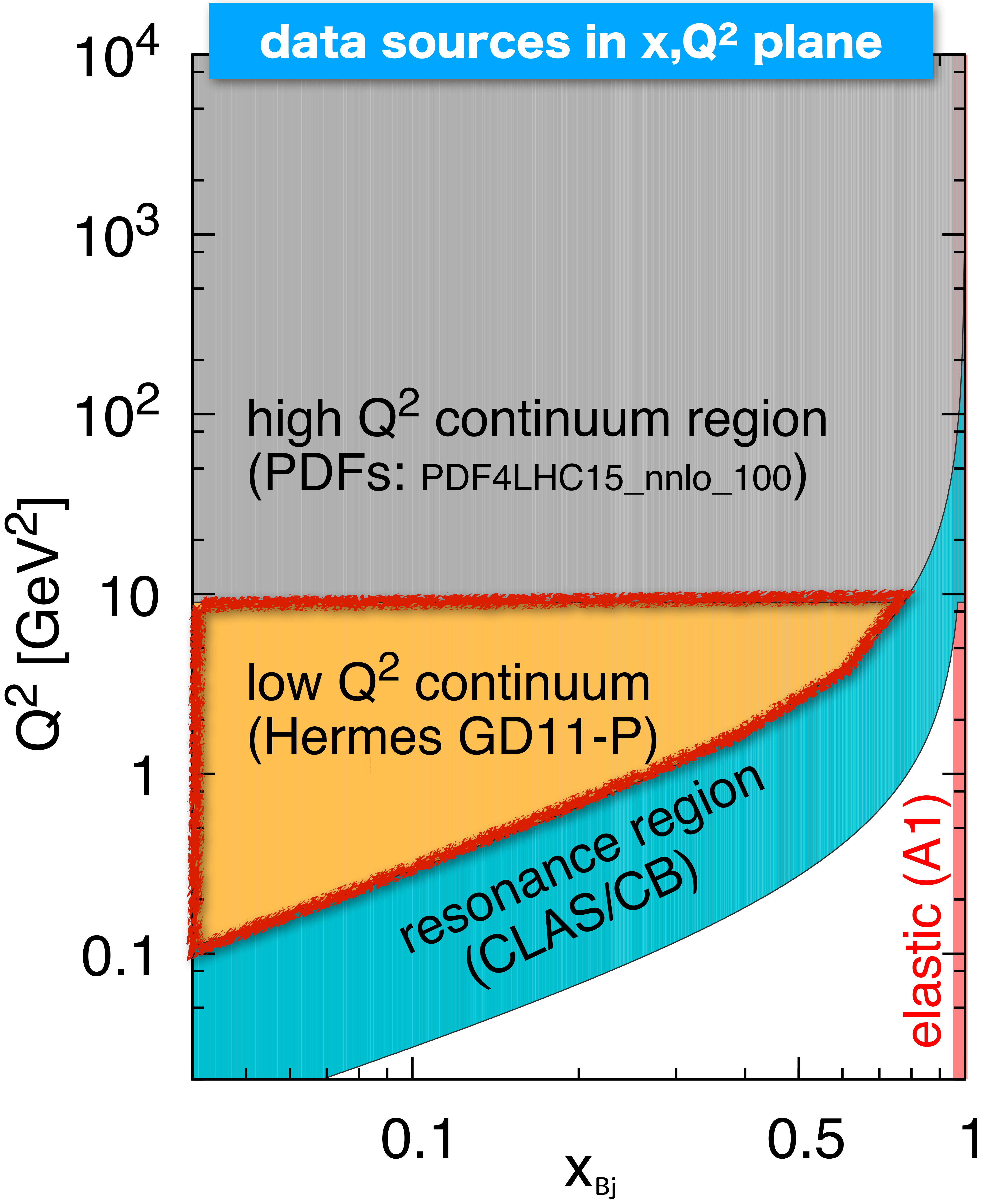


RESONANCE COMPONENT

- proton gets excited, e.g. to $\Delta \to p\pi$ and higher resonances
- relevant for $(m_p + m_\pi)^2 < W^2 \lesssim 3.5 \text{GeV}^2$

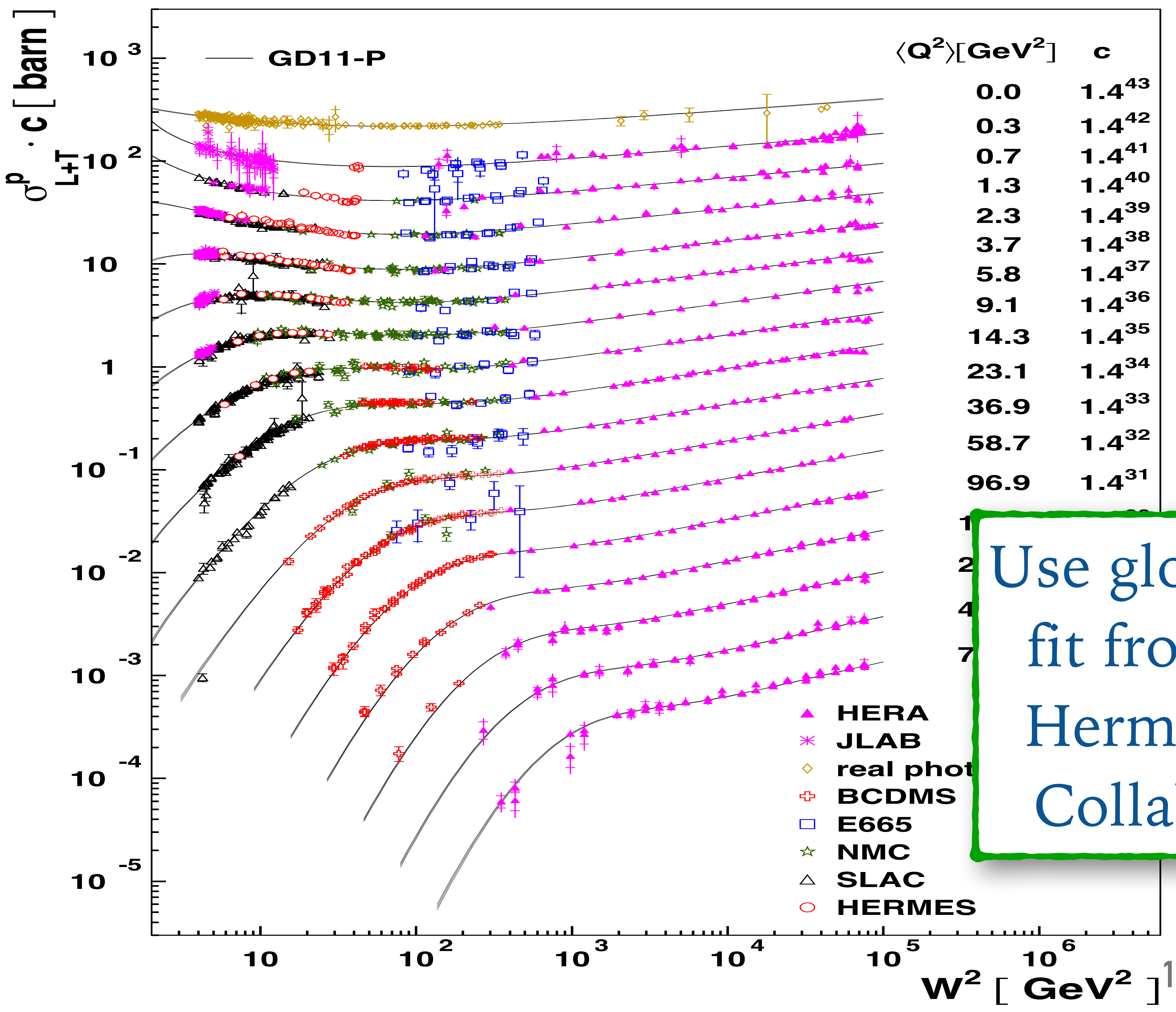


Use fits from CLAS expt. and Christy & Bosted

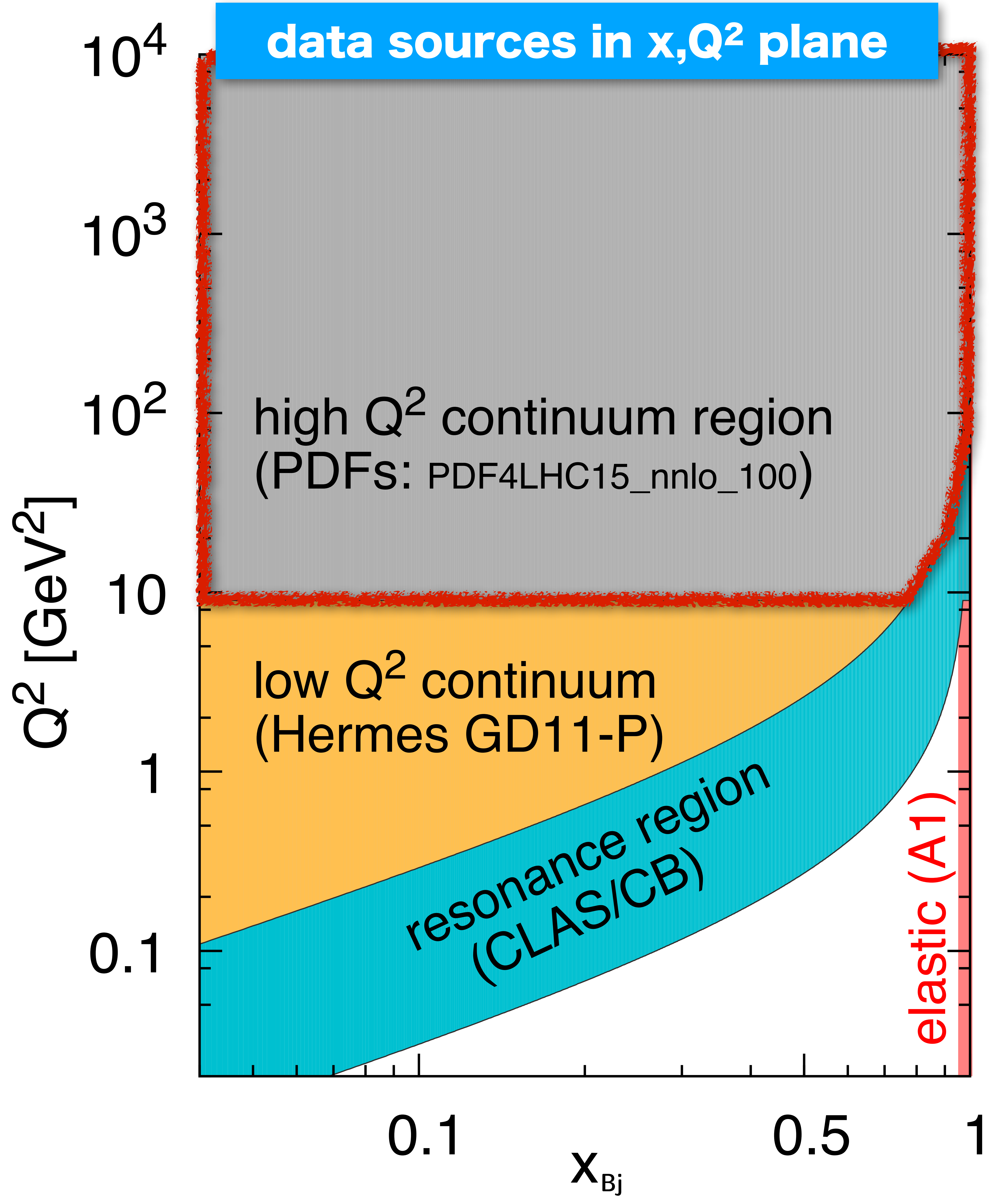


CONTINUUM COMPONENT

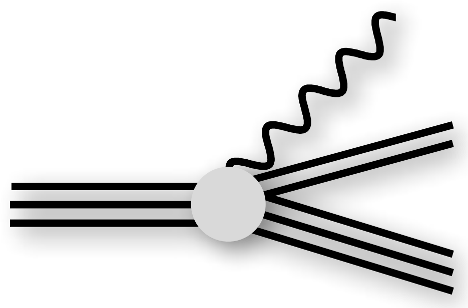
- ➤ Much data
- ➤ For $Q^2 \to 0$, $\sigma_{\gamma p}$ indep. of Q^2 at fixed W^2



Use global fit from Hermes Collab.



CONTINUUM COMPONENT

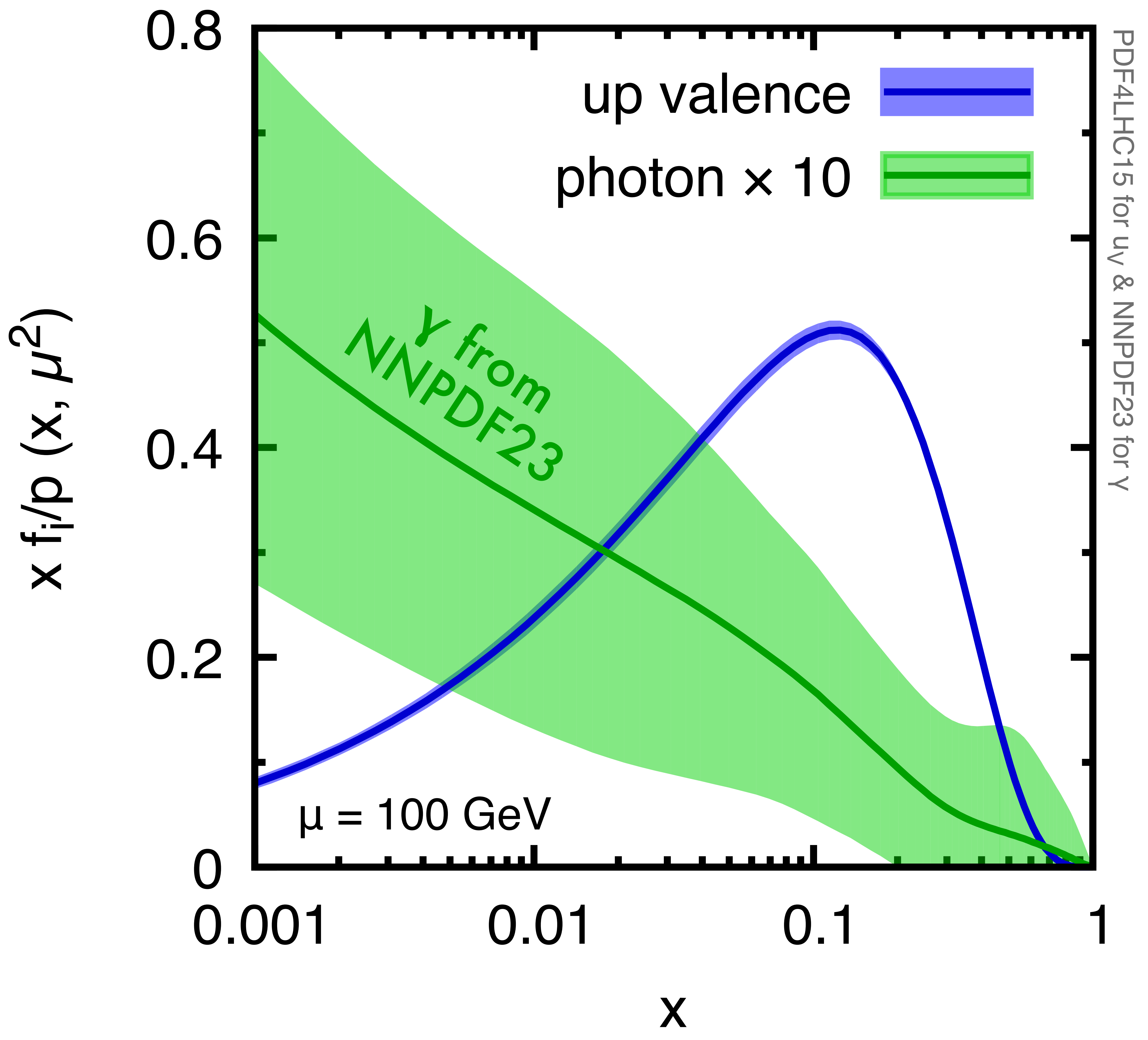


- Less direct data for F_2 and F_L at high Q^2
- But we can reliably use PDFs and coefficient functions to calculate them
- We use NNLO coefficient functions in a zero-mass variable flavour-number scheme

