Measuring a_μ^{HLO} in the space-like region

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(Work done in collaboration with G. Abbiendi, C.M.Carloni Calame, U. Marconi, C. Matteuzzi, G. Montagna, O. Nicrosini, M. Passera, F. Piccinini, R. Tenchini, L. Trentadue)

Seminar at CPT, Marseille, 7 November 2016

Outline

- α_{em} running and the Vacuum Polarization
- The muon anomaly a_μ =(g-2)/2
- Current status of a_μ^{HLO} (dispersive approach)
- New proposal to compute a_μ^{HLO} in the spacelike region:
  - Using Bhabha at low energy e+e- machines (VEPP2000/DAFNE, τ/charm, B-factories)
  - Using a high energy muon beam on e- target at CERN
- Conclusion

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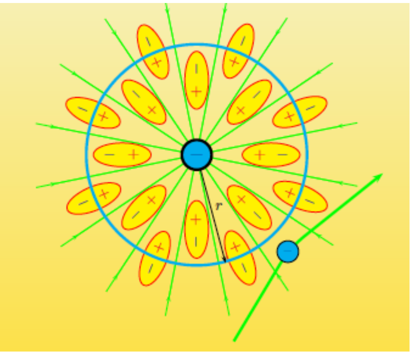
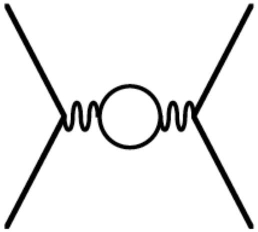
Based on:

1) *C.M. Carloni Calame, M. Passera, L. Trentadue, G.V.*, ""Measuring the leading order hadronic contribution to the muon g-2 in the space-like region", Phys.Lett. B746 (2015) 325-32
2) *G. Abbiendi, C.M.Carloni Calame, U. Marconi, C. Matteuzzi, G. Montagna, O. Nicrosini, M. Passera, F. Piccinini, R. Tenchini, L. Trentadue, G.V.*, "Measuring the leading hadronic contribution to the muon g-2 via μe scattering", arXiv: 1609.08987, submitted to EPJC

α_{em} running and the Vacuum Polarization



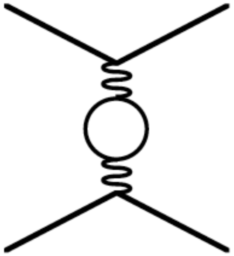
- Due to Vacuum Polarization effects $\alpha_{em}(q^2)$ is a running parameter from its value at vanishing momentum transfer to the effective q^2.
- The "Vacuum Polarization" function $\Pi(q^2)$ can be "absorbed" in a redefinition of an effective charge:



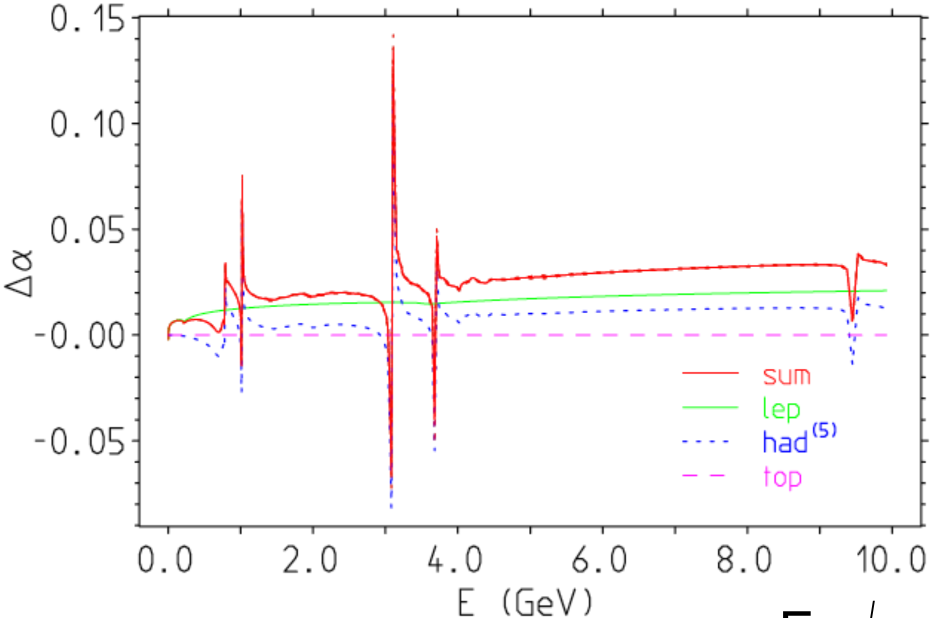
$$e^2 \to e^2(q^2) = \frac{e^2}{1+(\Pi(q^2)-\Pi(0))} \qquad \alpha(q^2) = \frac{\alpha(0)}{1-\Delta\alpha}; \quad \Delta\alpha = -\Re e\left(\Pi(q^2)-\Pi(0)\right)$$

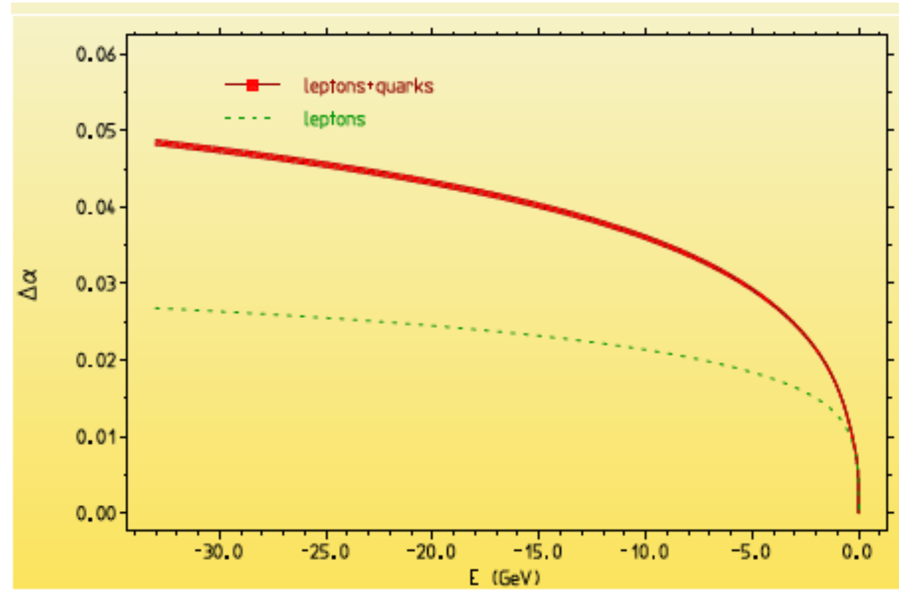
$$\Delta\alpha = \Delta\alpha_l + \Delta\alpha^{(5)}_{had} + \Delta\alpha_{top}$$

- Δa takes a contribution by non perturbative hadronic effects ($\Delta a^{(5)}_{had}$) which exibits a different behaviour in time-like and space-like region



Running of α_{em}

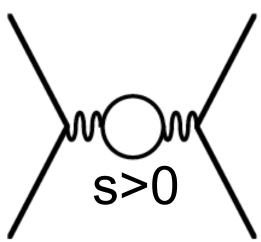




E=√s

Time-like

Behaviour characterized by the opening of resonances

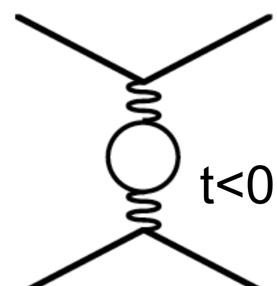


$$\Delta\alpha_{had}^{(5)}(M_Z^2) = -\frac{\alpha M_Z^2}{3\pi}\mathrm{Re}\int_{4m_\pi^2}^{\infty} ds \frac{R(s)}{s(s-M_Z^2-i\varepsilon)}$$

E=-√-t

Space-like

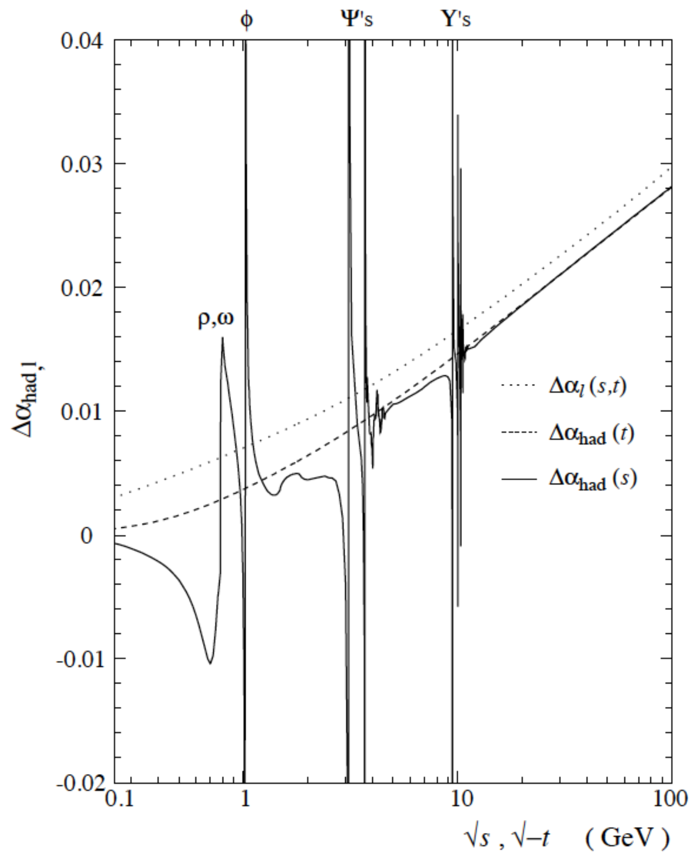
Very smooth behaviour



$$\Delta\alpha_{had}^{(5)}(-q_0^2) = -\frac{\alpha M_Z^2}{3\pi}\mathrm{Re}\int_{4m_\pi^2}^{\infty} ds \frac{R(s)}{s(s+q_0^2)}$$

Measurement of α_{em} running

- Measurements of $\alpha_{em}(q^2)$ in space/time like region can prove the running of α_{em}
- It can provide a test of "duality" (fare way from resonances)
- It has been done in past by few experiments at e^+e^- colliders by comparing a "well-known" QED process with some reference (obtained from data or MC)

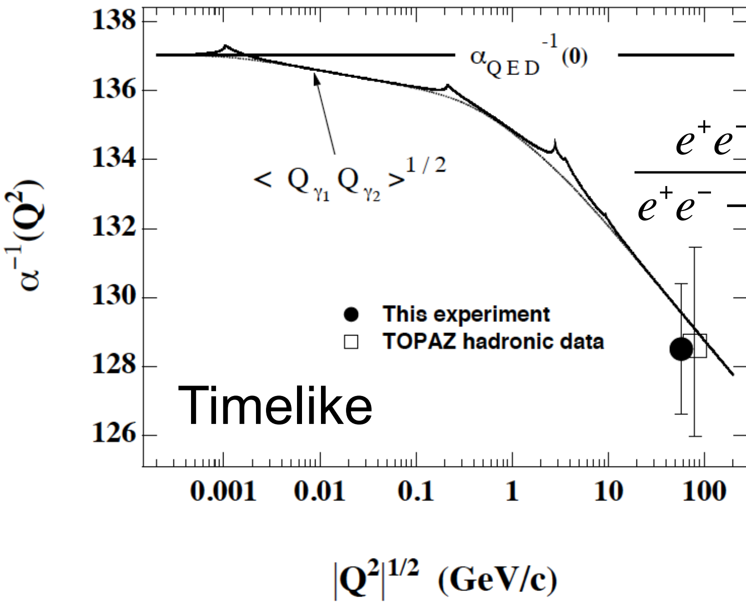


$$\left(\frac{\alpha(q^2)}{\alpha(q_0^2)}\right)^2 \sim \frac{N_{signal}(q^2)}{N_{norm}(q_0^2)}$$

N_{signal} can be Bhabha process, muon pairs, etc…
N_{signal} can be Bhabha process, $\gamma\gamma$ pairs, Theory, etc…

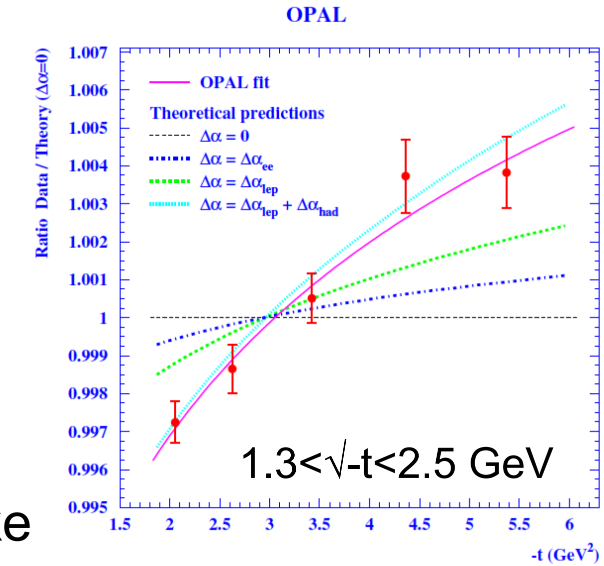
Direct measurement of α_{em} running

e+e- collider TRISTAN at √s=57.8 GeV,

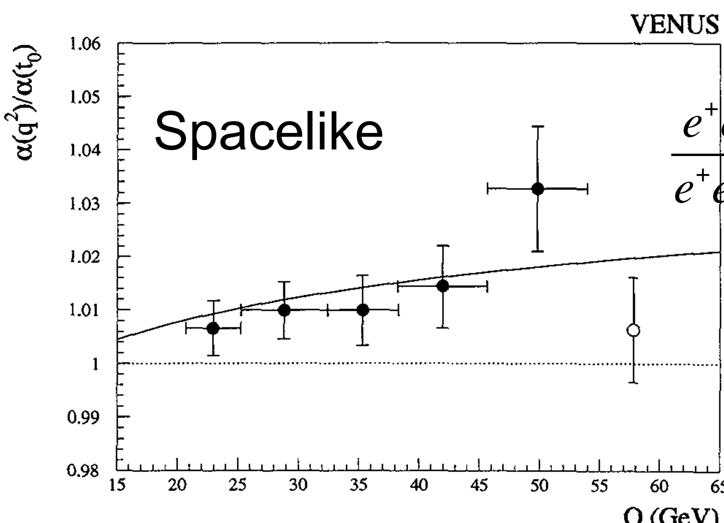


$$\frac{e^+e^- \to \mu^+\mu^-}{e^+e^- \to e^+e^-\mu^+\mu^-}$$

e+e- collider LEP at √s=189 GeV, using **Bhabha** events

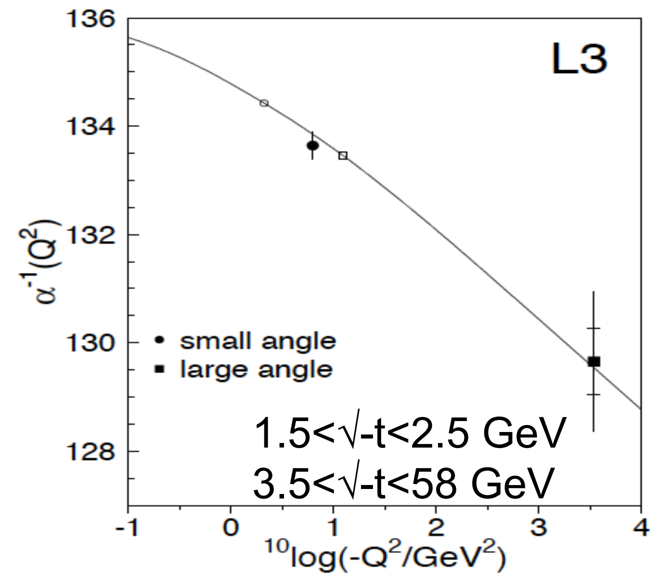


Spacelike



10<√-t<54 GeV

$$\frac{e^+e^- \to e^+e^-}{e^+e^- \to \mu^+\mu^-}$$



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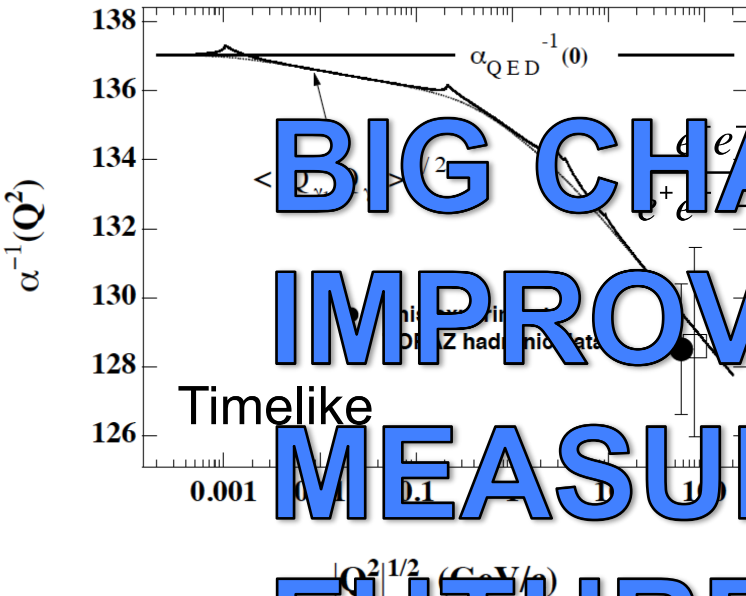

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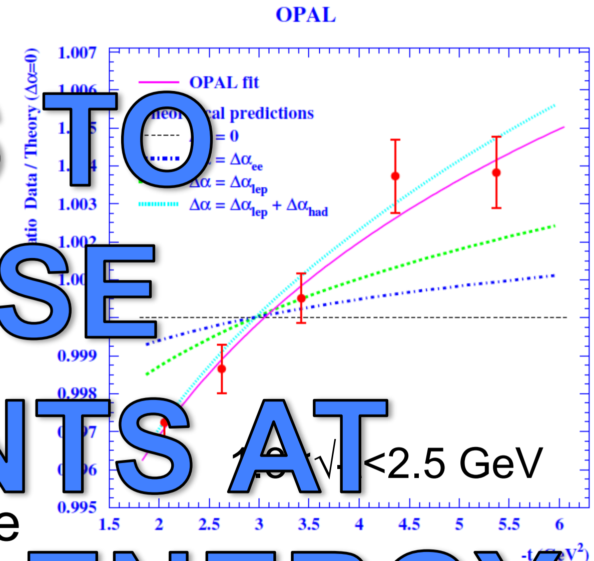
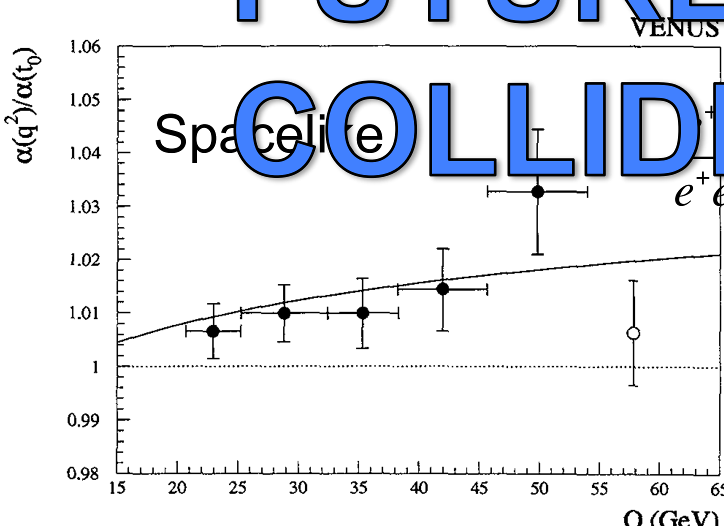


Spacelike



Spacelike

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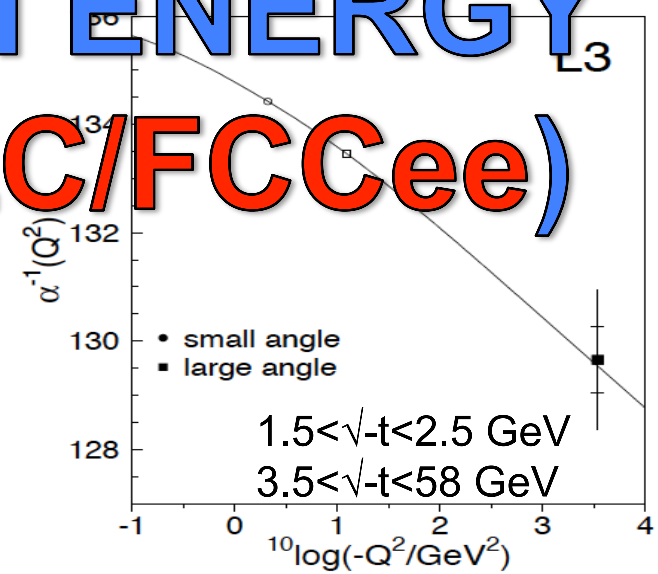


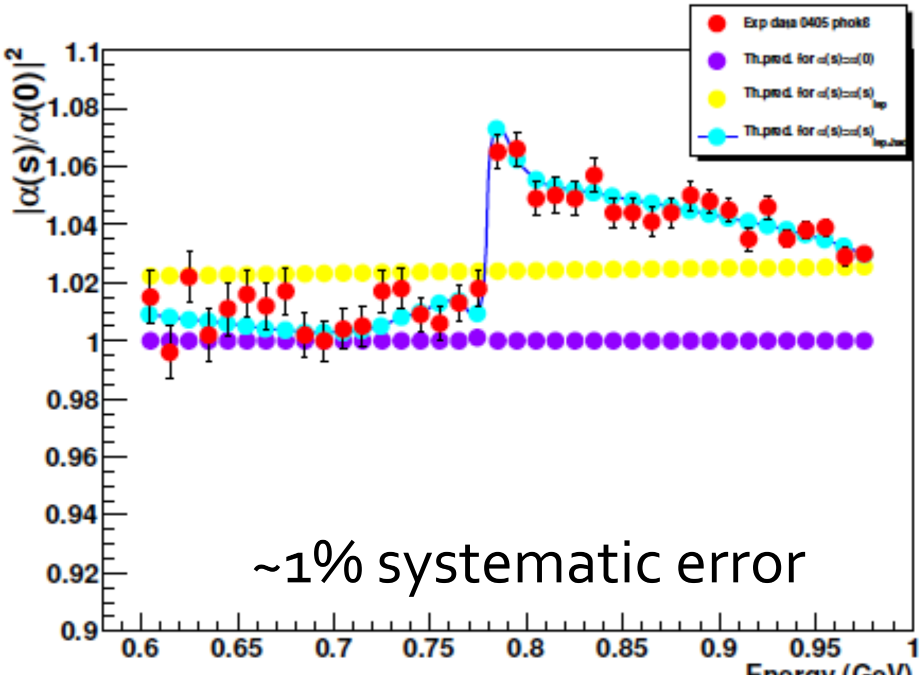
1.5<√-t<2.5 GeV
3.5<√-t<58 GeV

BIG CHANCES TO IMPROVE THESE MEASUREMENTS AT FUTURE HIGH ENERGY COLLIDER (ILC/FCCee)

arXiv:**1609.06631**: submitted to PLB

A new KLOE measurement (from $\mu\mu\gamma$ with 1.7 fb^{-1})!

Measurement of the effective α_{QED} coupling constant between 600 and 980 MeV



$$\left|\frac{\alpha_{QED}(s)}{\alpha_{QED}(0)}\right|^2 = \frac{\frac{d\sigma^{ISR}}{dM_{\mu\mu}}}{\frac{d\sigma^{MC}}{dM_{\mu\mu}}}$$

$\frac{d\sigma^{MC}}{dM_{\mu\mu}}$ with the VP contribution removed.

$$\left|\frac{\alpha(s)}{\alpha(0)}\right|^2 = 1/(1-\Delta\alpha(s))$$

$$\Delta\alpha(s) = \Delta\alpha_{lep} + \Delta\alpha_{had}$$

(we neglect the top contribution)

Good agreement with data based compilation (F. Jegerlehner)
>5σ evidence of hadronic contribution ρ,ω to $\alpha(s)$

$$\Delta\alpha_{had}(s) = -\left(\frac{\alpha(0)s}{3\pi}\right) Re \int_{m_\pi^2}^{\infty} ds' \frac{R(s')}{s'(s'-s-i\epsilon)} \qquad R(s) = \frac{\sigma_{tot}(e^+e^-\to\gamma*\to hadrons)}{\sigma_{tot}(e^+e^-\to\gamma*\to\mu^+\mu^-)}$$

Muon g-2: summary of the present status

- E821 experiment at BNL has generated enormous interest:

$$a_\mu^{E821} = 11659208.9(6.3)\times 10^{-10} \quad \text{(0.54 ppm)}$$

- Tantalizing ~3σ deviation with SM (persistent since >10 years):

$$a_\mu^{SM} = 11659180.2(4.9)\times 10^{-10}\,(DHMZ)$$

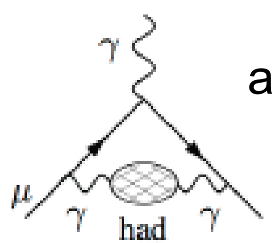
M. Davier, A. Hoecker, B. Malaescu and Z. Zhang, Eur. Phys. J. C71 (2011)

$$a_\mu^{E821} - a_\mu^{SM} \sim (28\pm 8)\times 10^{-10}$$

- Current discrepancy limited by:
  - Experimental uncertainty → New experiments at FNAL and J-PARC **x4** accuracy
  - Theoretical uncertanty → limited by hadronic effects

$$a_\mu^{SM} = a_\mu^{QED} + \boxed{a_\mu^{HAD}} + a_\mu^{Weak}$$

Hadronic Vacuum polarization (HLO)



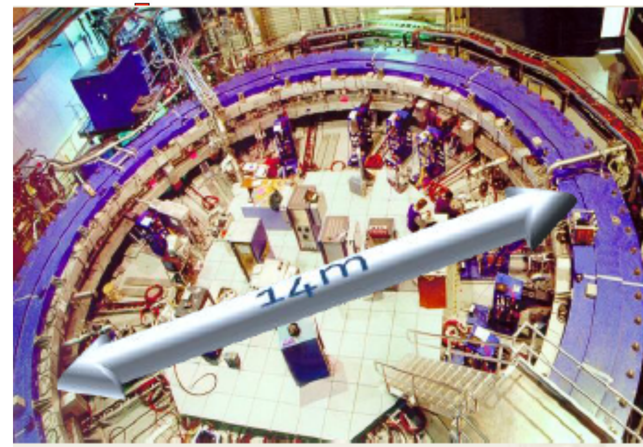
$a_\mu^{HLO} = (692.3\pm 4.2)10^{-10}$

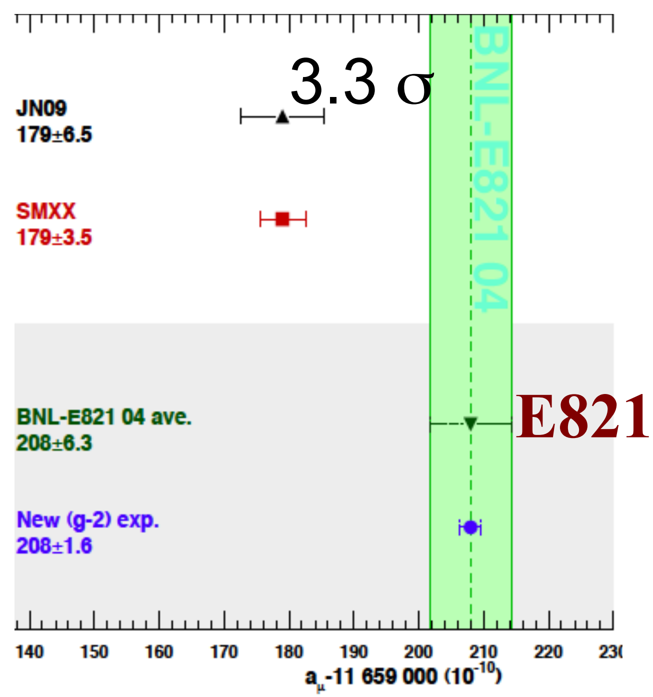
$\delta a_\mu / a_\mu \sim 0.6\%$

$(g-2)_\mu$: a new experiment at FNAL (E989)

- New experiment at FNAL (E989) at magic momentum, consolidated method. **20 x stat.** w.r.t. E821. Relocate the BNL storage ring to FNAL.

