Gravity Perturbations from Earthquakes

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Halfspace Model

Perturbation of gravity potential:

$$\delta\phi(z_0) = -2\pi G\rho_0\left[\mathrm{e}^{-k_\varrho|z_0|}\left(\mathrm{sgn}(z_0)\phi_\mathrm{s}(0) + k_\varrho\psi_\mathrm{s}(0)\right) + 2\phi_\mathrm{s}(z_0)\right]$$

Previous equation + elastodynamic equations:

$$\delta\tilde{\phi}_\infty(\vec{k}_\varrho, z_0, s) = 2\pi G\mathrm{e}^{-k_\varrho(z_0-z_\mathrm{s})}\frac{1}{k_\varrho s^2}\cdot\left(k_x, k_y, \mathrm{i}k_\varrho\right)\cdot\mathbf{M}\cdot\begin{pmatrix}k_x\\k_y\\\mathrm{i}k_\varrho\end{pmatrix}$$

$$\delta\tilde{\phi}_\alpha(\vec{k}_\varrho, z_0, s) = -4\pi G\mathrm{e}^{-k_\varrho z_0+\nu_\alpha z_\mathrm{s}}\frac{1}{s^2}\frac{k_\varrho(k_\varrho-\nu_\beta)^2}{(\nu_\beta^2+k_\varrho^2)^2-4\nu_\alpha\nu_\beta k_\varrho^2}\cdot\left(k_x, k_y, \mathrm{i}\nu_\alpha\right)\cdot\mathbf{M}\cdot\begin{pmatrix}k_x\\k_y\\\mathrm{i}\nu_\alpha\end{pmatrix}$$

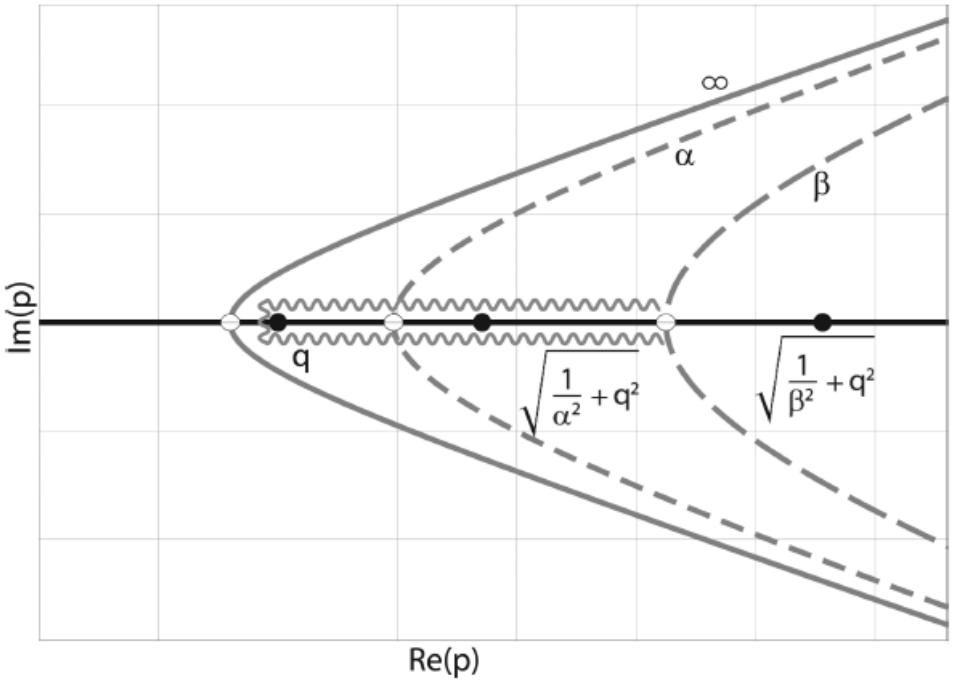
$$\delta\tilde{\phi}_\beta(\vec{k}_\varrho, z_0, s) = -4\pi G\mathrm{e}^{-k_\varrho z_0+\nu_\beta z_\mathrm{s}}\frac{1}{s^2}\frac{k_\varrho^2-2k_\varrho\nu_\alpha+\nu_\beta^2}{(\nu_\beta^2+k_\varrho^2)^2-4\nu_\alpha\nu_\beta k_\varrho^2}\cdot\left(k_x\nu_\beta, k_y\nu_\beta, \mathrm{i}k_\varrho^2\right)\cdot\mathbf{M}\cdot\begin{pmatrix}k_x\\k_y\\\mathrm{i}\nu_\beta\end{pmatrix}$$