As a PDF we use
PDF4LHC15_nnlo_100
from LHAPDF

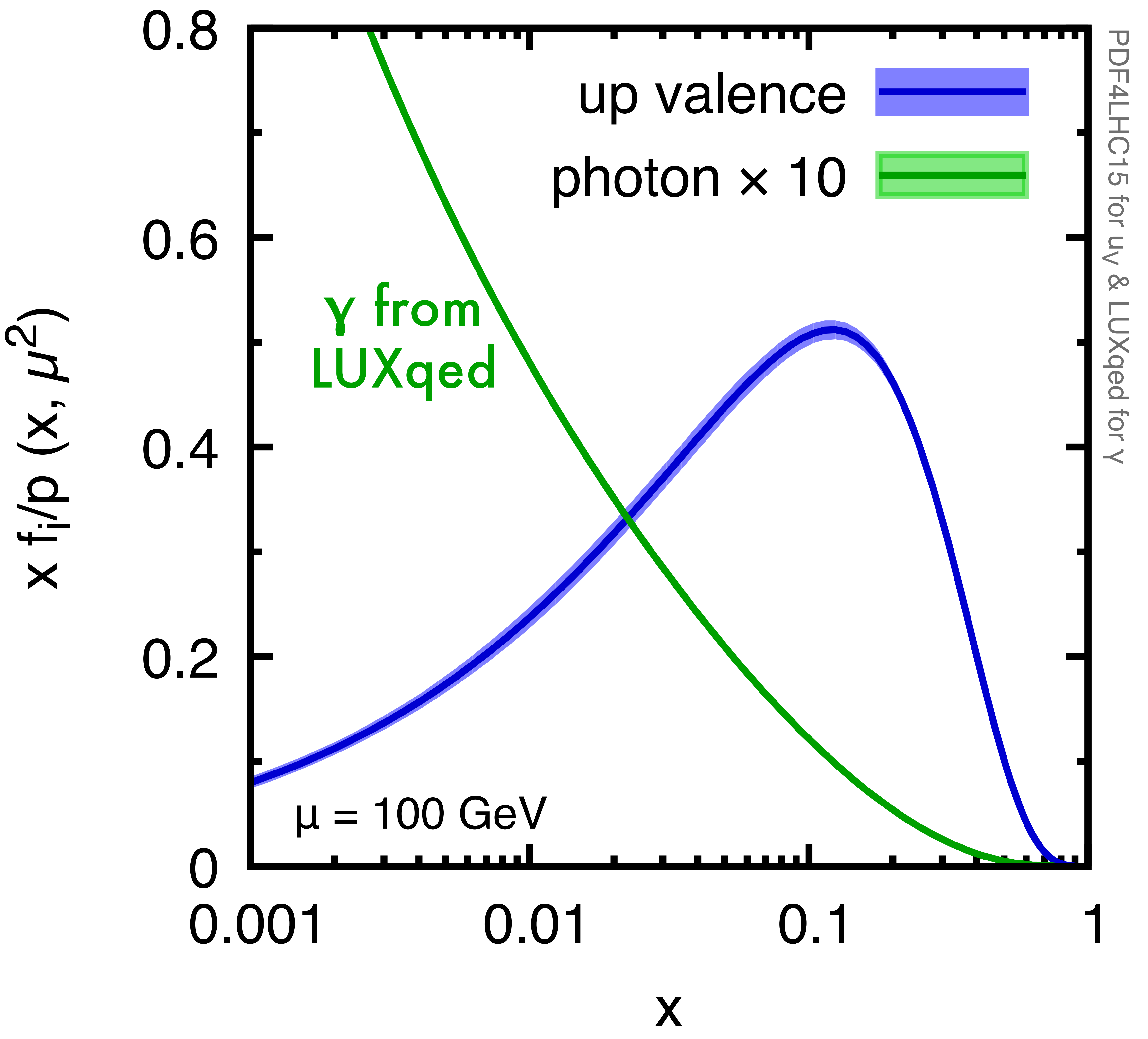
photon PDF results

- Model-independent uncertainty (NNPDF) was 50–100%



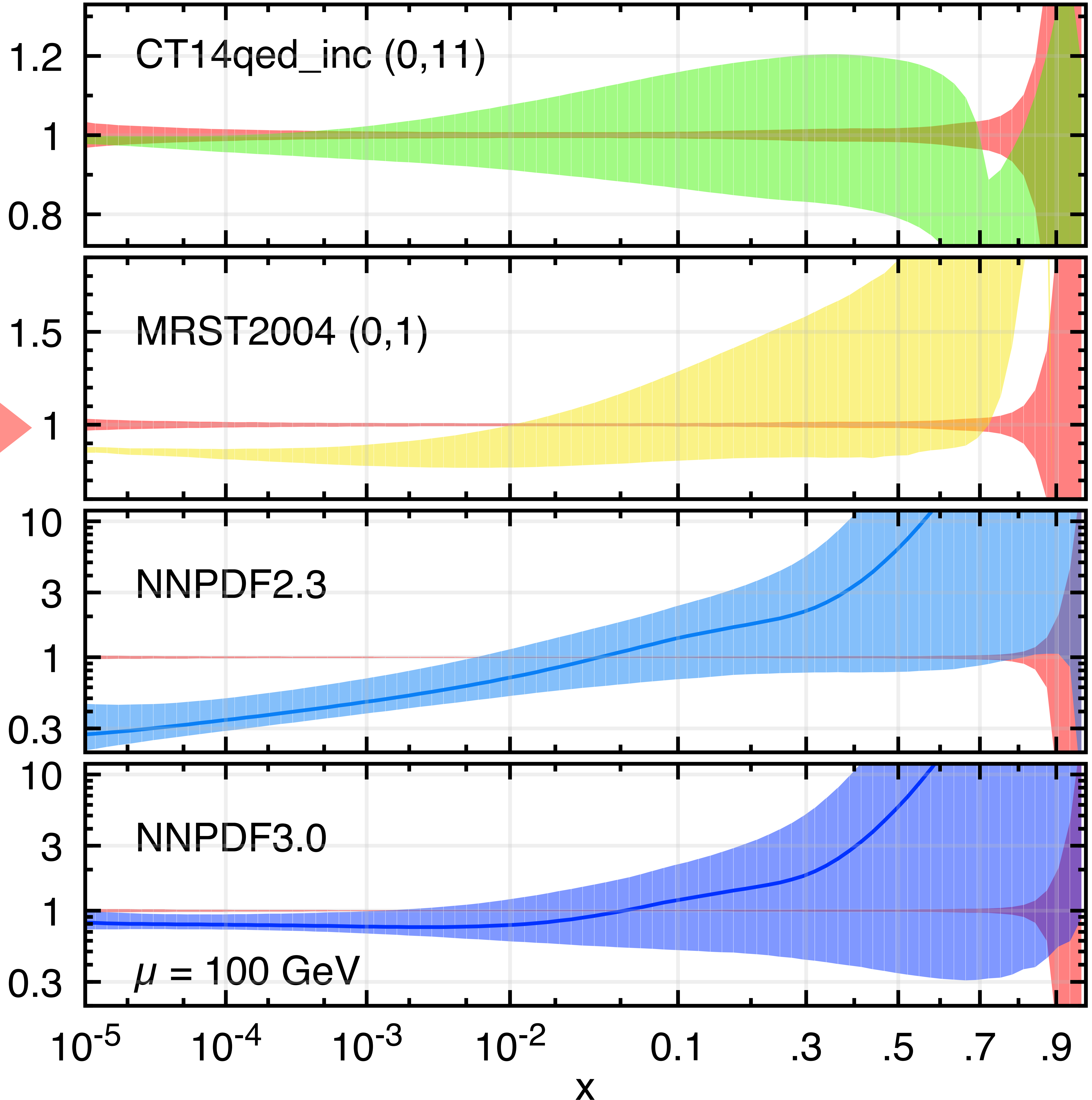
photon PDF results

- ➤ Model-independent uncertainty (NNPDF) was 50–100%
- ➤ Goes down to O(1%) with LUXqed determination



LUXqed v. other photon PDFs

LUXqed is the red band



Impact for Higgs + W production

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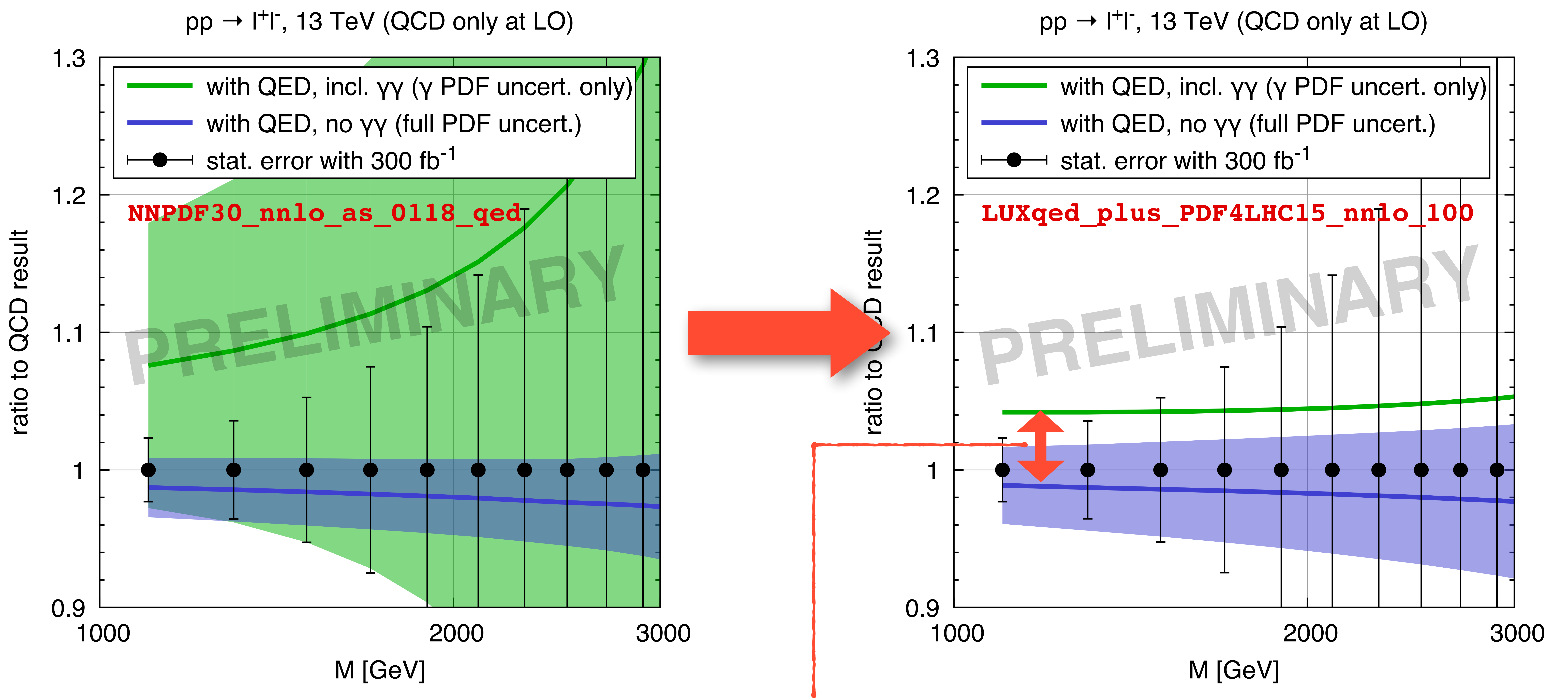
$pp \to H W^+ (\to l^+ \nu) + X$ at 13 TeV	
non-photon induced contributions	91.2 ± 1.8 fb
photon-induced contribs (NNPDF23)	$6.0\,^{+4.4}_{-2.9}$ fb
photon-induced contribs (LUXqed)	4.4 ± 0.1 fb

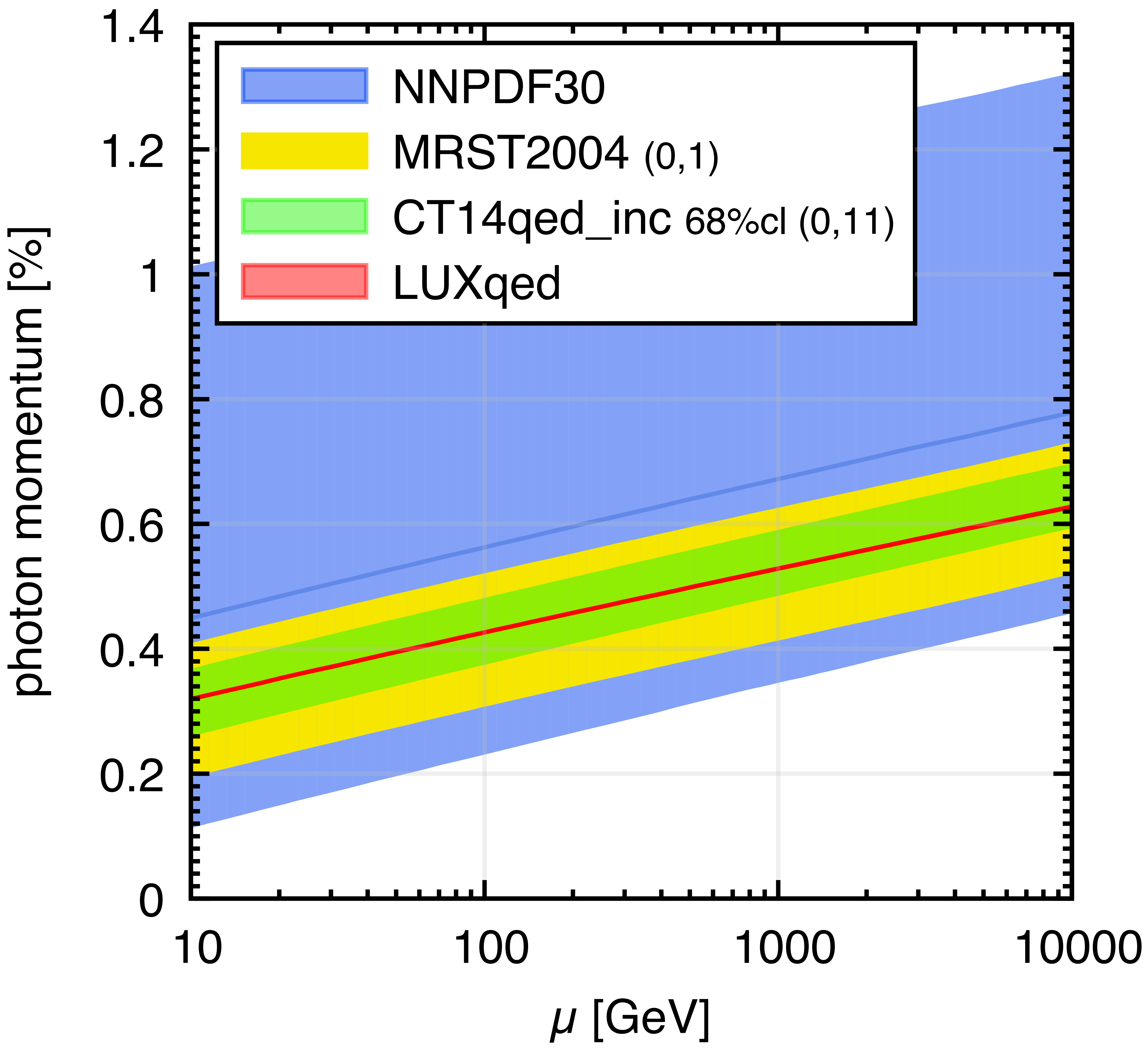
di-lepton spectrum



γγ component has few-% effect on Drell-Yan spectrum; negligible uncertainty

How much bright is the proton? [γ momentum fraction]

% of proton's momentum carried by photon



momentum (μ = 100 GeV)	
gluon	46.8 ± 0.4%
up valence	18.2 ± 0.3%
down valence	7.5 ± 0.2%
light sea quarks	20.7 ± 0.4%
charm	4.0 ± 0.1%
bottom	2.5 ± 0.1%
photon	0.426 ± 0.003%

`LUXqed_plus_PDF4LHC15_nnlo_100`

(1+107 members, symmhessian, errors handled by LHAPDF out of the box, valid for μ > 10 GeV)

CONCLUSIONS

Summary

- The photon content of the proton matters starts to matter in many places at LHC

- Electron-proton scattering expts. (→ structure functions F_2 and F_L) have effectively been measuring proton's γ content for 50 years…

- Photon distribution can be determined from that data to within 1–2% — i.e. as precise as any "QCD" parton

- Available through LHAPDF as `LUXqed_plus_PDF4LHC15_nnlo_100`

EXTRAS

physical picture

photon distribution from fast-moving charged particle

Point-like particle, e.g. electrons

- Fermi, Z. Phys. 1924 ; von Weizsäcker, Z. Phys 1924; Williams, Phys.Rev. 1934

$$f_{\gamma/e}(x,\mu^2) = \frac{\alpha}{2\pi}\left[\frac{1+(1-x)^2}{x}\log\left(\frac{1-x}{x^2}\frac{\mu^2}{m_e^2}\right) - 2\frac{1-x-x^2\frac{m_e^2}{\mu^2}}{x}\right]$$