→ δa_μx4 improvement (0.14ppm)





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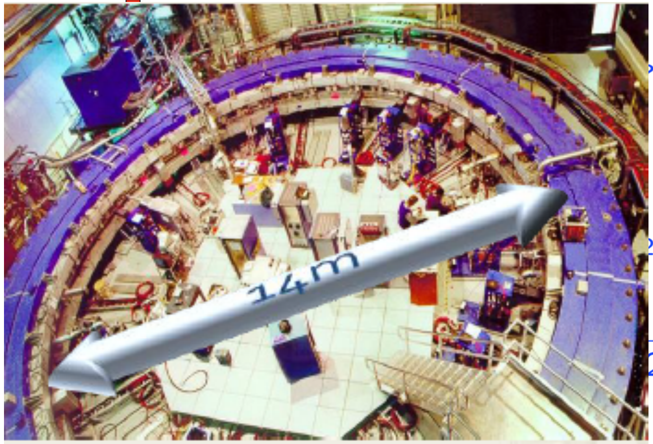
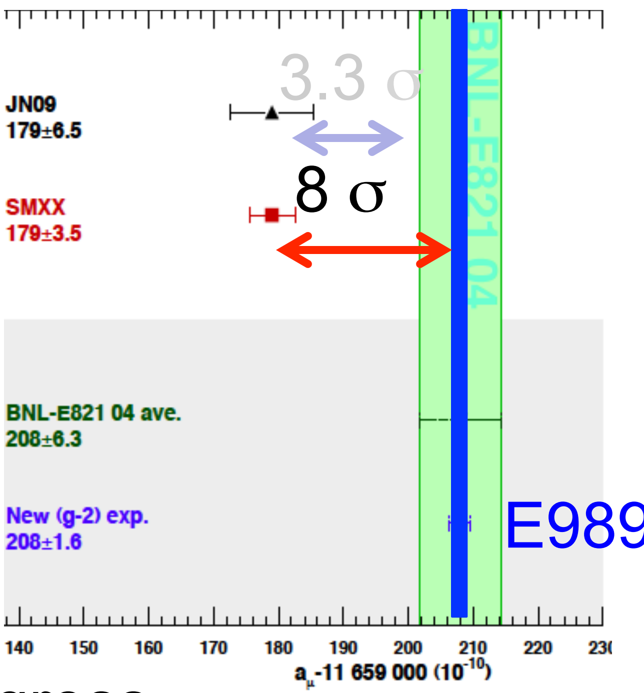


→ δa_μx4 improvement (0.14ppm)

If the central value remains the same ⇒ 5-8σ from SM* (enough to claim discovery of New Physics!)



***Depending on the progress on Theory**

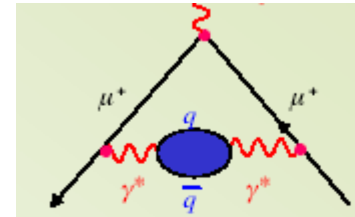
Thomas Blum; Achim Denig; Ivan Logashenko; Eduardo de Rafael; Lee oberts, B.; Thomas Teubner; Graziano Venanzoni (2013). "The Muon (g-2) heory Value: Present and Future". arXiv:1311.2198 [hep-ph].

Complementary proposal at J-PARC in progress

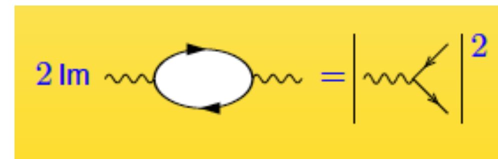
a_μ^{HLO} calculation, traditional way: time-like data

C. Bouchiat, L. Michel, Bouchiat, 1961;
M. Gourdin, E. de Rafael, 1969

$a_\mu = (g-2)/2$



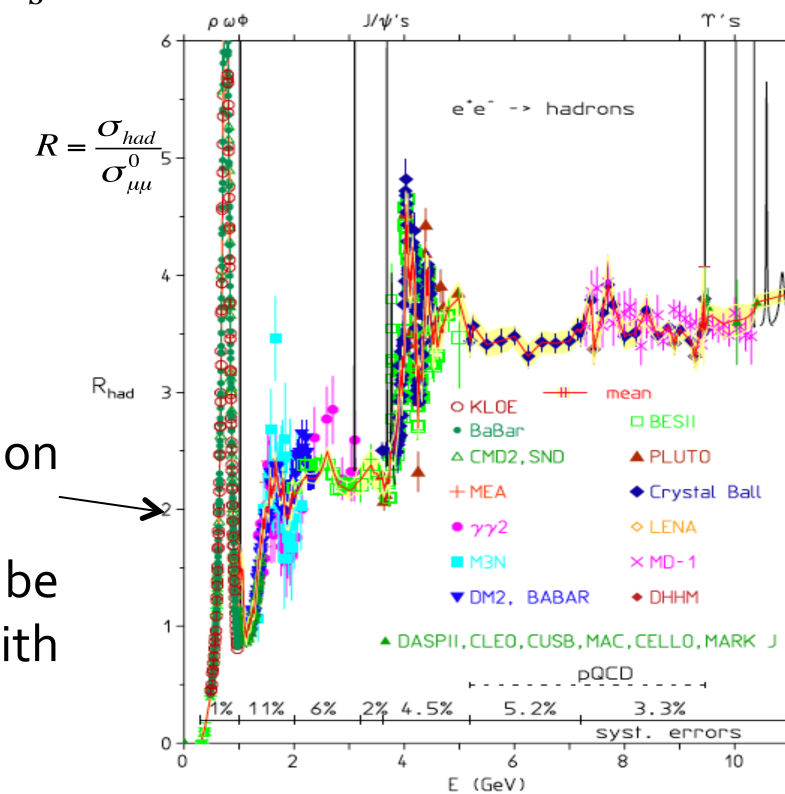
$$a_\mu^{HLO} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} \sigma_{e^+e^- \to hadr}(s) K(s)\, ds$$



$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)(s/m^2)} \sim \frac{1}{s} \qquad \sigma_{e^+e^- \to hadr}(s) = \frac{4\pi}{s} \mathrm{Im}\,\Pi_{had}(s)$$

Traditional way: based on precise experimental (time-like) data:

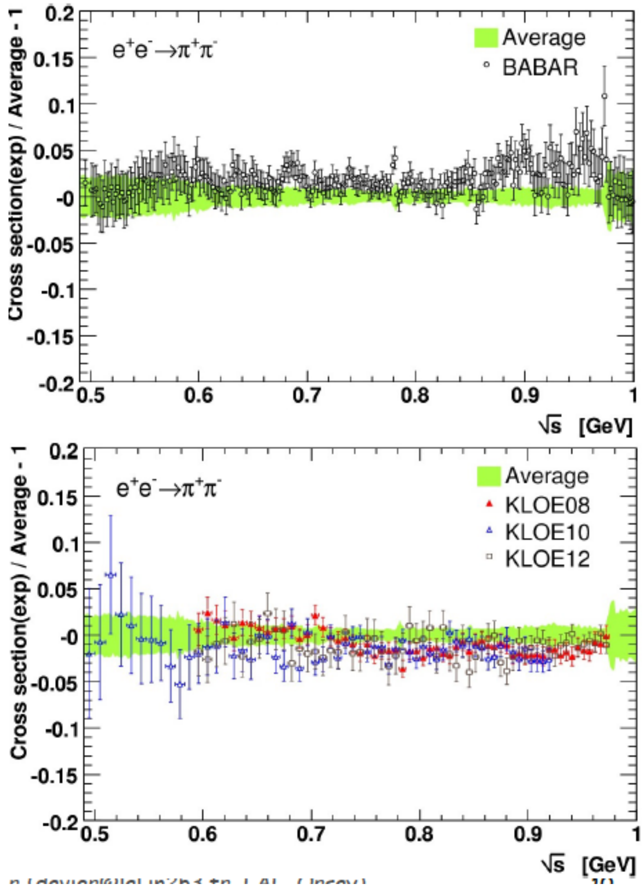
$a_\mu^{HLO} = (692.3 \pm 4.2) 10^{-10}$ (DHMZ)



- Main contribution in the low energy region (**highly fluctuating!**)
- Current precision at 0.6% → needs to be reduced by a factor **~2** to be competitive with the new **g-2** experiments

Timelike data aiming at 0.2% on a_μ^{HLO}?

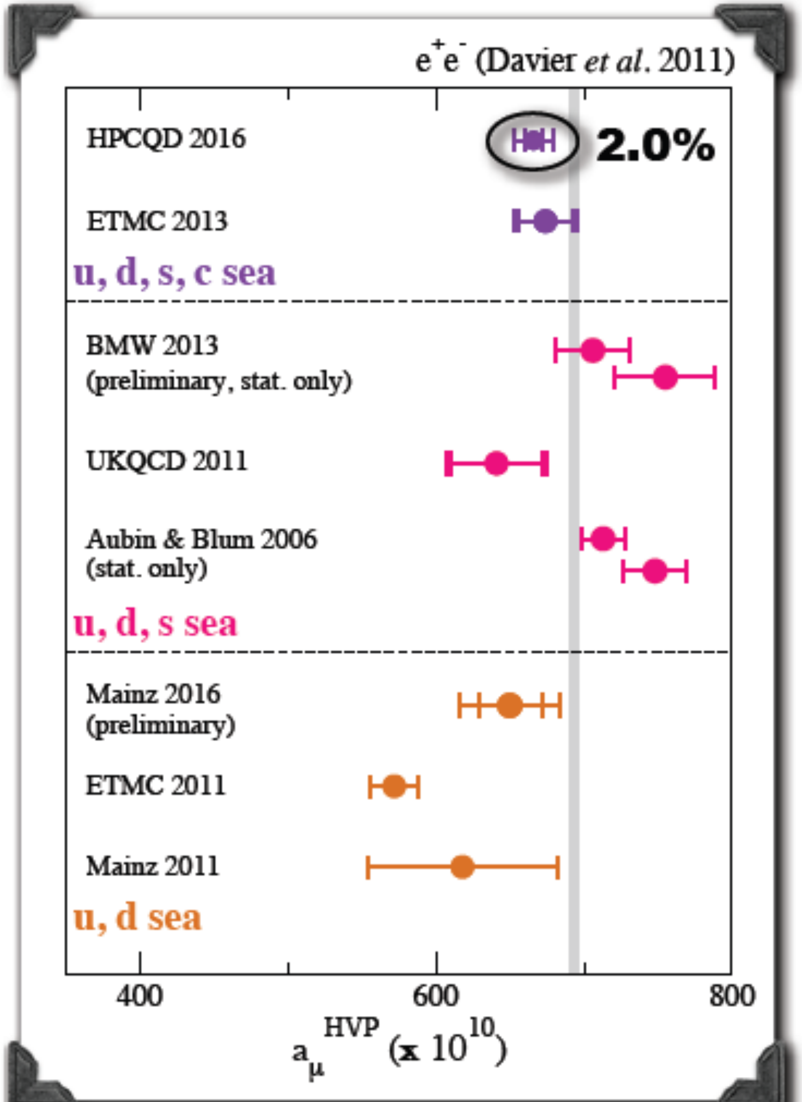
- Not an easy task!
  - >30 channels to keep under control (at (sub)percent level)
  - local discrepancies in main channels (2π (KLOE/Babar), K^+K^- CMD2/Babar)
  - Isospin corrections for not measured channels
  - Treatment of narrow resonances? (See F. Jegerlehner, ArXiv:1511.04473)



M. Davier, TAU16 WS

An independent/complementary approach is highly desirable!

Lattice-QCD progress on a_μ^{HVP}



- Can calculate nonperturbative vacuum polarization function $\Pi(Q^2)$ directly in lattice QCD from simple 2-point correlation function of EM quark current [Blum, PRL 91 (2003) 052001]
- Several ongoing lattice efforts yielding new results since ICHEP 2014 including:

(1) First calculation of quark-disconnected contribution [RBC/UKQCD, PRL116, 232002 (2016)]

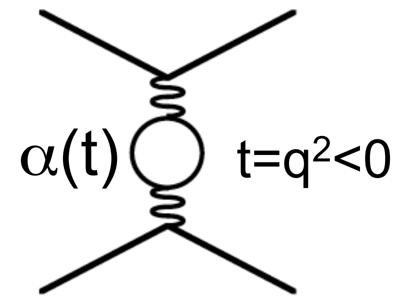
(2) Second complete calculation of leading-order a_μ^{HVP} [HPQCD, arXiv:1601.03071]

- First to reach precision needed to observe significant deviation from experiment

- ~1% total uncertainty by 2018 possible
- Sub-percent precision will require inclusion of isospin breaking & QED, and hence take longer

Alternative approach: a_μ^{HLO} from space-like region

[*C.M. Carloni Calame, M. Passera, L. Trentadue, G. Venanzoni* Phys.Lett. B746 (2015) 325-32]

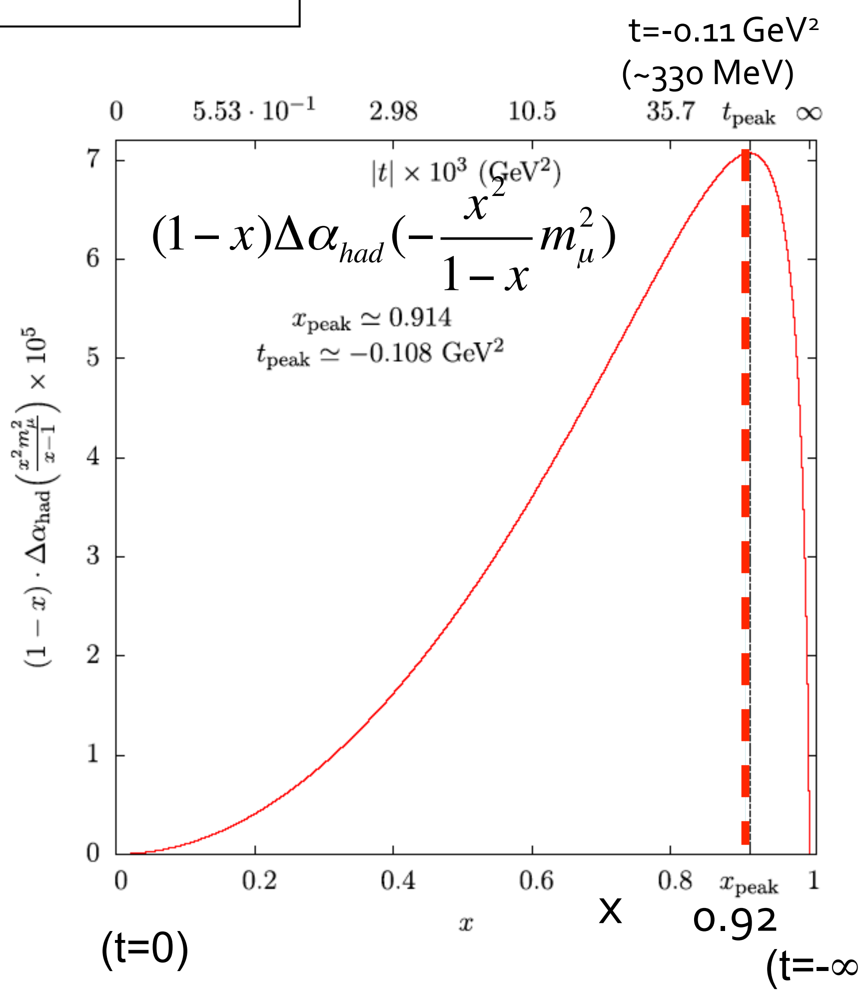


$$a_\mu^{HLO} = -\frac{\alpha}{\pi}\int_0^1 (1-x)\Delta\alpha_{had}\left(-\frac{x^2}{1-x}m_\mu^2\right)dx$$

$$t = \frac{x^2 m_\mu^2}{x-1} \qquad 0 \le -t < +\infty$$

$$x = \frac{t}{2m_\mu^2}\left(1-\sqrt{1-\frac{4m_\mu^2}{t}}\right); \qquad 0 \le x < 1;$$

- **a_μ^{HLO} is given by the integral of the curve** (smooth behaviour)
- **It requires a measurement of** the hadronic contribution to the effective electromagnetic coupling in the space-like region **$\Delta\alpha_{had}$(t) (t=q²<0)**
- **It enhances** the contribution from **low** q² region (below 0.11 GeV²)
- **Its precision** is determined by the **uncertainty** on $\Delta\alpha_{had}$ (t) in this region



Two experimental approaches:

1) Bhabha scattering at flavors factories
2) High energy muon beam on e- target

Bhabha scattering at flavour factories: experimental considerations

Using Bhabha at small angle (to emphasize t-channel contribution) to extract $\Delta\alpha$:

$$\left(\frac{\alpha(t)}{\alpha(0)}\right)^2 \sim \frac{d\sigma_{ee\to ee}(t)}{d\sigma^0_{MC}(t)}$$

Where $d\sigma^o_{MC}$ is the MC prediction for Bhabha process with α(t)=α(o), and there are corrections due to RC…

$$\Delta\alpha_{had}(t) = 1 - \left(\frac{\alpha(t)}{\alpha(0)}\right)^{-1} - \Delta\alpha_{lept}(t)$$

$\Delta\alpha_{lep}(t)$ theoretically well known!

Which experimental accuracy we are aiming at?
$\delta\Delta\alpha_{had}$~1/2 fractional accuracy on $d\sigma(t)/d\sigma^0_{MC}(t)$.

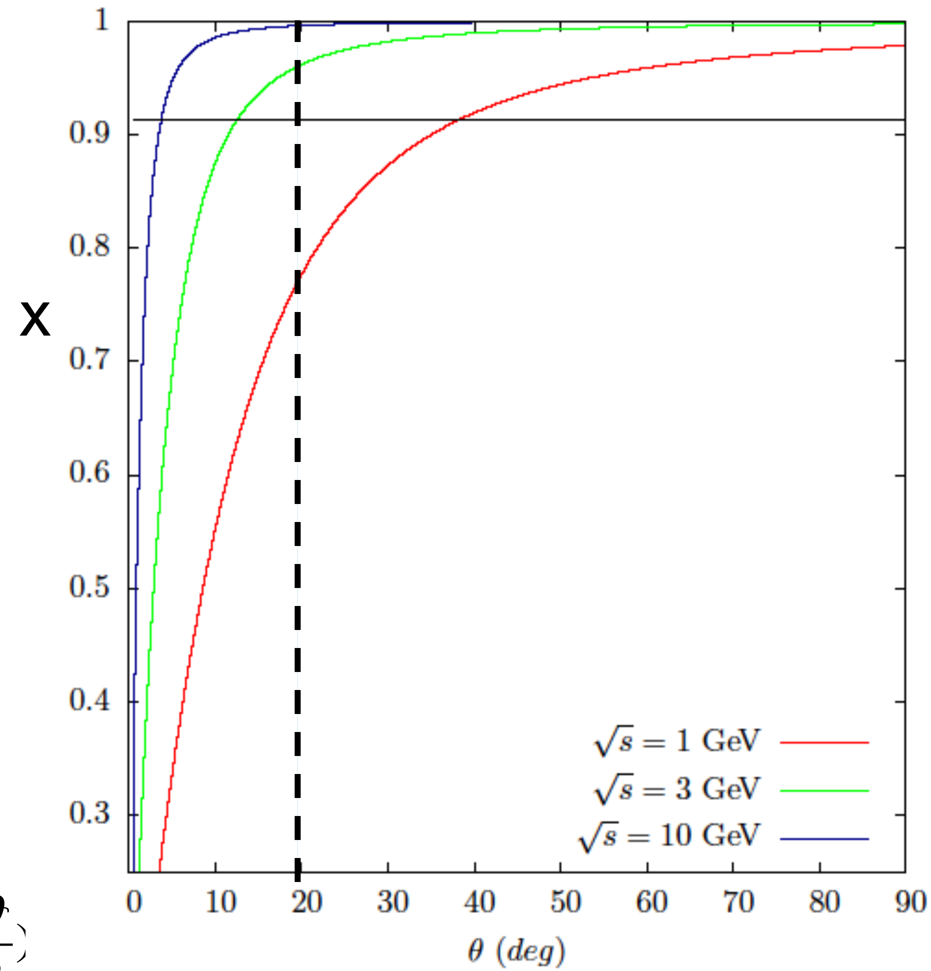
If we assume to measure $\delta\Delta\alpha_{had}$ at 5% at the peak of the integrand ($\Delta\alpha_{had}$ ~10^{-3} at x=0.92) → fractional accuracy on $d\sigma(t)/d\sigma^0_{MC}(t)$ ~ 10^{-4} !
Very challenging measurement (one order of magnitude improvement respect to date) for systematic error

Experimental considerations - II

Most of the region (up to x~0.98) can be covered with a low energy machine (like Dafne/VEPP-2000 or tau/charm-B-factories)

Example:
Covering up to 60° at $\sqrt{s}$=1 GeV can arrive at x= 0.95(!)

A different situation can be obtained at tau/charm/ B-factories (and at future ILC/ FCCee machines) where smaller angles (below 20°) are needed



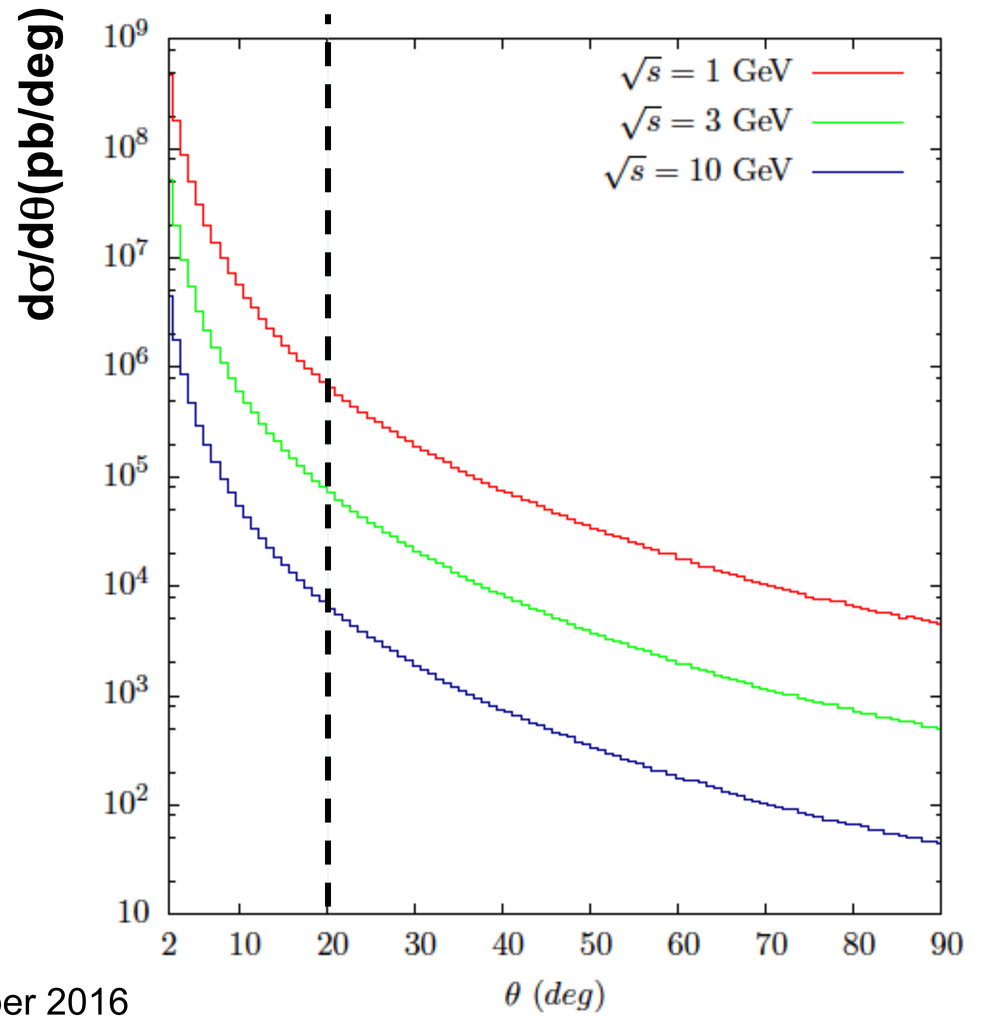
$$t = -s\sin^2(\frac{\vartheta}{2})$$

Statistical consideration

10^{-4} accuracy on Bhabha cross section requires at least 10^8 events which at 20^o mean at least:

- O(1) fb^{-1} @ 1 GeV
- O(10) fb^{-1} @ 3 GeV
- O(100) fb^{-1} @ 10 GeV

These luminosities are within reach at flavour factories!



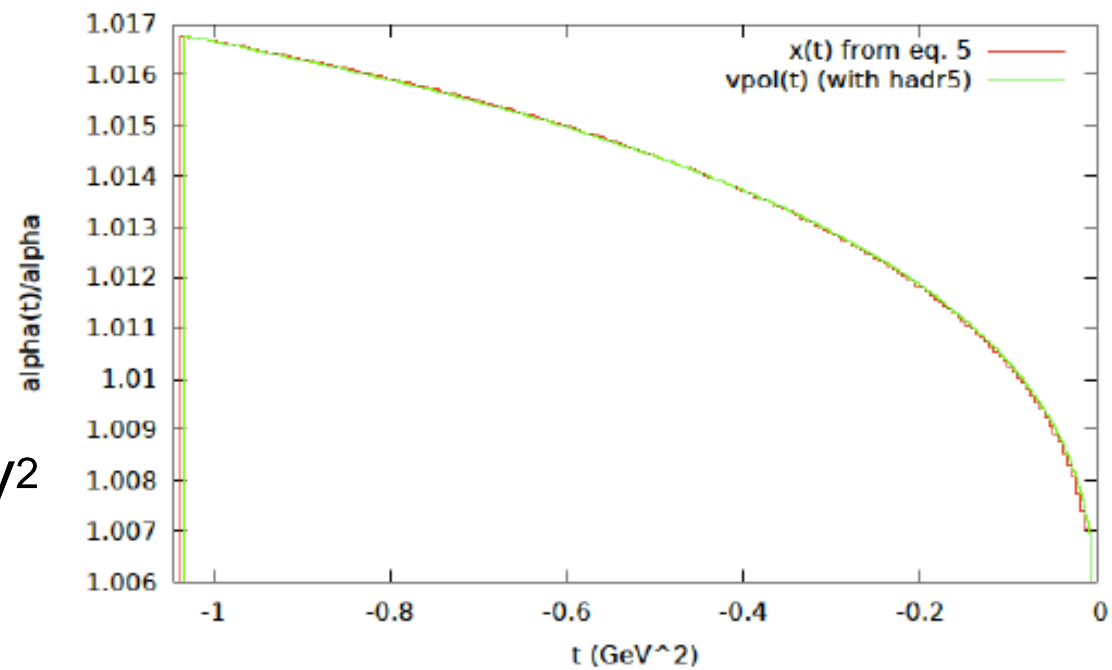
Additional considerations: s-channel

At low energy (<10 GeV) above 10° there is still a sizeable contribution from s-channel.
At LO no difficulty to deconvolute the cross section for the s-channel

Test with Babayaga:

$s=1$ GeV
$10^{o}<\theta<170^{o}$

$d\sigma_{born}/dt=1.52$ mb/GeV^2



However this picture changes with Rad. Corr.

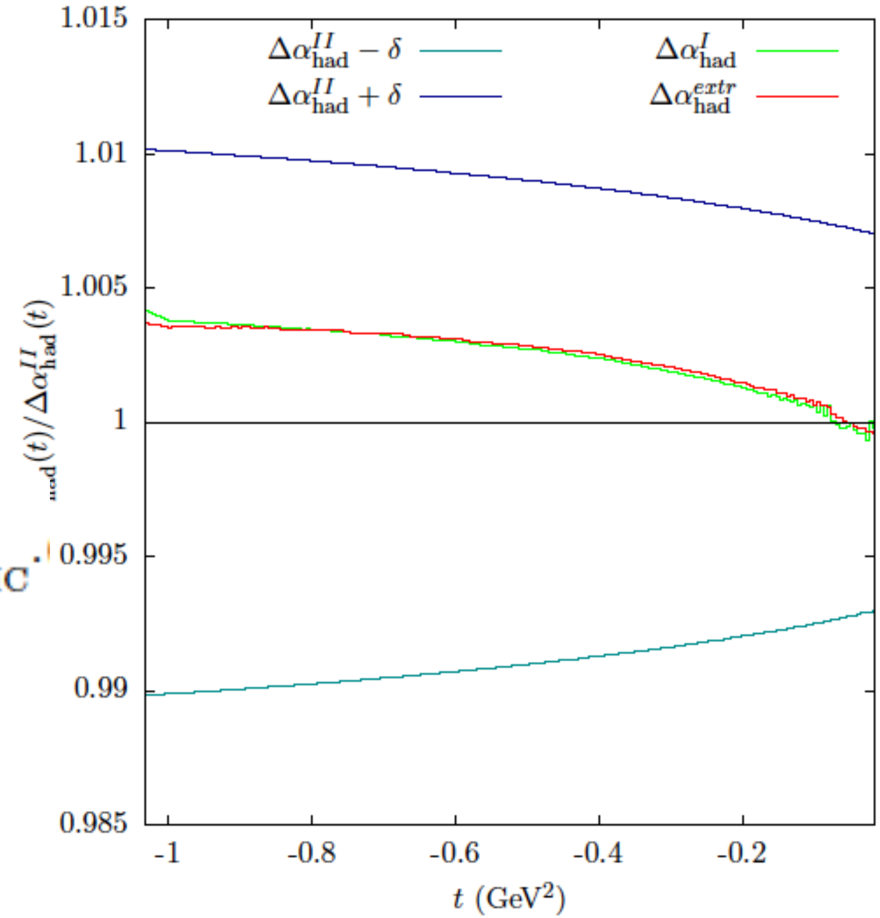
Additional considerations: Rad. Corr.

A Monte Carlo procedure has been developed to check if $\Delta\alpha_{had}(t)$ can be obtained by a minimization procedure with a different $\Delta\alpha_{had}(t)'$ inside

$$\left.\frac{d\sigma}{dt}\right|_{\rm data} = \frac{d\sigma}{dt}\Big(\alpha(t),\alpha(s)\Big)\Big|_{\rm MC},$$

→

$$\left.\frac{d\sigma}{dt}\right|_{j,\rm data} = \frac{d\sigma}{dt}\Big(\bar{\alpha}(t)+\frac{i_j}{N}\delta(t),\alpha(s)\Big)\Big|_{j,\rm MC}.$$



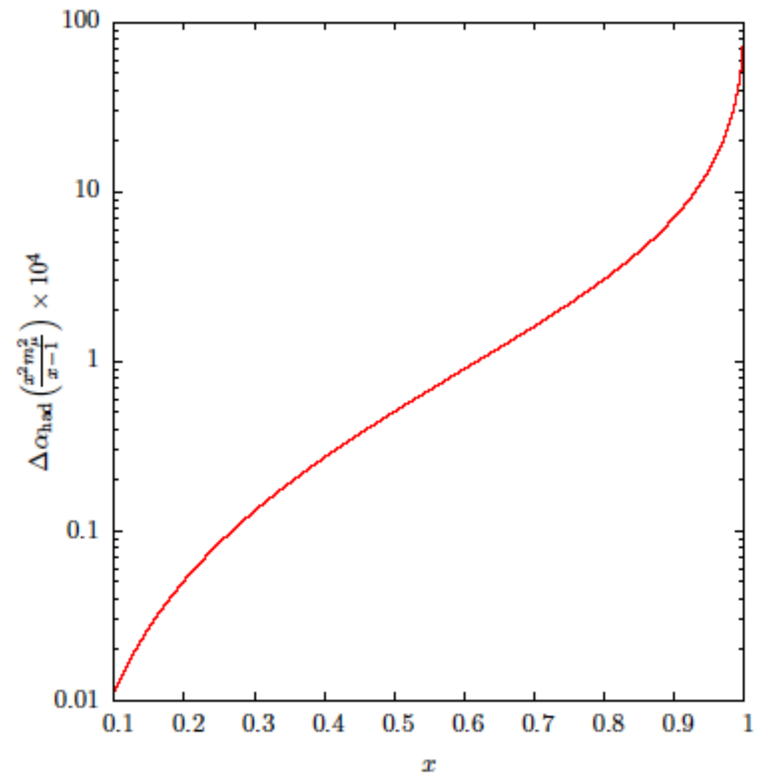
$\Delta\alpha_{had}(t)$ is obtained with $<10^{-4}$ error !

Additional consideration: Normalization

To compare Bhabha absolute cross section from data with MC we need Luminosity of the machine.

Two possibilities:

1) Use Bhabha at very small angle where the uncertainty on $\Delta\alpha_{had}$ can be neglected (for example at E_{beam}=1 GeV and θ=5°, $\Delta\alpha_{had}$ ~10^{-5}).
2) Use a process with $\Delta\alpha_{had}$=0, like $e^+e^- \rightarrow \gamma\gamma$. However very difficult to determine it at 10^{-4} accuracy.

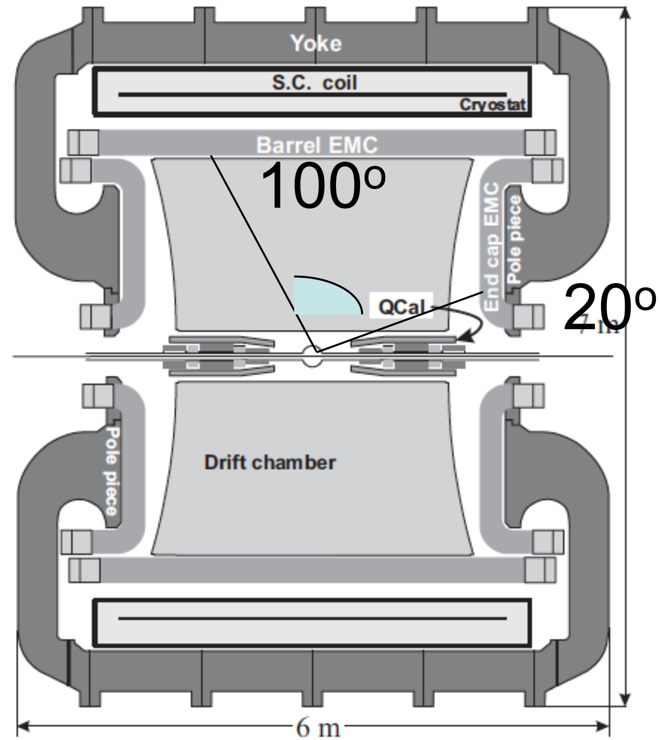


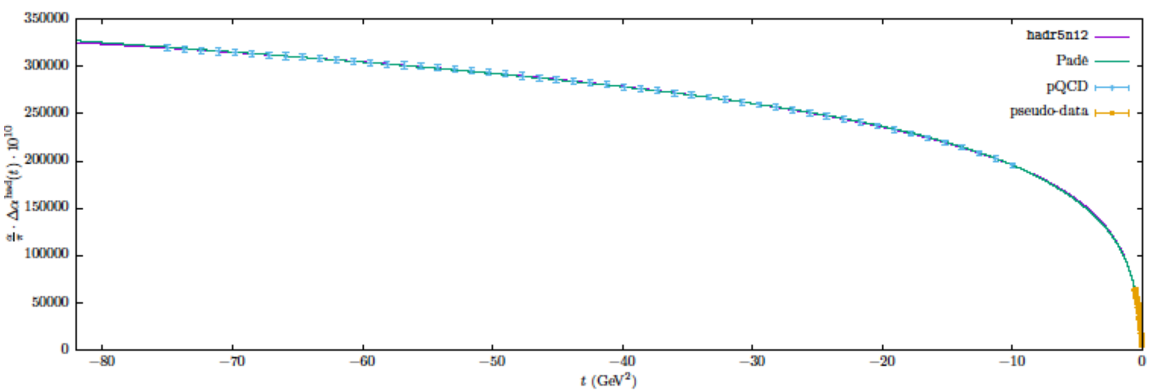
Option 1) looks better to us as some of the common systematics cancel in the measurement !

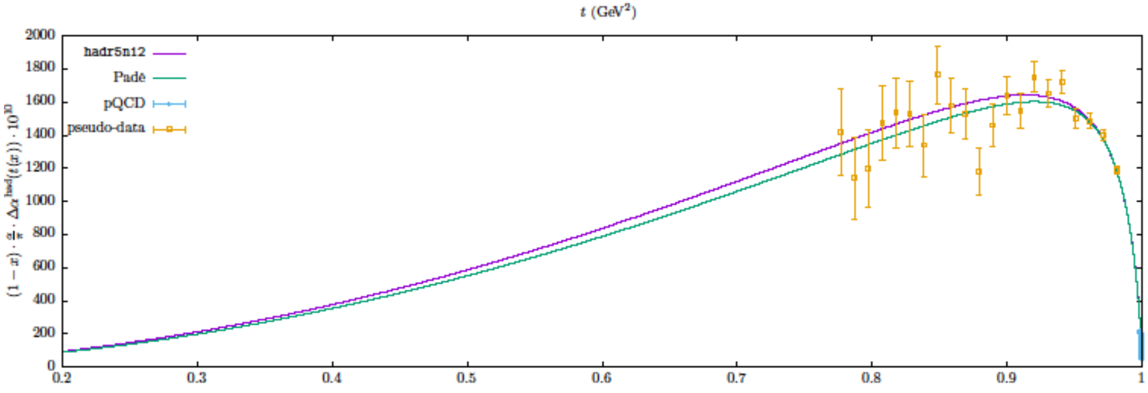
What can be done a KLOE/KLOE2?

We did the following simulation:

- 20 points between $20^o<\theta<100^o$ ($0.03<-t<0.59$ GeV^2; $0.78<x<0.98$) @ $\sqrt{s}=1$ GeV
- For each point $\delta\sigma_{e+e-}/\sigma_{e+e-} \sim 10^{-4}$ (stat and syst)
- We fit $\Delta\alpha_{had}(t)$ using our points+ pQCD for $-t>10$ GeV^2 with a polinomial function (like lattice)



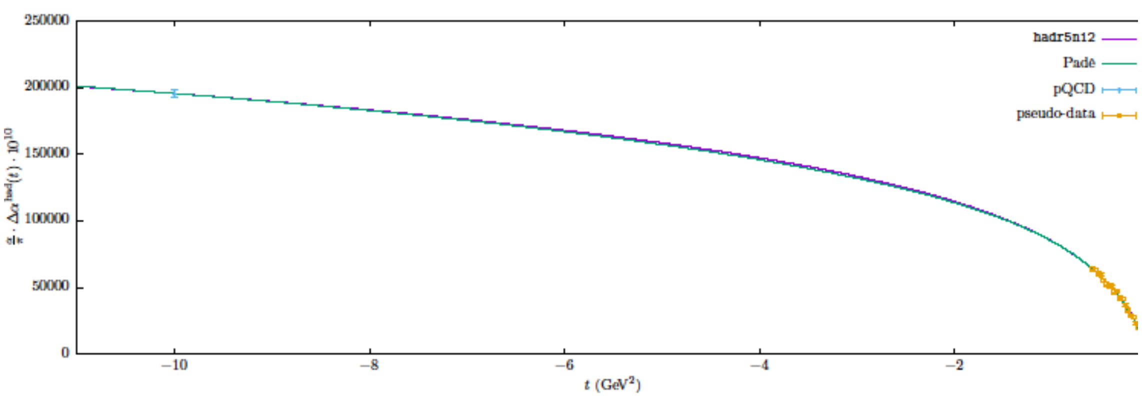


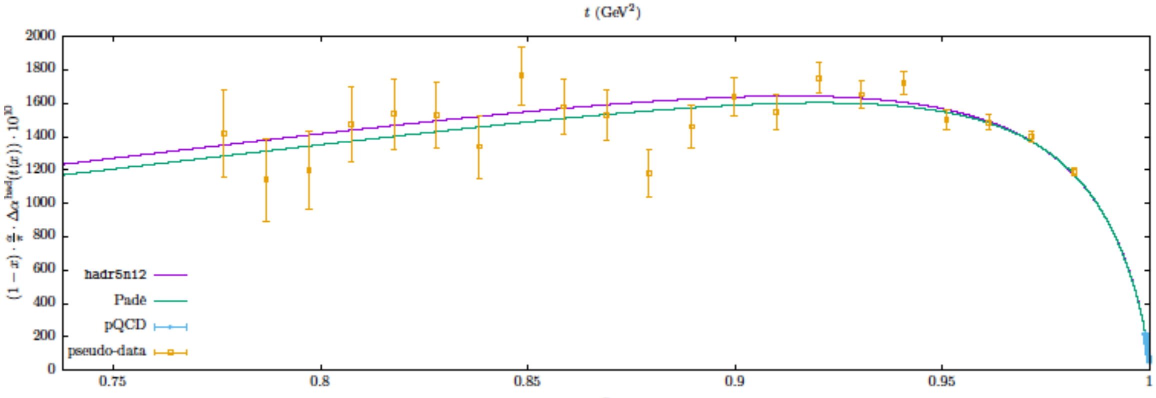


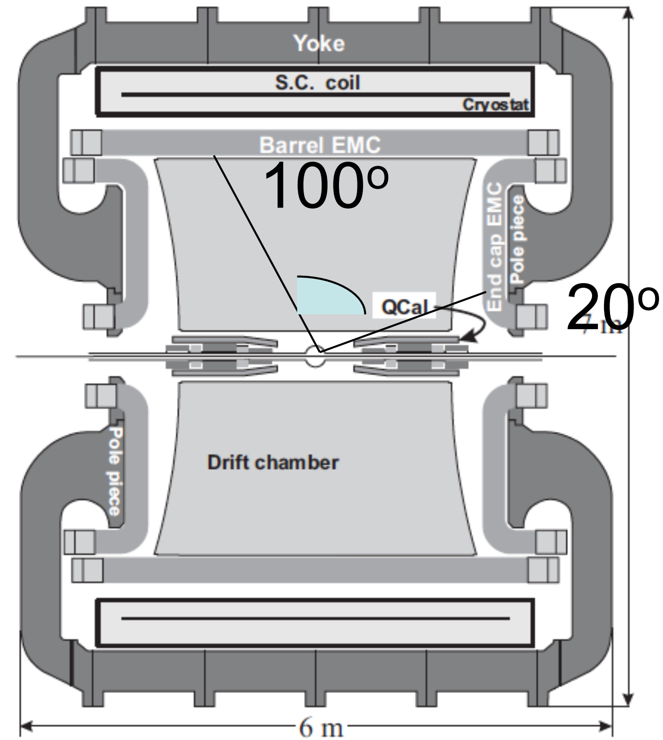
What can be done a KLOE/KLOE2?

We did the following simulation:

- 20 points between $20^o<\theta<100^o$ ($0.03<-t<0.59$ GeV^2; $0.78<x<0.98$) @ $\sqrt{s}=1$ GeV
- For each point $\delta\sigma_{e+e-}/\sigma_{e+e-}\sim10^{-4}$ (stat and syst)
- We fit $\Delta\alpha_{had}(t)$ using our points+ pQCD for $-t>10$ GeV^2 with a polinomial function (like lattice)





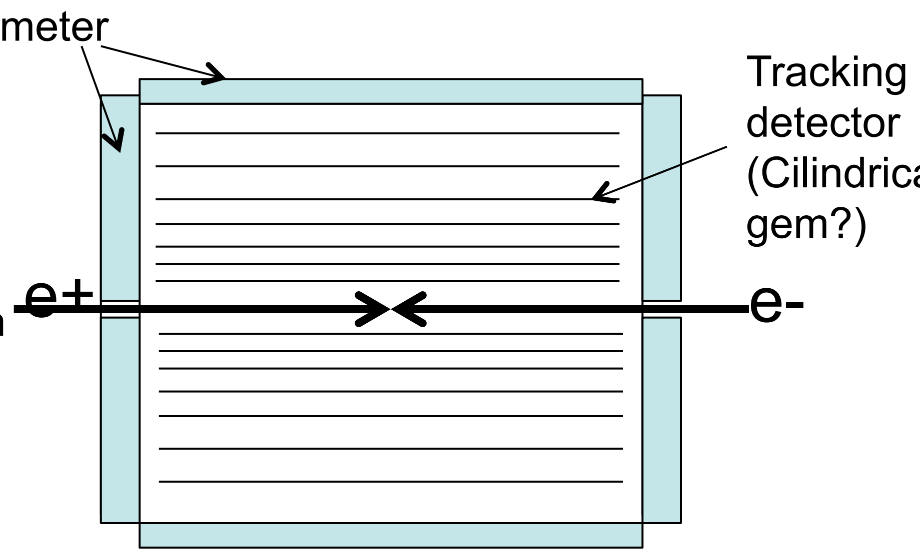


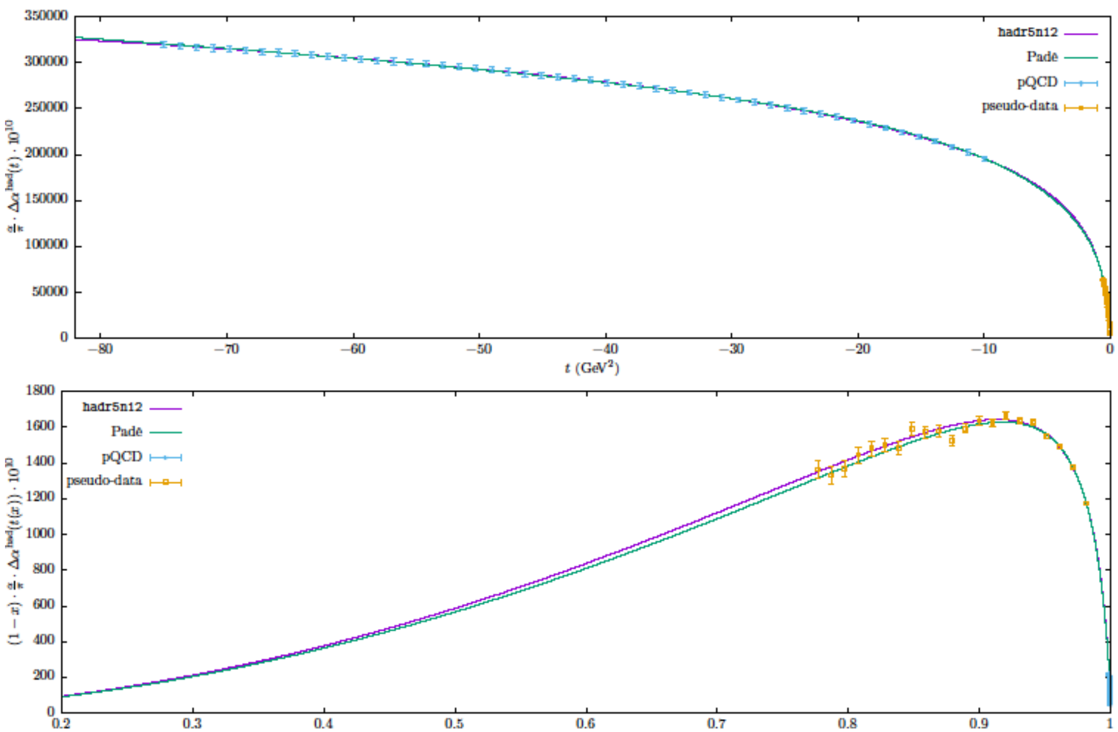
$\delta a_\mu^{HLO}\sim3\%_{stat}\oplus7\%_{syst}$

(preliminary)

What can be done with a dedicated detector (at 1-2 GeV)

- A dedicated detector with a coverage at small angle (< 5°) would allow to use small angle Bhabha for the normalization (N_o).
- The running of a can be obtained as "simple" ratio Ni/N_o where Ni is the Bhabha events in the $\Delta\theta_i$ bin.
- One can achieve an error ~10^{-5} (stat+syst) on this ratio





Same simulation as in KLOE with 20 points and 10^{-5} (stat and syst) error for each point

What can be done with a dedicated detector (at 1-2 GeV)