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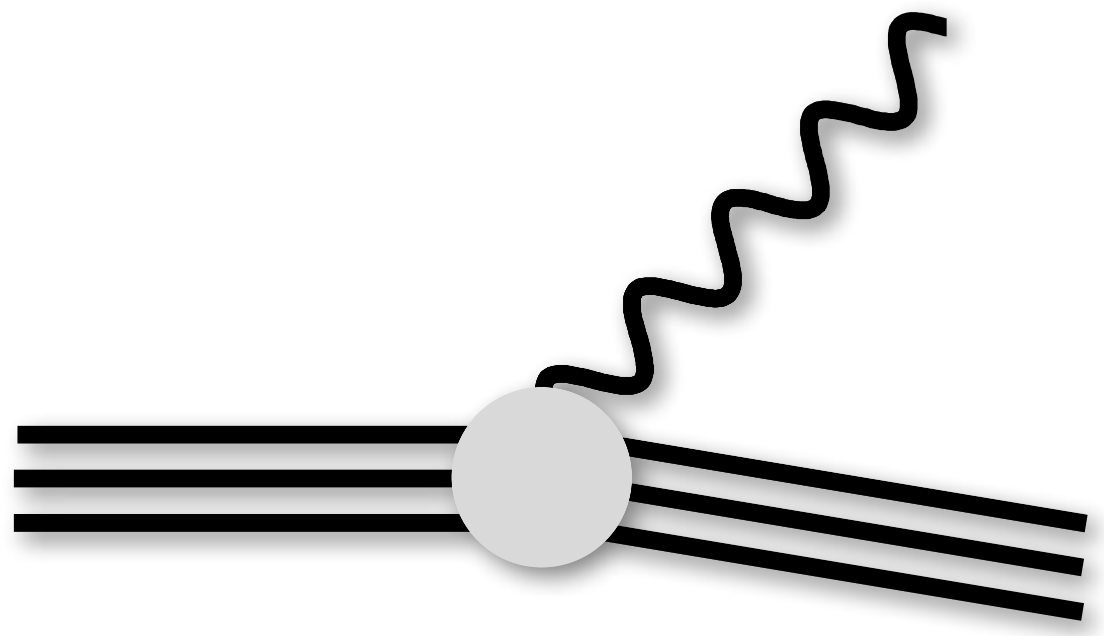
But protons are not point-like…

- Budnev, Ginzburg, Meledin & Serbo, Phys.Rept. 1974
  → an answer for the case where the proton remains intact after photon emission



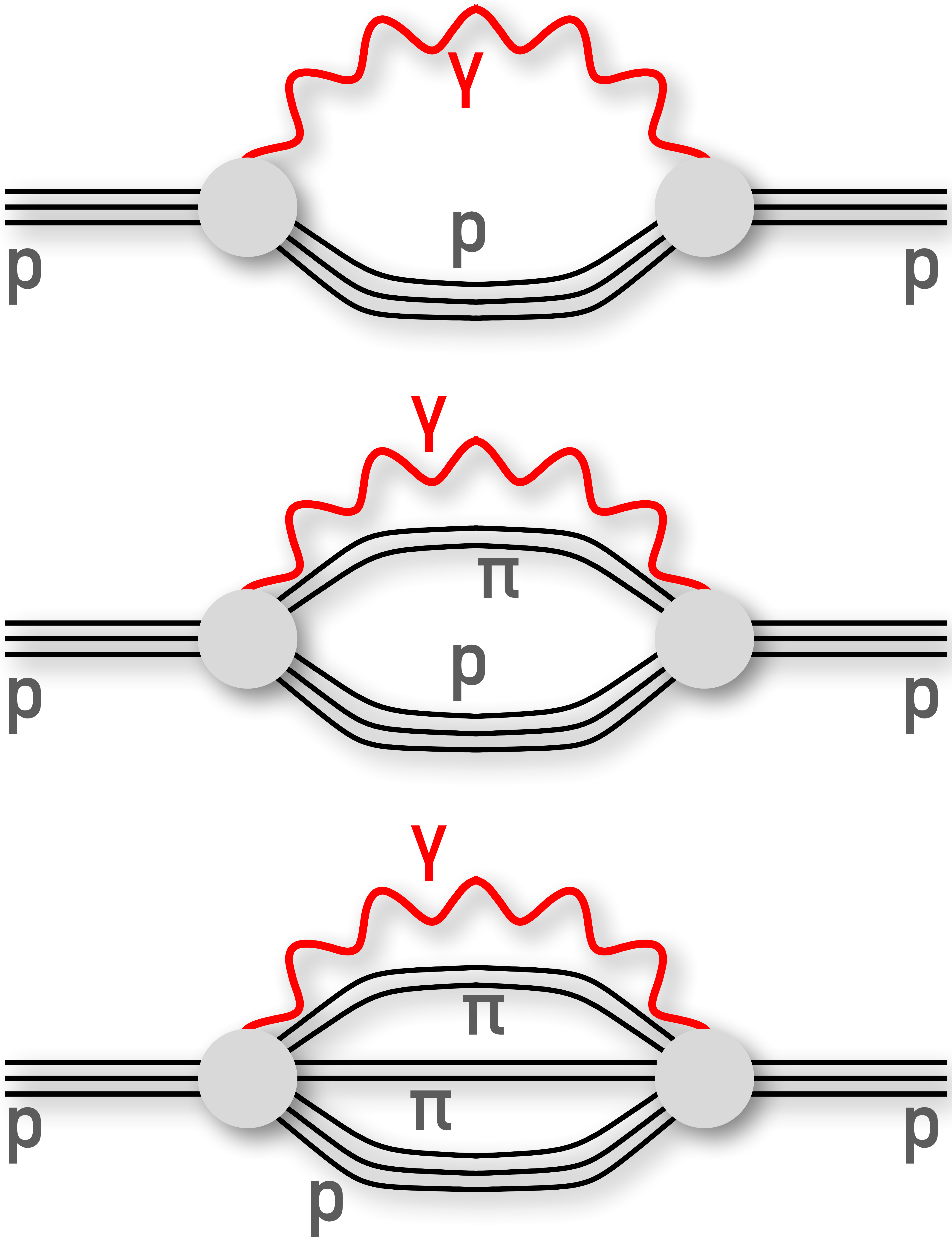
given in terms of "proton form factors" (measurable from elastic ep scattering)

"number of photons" inside a proton?

Proton constantly fluctuates in & out of different Fock states, some of which have a photon.

The states with extra pions (etc.) are called the **inelastic** component

Intrinsically non-perturbative.



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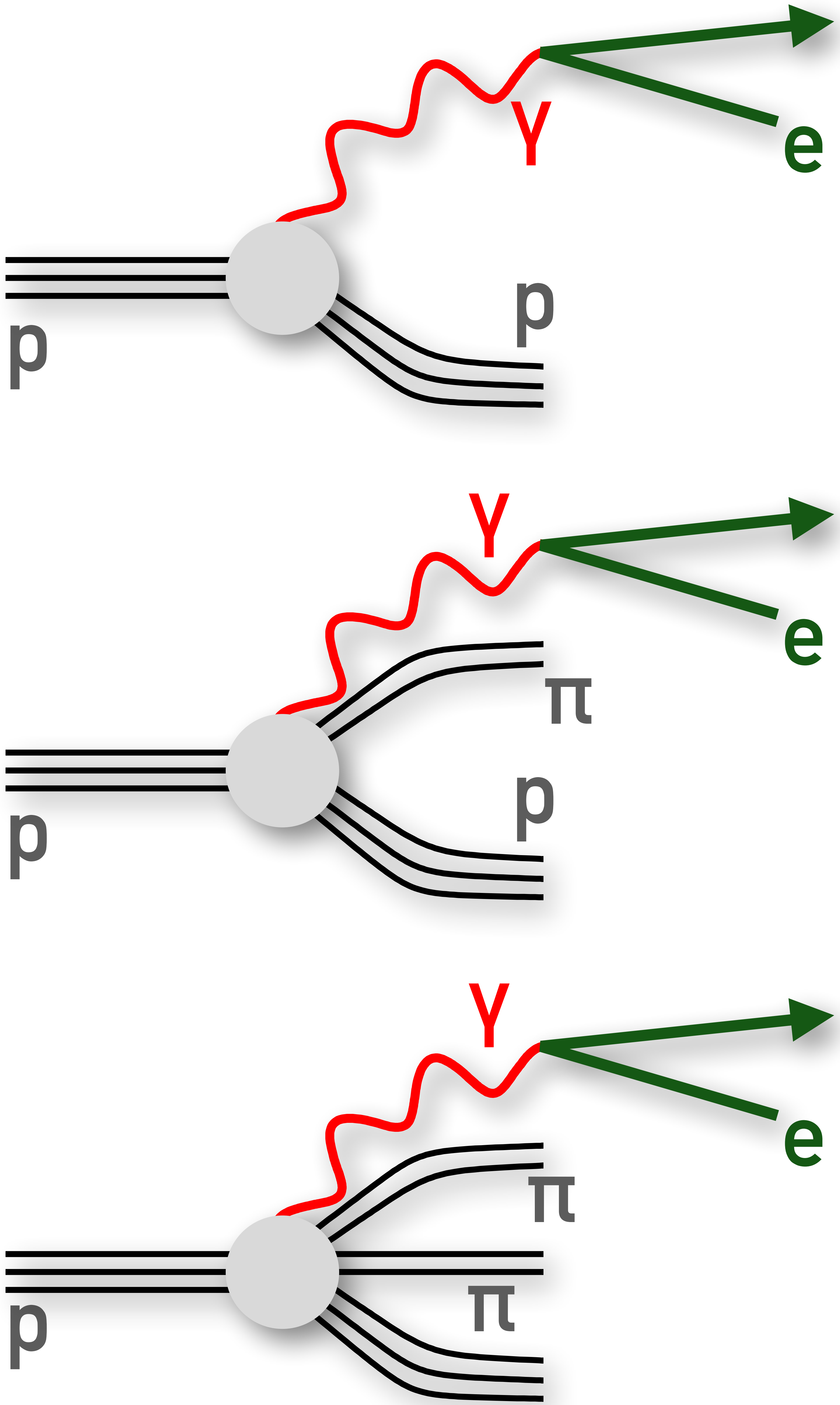
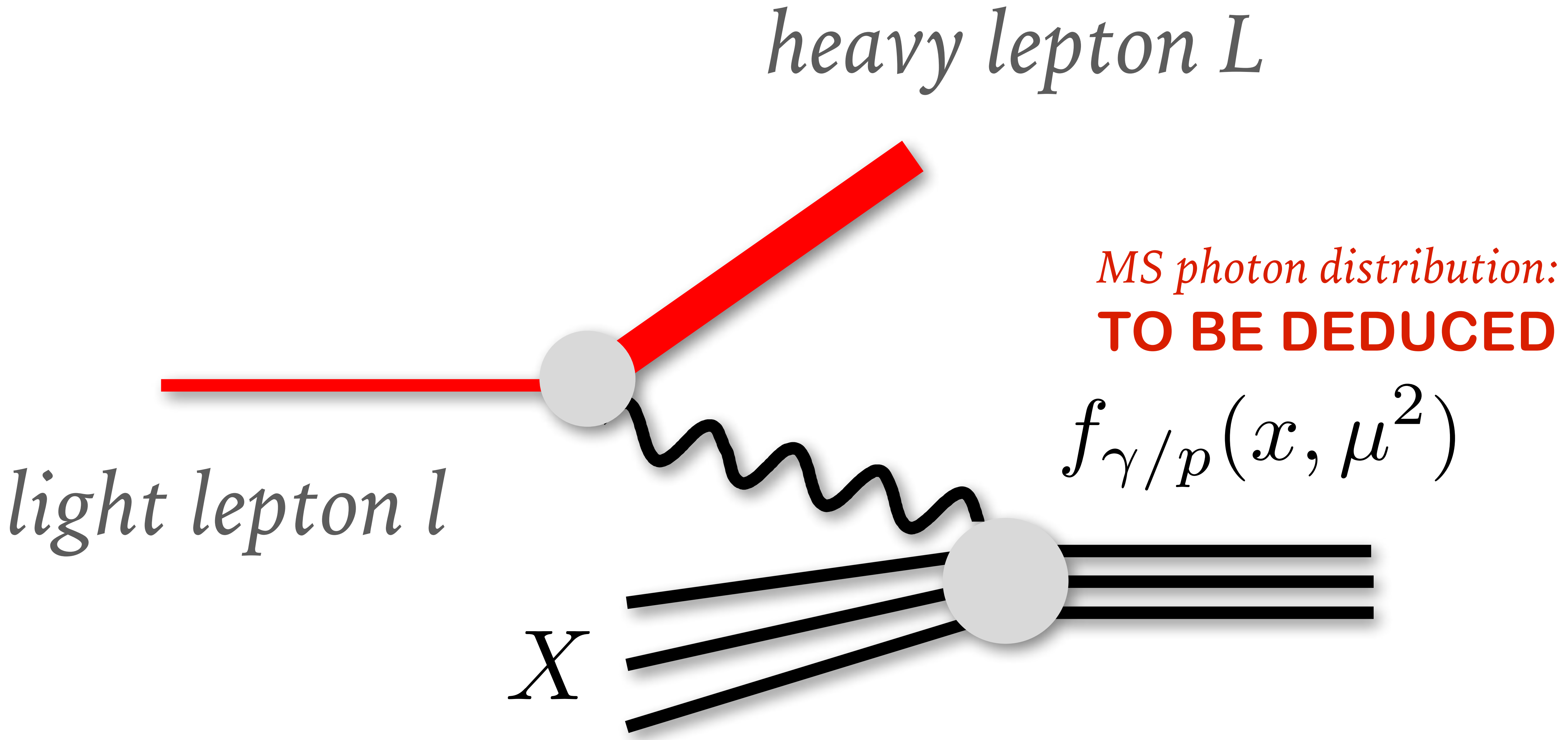


derivation of LUX master formula

How do you deduce photon distribution? Two approaches

Approach 1

- Imagine production of BSM heavy lepton (L), from a light neutral lepton l and a photon γ, i.e. lγ → L
- Calculate lp → L + X in terms of (known) structure functions
- Calculate lp → L + X in terms of (unknown) photon distribution
- Equate them to get the photon distribution



$$\mathcal{L}_{\rm int} = (e/\Lambda)\overline{L}\,\sigma^{\mu\nu}F_{\mu\nu}\,l$$

How do you deduce photon distribution? Two approaches

Approach 2

- exploits operator definition of the photon distribution
- is especially powerful for going to higher order

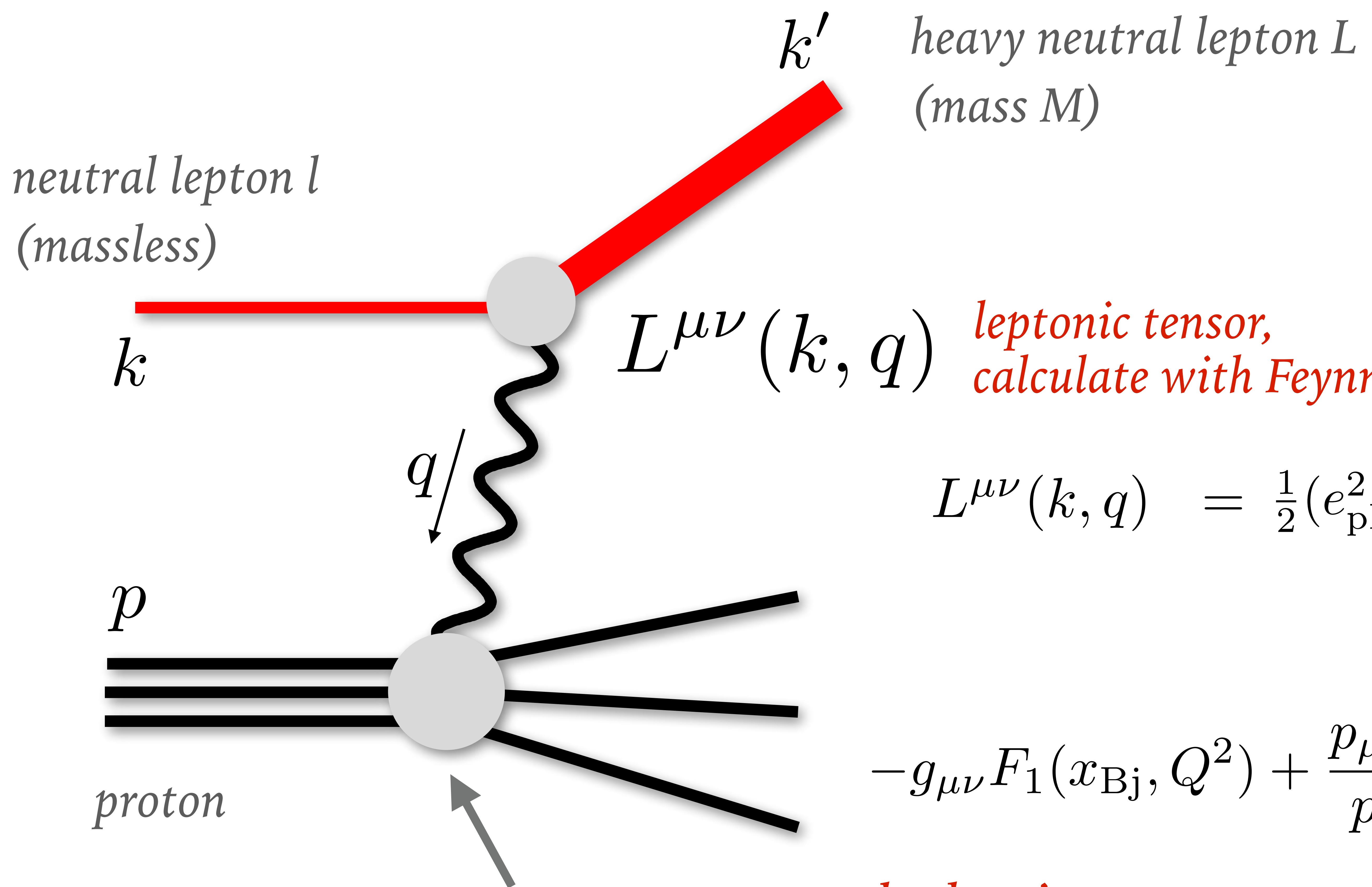
$$f_\gamma(x,\mu) = -\frac{1}{4\pi x p^+}\int_{-\infty}^{\infty} dw e^{-ixwp^+} \times$$
$$\langle p|F^{n\lambda}(wn)F^{n}{}_{\lambda}(0) + F^{n\lambda}(0)F^{n}{}_{\lambda}(wn)|p\rangle_c$$

Results with this approach are in progress

(and consistent with approach 1!)