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- The running of a can be obtained as "simple" ratio Ni/N_o where Ni is the Bhabha events in the $\Delta\theta_i$ bin.
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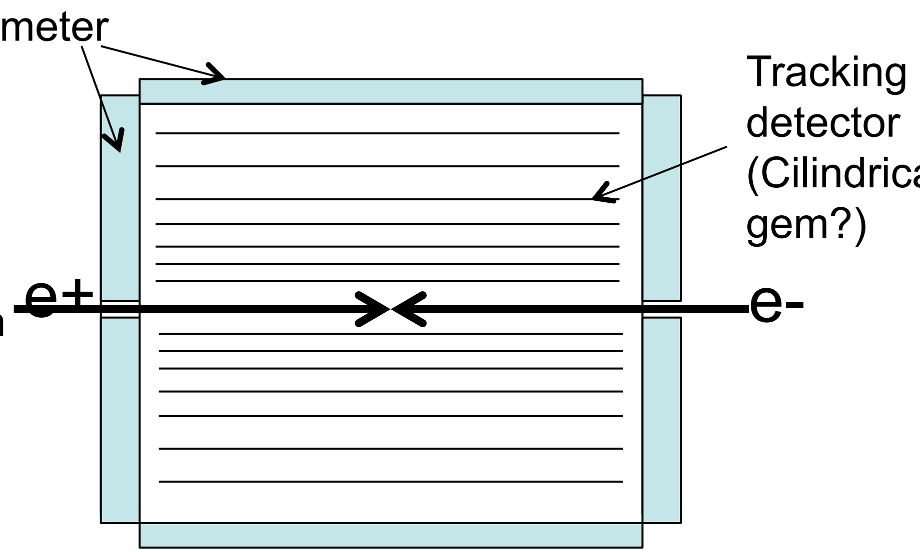
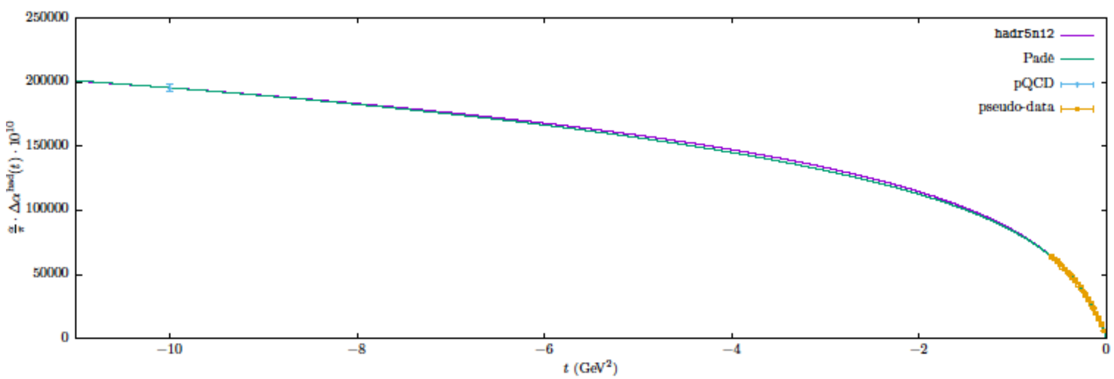
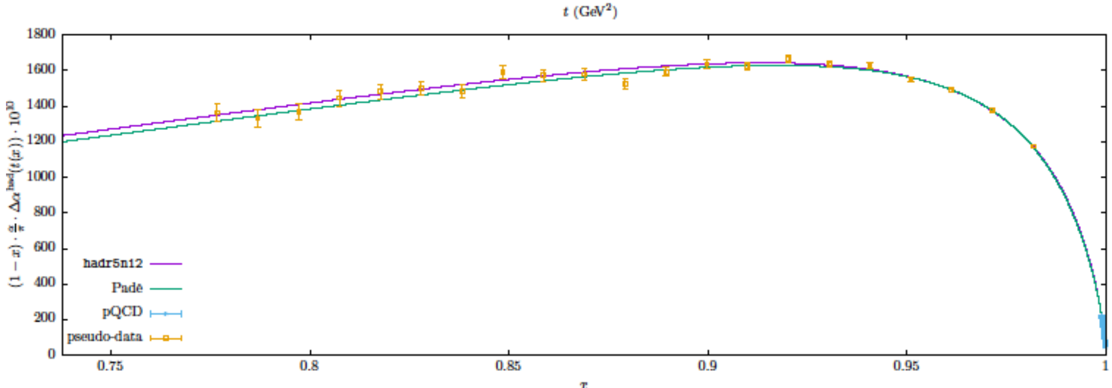




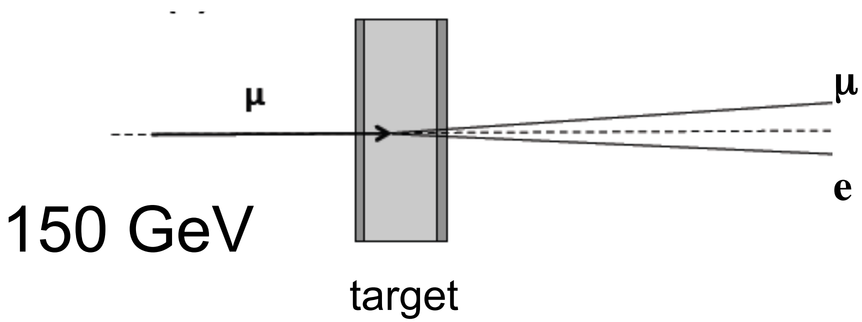


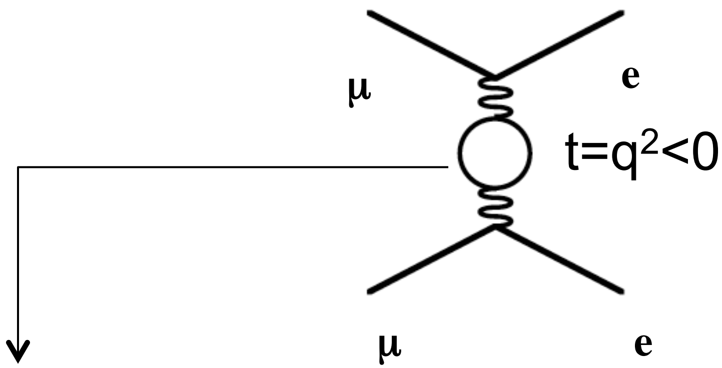
δa_μ^{HLO}~$0.3\%_{stat} \oplus 1\%_{syst}$

(preliminary)

High precision measurement of a_μ^{HLO} with a 150 GeV μ beam on Be target at CERN

(through the elastic scattering μe → μ e)





$$\left|\frac{\alpha(t)}{\alpha_0}\right|^2 = \left|\frac{1}{1-\Delta\alpha(t)}\right|^2$$

Why measuring $\Delta\alpha_{had}(t)$ with a 150 GeV μ beam on e^- target ?

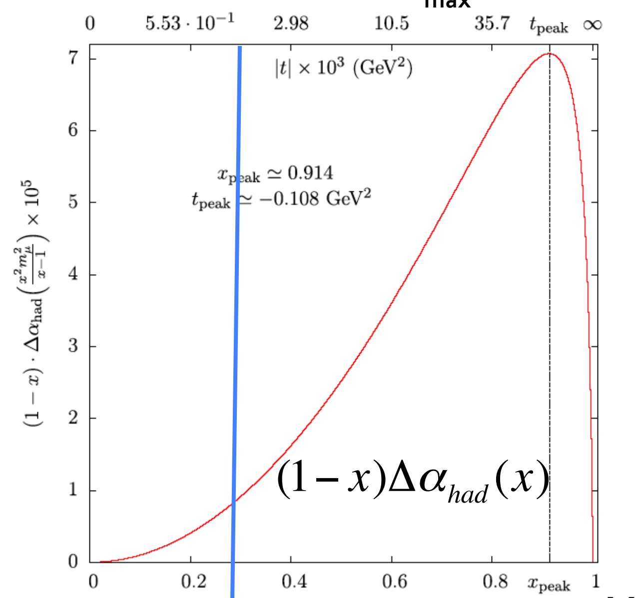
It looks an ideal process!

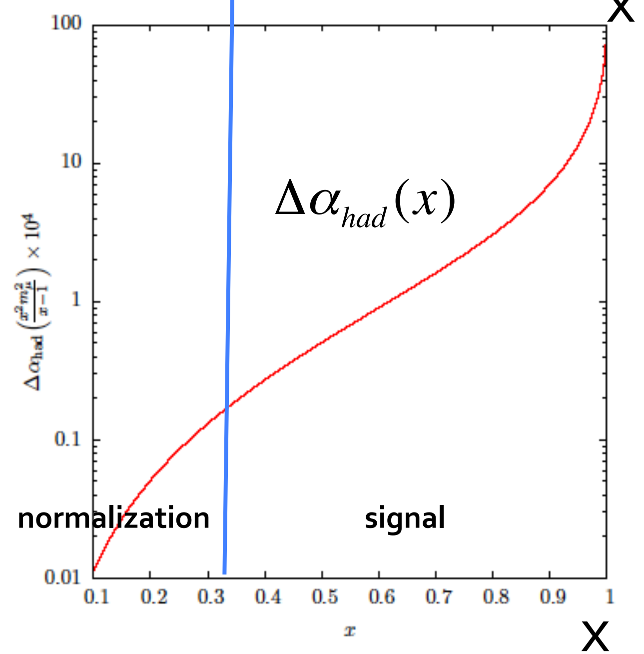
- $\mu\, e \rightarrow \mu\, e$ is pure t-channel (at LO)
- It gives $0<-t<0.161$ GeV^2 ($0<x<0.93$) →
- The kinematics is very simple: $t=-2m_eE_e$
- High boosted system gives access to all angles (t) in the cms region

  $\theta_e^{LAB}<32$ mrad ($E_e>1$ GeV)

  $\theta_\mu^{LAB}<5$ mrad
- It allows using the same detector for signal and normalization
- Events at $x\sim0.3$ ($t\sim-10^{-3}$ GeV^2) can be used as normalization ($\Delta\alpha_{had}(t)<10^{-5}$) →

$t_{max}=-0.11$GeV^2





Detector considerations I

- In order to be competitive with a_μ^{HLO} from time-like data (0.6% error) a subpercent uncertainty on a_μ^{HLO} is required

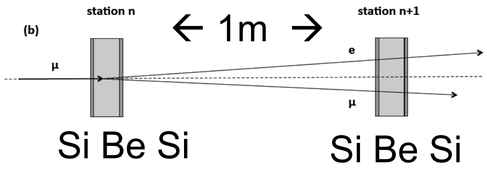
$$R^{MC} = \frac{d\sigma_0^{MC}(t)}{d\sigma_0^{MC}(t_0)}$$

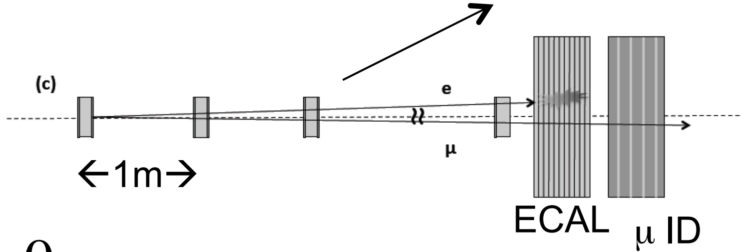
$$\delta\Delta\alpha_{had}(t) \sim 0.5\sqrt{\left(\frac{\delta N^{data}(t)}{N^{data}(t)}\right)^2 + \left(\frac{\delta N^{norm}(t_0)}{N^{norm}(t_0)}\right)^2 + \left(\frac{\delta R^{MC}}{R^{MC}}\right)^2} + corr.\,terms$$

- $\delta\Delta\alpha_{had}/\Delta\alpha_{had}$ at 0.5% at peak region (x=0.92, $\Delta a_{had}\sim 10^{-3}$)→
  $\delta N(t)/N(t)\sim 10^{-5}$

- Such an accuracy demands high **statistics** keeping **low systematic** errors!
- **Dense** (active) target would provide the required statistics at a price of an unavoidable large multiple scattering and background process (pair production, bremsstrahlung, nuclear interaction)
- Our **choice** goes to light Z (Be) target with a modular apparatus which minimizes systematic errors

Detector considerations II





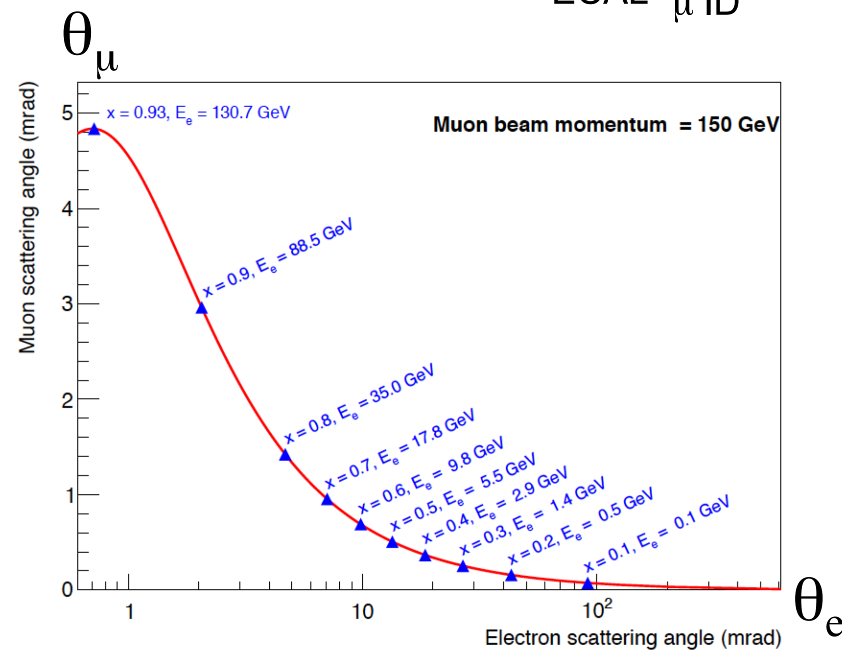
- Modular apparatus: 20 layers of 3 cm Be (target), each coupled to 1 m distant Si (0.3 mm) planes. It provides a 0.02 mrad resolution on the scattering angle

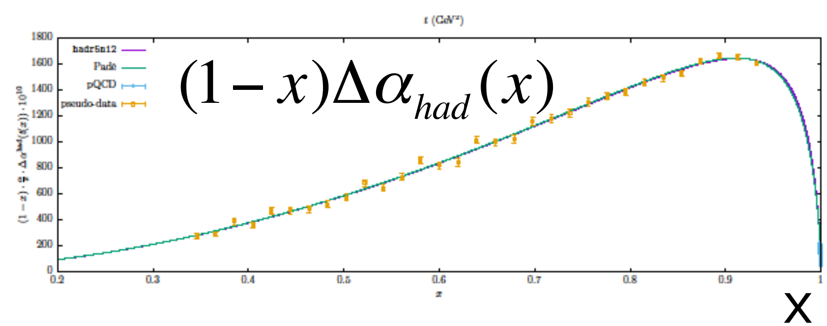
- The $t=q^2<0$ of the interaction is determined by the electron (or muon) scattering angle (a` la NA7)

- ECAL and μ Detector located downstream to solve PID ambiguity below 5 mrad. Above that, angular measurement gives correct PID

- It provides uniform full acceptance, with the potential to keep the systematic errors at 10^{-5} (main effect is the multiple scattering for normalization which can be studied by data)

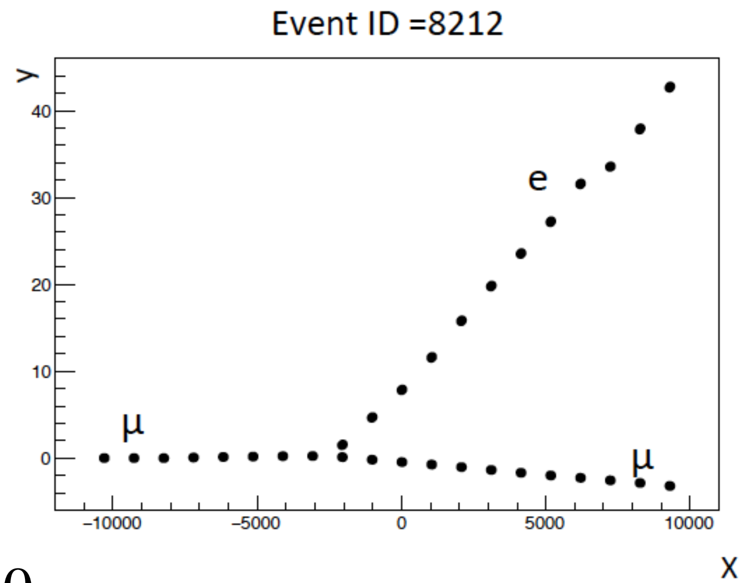
- **Statistical considerations show that a 0.3% error can be achieved on a_μ^{HLO} in 2 years of data taking with 2×10^7 μ/s**

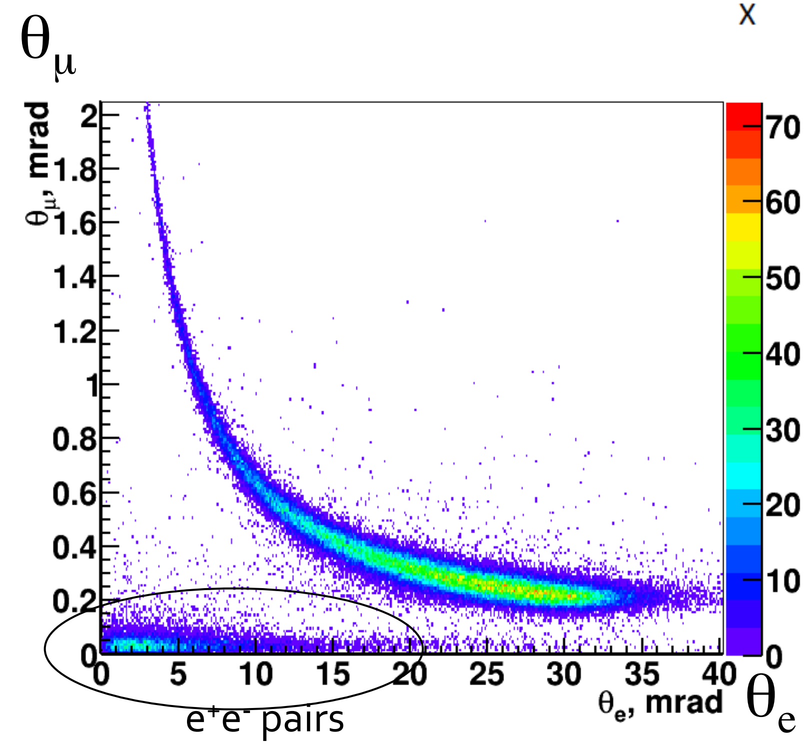




First simulation

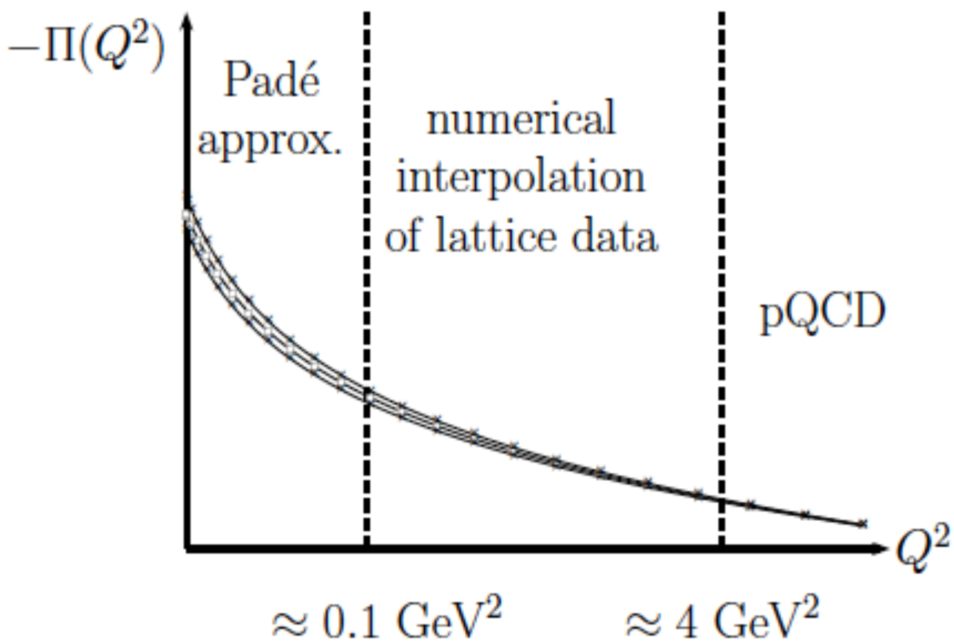
- A simulation of the detector based on GEANT-4 has started
- μ-e elastic scattering events have a clear topology
- Background events can be easily identified and rejected in the θ_μ vs θ_e plane
- Multiple scattering can be studied by data as it **breaks** the μ-e two-body angular correlation, moving events out of the kinematic constraint. It also causes **acoplanarity**, while two-body events are planar.
- Simulation will help to optimize the detector (i.e. additional thin layer(s) can be placed for luminosity)





Comparison with Lattice

- LQCD lattice in finite box: momenta are quantized $Q_{\min} = 2\pi/L$ where L is the lattice box length. $Q_{\min} \to 0 \Leftrightarrow L \to \infty$ infinite volume limit
- $Q_{\min} = 2\pi/L$ with $m_\pi a L \gtrsim 4$ for $m_\pi \sim 200$ MeV, such that $Q_{\min} \sim 314$ MeV
- about 44% of the low x contribution to $a_\mu^{\rm had}$ is not covered by data yet

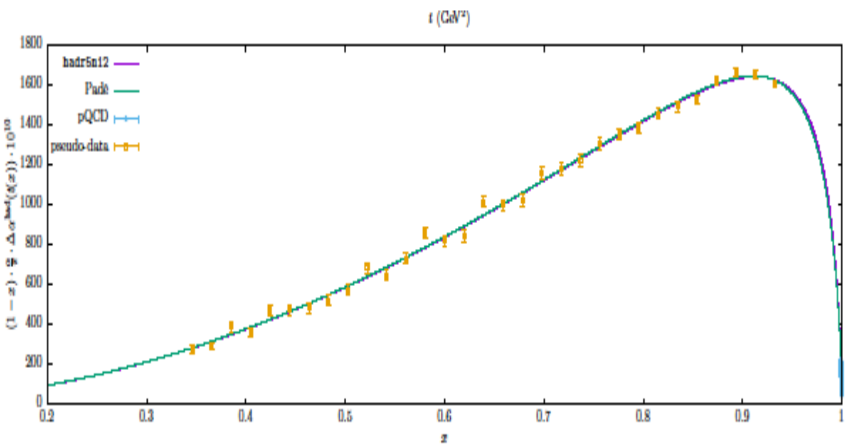


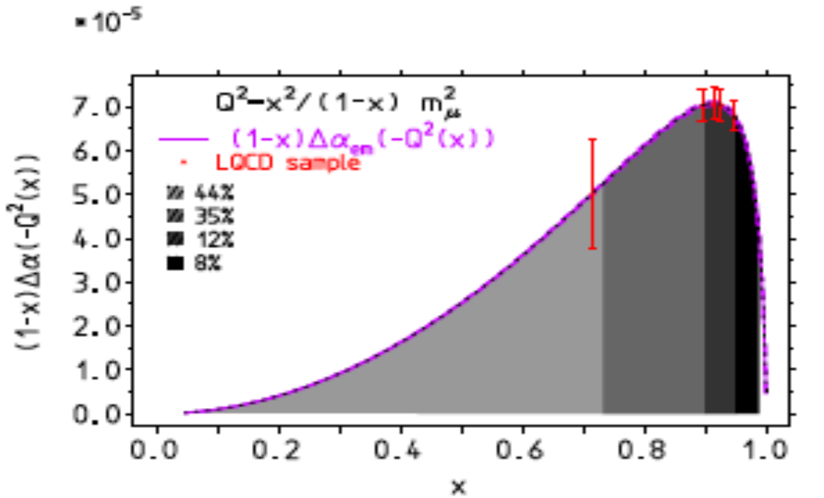
- lattice data: $Q^2 > (2\pi/L)^2$
- extrapolate to $Q^2 = 0$ via Padé's
- Note need $\Pi(0)$!
- required accuracy: needed LQCD data down to $Q^2_{\min} \approx 0.1$ GeV^2

F. Jegerlehner, talk at KLOE2 Physics Workshop, Frascati, 28/10/16

Comparison of our method with Lattice (as it is now)

$$a_\mu^{\text{had}} = \frac{\alpha}{\pi} \int_0^1 dx \, (1-x) \, \Delta\alpha^{\text{had}}\left(-Q^2(x)\right)$$





Interplay between our data and lattice calculation!

Conclusion

- A new approach to determine the full contribution to $\mathbf{a_\mu^{HLO}}$**,** based on the measurement of $\boldsymbol{\Delta\alpha_{had}(t)}$ in **space-like region** has been presented.

- Two experimental proposals with different systematics:
  - from Bhabha scattering at low energy colliders: difficult to control the systematic uncertainties at 10^{-5} in current configurations (it would need dedicated detectors at present flavour factories)
  - from the scattering $\mu\, e \rightarrow \mu\, e$ using a high energy muon beam (E~150 GeV) available in the North Area at CERN on electron target**:** very promising to reach the per mill goal! **A test with a single module could provide a proof-of-concept of the proposed method.**

- Theory side: high precision MC must be developed to control the systematics. The present knowledge of QED Radiative Corrections is at a few 10^{-4} level; work is in progress to extend MC used at flavour factories (BabaYaga) to μe scattering with expected accuracy at (better than) 10^{-5} on cross section ratios

THANKS!!!!

END

E989 Collaboration

Domestic Universities
- Boston
- Cornell
- Illinois
- James Madison
- Kentucky
- Massachusetts
- Michigan
- Michigan State
- Mississippi
- Northern Illinois University
- Northwestern
- Regis
- Virginia
- Washington
- York College

National Labs
- Argonne
- Brookhaven
- Fermilab

Italy
- Frascati,
- Roma 2,
- Udine
- Pisa
- Naples
- Trieste
- Lecce

China:
- Shanghai

The Netherlands:
- Groningen

Germany:
- Dresden

Russia:
- Dubna
- Novosibirsk

England
University College London

Liverpool

Oxford

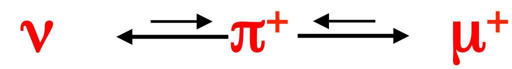
Korea
KAIST

D.W. Hertzog, Co-Spokesperson
B.L. Roberts, Co-Spokesperson
C. Polly, Project Manager

- 33 institution, 150 members

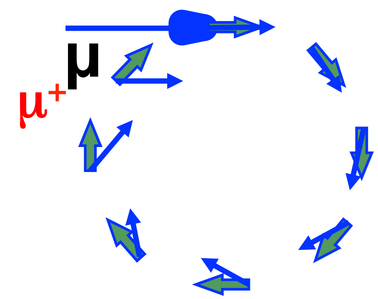
How to measure g-2 in a storage ring

(1) Polarized muons



~97% polarized for forward decays

(2) Precession proportional to (g-2)



$$\omega_a = \omega_{spin} - \omega_{cyclotron} = \left(\frac{g-2}{2}\right)\frac{eB}{mc}$$

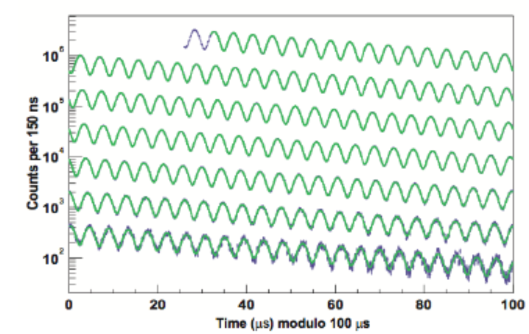
(3) P_μ magic momentum = 3.094 GeV/c

$$\vec{\omega}_a = \frac{e}{mc}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\vec{\beta}\times\vec{E}\right]$$

E field doesn't affect muon spin when γ = 29.3

(4) Parity violation in the decay gives average spin direction

$$\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$$



How to measure g-2 in a storage ring

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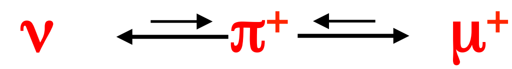

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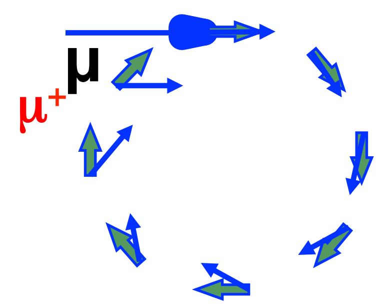

$$\omega_a = \omega_{spin} - \omega_{cyclotron} = \left(\frac{g-2}{2}\right)\frac{eB}{mc}$$

Measure 2 quantities

(3) P_μ magic momentum = 3.094 GeV/c

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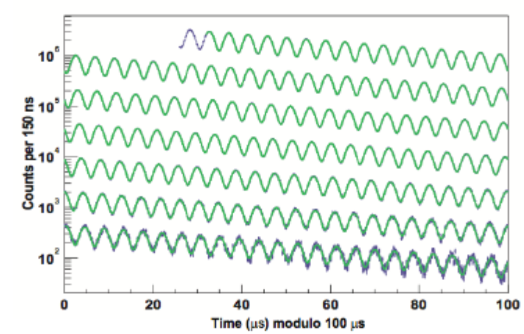


4 key elements for E989 at FNAL

- Consolidated method
- More muons (x20)
- Reduced systematics (ring and detector)
- New crew

- **E821 at Brookhaven**

$$\left.\begin{aligned}\sigma_{\mathrm{stat}} &= \pm 0.46\ \mathrm{ppm}\\ \sigma_{\mathrm{syst}} &= \pm 0.28\ \mathrm{ppm}\end{aligned}\right\}\ \sigma = \pm 0.54\ \mathrm{ppm}$$

- **E989 at Fermilab**

$$\left.\begin{aligned}\sigma_{\mathrm{stat}} &= \pm 0.1\ \mathrm{ppm}\\ \sigma_{\mathrm{syst}} &= \pm 0.1\ \mathrm{ppm}\end{aligned}\right\}\ \sigma = \pm 0.14\ \mathrm{ppm}$$

$\hookrightarrow$ $0.7\omega_a \oplus 0.7\omega_p$

What we need to do…

- 4 Major Steps
  - Transport BNL storage ring and associated equipment to Fermilab ✔
  - Construct a new experimental hall to house the storage ring ✔
  - Modify anti-proton complex to provide a high-purity, intense beam of 3.094 GeV/c muons 70% complete
  - Upgrade various subsystems (injection devices, field monitoring, detectors & DAQ) to meet requirements for rates and systematics 60% complete

- Overall plan to achieve a factor of four improvement in precision
  - Increase statistics by x 21 to reduce stat error from 0.46 ppm to 0.1 ppm
  - Reduce systematics on ω_a from 0.2 ppm to 0.07 ppm
  - Reduce systematics on ω_p from 0.17 ppm to 0.07 ppm

Fermilab Muon Campus Vision, circa 2012

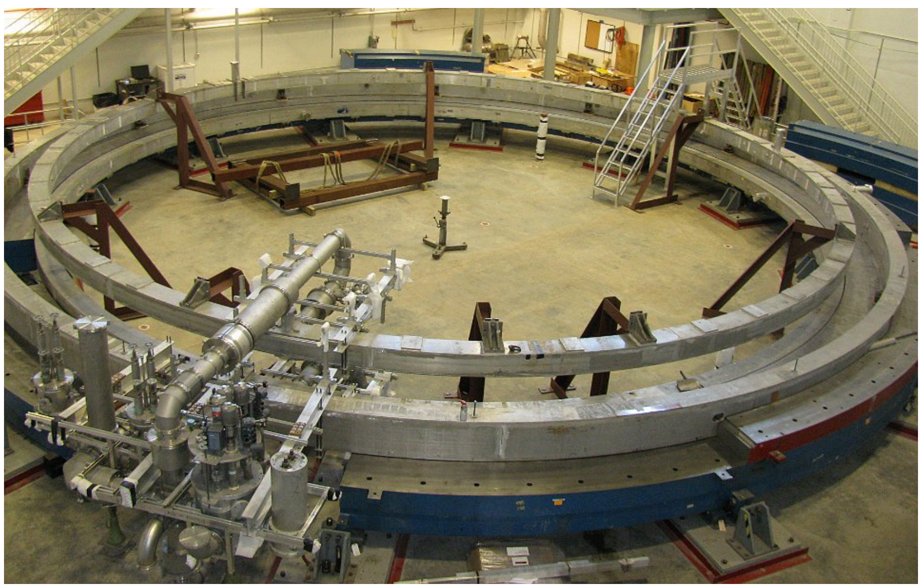
- **Convert FNAL anti-proton source to produce customized muon beams for experiments like Muon g-2 and Mu2e**

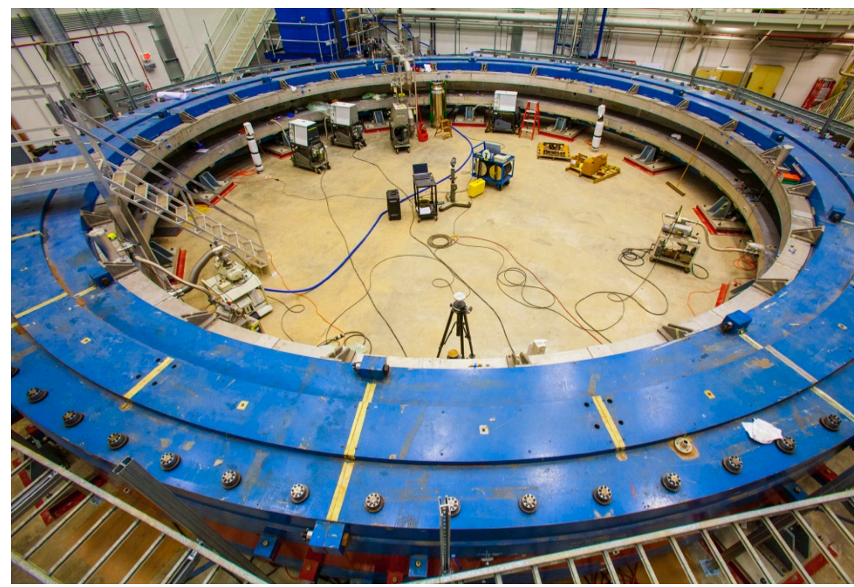
Muon Campus Reality – View from Wilson Hall Today

- Muon g-2 hall complete, BNL storage ring installed and operational
- Mu2e civil construction complete, building outfitting underway
- Conversion of accelerator complex to muon source nearing completion

Ring Reassembling (July 2014 – June 2015)

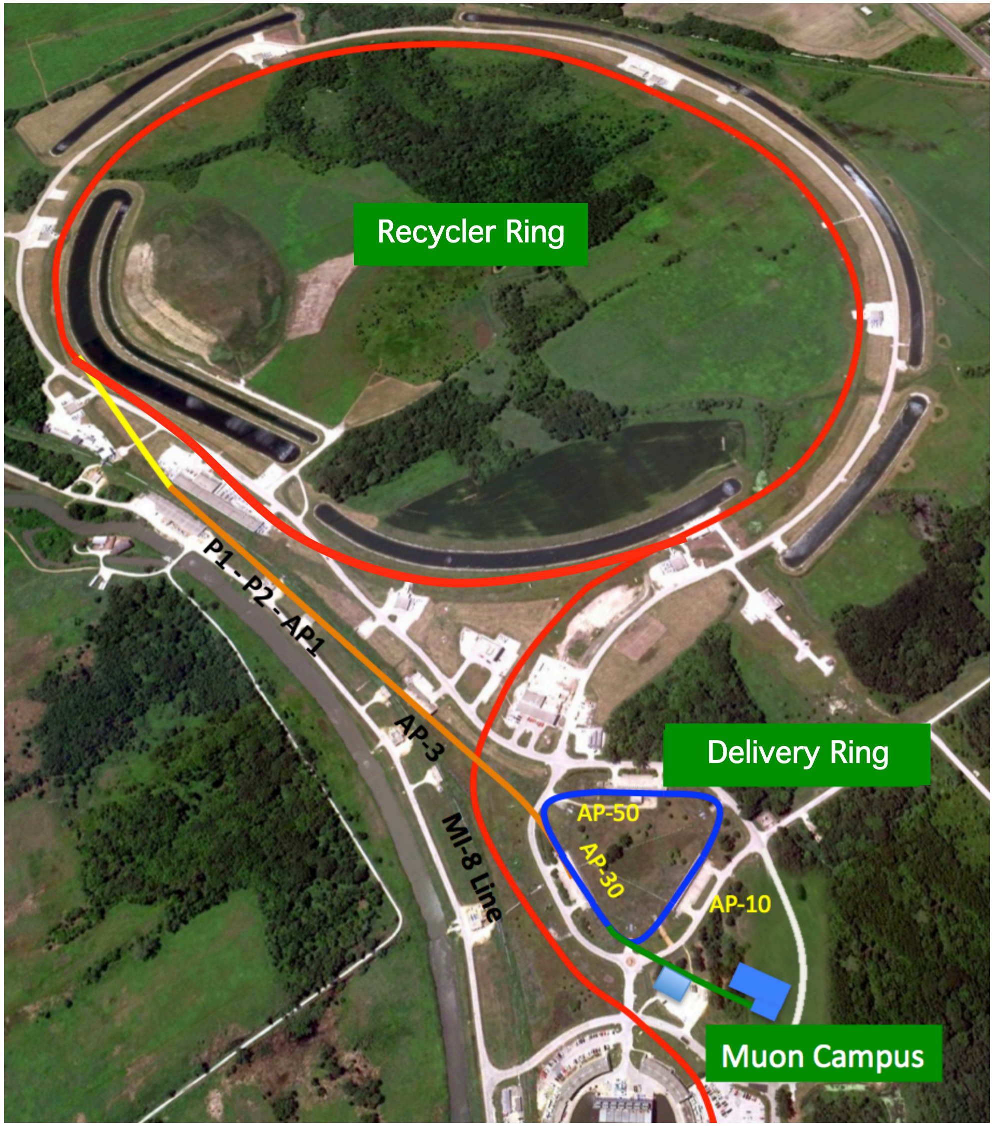
μ g–2





Achieved full power in September 2015

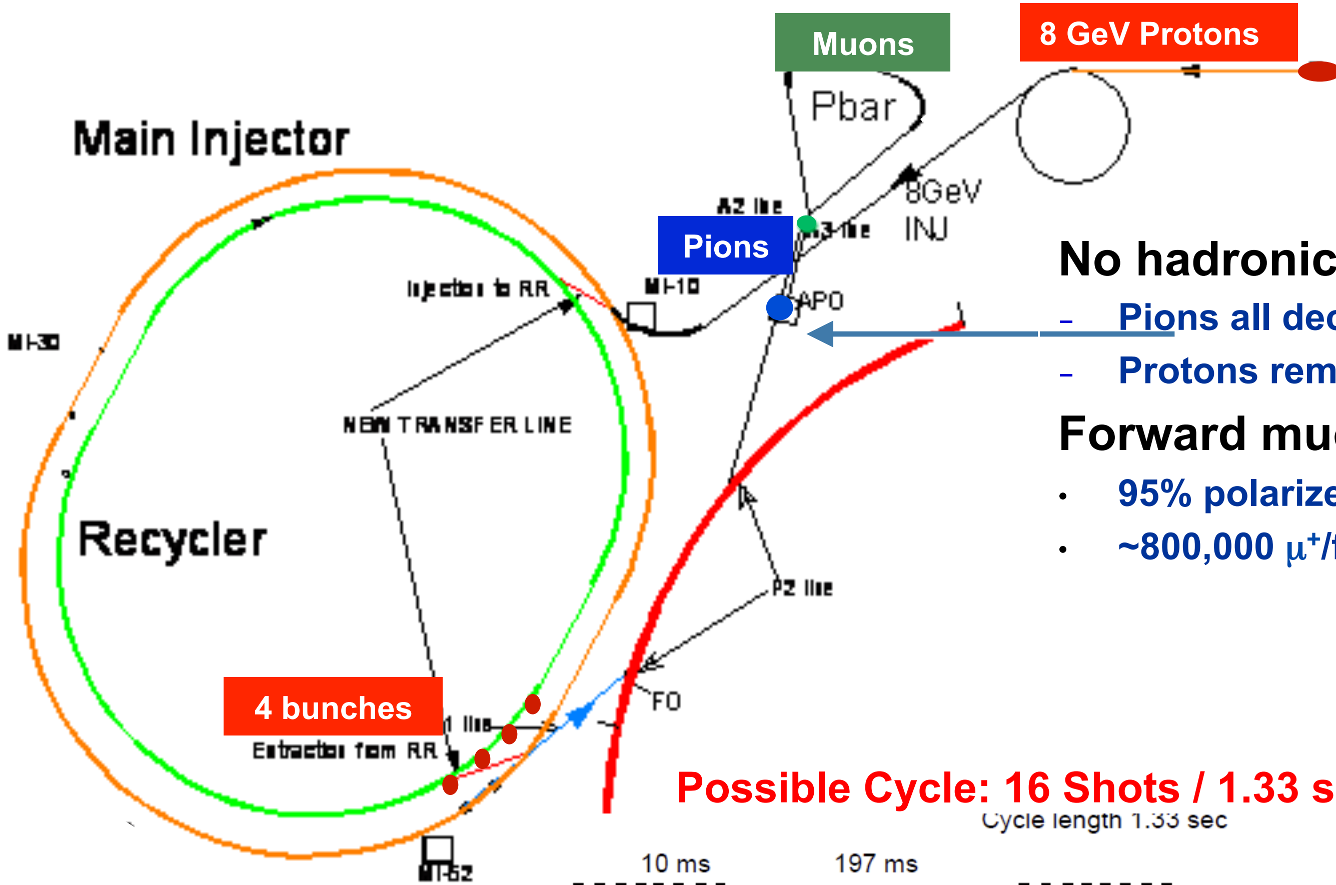
First challenge - statistics

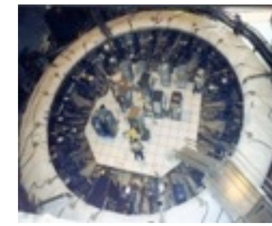


Achieving required statistics is a primary concern
- Need a factor 21 more statistics than BNL
- Beam power reduced by 4

Need a factor of 85 improvement in integrated beam coming from many other factors
- Collection of pions from Li lens
- Capture of decay muons in FODO channel
- p_{π} closer to magic momentum
- Longer decay channel
- Increased injection efficiency
- Earlier start time of fits
- Longer runtime (~18 months for production running + systematics)

A pure muon beam of 3.1 GeV





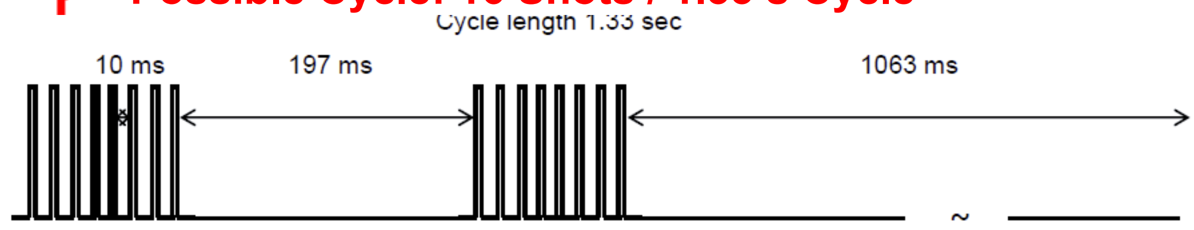
No hadronic flash

- Pions all decay
- Protons removed by a DR kicker

Forward muons collected

- 95% polarized
- ~800,000 μ^+/fill ($\delta P/P$ 2%) at ring

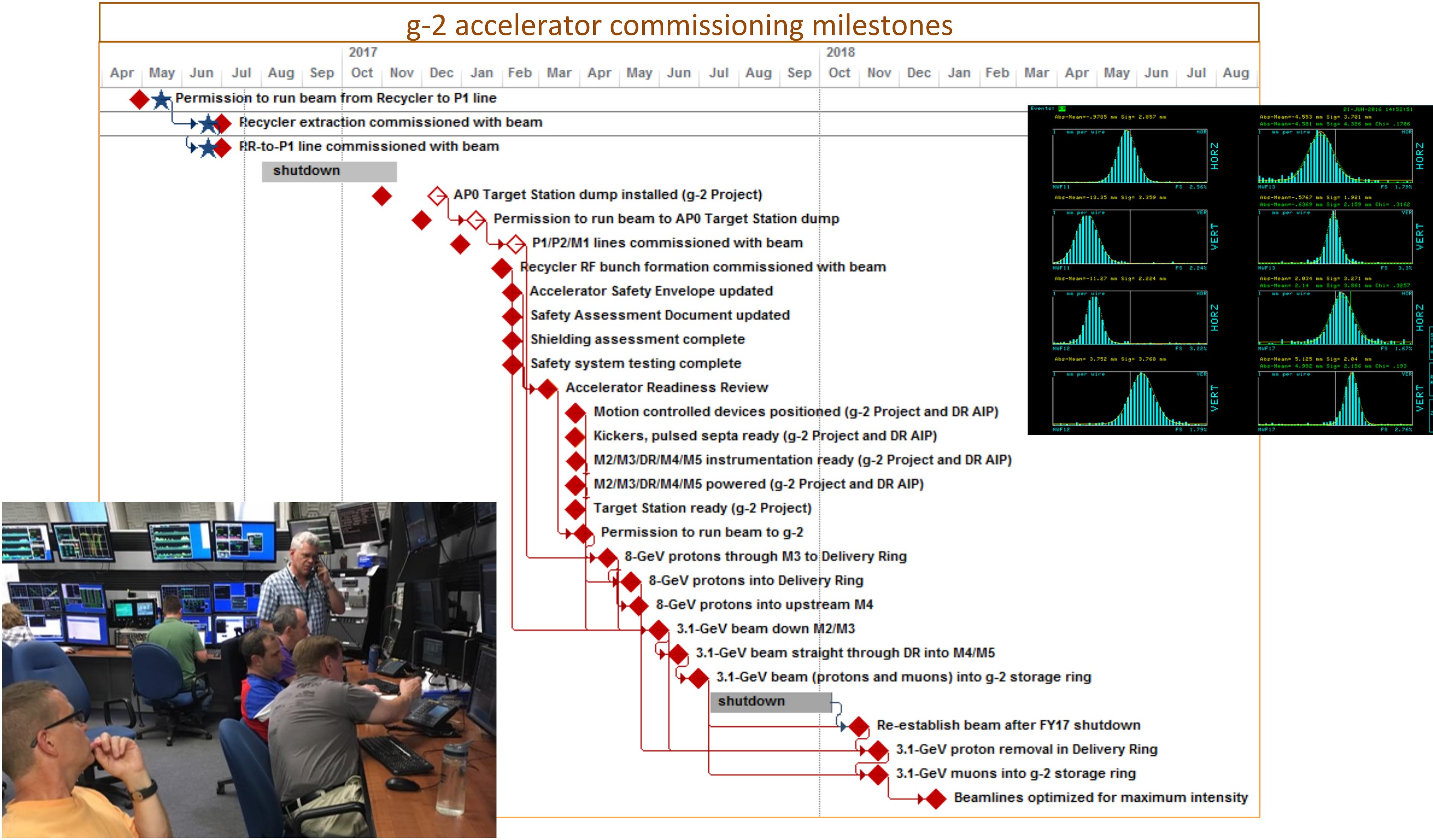
Possible Cycle: 16 Shots / 1.33 s Cycle



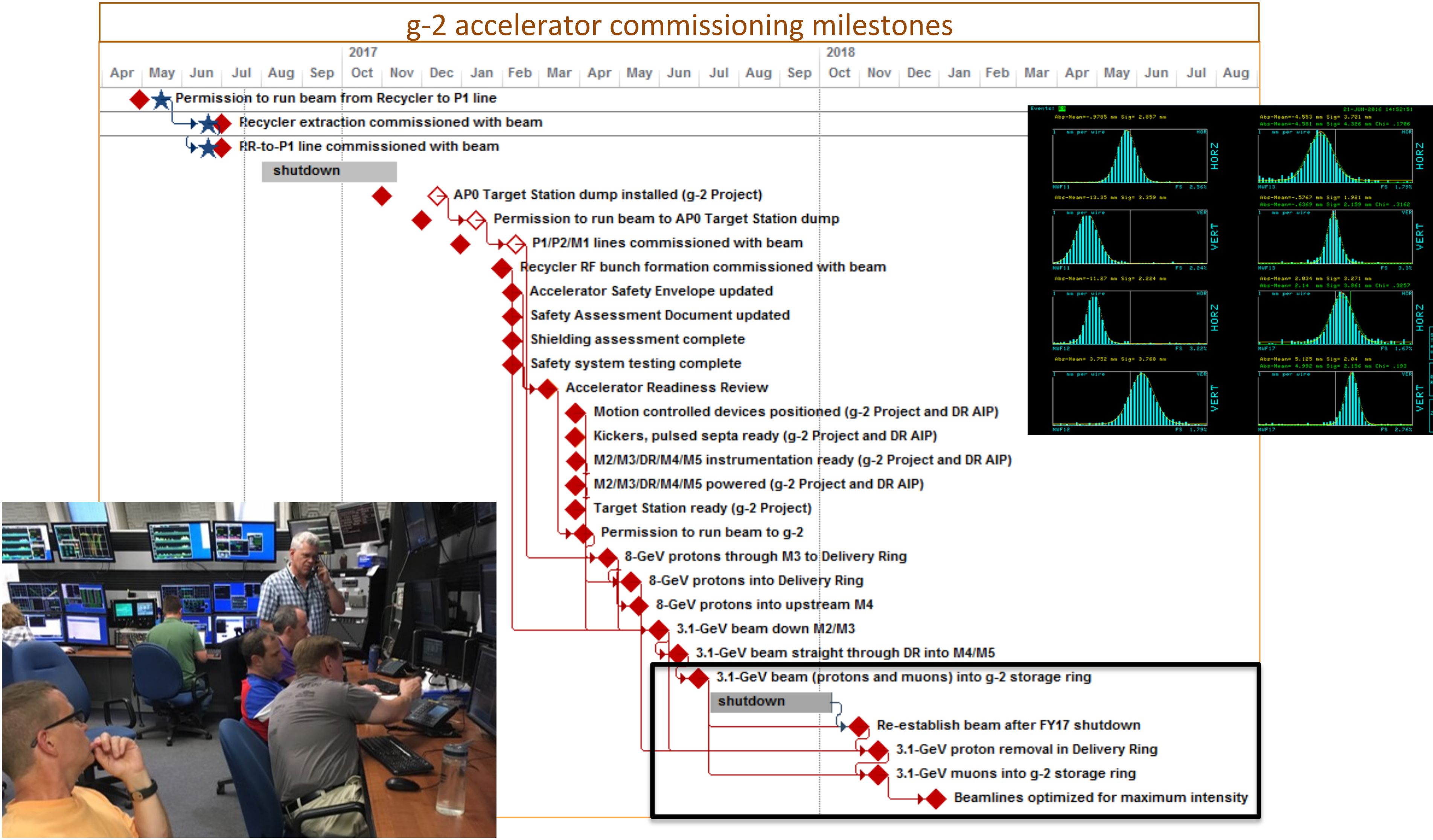
All protons are "excess" from those used for NOvA ν B…

Beam transported from Recycler thru P1, P2 lines!