STEP 1

work out a cross section (exact) in terms of F_2 and F_L struct. fns.



$L^{\mu\nu}(k,q)$ *leptonic tensor, calculate with Feynman diag.*

$$\mathcal{L}_{\rm int} = (e/\Lambda)\overline{L}\,\sigma^{\mu\nu}F_{\mu\nu}\,l$$

$$L^{\mu\nu}(k,q) \;=\; \tfrac{1}{2}(e_{\rm ph}^2(q^2)/\Lambda^2){\rm Tr}\left(\not{k}'\,[\not{q},\gamma^\mu]\,(\not{k}'+M)\,[\gamma^\nu,\not{q}]\right)$$

$$-g_{\mu\nu}F_1(x_{\rm Bj},Q^2) + \frac{p_\mu p_\nu}{p.q}F_2(x_{\rm Bj},Q^2) + \cdots$$

$$Q^2 = -q^2$$

$$x_{\rm Bj} = \frac{Q^2}{2p.q}$$

$W_{\mu\nu}(p,q)$ *hadronic tensor, known in terms of F_2 and F_L*

$$\sigma = \frac{1}{4p\cdot k}\int\frac{d^4q}{(2\pi)^4 q^4}e_{\rm ph}^2(q^2)\left[4\pi W_{\mu\nu}\,L^{\mu\nu}(k,q)\right]\times 2\pi\delta((k-q)^2-M^2)$$

Cross section in terms of structure functions

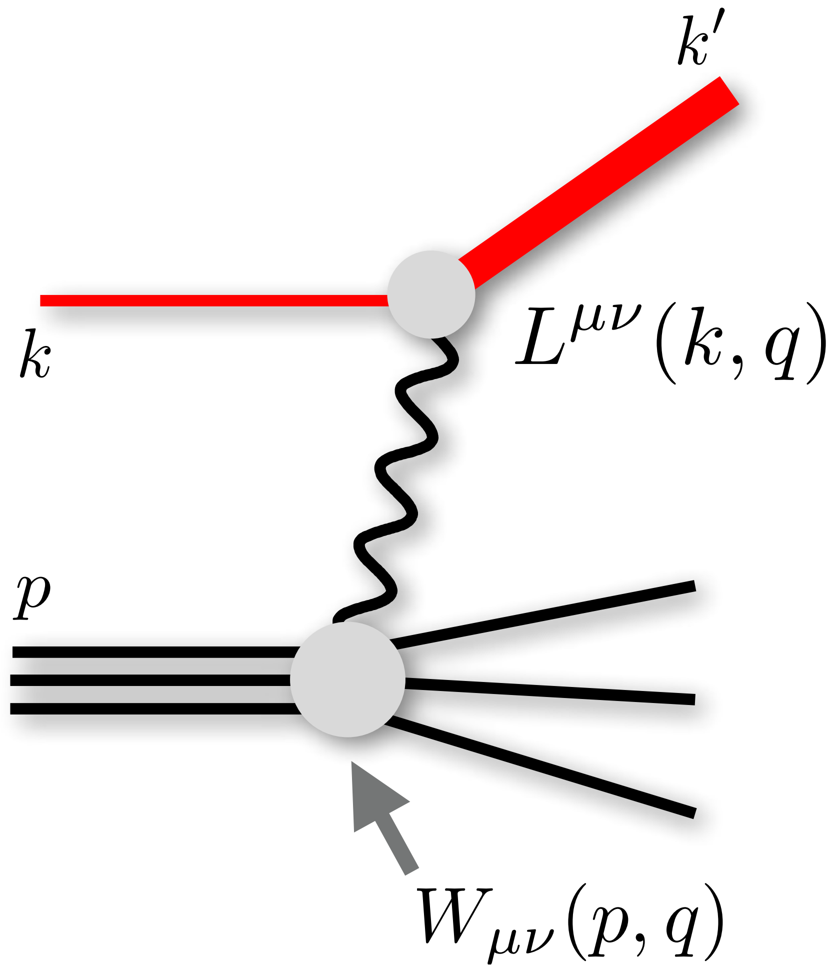
- Lagrangian of interaction: (magnetic moment coupling)

$$\mathcal{L}_{\rm int} = (e/\Lambda)\overline{L}\,\sigma^{\mu\nu}F_{\mu\nu}\,l$$

- Using leptons neutral and taking Λ large, ensure that only single-photon exchange is relevant
- **Answer is exact up to $1/\Lambda$ corrections**

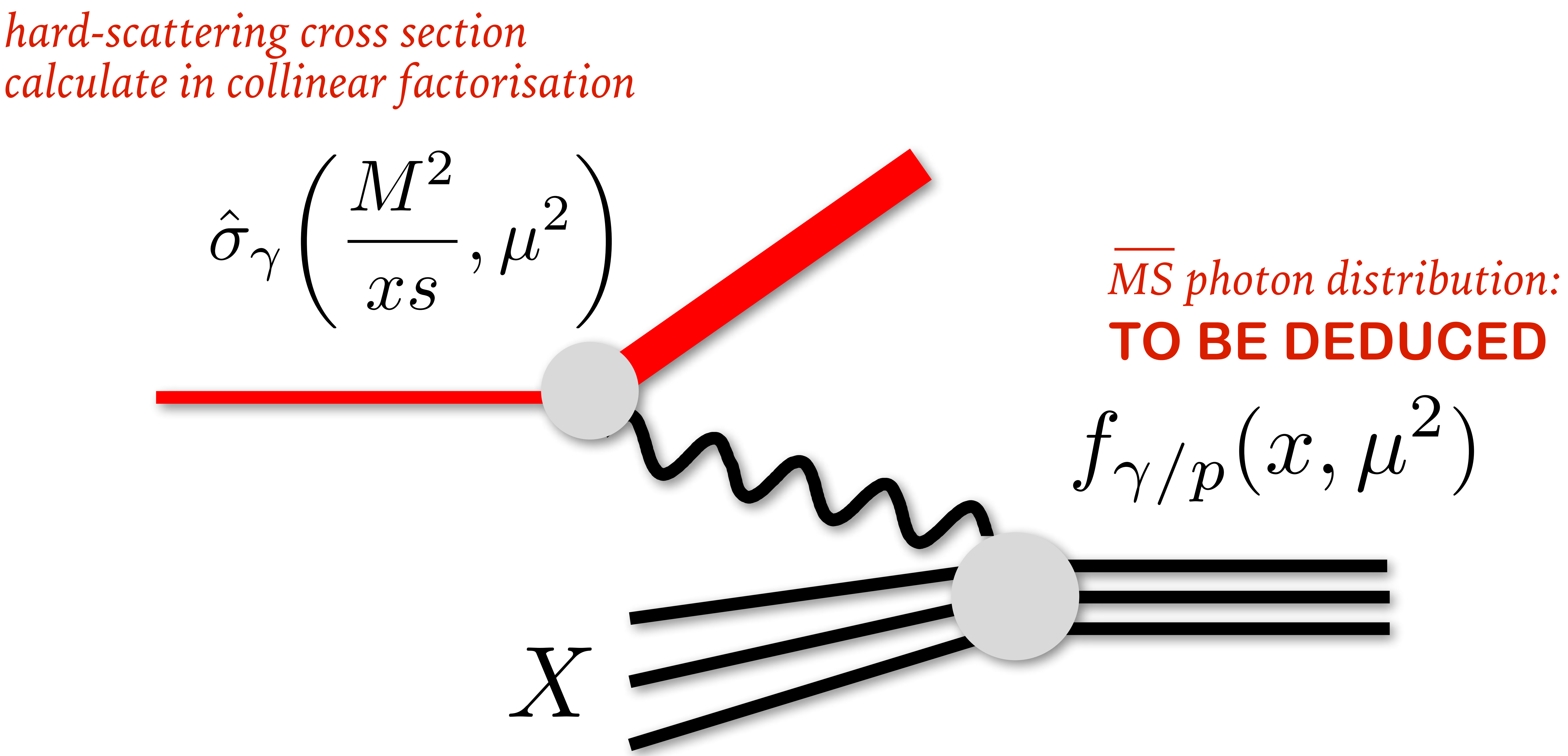
$$\sigma = \frac{c_0}{2\pi}\int_x^{1-\frac{2xm_p}{M}}\frac{dz}{z}\int_{Q^2_{\rm min}}^{Q^2_{\rm max}}\frac{dQ^2}{Q^2}\alpha^2_{\rm ph}(-Q^2)\Bigg[\Bigg(2-2z+z^2 + \frac{2x^2m_p^2}{Q^2}+\frac{z^2Q^2}{M^2}-\frac{2zQ^2}{M^2}-\frac{2x^2Q^2m_p^2}{M^4}\Bigg)F_2(x/z,Q^2) + \left(-z^2-\frac{z^2Q^2}{2M^2}+\frac{z^2Q^4}{2M^4}\right)F_L(x/z,Q^2)\Bigg]$$

$$c_0 = 16\pi^2/\Lambda^2$$



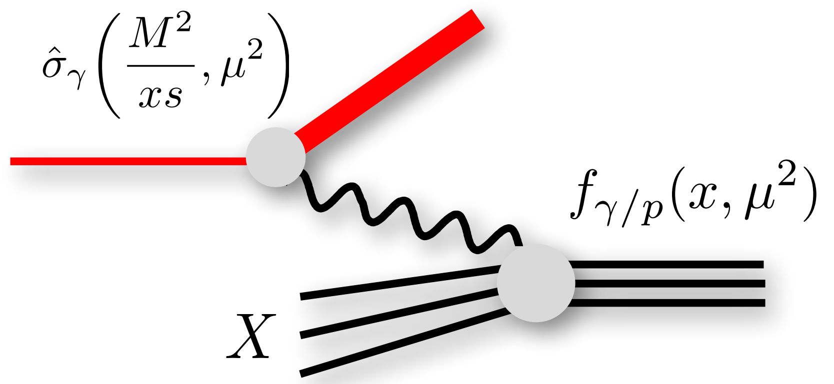
STEP 2

work out same cross section in terms of a photon distribution



$$\sigma = c_0 \sum_a \int \frac{dx}{x} \hat{\sigma}_a\left(\frac{M^2}{xs}, \mu^2\right) x f_{a/p}\left(x, \mu^2\right)$$

Cross section in terms of structure functions



➤ Hard cross section driven by the photon distribution at LO

$$\hat{\sigma}_a(z,\mu^2) = \alpha(\mu^2)\delta(1-z)\delta_{a\gamma} + \frac{\alpha^2(\mu^2)}{2\pi}\left[-2+3z+ + zp_{\gamma q}(z)\ln\frac{M^2(1-z)^2}{z\mu^2}\right]\sum_{i\in\{q,\bar{q}\}} e_i^2\delta_{ai} + \ldots, \quad (5)$$

➤Quarks and gluons come in at higher orders

Accuracy aim

- Take quark and gluon distributions $\sim O(1)$
- α is QED coupling, α_s is QCD coupling, $L = \ln \mu^2/m_p^2$
  - Take $L \sim 1/\alpha_s$, so all $(\alpha_s L)^n \sim 1$
  - Think of $\alpha \sim (\alpha_s)^2$
- To first order, photon distribution $\sim (\alpha L)$
- we aim to control all terms:
  - $\alpha L (\alpha_s L)^n$ [LO]
  - $\alpha_s \alpha L (\alpha_s L)^n \equiv \alpha (\alpha_s L)^n$ [NLO — extra α_s or $1/L$]
  - $\alpha^2 L^2 (\alpha_s L)^n$ [NLO — extra αL]
- Matching done at large M^2 and μ^2 to eliminate higher twists

Cross checks & literature comparisons

- Repeat calculation for a different process (γp→H+X, via γγ→H). Intermediate results differ, **final photon distribution is identical**.
- Substitute elastic-scattering component of F_2 and F_L:

$$F_2^{\rm el} = \frac{[G_E(Q^2)]^2 + [G_M(Q^2)]^2\tau}{1+\tau}\delta(1-x)\,,$$

$$F_L^{\rm el} = \frac{[G_E(Q^2)]^2}{\tau}\delta(1-x)\,, \qquad \tau = Q^2/(4m_p^2)$$

and reproduce widely-used **Equivalent Photon Approximation** with electric (G_E) and magnetic (G_M) Sachs proton form factors

Budnev et al., Phys.Rept.15(1975)181

Cross checks & literature comparisons

- μ^2 derivative of our answer should reproduce known DGLAP QCD-QED splitting functions
- At LO, this is trivial.
- At NLO we get relations between QED-QCD splitting functions (P) and DIS coefficient functions (C)

$$P^{(1,1)}_{\gamma q} = e_q^2 \left[p_{\gamma q} \otimes C_{2q} - h \otimes C_{Lq} + (\bar{p}_{\gamma q} - h) \otimes P^{(1,0)}_{qq}\right] ,$$

$$P^{(1,1)}_{\gamma g} = \sum_{q,\bar{q}} e_q^2 \left[p_{\gamma q} \otimes C_{2g} - h \otimes C_{Lg} + (\bar{p}_{\gamma q} - h) \otimes P^{(1,0)}_{qg}\right] ,$$

$$P^{(1,1)}_{\gamma\gamma} = (2\pi)^2 b^{(1,2)}_{\alpha} \delta(1-x) = -C_F N_C \sum_q e_q^2 \delta(1-x)$$

$h(z) \equiv z$ and $\bar{p}_{\gamma q}(z) \equiv p_{\gamma q}(z) \ln \frac{1}{1-z}$