Muon g-2



Data taking will start in ~1 year from now



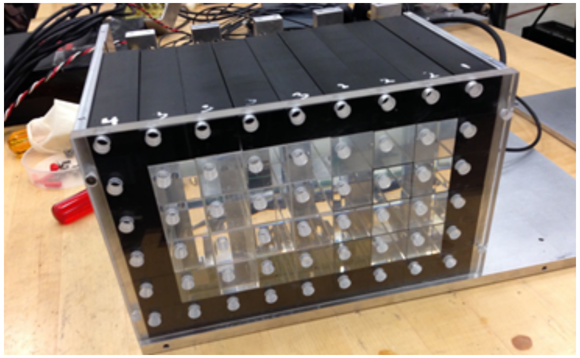
Inizio presa dati autunno 2017

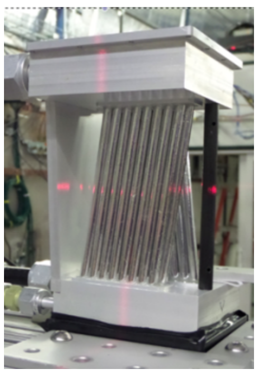
Second challenge – ω_a systematics

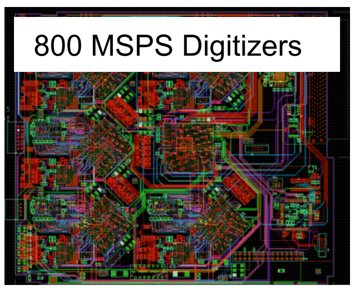
Category	E821 [ppb]	E989 Improvement Plans	Goal [ppb]
Gain changes	120	Better laser calibration	
		low-energy threshold	20
Pileup	80	Low-energy samples recorded	
		calorimeter segmentation	40
Lost muons	90	Better collimation in ring	20
CBO	70	Higher n value (frequency)	
		Better match of beamline to ring	< 30
E and pitch	50	Improved tracker	
		Precise storage ring simulations	30
Total	180	Quadrature sum	70

- Tackling each of the major systematic errors with knowledge gained from BNL E821 and improved hardware

New detector systems to be installed by March 2017

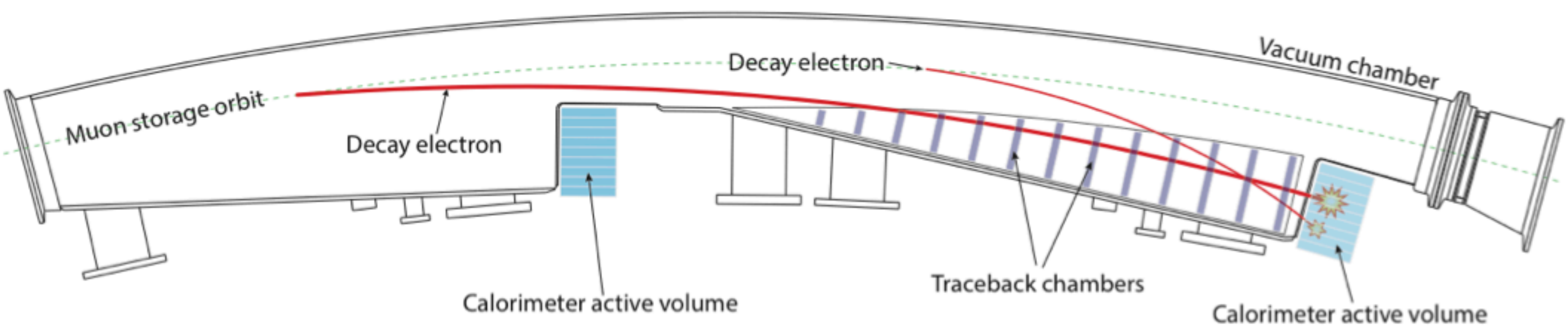




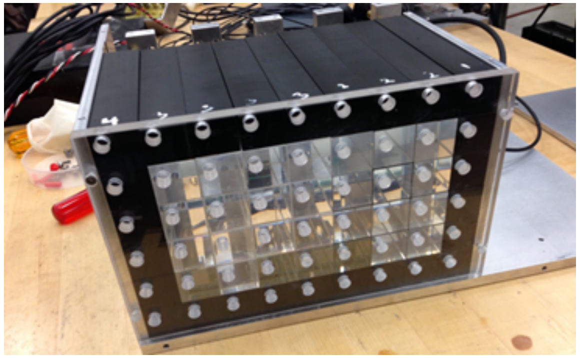


- Calorimeters 24 6x9 PbF_2 crystal arrays with SiPM readout, segmentation to reduce pileup
- New electronics and DAQ, 800MHz WFDs and a greatly reduced threshold
- Three 1500 channel straw trackers to precisely monitor properties of stored muon beam via tracking of Michel decay positrons, significant UK contributions
- New laser calibration system from INFN crucial for untangling gain from other systematics

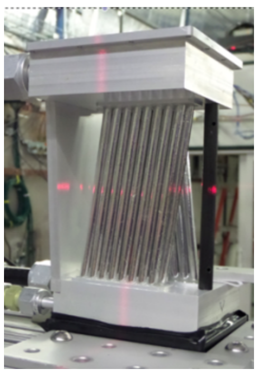
Top view of 1 of 12 vacuum chambers

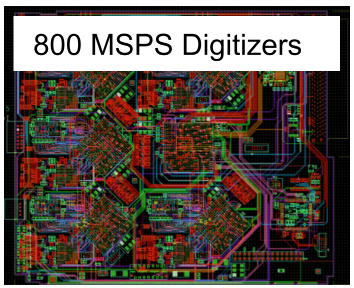


New detector systems to be installed by March 2017

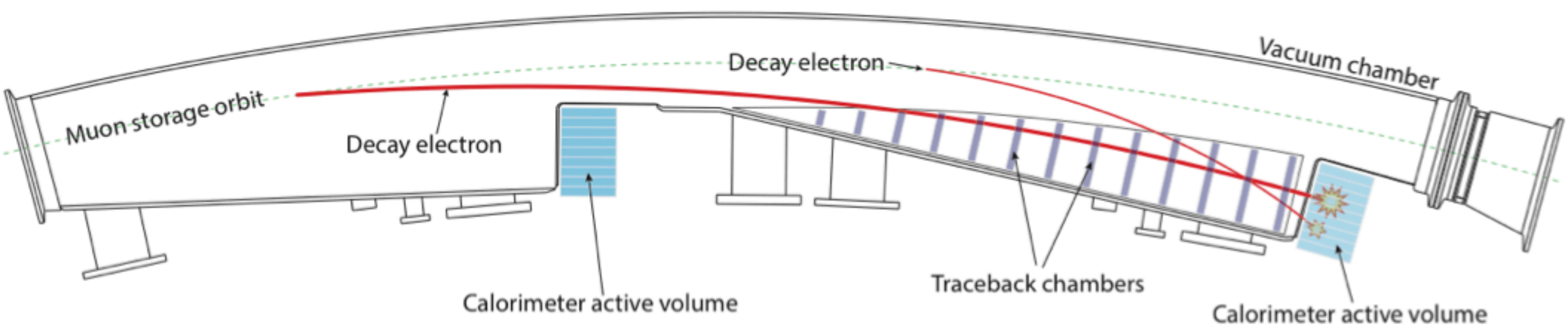


- Calorimeters 24 6x9 PbF_2 crystal arrays with SiPM readout, segmentation to reduce pileup
- New electronics and DAQ, 800MHz WFDs and a greatly reduced threshold
- Three 1500 channel straw trackers to precisely monitor properties of stored muon beam via tracking of Michel decay positrons, significant UK contributions
- New laser calibration system from INFN crucial for untangling gain from other systematics

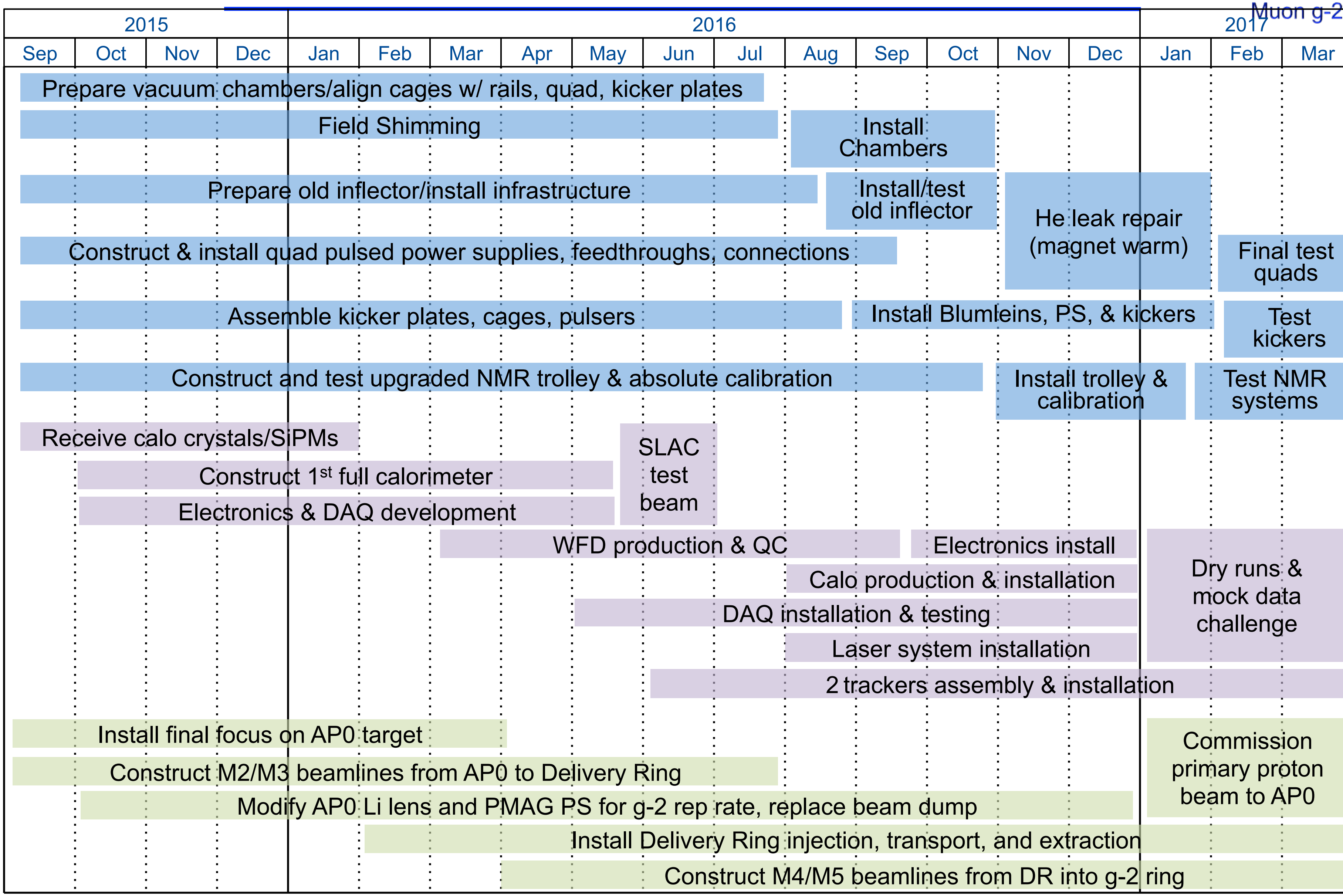




Top view of 1 of 12 vacuum chambers



Schedule

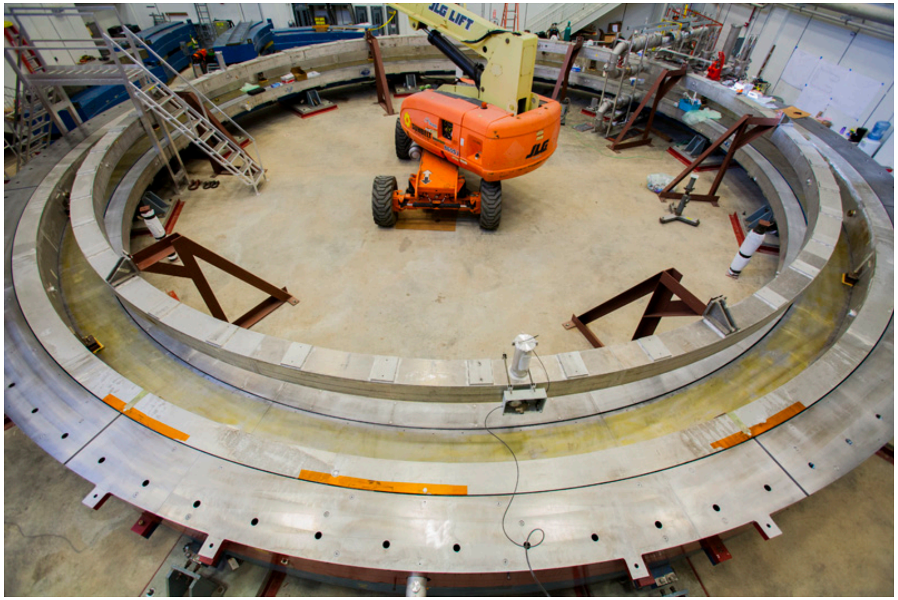

2015
2016
2017
Sep
Oct
Nov
Dec
Jan
Feb
Mar
Apr
May
Jun
Jul
Aug
Sep
Oct
Nov
Dec
Jan
Feb
Mar
Prepare vacuum chambers/align cages w/ rails, quad, kicker plates
Field Shimming
Install Chambers
Prepare old inflector/install infrastructure
Install/test old inflector
He leak repair (magnet warm)
Construct & install quad pulsed power supplies, feedthroughs, connections
Final test quads
Assemble kicker plates, cages, pulsers
Install Blumleins, PS, & kickers
Test kickers
Construct and test upgraded NMR trolley & absolute calibration
Install trolley & calibration
Test NMR systems
Receive calo crystals/SiPMs
SLAC test beam
Construct 1st full calorimeter
Electronics & DAQ development
WFD production & QC
Electronics install
Dry runs & mock data challenge
Calo production & installation
DAQ installation & testing
Laser system installation
2 trackers assembly & installation
Install final focus on AP0 target
Commission primary proton beam to AP0
Construct M2/M3 beamlines from AP0 to Delivery Ring
Modify AP0 Li lens and PMAG PS for g-2 rep rate, replace beam dump
Install Delivery Ring injection, transport, and extraction
Construct M4/M5 beamlines from DR into g-2 ring

Third challenge – ω_p systematics

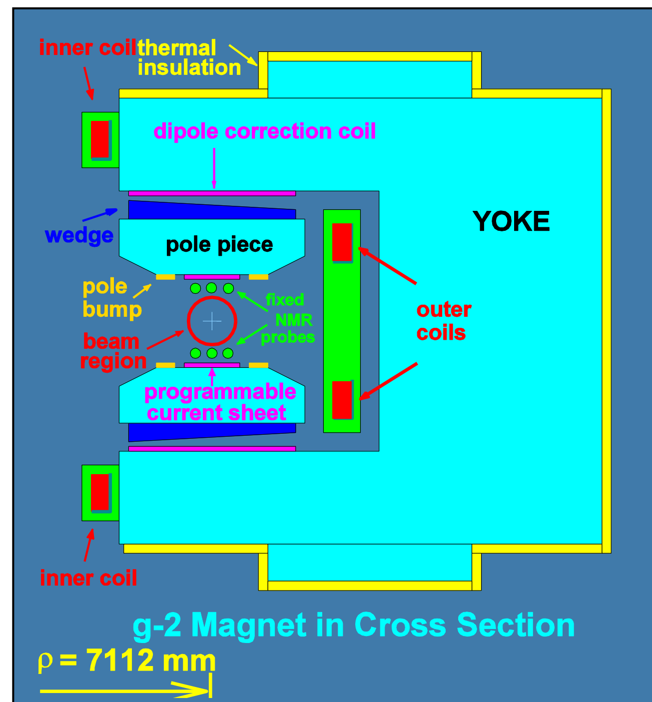
Category	E821 [ppb]	Main E989 Improvement Plans	Goal [ppb]
Absolute field calibration	50	Improved T stability and monitoring, precision tests in MRI solenoid with thermal enclosure, new improved calibration probes	35
Trolley probe calibrations	90	3-axis motion of plunging probe, higher accuracy position determination by physical stops/optical methods, more frequent calibration, smaller field gradients, smaller abs cal probe to calibrate all trolley probes	30
Trolley measurements of B_0	50	Reduced/measured rail irregularities; reduced position uncertainty by factor of 2; stabilized magnet field during measurements; smaller field gradients	30
Fixed probe interpolation	70	Better temp. stability of the magnet, more frequent trolley runs, more fixed probes	30
Muon distribution	30	Improved field uniformity, improved muon tracking	10
External fields	–	Measure external fields; active feedback	5
Others †	100	Improved trolley power supply; calibrate and reduce temperature effects on trolley; measure kicker field transients, measure/reduce O_2 and image effects	30
Total syst. unc. on ω_p	170		70

- Need to know the average field observed by a muon in the storage ring absolutely to better than 70 ppb, many hardware improvements
- Very challenging…first major step is making the field as uniform as possible
  - Has been our main thrust over the last 9 months

Field stability and uniformity improvements



- Environmental
  - 2’9” heavily-reinforced floor installed on 12’ deep excavation of undisturbed soil
  - Temperature control to +/- 1C

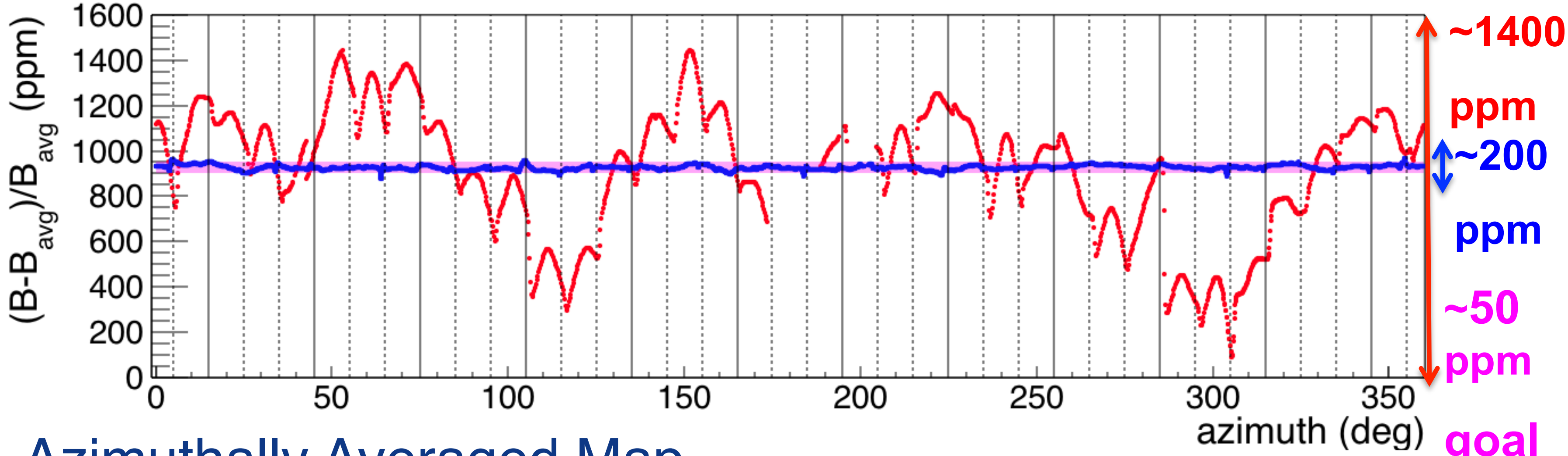


- **Construction tolerances**
  - **26 ton pieces of yoke steel (30 of them) placed to 125 micron tolerance**
  - **Pole pieces aligned to 25 micron**
- **10 months of interactively shimming B-field with bits of steel and current loops (just ended last month)**

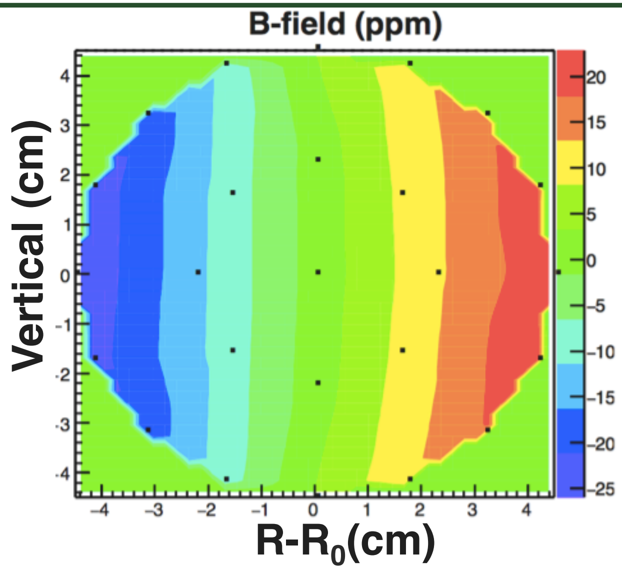
Progress on Field

Oct 2015 → Aug 2016

Goal

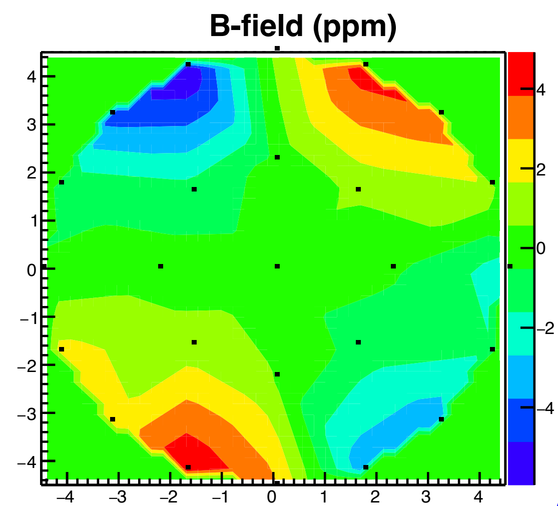


Azimuthally Averaged Map



	Norm	Skew
Quad	25.13	-0.53
Sext	-1.99	-0.11
Octu	-1.16	-0.31
Decu	0.95	-0.07

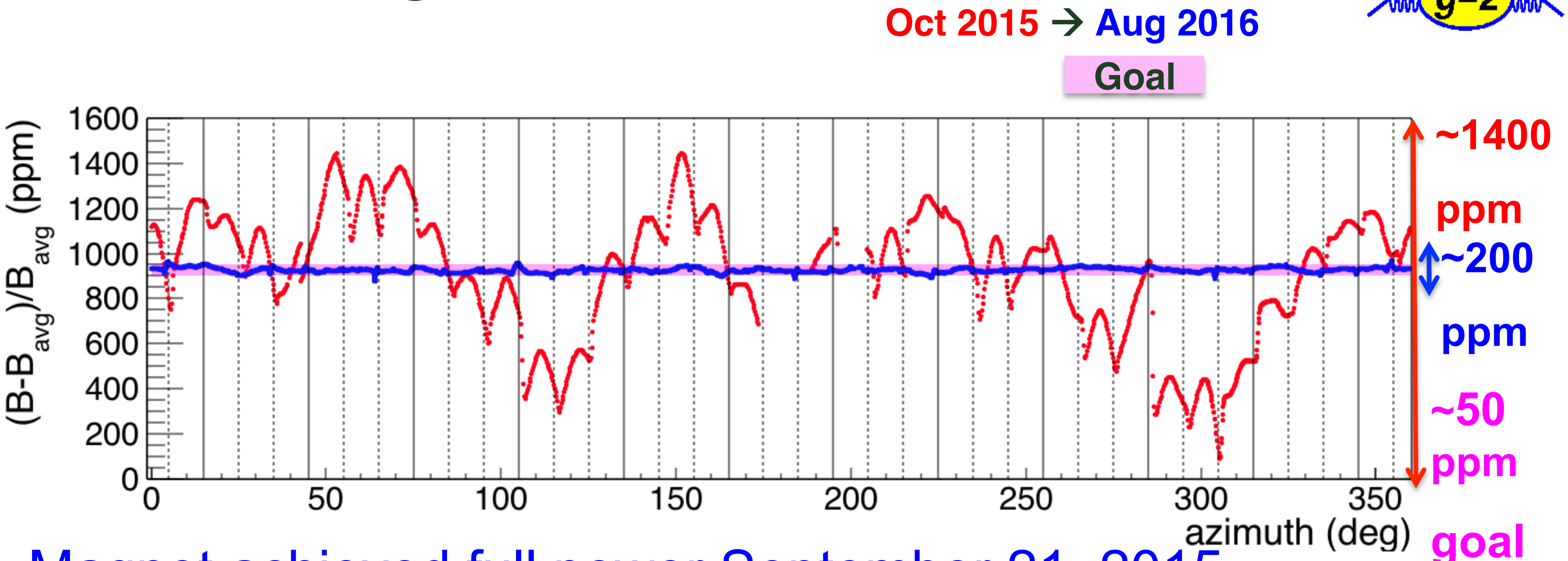
Oct 2015



	Norm	Skew
Quad	-0.02	-0.57
Sext	-0.70	3.84
Octu	-0.76	0.56
Decu	0.44	-1.61

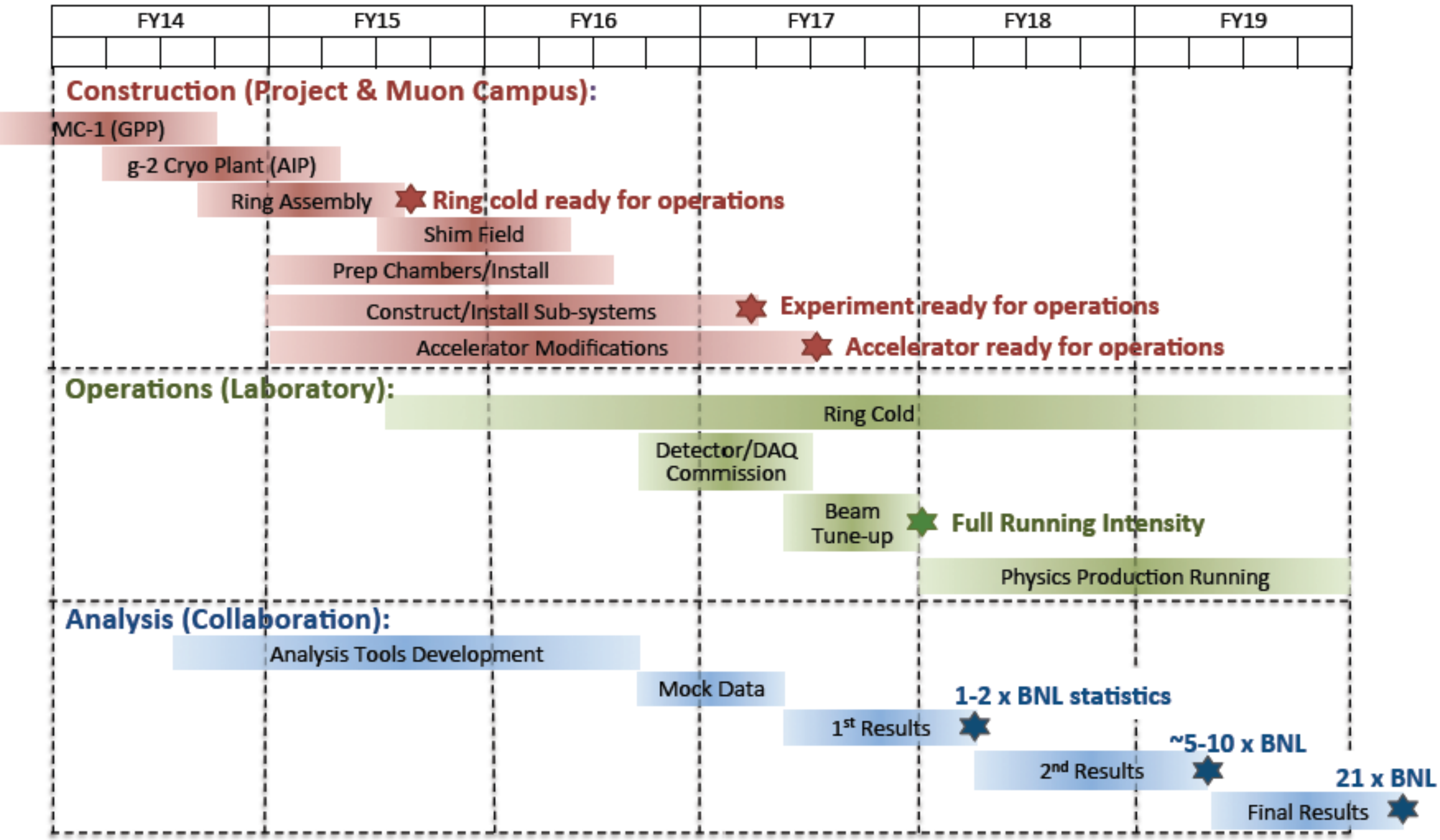
Aug 2016

Progress on Field



- Magnet achieved full power September 21, 2015
- Field started out with a peak variation of 1400 ppm
- June 2016 peak to peak variation was reduced to 200 ppm
- The goal of shimming is 50 ppm with a muon weighted systematic uncertainty of 70 ppb
- BNL achieved 100 ppm with an averaged field uniformity of +- 1ppm. They estimated their systematic uncertainty of 140 ppb. We would like to improve of a factor 2!