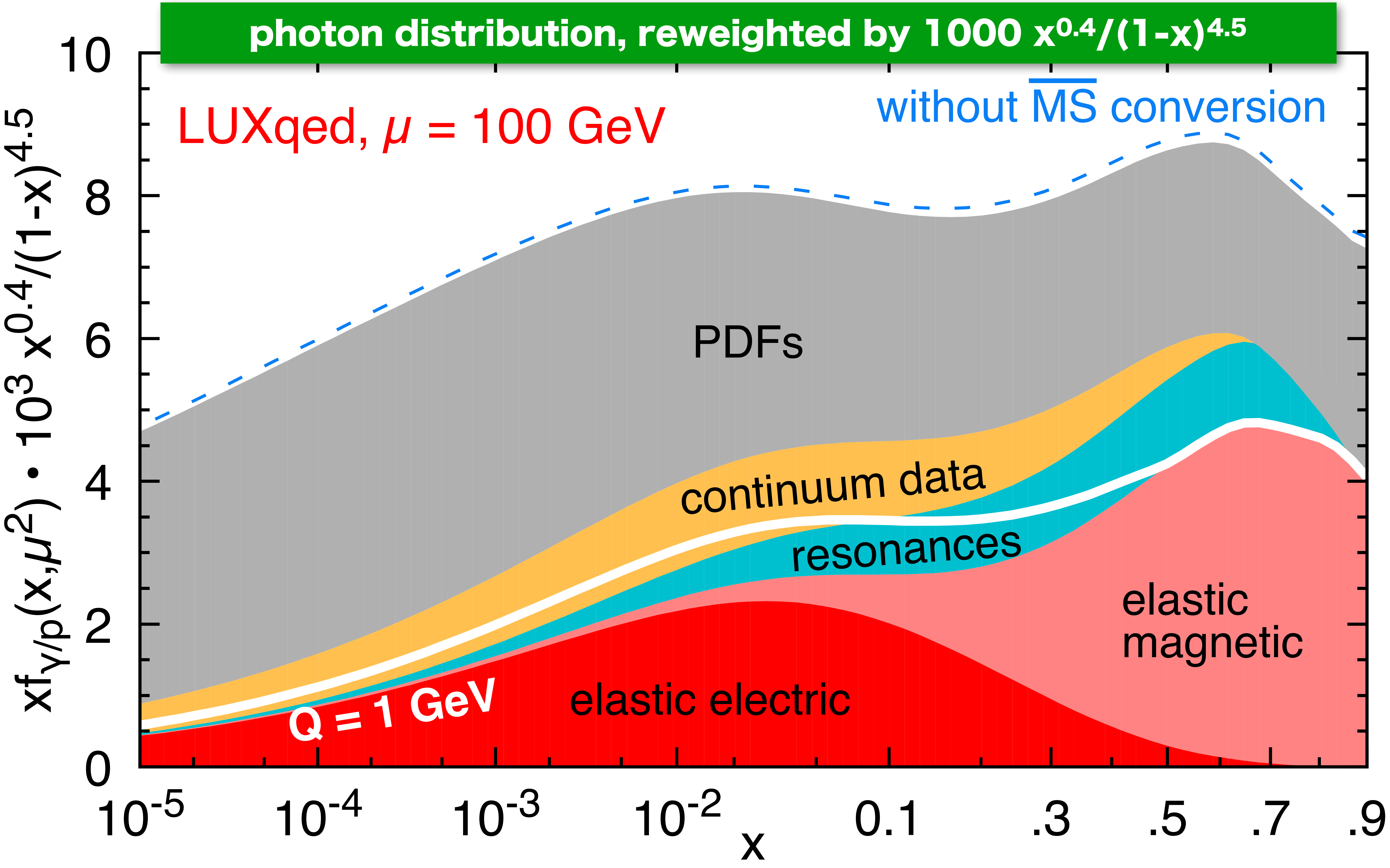
- These **agree with de Florian, Sborlini & Rodrigo results**

for $O(\alpha\, \alpha_s)$ terms, arXiv:1512.00612

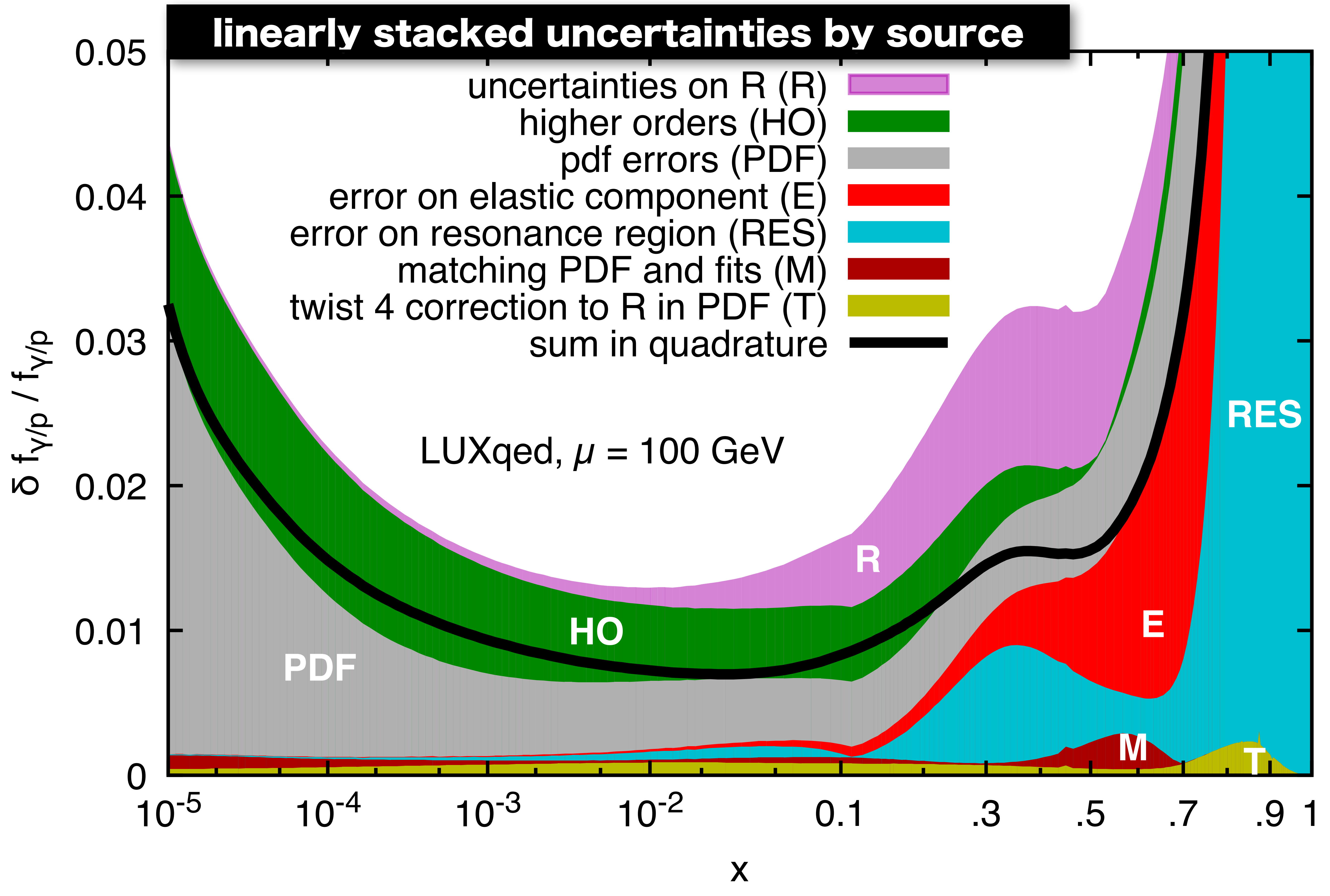
contributions & uncertainties

F_2 and F_L in our parametrisation

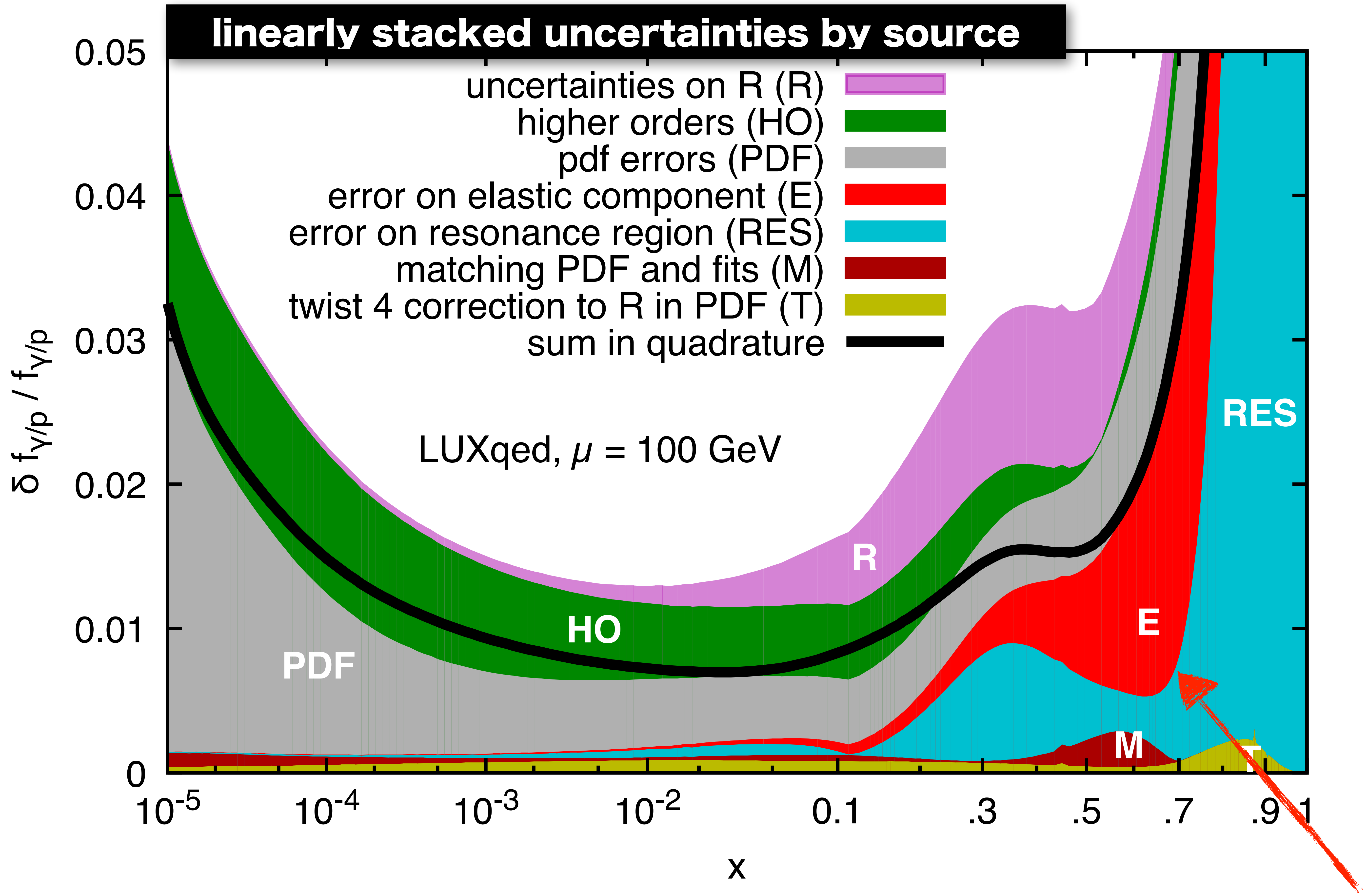
SEPARATE CONTRIBUTIONS TO PHOTON PDF



photon uncertainties (aim to be conservative & pragmatic)

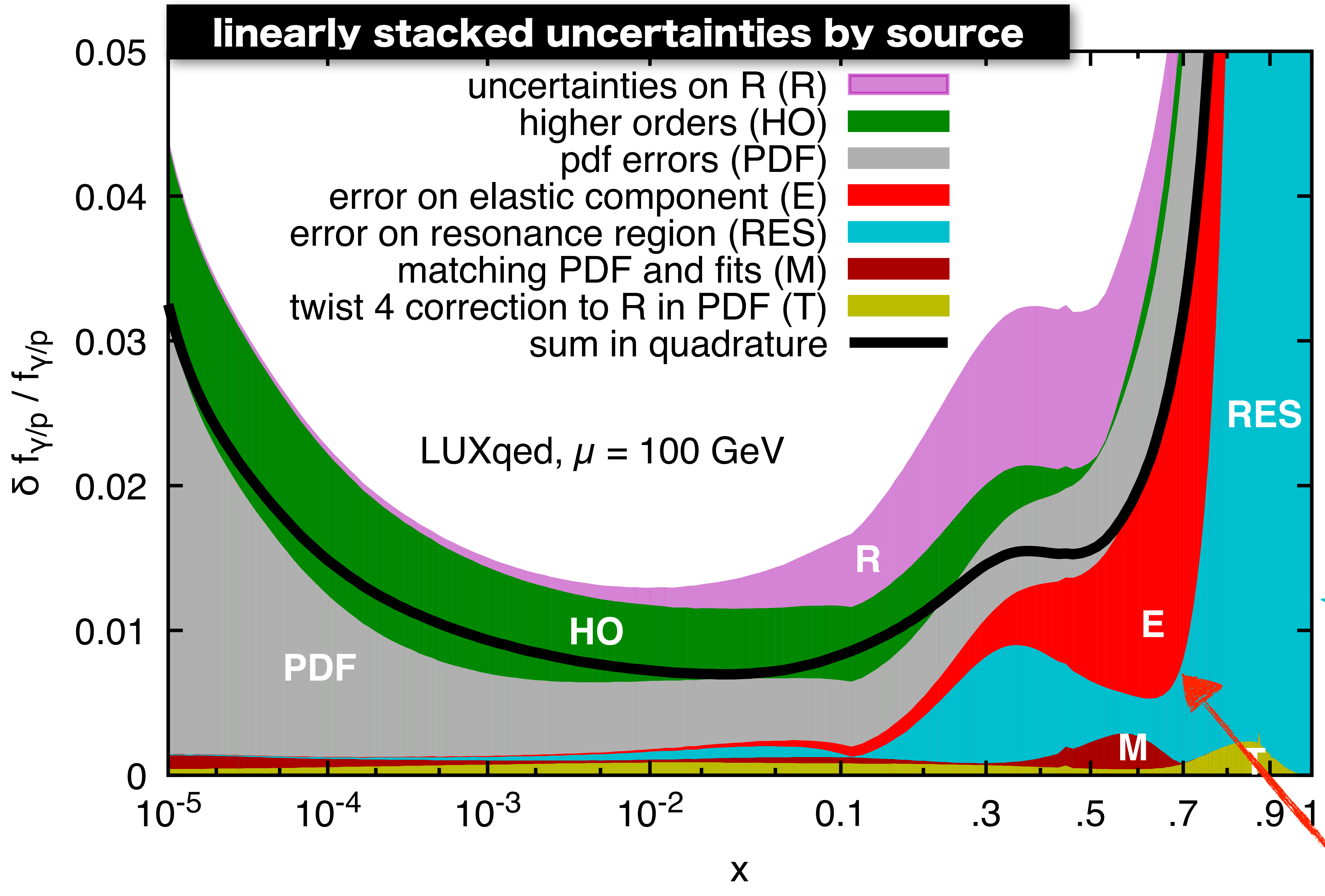


photon uncertainties (aim to be conservative & pragmatic)



uncertainty on elastic component (quoted ⊕ unpol./pol.)

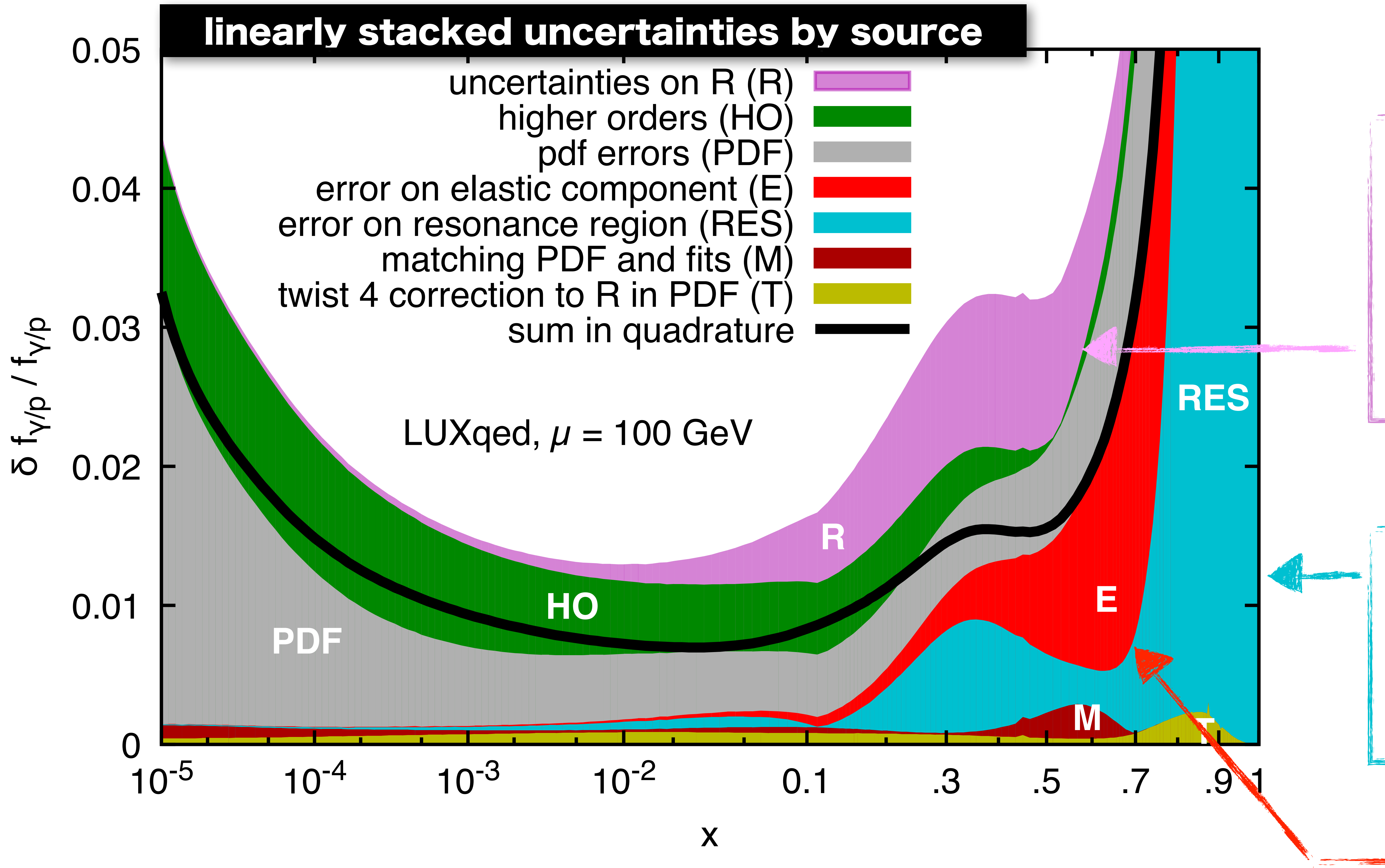
photon uncertainties (aim to be conservative & pragmatic)



replace CLAS resonance fit with Christy-Bosted

uncertainty on elastic component (quoted ⊕ unpol./pol.)

photon uncertainties (aim to be conservative & pragmatic)

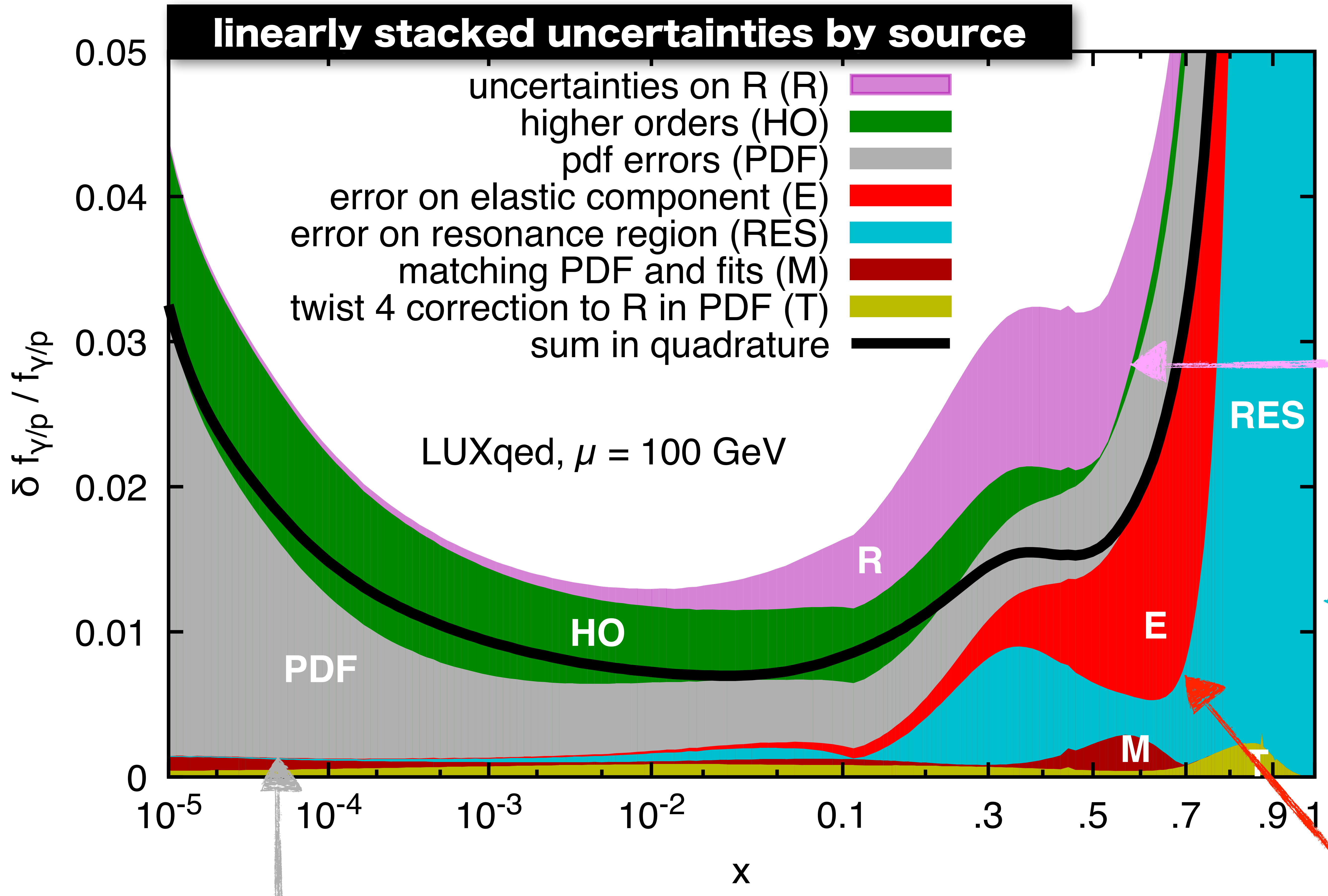


±50% on R ($\sim F_L/(F_2 - F_L)$) in low-Q^2 continuum and resonance regions

replace CLAS resonance fit with Christy-Bosted

uncertainty on elastic component (quoted ⊕ unpol./pol.)

photon uncertainties (aim to be conservative & pragmatic)



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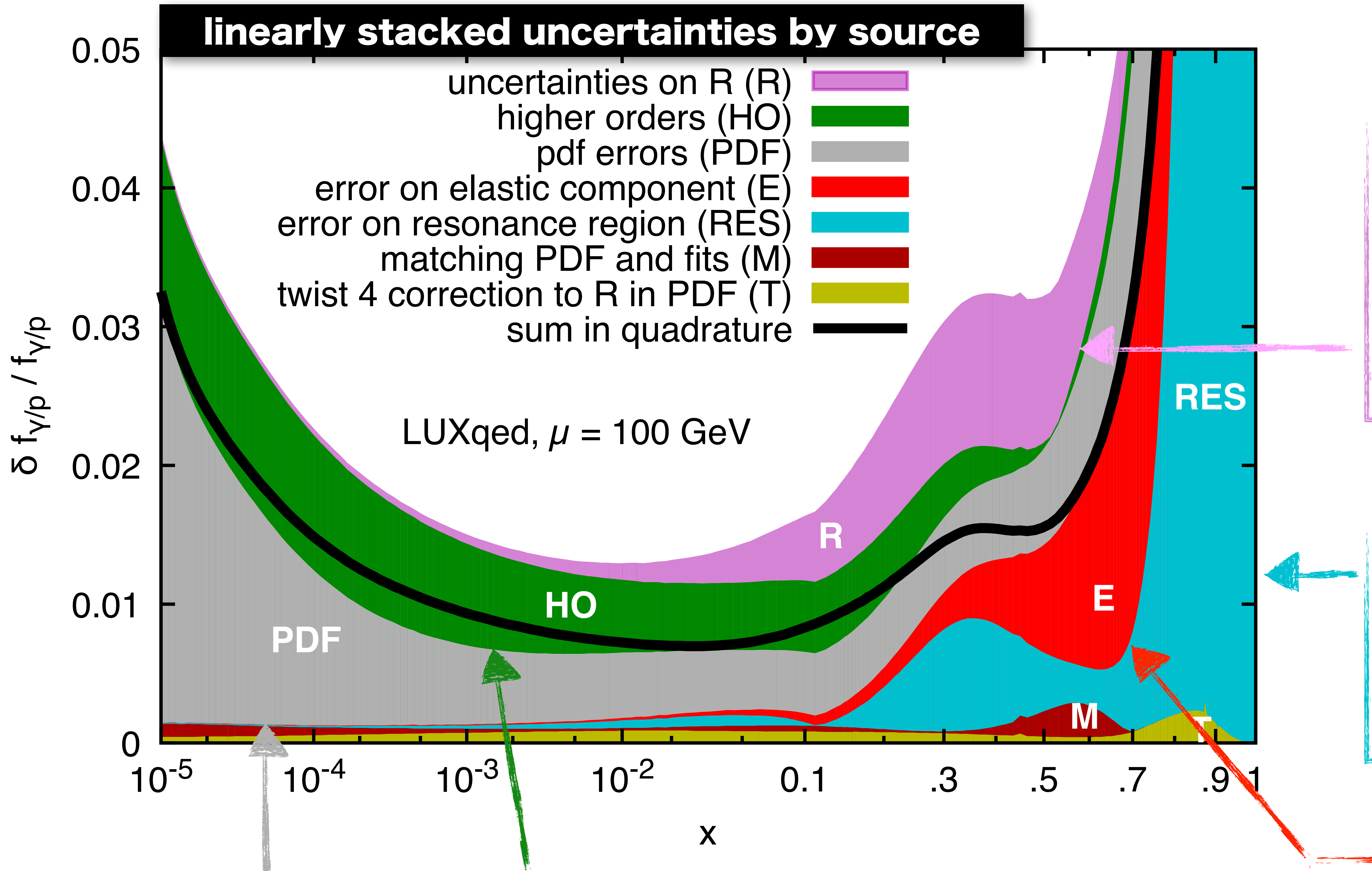
standard PDF uncertainty

photon uncertainties (aim to be conservative & pragmatic)



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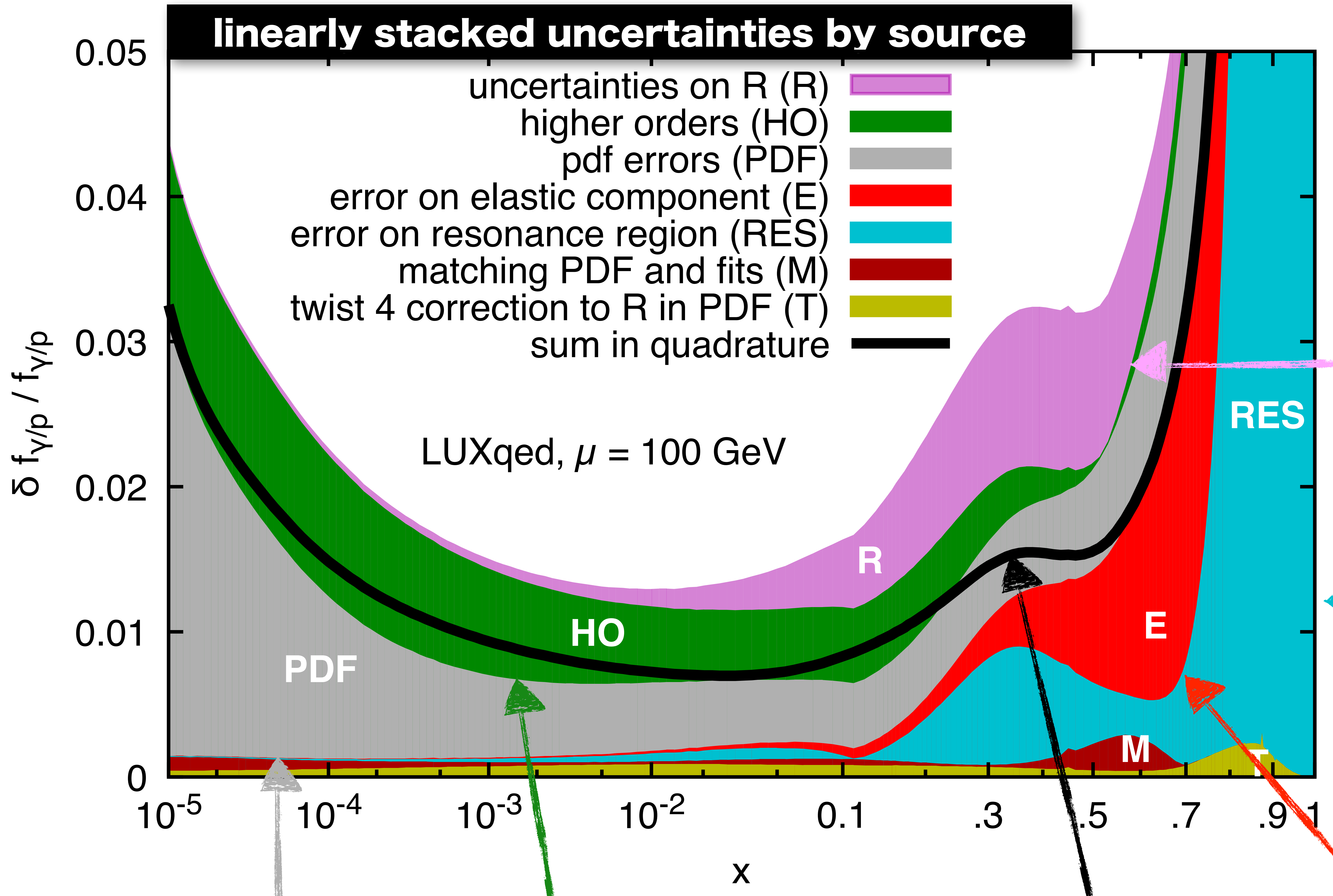
standard PDF uncertainty

treatment of upper limit of Q^2 integral ($\mu^2/(1-z)$ v. μ^2)

photon uncertainties (aim to be conservative & pragmatic)