Schedule overview



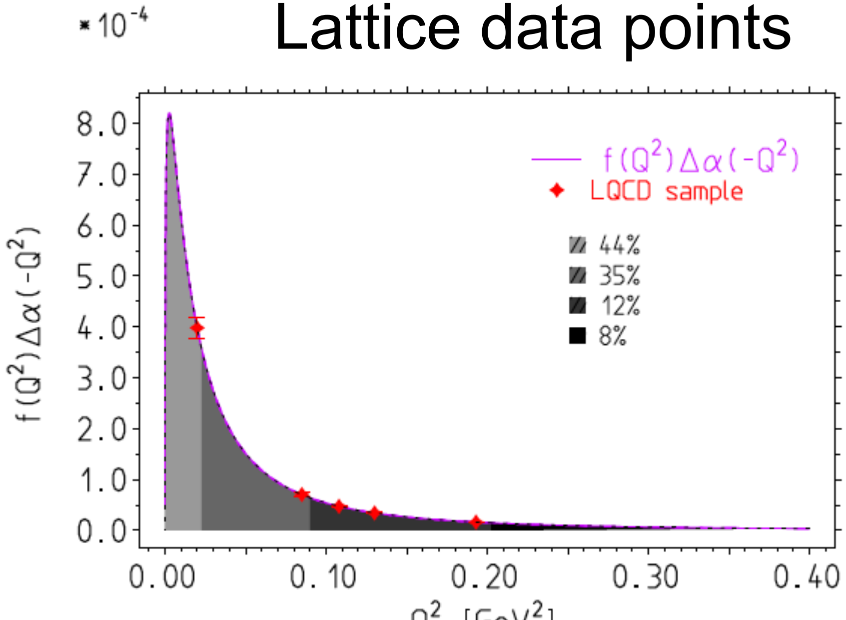
Thiws new approach may become a path within an unexplored region of the field theoretical dynamics

It may lead to a possibly long series of phenomenological results

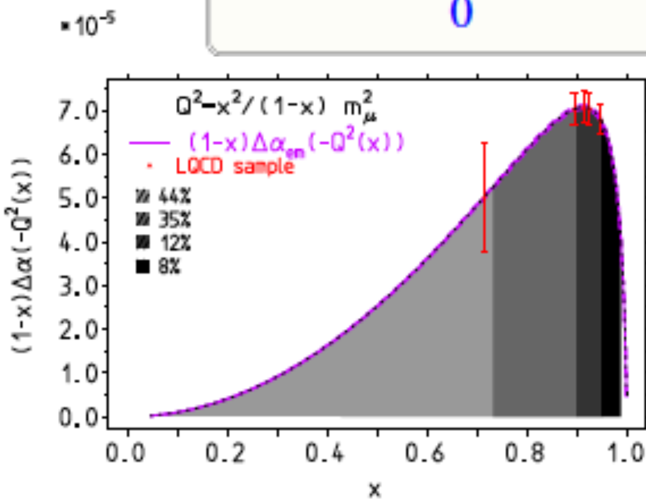
The (crossed) t-channel dynamics, as complementary and independent with respect to the s-channel one will permit an alternative new approach to the Standard Model precision physics

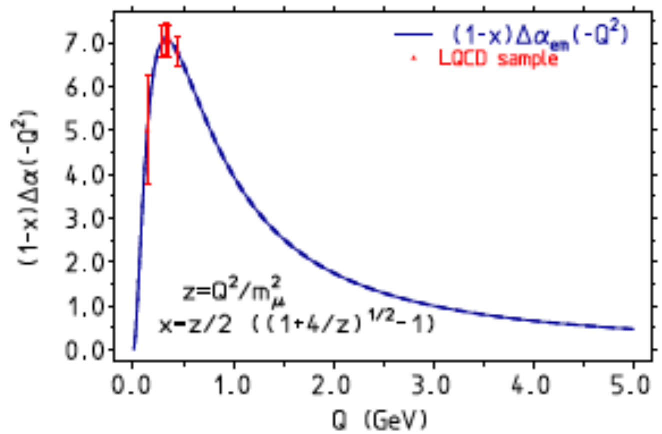
L. Trentadue, Kloe2 Physics Workshop, Frascati, 28/10/16

Lattice data points



$$a_\mu^{\rm had} = \frac{\alpha}{\pi} \int_0^1 {\rm d}x\,(1-x)\,\Delta\alpha^{\rm had}\left(-Q^2(x)\right)$$

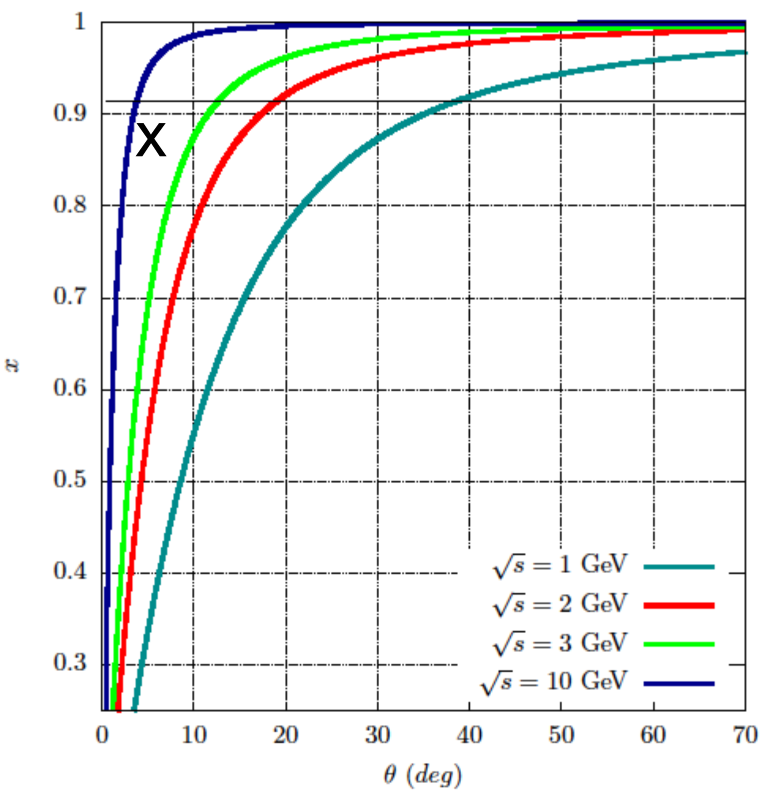


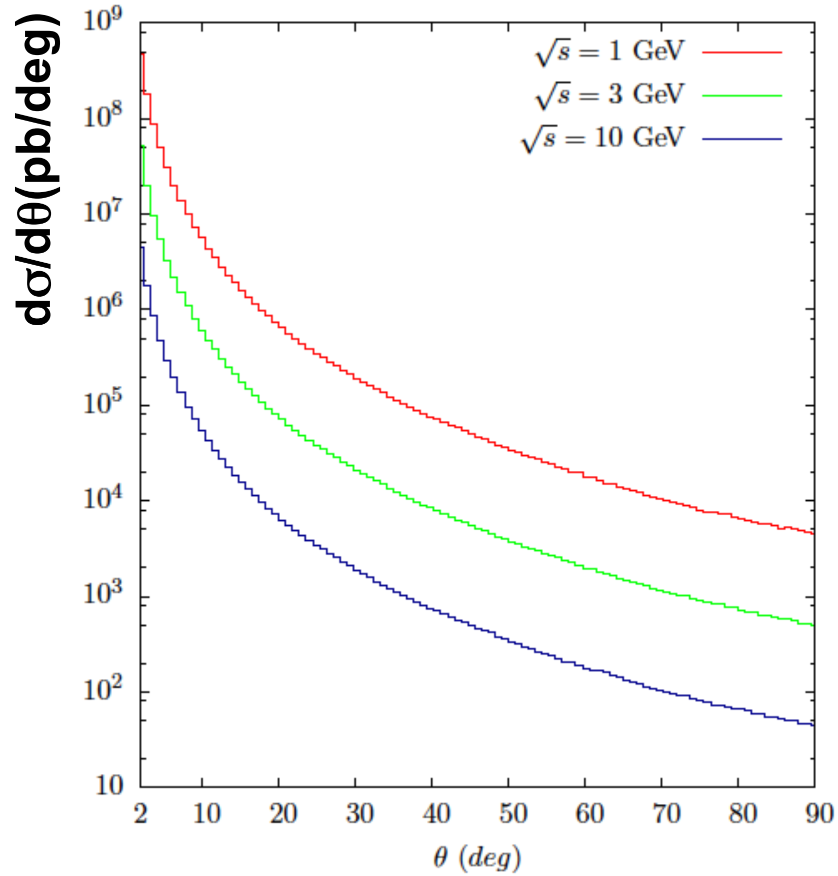


The integrand of $a_\mu^{\rm had}$ integral as functions of x and Q. Strongly peaked at about

Considerations on a dedicated detector

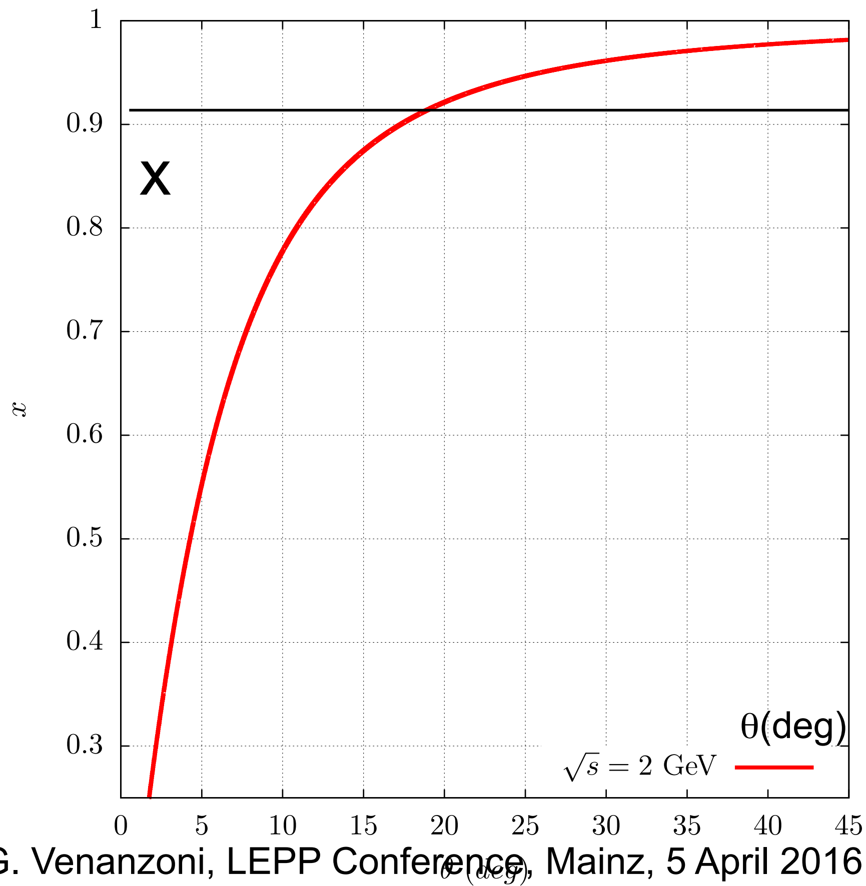
- The detector should be hermetic with a very good momentum resolution and rejection of background (γγ, μμ, hadrons). It should allow to identify the Bhabha with an accuracy < 10^{-4}.
- The luminosity shouldn't be a problem. The design of the detector should depend on the energy of the machine

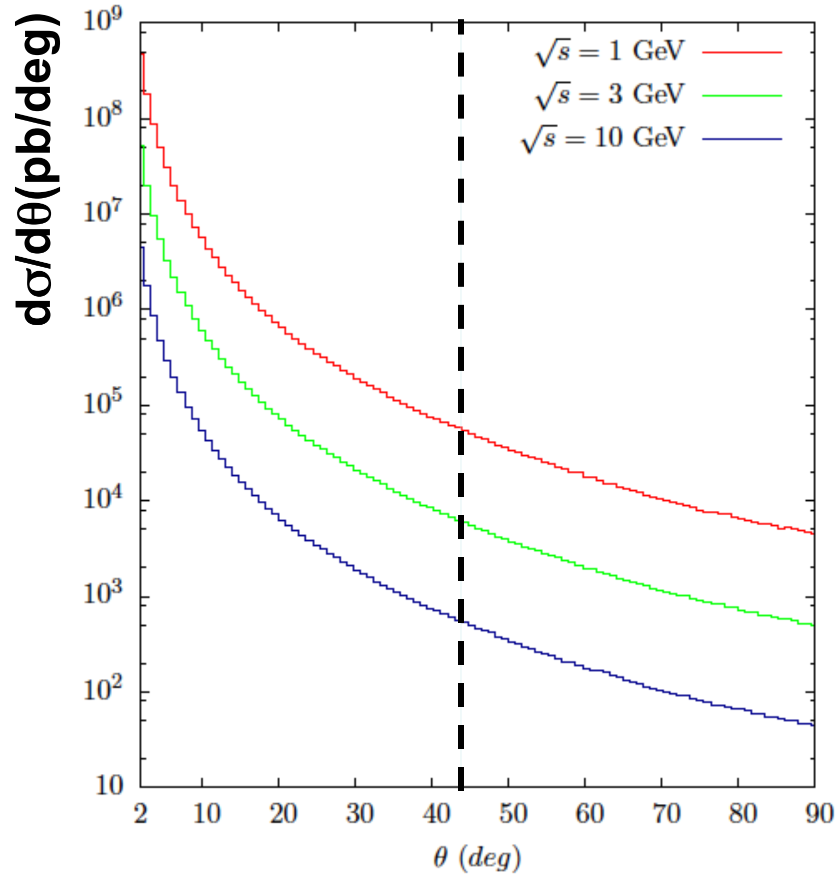




Example: measurement at √s=2 GeV

- The region $0.2<x<0.98$ can be explored at √s=2 GeV with $2^o<\theta<45^o$ (for $x>0.98$ pQCD could be used)
- Normalization can be provided by Bhabha at very small angle ($2^o<\theta<5^o$) where $\Delta\alpha^{had}<10^{-5}$ (1% of the $\Delta\alpha^{had}(x=0.92)$) and statistics is large
- $L=10^{32}$ would allow to do a measurement of $a_\mu^{HLO}<1\%$ within 1 year (statistically)





Muon anomaly

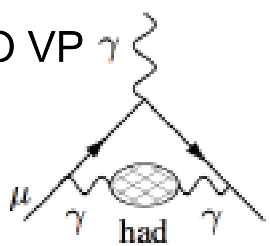
$$a_\mu = \frac{(g_\mu - 2)}{2}$$

- Long established discrepancy (>3σ) between SM prediction and BNL E821 exp.

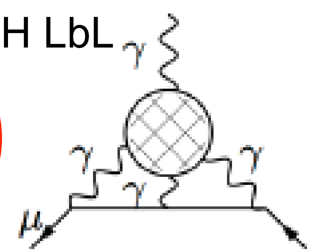
- Theoretical error δa_μ^{SM} (~6×10^{-10}) dominated by HLO VP ($4\div5\times10^{-10}$) and HLbL ($[2.5\div4]\times10^{-10}$). A twofold improvement on δa_μ^{SM} from 2001 (thanks to new e^+e^- measurements)!

- Experimental error δa_μ^{EXP} ~6×10^{-10} (E821). Plan to reduce it to $1.5\ 10^{-10}$ by the new g-2 experiments at FNAL and J-PARC.

HLO VP



H LbL



$a_\mu^{HLO} = (690.9\pm4.4)10^{-10}$ [Eidelman, TAU08]

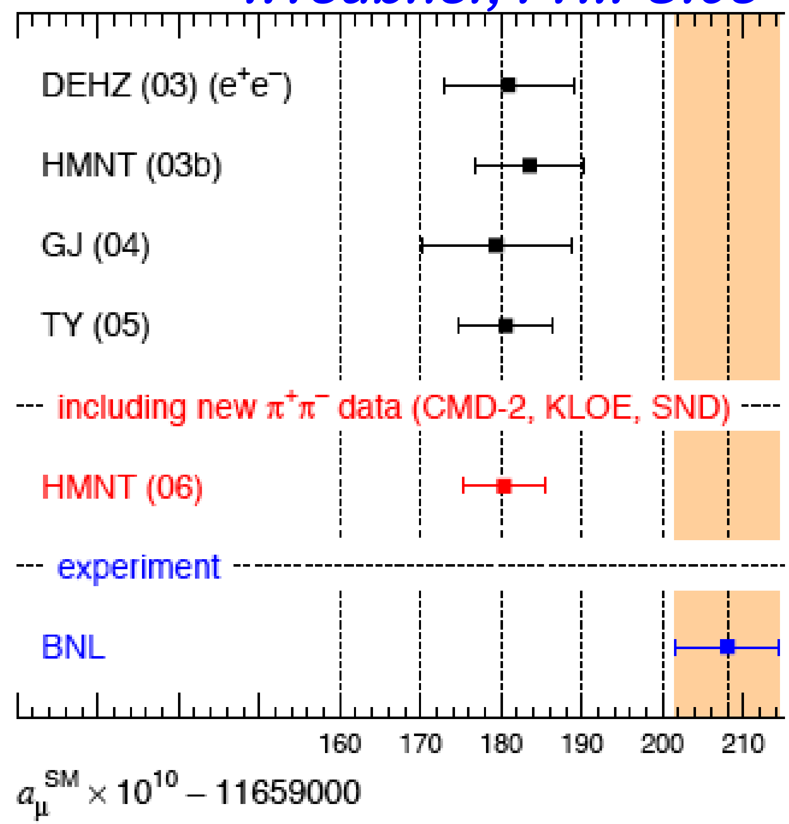
δa_μ^{HLO} ~0.7%

$a_\mu^{HLbL} = (10.5\pm2.6)10^{-10}$ [Prades, dR&V. 08]
$(11\pm4)10^{-10}$ (Jegerlehner, Nyffler)

δa_μ^{HLbL} ~25-40%

a_μ^{SM} compared to BNL world av.

T.Teubner, PHIPSI08



$$a_\mu^{EXP} - a_\mu^{TH} = (27.6 \pm 8.1)\cdot 10^{-10}, \ \sim 3.4\sigma$$

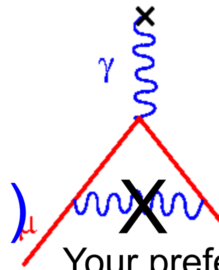
In 2001 $a_\mu^{EXP} - a_\mu^{TH} = (23\pm16)\cdot10^{-10}$

New Physics?

$$a_\mu^{TH} = a_\mu^{QED} + a_\mu^{HAD} + a_\mu^{Weak} + a_\mu^{???}$$

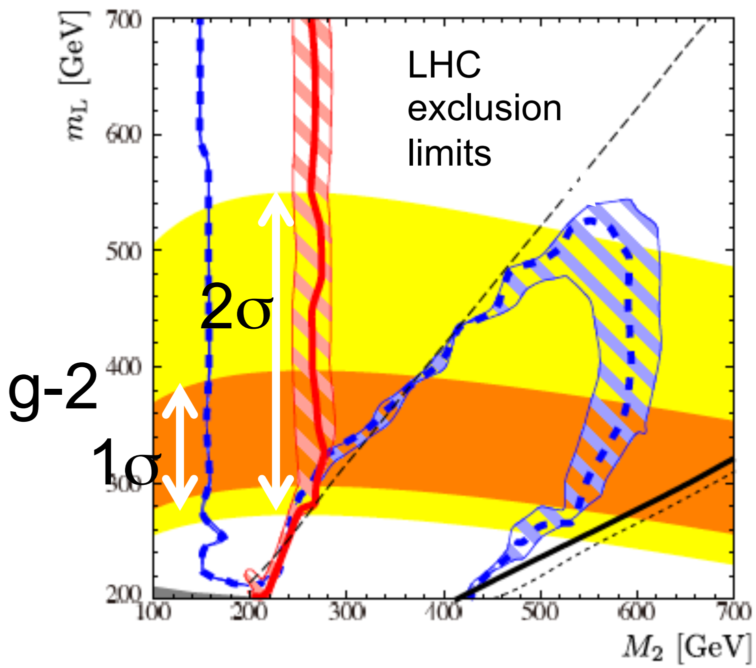
$\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = 288(63)(49)\times10^{-11}$ ~2 a_μ^{Weak} (154×10^{-11})

(BNL) (SM)



Your preferred BSM theory

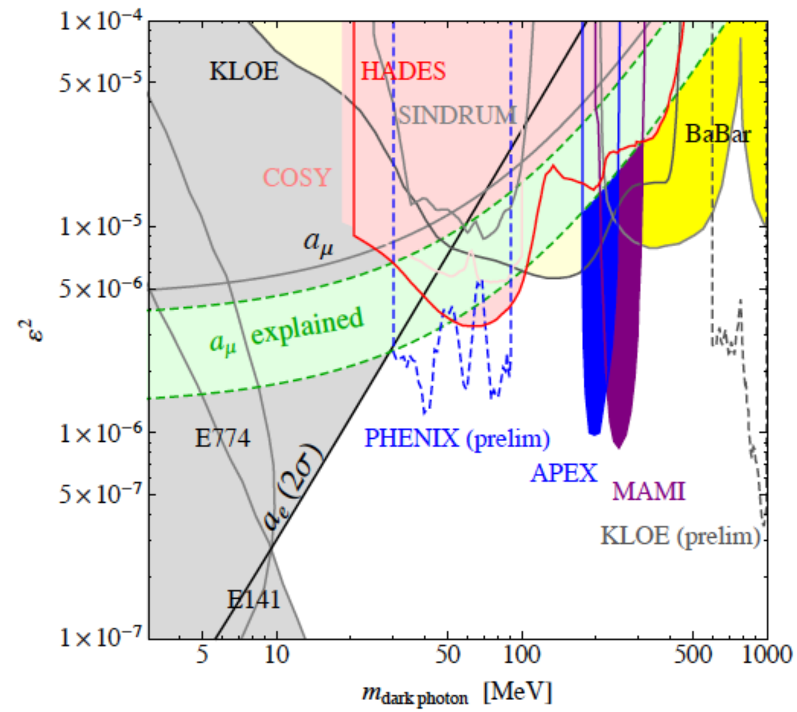
SUSY?



(d) $\mu = 2\,\mathrm{TeV}$, $m_R = 1.5 m_L$

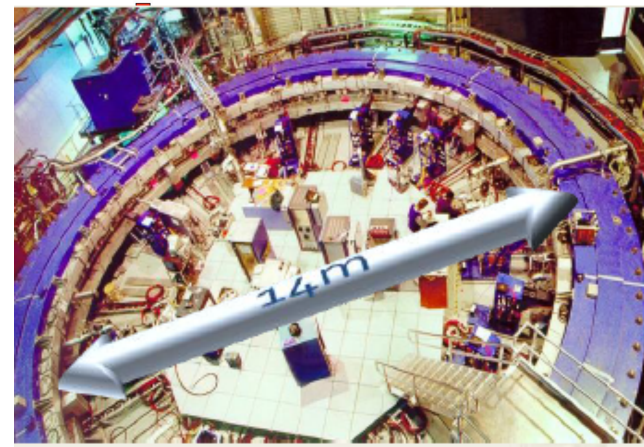
[Endo,Hamaguchi,Iwamoto,Yoshinaga '13]

Dark Photons?



$(g-2)_\mu$: a new experiment at FNAL (E989)

- New experiment at FNAL (E989) at magic momentum, consolidated method. **20 x stat**. w.r.t. E821. Relocate the BNL storage ring to FNAL.