±50% on R ($\sim F_L/(F_2 - F_L)$) in low-Q^2 continuum and resonance regions

replace CLAS resonance fit with Christy-Bosted

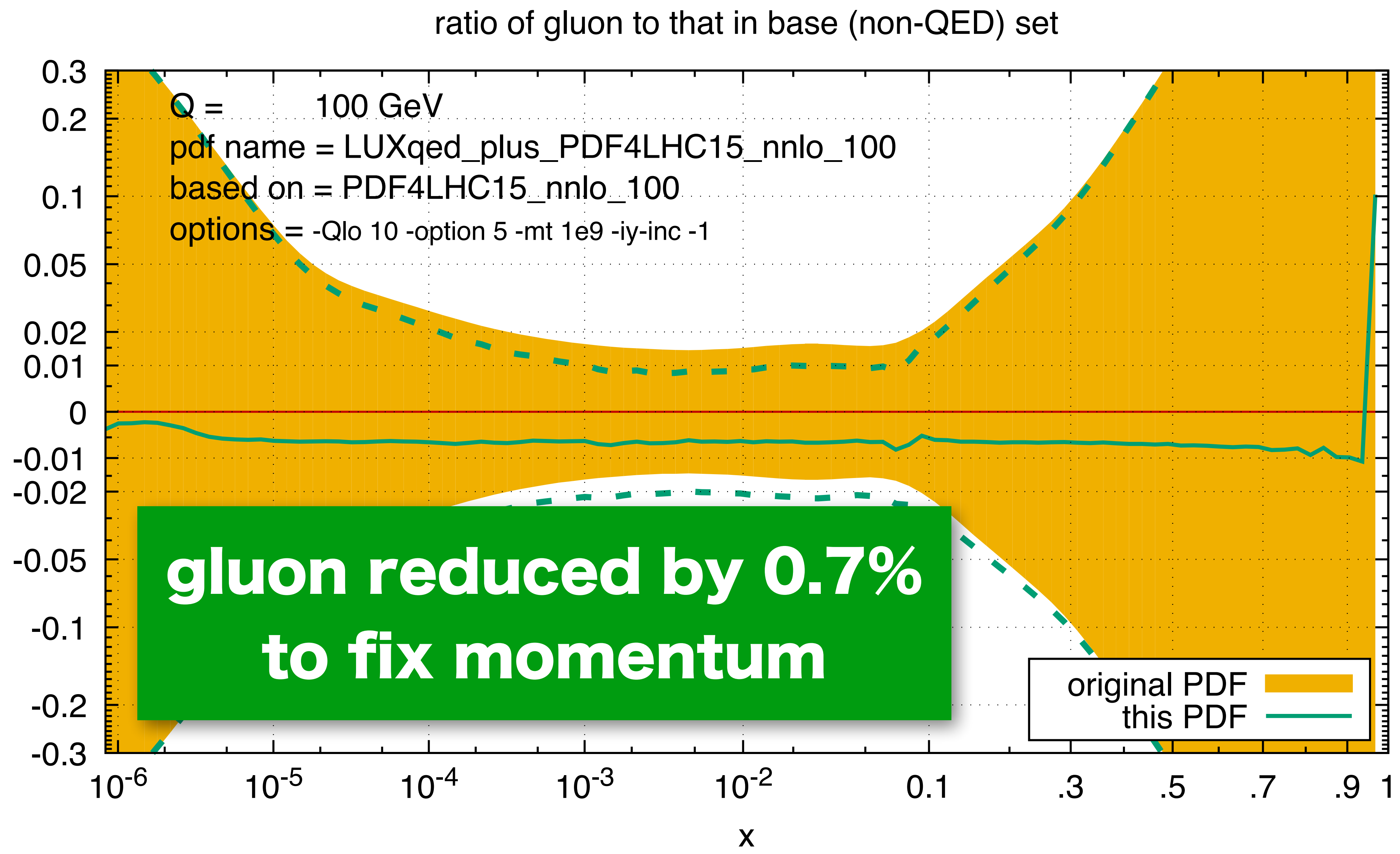
standard PDF uncertainty

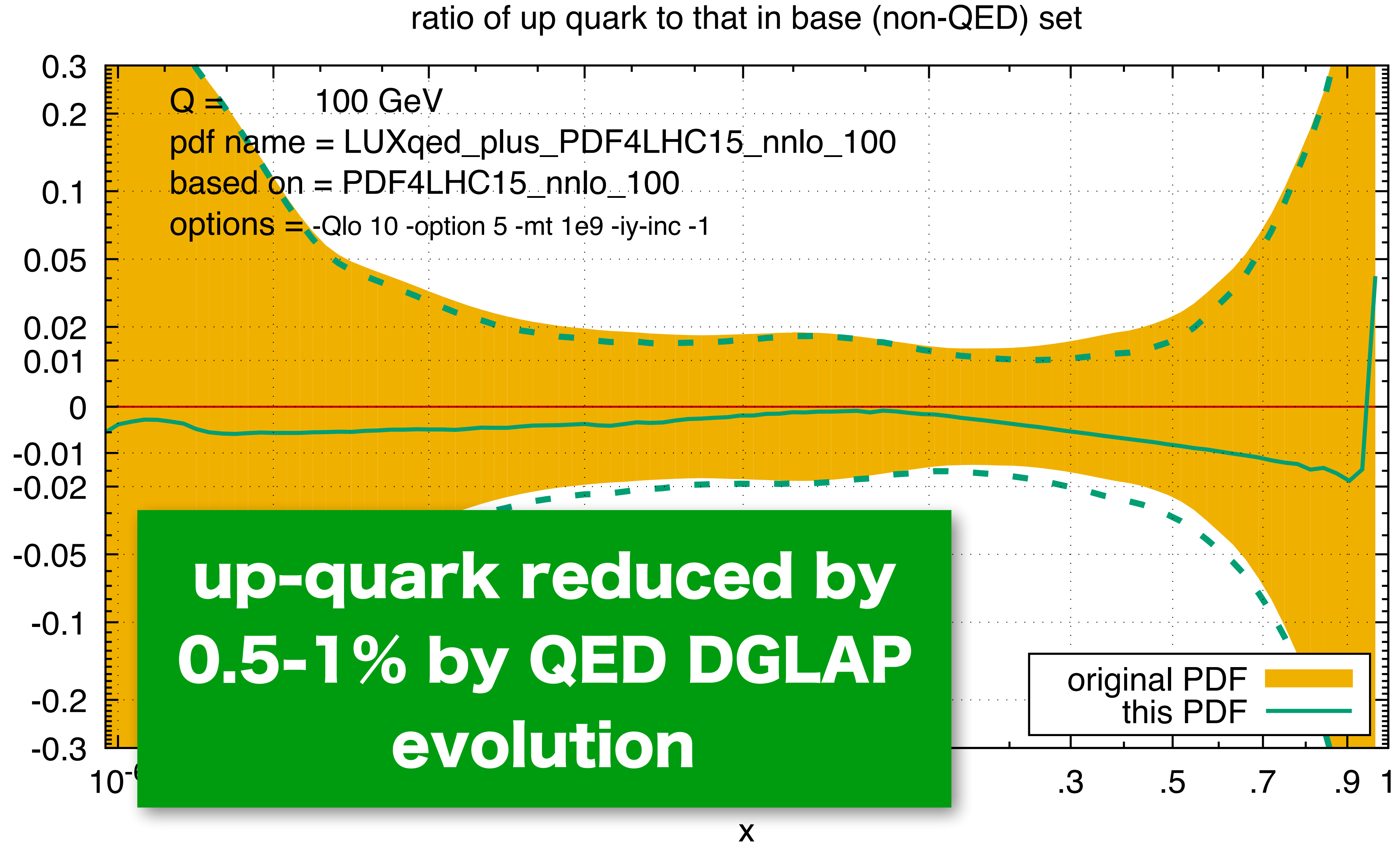
treatment of upper limit of Q^2 integral ($\mu^2/(1-z)$ v. μ^2)

final total uncertainty ~ 1 – 2% **(sum in quad. of all sources)**

uncertainty on elastic component (quoted ⊕ unpol./pol.)

matching procedure for full set of partons

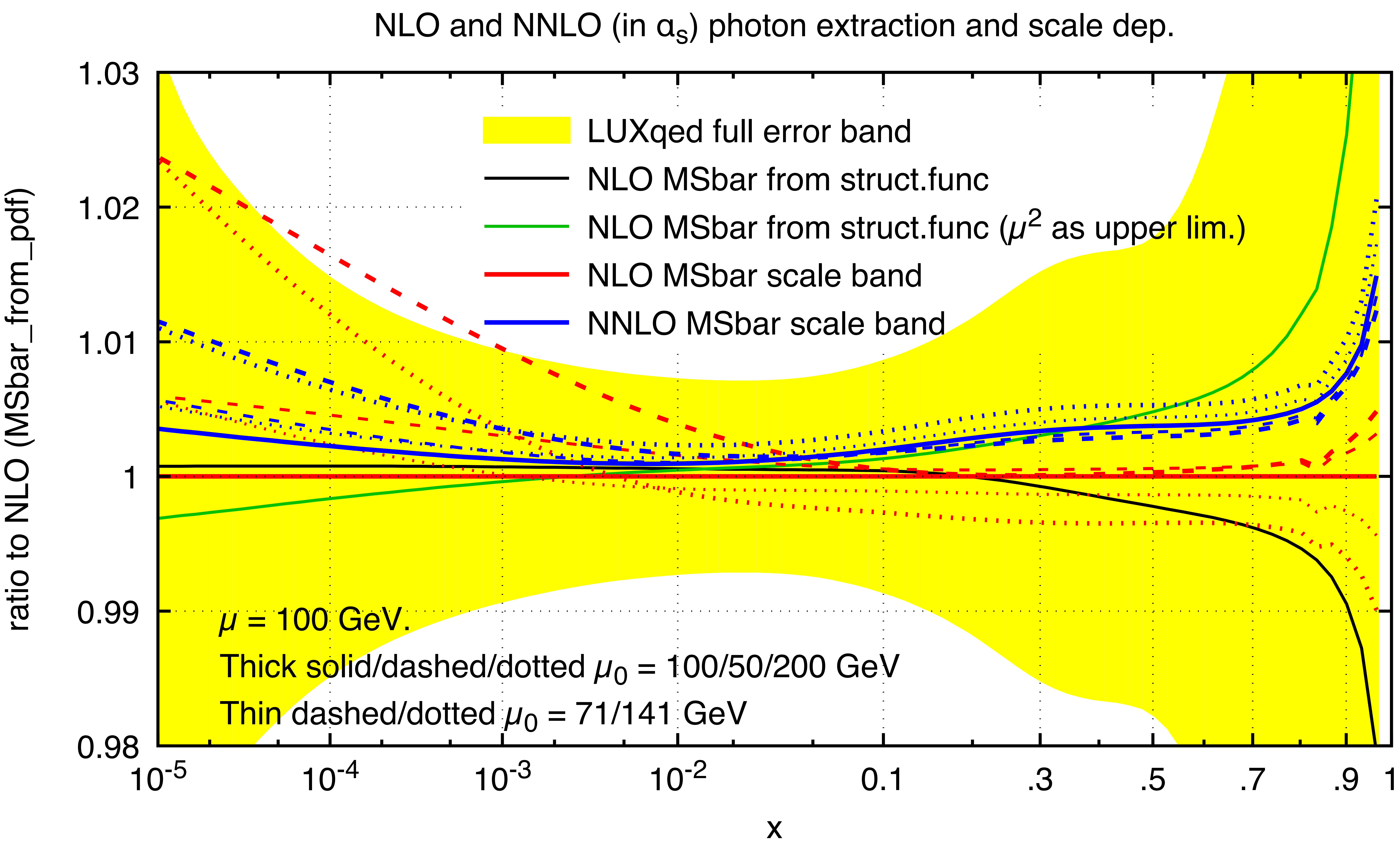




- evaluate master eqn. for $\mu = 100$ GeV (with default PDF4LHC15_nnlo partons)
- Do $O(\alpha\alpha_s)$ photon evolution down to $\mu = 10$ GeV (other partons: pure QCD evln.)
- Adjust momentum sum-rule by rescaling gluon $g(x) \to 0.993 g(x)$
- Evolve back up with NNLO-QCD & $O(\alpha\alpha_s)$ QED for all partons

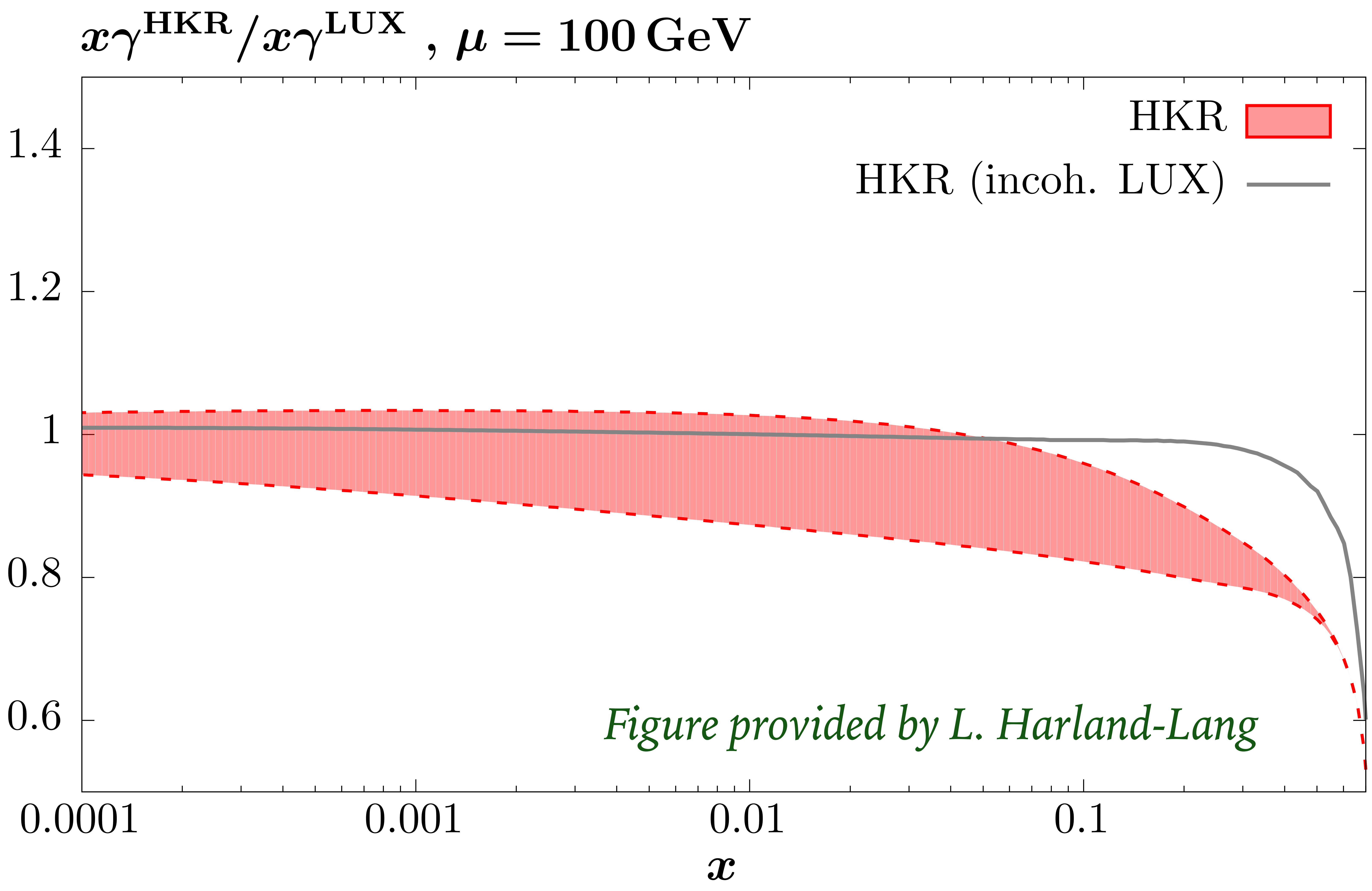
better approach would be full PDF re-fit for QCD partons incl. EW/QED corrections & LUXqed photon

higher-order contributions



LUXqed v. others

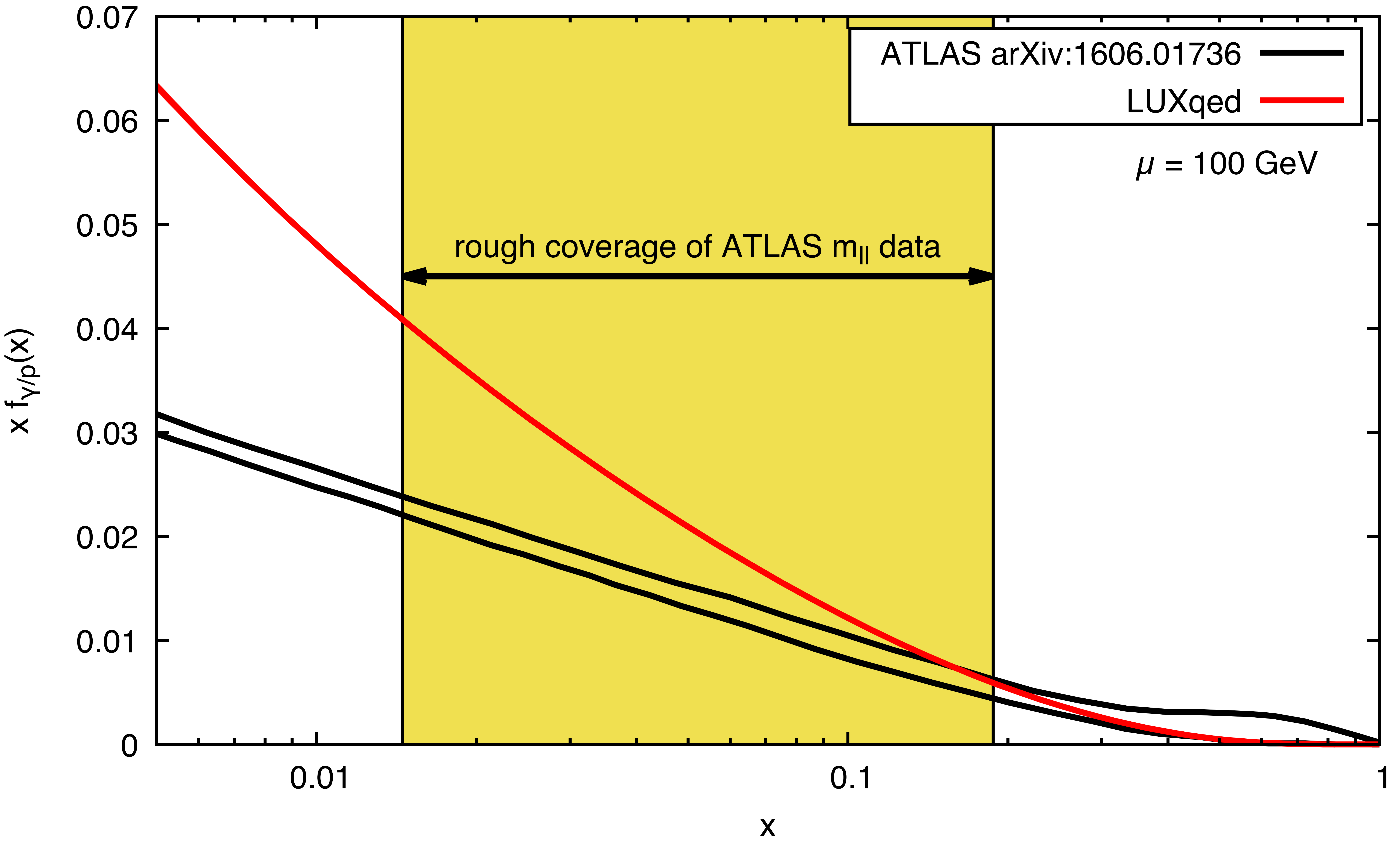
ratio of HKR (1607.04635) to LUXqed



HKR based on elastic contribution (dipole approx)

+ model for inelastic part + evolution

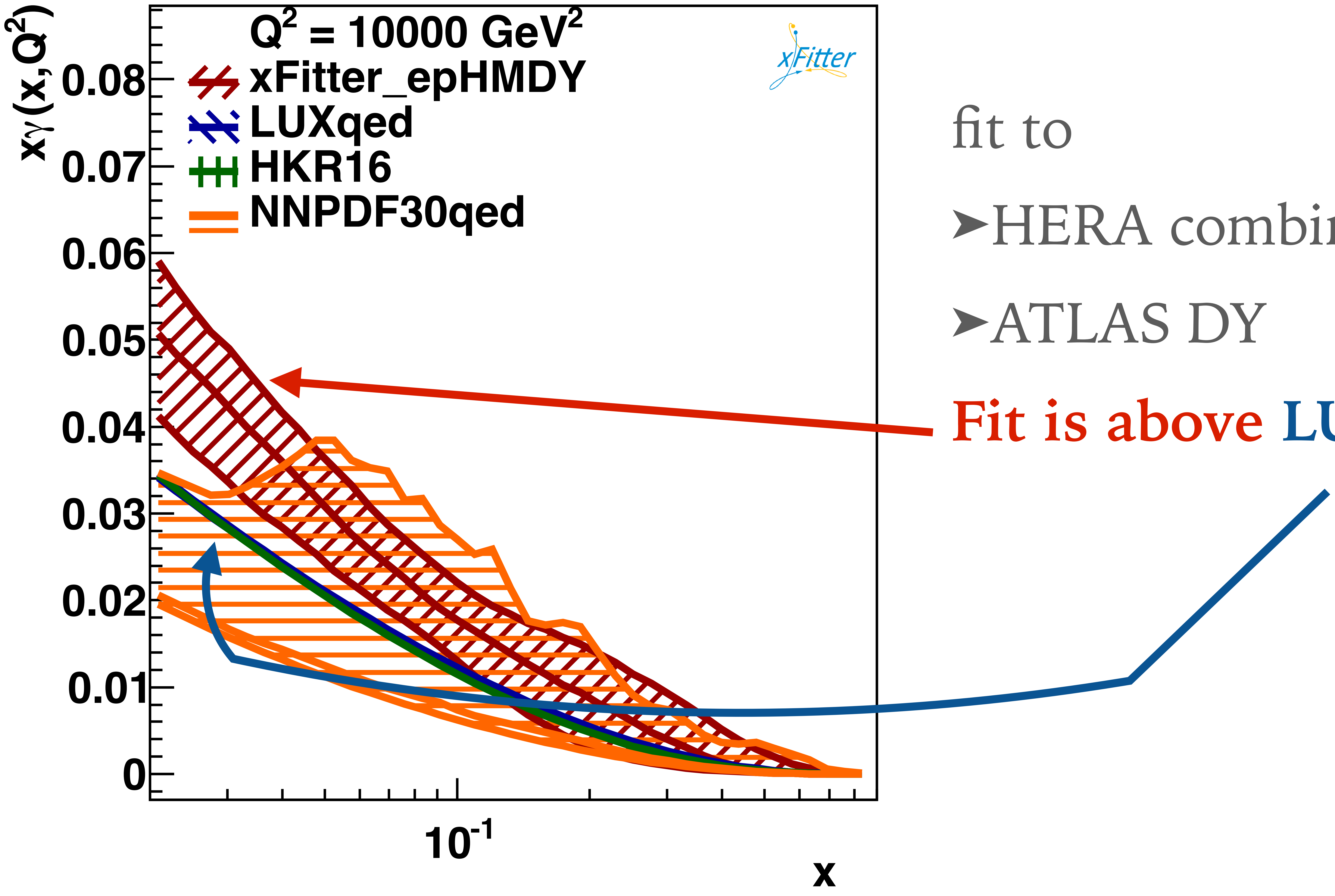
ratio of ATLAS photon (1606.01736) to LUXqed



ATLAS result based on reweighting of NNPDF23 with high-mass (Mll > 116 GeV) data.

Fit is below LUXqed

later fit (1701.08553) to same data



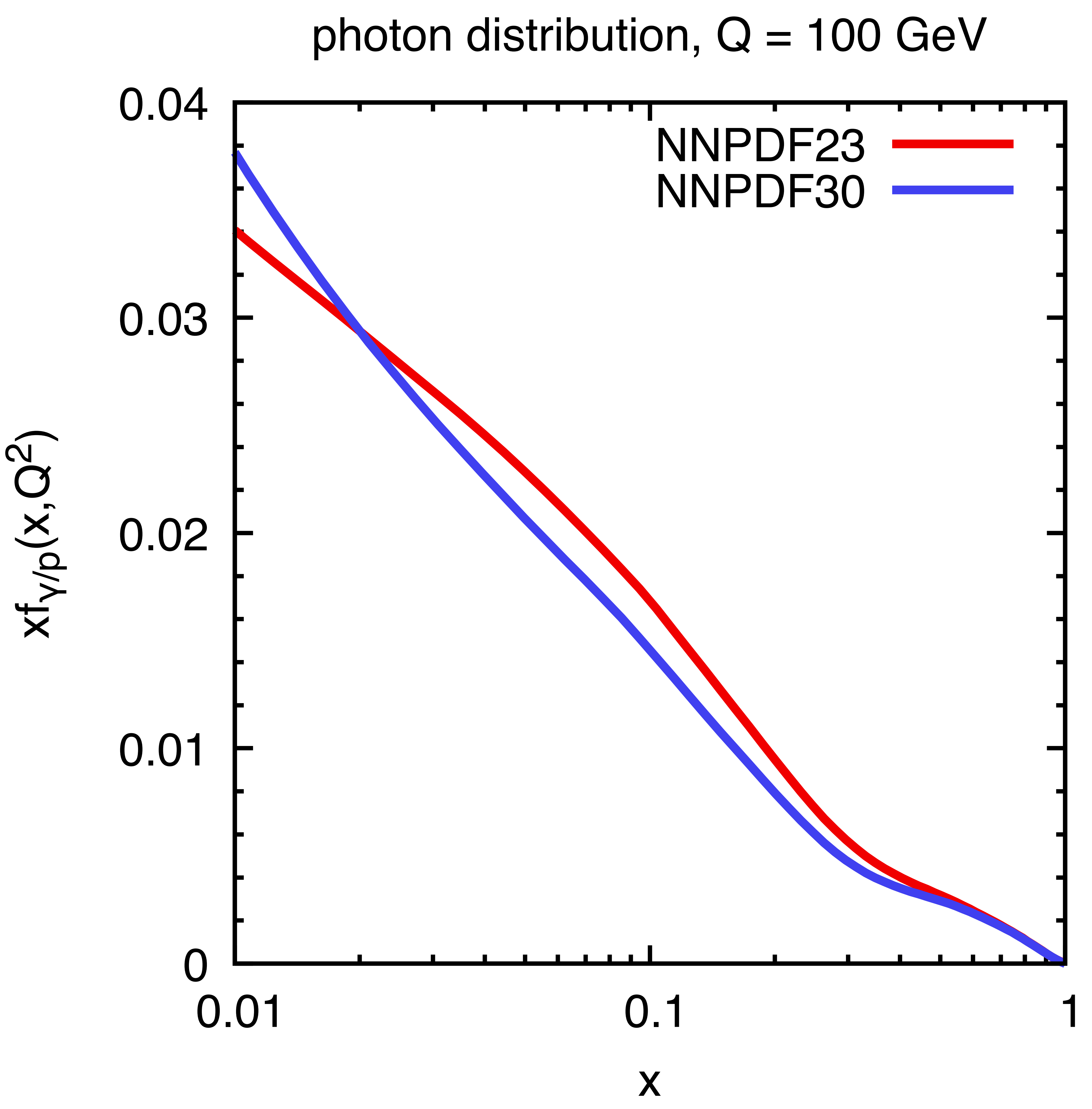
fit to

➤HERA combined data

➤ATLAS DY

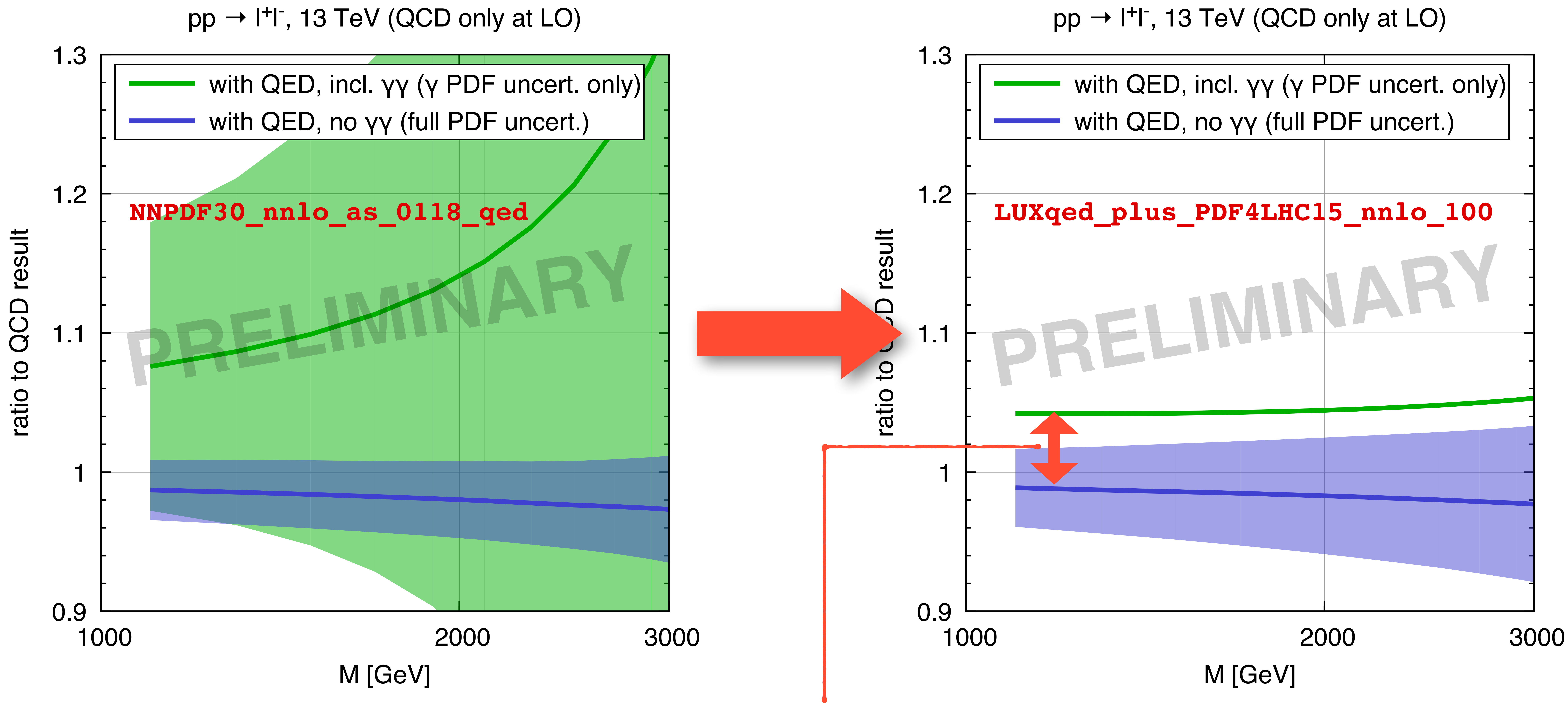
Fit is above LUXqed

explanation does not lie with NNPDF23 v. 30 evolution differences



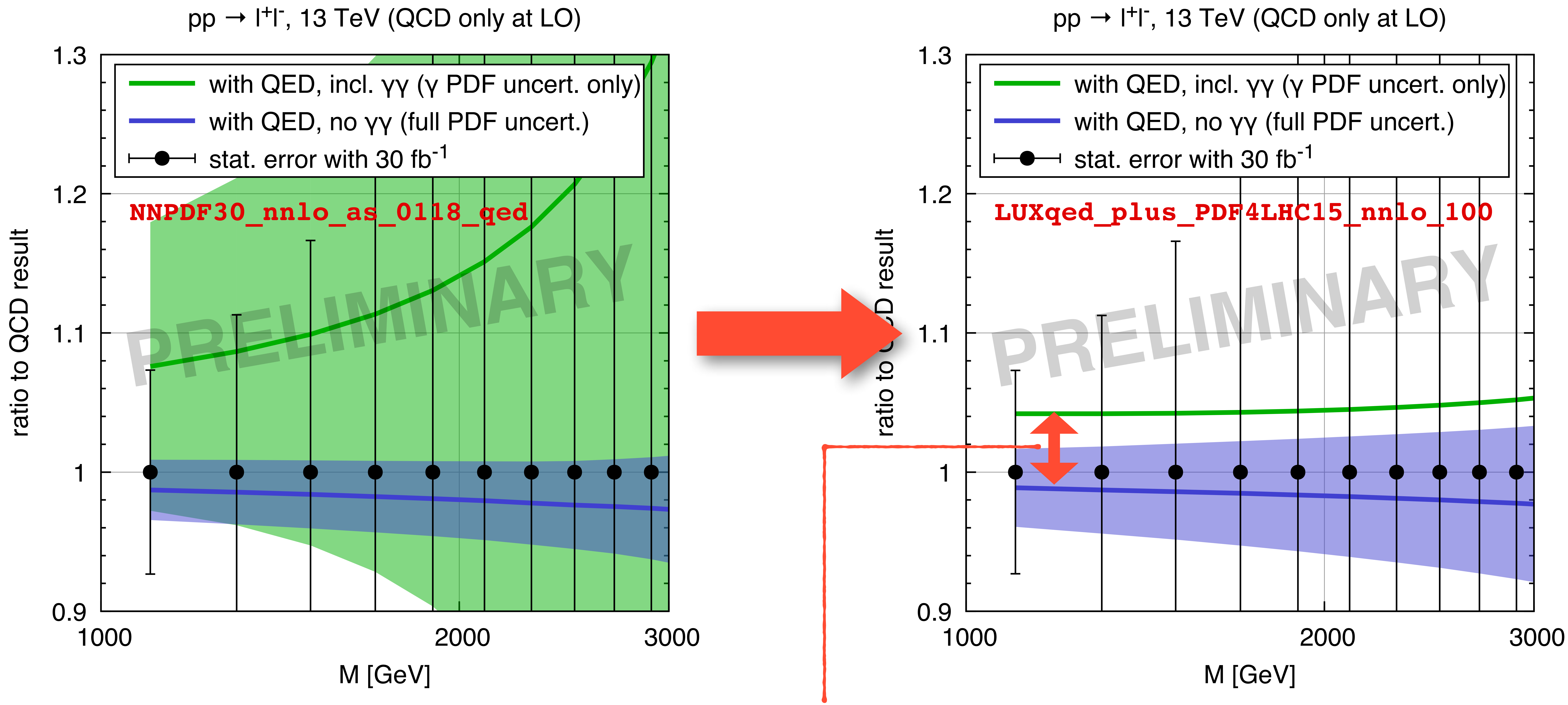
extra dilepton plots

di-lepton spectrum



γγ component has few-% effect on Drell-Yan spectrum; negligible uncertainty

di-lepton spectrum



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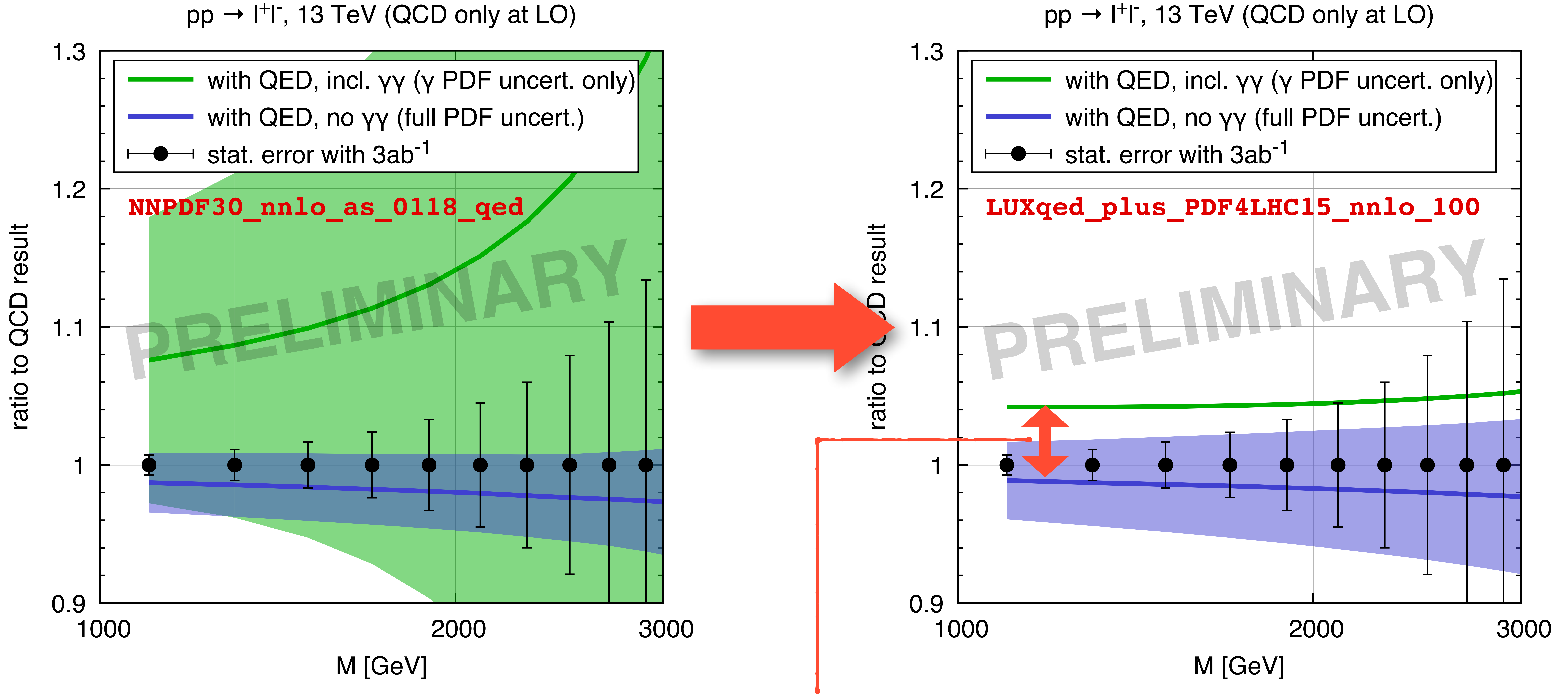

γγ component has few-% effect on Drell-Yan spectrum; negligible uncertainty