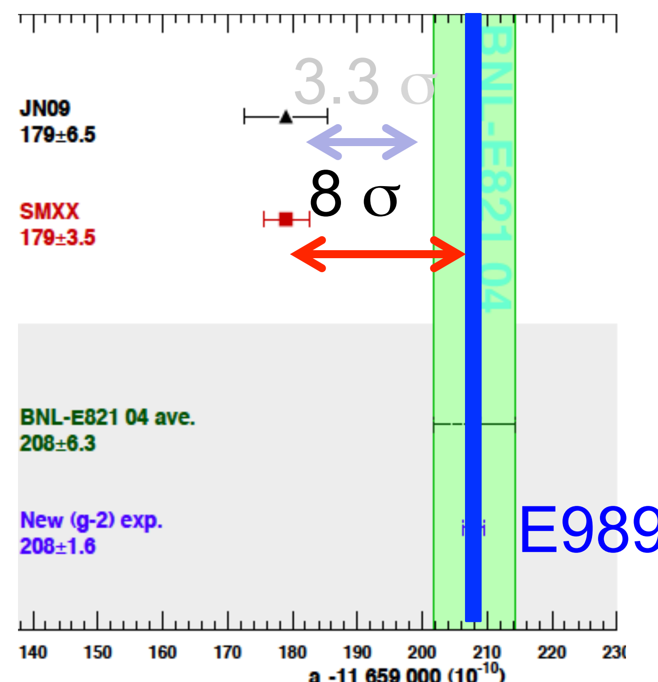


→ δa_μx4 improvement (0.14ppm)

If the central value remains the same ⇒ 5-8σ from SM* (enough to claim discovery of New Physics!)

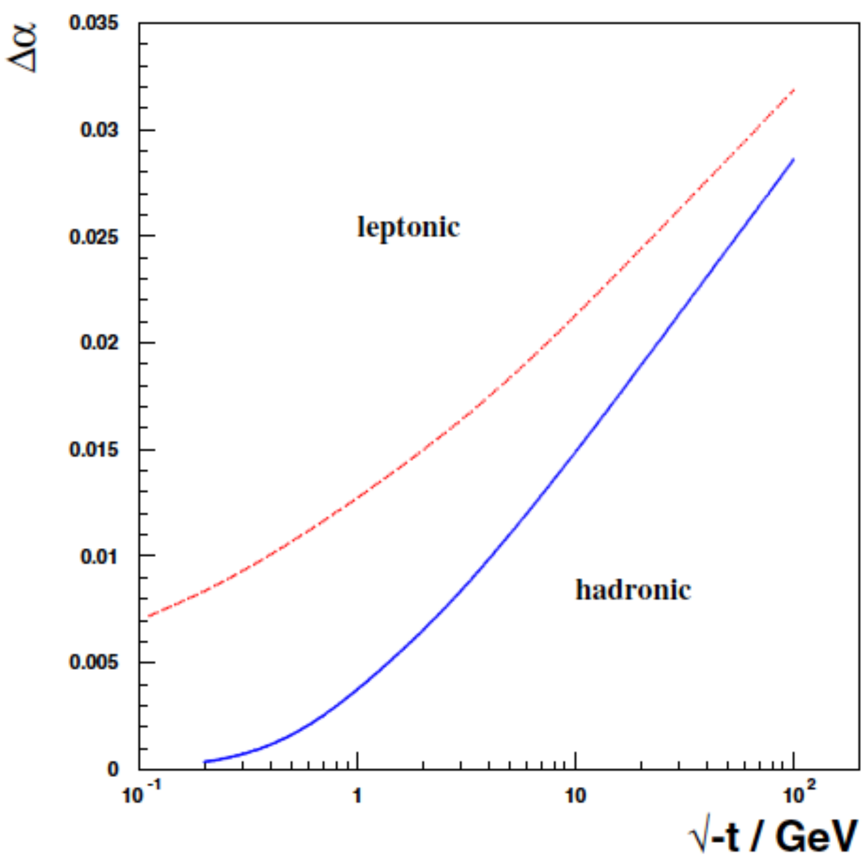
*Depending on the progress on Theory

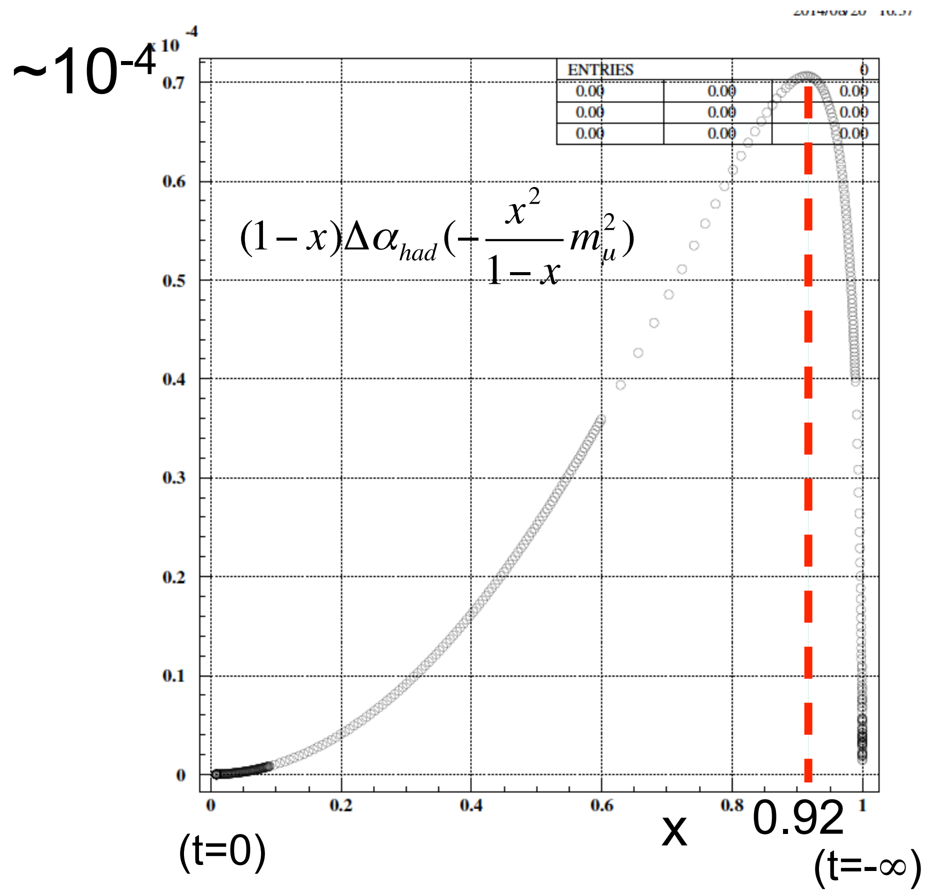
Thomas Blum; Achim Denig; Ivan Logashenko; Eduardo de Rafael; Lee oberts, B.; Thomas Teubner; Graziano Venanzoni (2013). "The Muon (g-2) heory Value: Present and Future". arXiv:1311.2198 [hep-ph].



Alternative proposal at JPARC in progress [H. Iinuma JPC 295 (2011) 012032

Behaviors



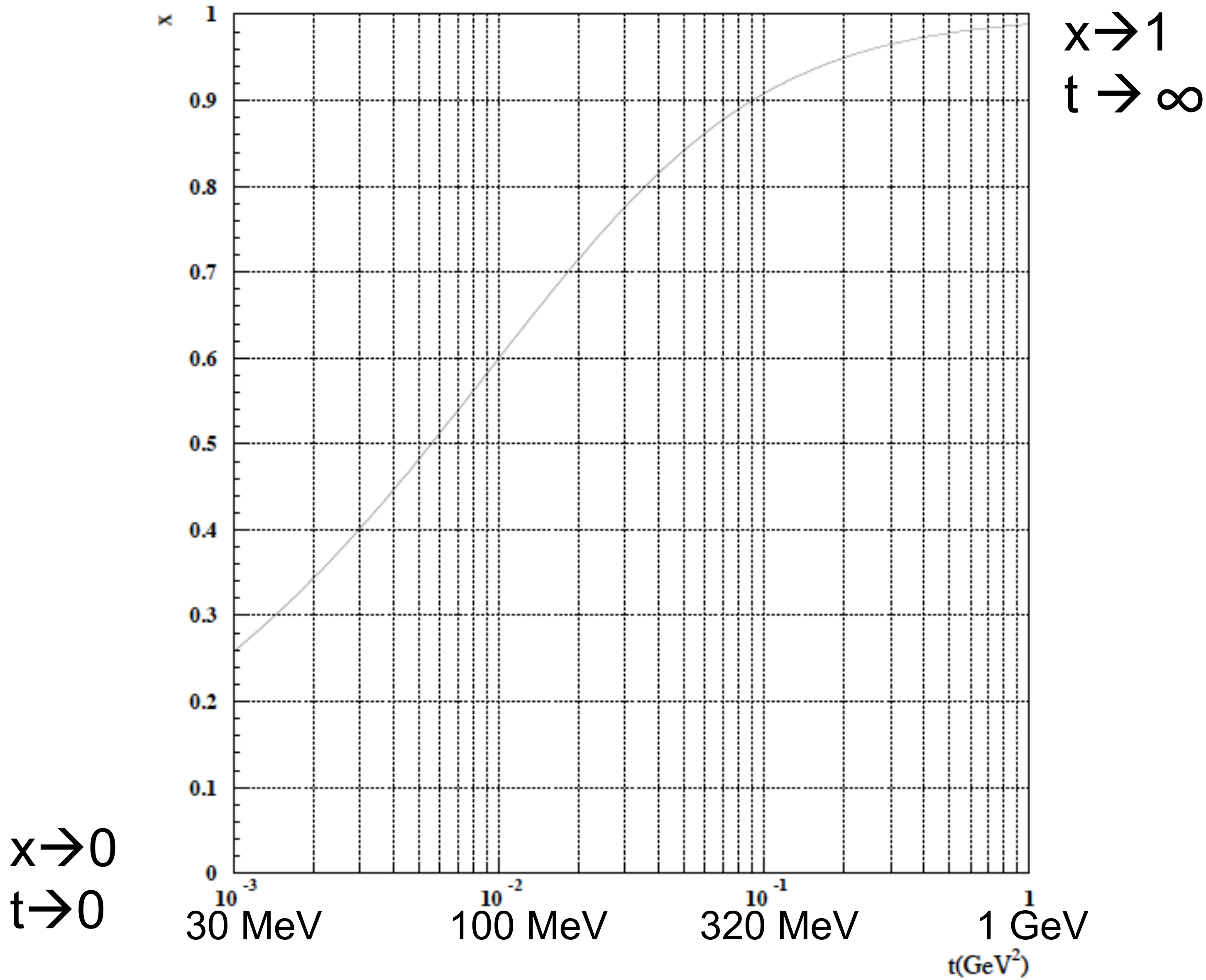


$\Delta\alpha$~log(-t)
Dominated at low |t| by leptonic contribution

A. Arbuzov et al., Eur. Phys. J. C 34 (2004) 267

High |t|-values are depressed by 1-x
(a kind of analogy with time-like region)
The integrand is peaked at ~x=0.92
→t=-0.11 GeV^2 (~0.33 GeV) for which
$\Delta\alpha_{had}$(0.92)~ 10^{-3}

x vs t behaviour



Measurement of DAFNE Luminosity with KLOE/KLOE-2 at 10^{-4}?

F. Ambrosino et al [KLOE] Eur. Phys. J. C 47, 589–596 (2006)

Table 2. Summary of the corrections and systematic errors in the measurement of the luminosity

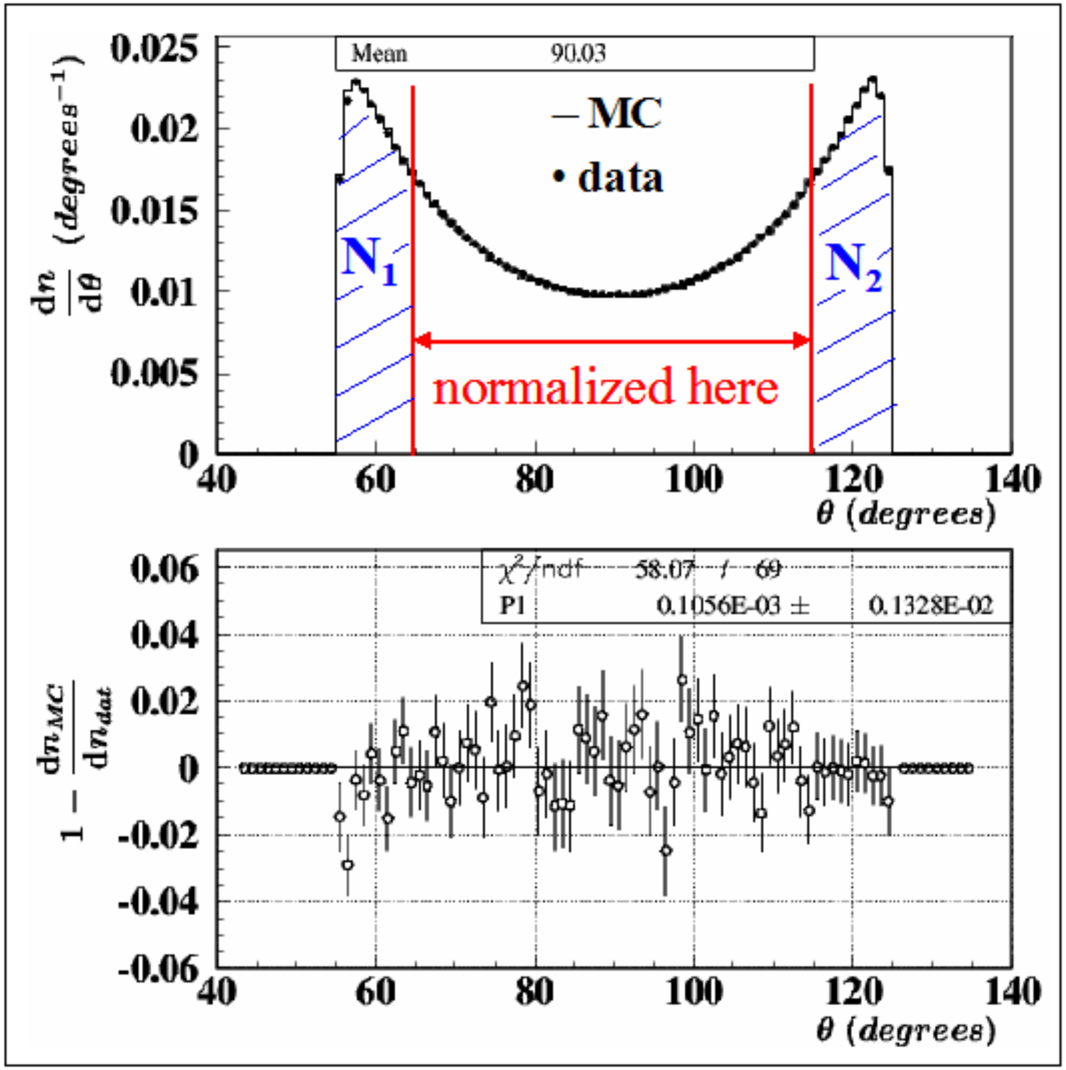
	correction (%)	systematic error (%)
angular acceptance	+0.25	0.25
tracking	–	0.06
clustering	+0.14	0.11
background	−0.62	0.13
cosmic veto	+0.40	–
energy calibration	–	0.10
center of mass energy	+0.10	0.10
	+0.34	0.32

Adding in quadrature: 0.3 %

(can be improved by a factor 10?)

From F. Nguyen 2006

Polar angle systematics



✓ global agreement is very good

but the cut occurs in a steep region of the distributions

⇒ estimate of border mismatches

✓ after normalizing MC to make it coincide with data in the region $65^o < \theta < 115^o$, we estimate as a systematic error:

$$\frac{N^{dat}_{[55:65]+[115:125]} - N^{MC}_{[55:65]+[115:125]}}{N^{dat}_{TOT}} \sim 0.25\%$$

Can be improved at 10^{-4}?

A measurement of the Luminosity at 10^{-4} at LEP

Giovanni Abbiendi

INFN - Bologna

Eur. Phys. J. C 45, 1–21 (2006)
Digital Object Identifier (DOI) 10.1140/epjc/s2005-02389-3

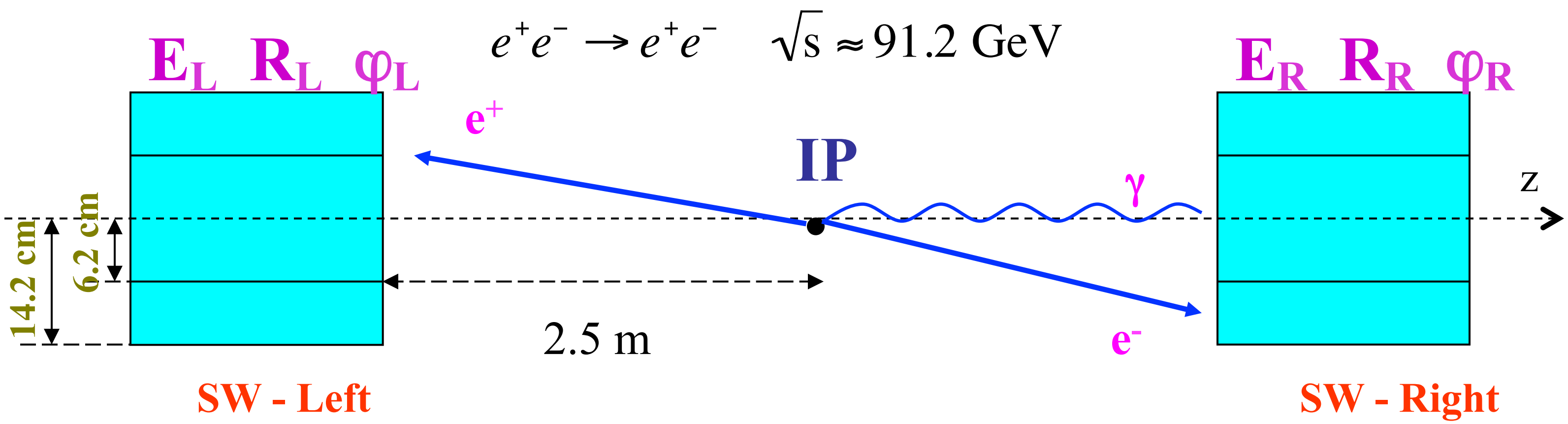
THE EUROPEAN PHYSICAL JOURNAL C

Measurement of the running of the QED coupling in small-angle Bhabha scattering at LEP

The OPAL Collaboration

G. Abbiendi[2], C. Ainsley[5], P.F. Åkesson[3,y], G. Alexander[22], G. Anagnostou[1], K.J. Anderson[9], S. Asai[23], D. Axen[27], I. Bailey[26], E. Barberio[8,p], T. Barillari[32], R.J. Barlow[16], R.J. Batley[5], P. Bechtle[25], T. Behnke[25], K.W. Bell[20], P.J. Bell[1], G. Bella[22], A. Bellerive[6], G. Benelli[4], S. Bethke[32], O. Biebel[31], O. Boeriu[10], P. Bock[11], M. Boutemeur[31], S. Braibant[2], R.M. Brown[20], H.J. Burckhart[8], S. Campana[4], P. Capiluppi[2], R.K. Carnegie[6], A.A. Carter[13], J.R. Carter[5], C.Y. Chang[17], D.G. Charlton[1], C. Ciocca[2], A. Csilling[29], M. Cuffiani[2], S. Dado[21], G.M. Dallavalle[2], A. De Roeck[8], E.A. De Wolf[8,s], K. Desch[25], B. Dienes[30], J. Dubbert[31], E. Duchovni[24], G. Duckeck[31], I.P. Duerdoth[16], E. Etzion[22], F. Fabbri[2], P. Ferrari[8], F. Fiedler[31], I. Fleck[10], M. Ford[16], A. Frey[8], P. Gagnon[12], J.W. Gary[4], C. Geich-Gimbel[3], G. Giacomelli[2], P. Giacomelli[2], R. Giacomelli[2], M. Giunta[4], J. Goldberg[21], E. Gross[24], J. Grunhaus[22], M. Gruwé[8], P.O. Günther[3], A. Gupta[9], C. Hajdu[29], M. Hamann[25], G.G. Hanson[4], A. Harel[21], M. Hauschild[8], C.M. Hawkes[1], R. Hawkings[8], R.J. Hemingway[6], G. Herten[10], R.D. Heuer[25], J.C. Hill[5], D. Horváth[29,c], P. Igo-Kemenes[11], K. Ishii[23], H. Jeremie[18], P. Jovanovic[1], T.R. Junk[6,i], J. Kanzaki[23,u], D. Karlen[26], K. Kawagoe[23], T. Kawamoto[23], R.K. Keeler[26], R.G. Kellogg[17], B.W. Kennedy[20], S. Kluth[32], T. Kobayashi[23], M. Kobel[3], S. Komamiya[23], T. Krämer[25], P. Krieger[6,l], J. von Krogh[11], T. Kuhl[25], M. Kupper[24], G.D. Lafferty[16], H. Landsman[21], D. Lanske[14], D. Lellouch[24], J. Letts[o], L. Levinson[24], J. Lillich[10], S.L. Lloyd[13], F.K. Loebinger[16], J. Lu[27,w], A. Ludwig[3], J. Ludwig[10], W. Mader[3,b], S. Marcellini[2], A.J. Martin[13], T. Mashimo[23], P. Mättig[m], J. McKenna[27], R.A. McPherson[26], F. Meijers[8], W. Menges[25], F.S. Merritt[9], H. Mes[6,a], N. Meyer[25], A. Michelini[2], S. Mihara[23], G. Mikenberg[24], D.J. Miller[15], W. Mohr[10], T. Mori[23], A. Mutter[10], K. Nagai[13], I. Nakamura[23,v], H. Nanjo[23], H.A. Neal[33], R. Nisius[32], S.W. O'Neale[1,*], A. Oh[8], M.J. Oreglia[9], S. Orito[23,*], C. Pahl[32], G. Pásztor[4,g], J.R. Pater[16], J.E. Pilcher[9], J. Pinfold[28], D.E. Plane[8], O. Pooth[14], M. Przybycień[8,n], A. Quadt[3], K. Rabbertz[8,r], C. Rembser[8], P. Renkel[24], J.M. Roney[26], A.M. Rossi[2], Y. Rozen[21], K. Runge[10], K. Sachs[6], T. Saeki[23], E.K.G. Sarkisyan[8,j], A.D. Schaile[31], O. Schaile[31], P. Scharff-Hansen[8], J. Schieck[32], T. Schörner-Sadenius[8,z], M. Schröder[8], M. Schumacher[3], R. Seuster[14,f], T.G. Shears[8,h], B.C. Shen[4], P. Sherwood[15], A. Skuja[17], A.M. Smith[8], R. Sobie[26], S. Söldner-Rembold[16], F. Spano[9], A. Stahl[3,x], D. Strom[19], R. Ströhmer[31], S. Tarem[21], M. Tasevsky[8,s], R. Teuscher[9], M.A. Thomson[5], E. Torrence[19], D. Toya[23], P. Tran[4], I. Trigger[8], Z. Trócsányi[30,e], E. Tsur[22], M.F. Turner-Watson[1], I. Ueda[23], B. Ujvári[30,e], C.F. Vollmer[31], P. Vannerem[10], R. Vértesi[30,e], M. Verzocchi[17], H. Voss[8,q], J. Vossebeld[8,h], C.P. Ward[5], D.R. Ward[5], P.M. Watkins[1], A.T. Watson[1], N.K. Watson[1], P.S. Wells[8], T. Wengler[8], N. Wermes[3], G.W. Wilson[16,k], J.A. Wilson[1], G. Wolf[24], T.R. Wyatt[16], S. Yamashita[23], D. Zer-Zion[4], L. Zivkovic[24]

Small-angle Bhabha scattering in OPAL



2 cylindrical calorimeters encircling the beam pipe at ± 2.5 m from the Interaction Point

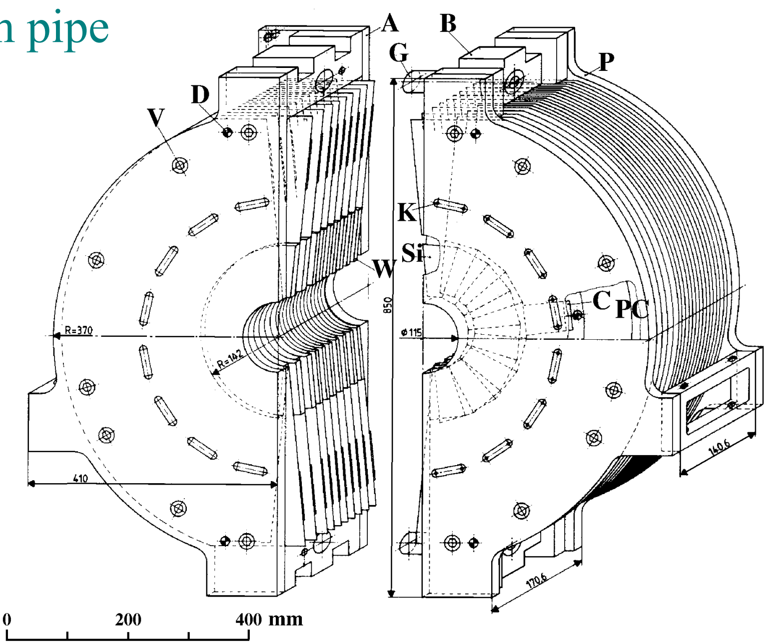
19 Silicon layers

18 Tungsten layers

Total Depth 22 X_0
(14 cm)

Each detector layer divided into 16 overlapping wedges

Sensitive radius: **6.2 – 14.2 cm**, corresponding to **scattering angle** of **25 – 58 mrad** from the beam line



Final Error on Luminosity

After all the effort on Radial reconstruction the dominant systematic error is related to Energy (mostly tail in the E response and nonlinearity) Quantitatively: (OPAL Collaboration, Eur.Phys.J. C14 (2000) 373)

	Systematic Error ($\times 10^{-4}$)
Energy	1.8
Inner Anchor	1.4
Radial Metrology	1.4

Total Experimental Systematic Error : 3.4×10^{-4}

Theoretical Error on Bhabha cross section: 5.4×10^{-4}