Cagniard – de Hoop

Basic idea:
Transform integrals on the left into form on the right.

$$\delta\bar{\phi}(\vec{\varrho}_0, s) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \mathrm{d}k_x \int_{-\infty}^{\infty} \mathrm{d}k_y \, \mathrm{e}^{\mathrm{i}\vec{k}_\varrho \cdot \vec{\varrho}_0} \delta\tilde{\phi}(\vec{k}_\varrho, s)$$

$$\delta\bar{\phi}(\vec{\varrho}_0, s) = \int_0^{\infty} \mathrm{d}t \, \mathrm{e}^{-st} \delta\phi(\vec{\varrho}_0, t)$$

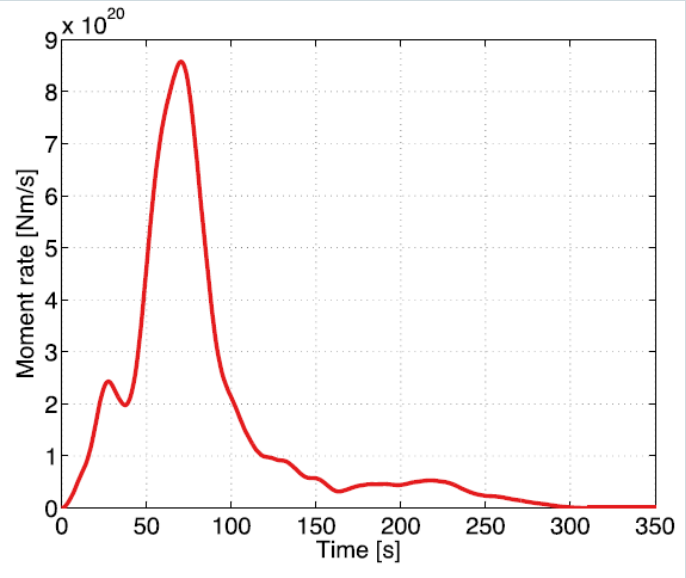


Remaining problem:
Change of integration variables then makes it necessary to perform a rather complicated integration in a complex plane.

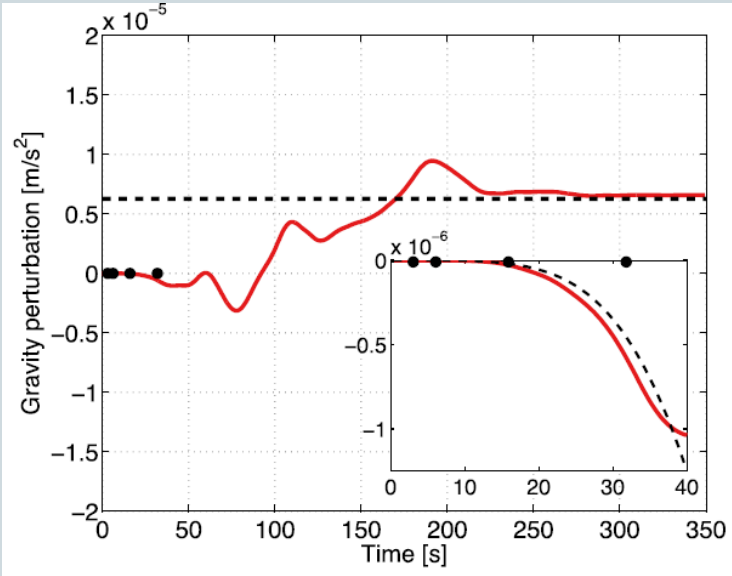
Cagniard – de Hoop method suggests a certain integration path.

Tohoku-Oki Earthquake

Moment rate function

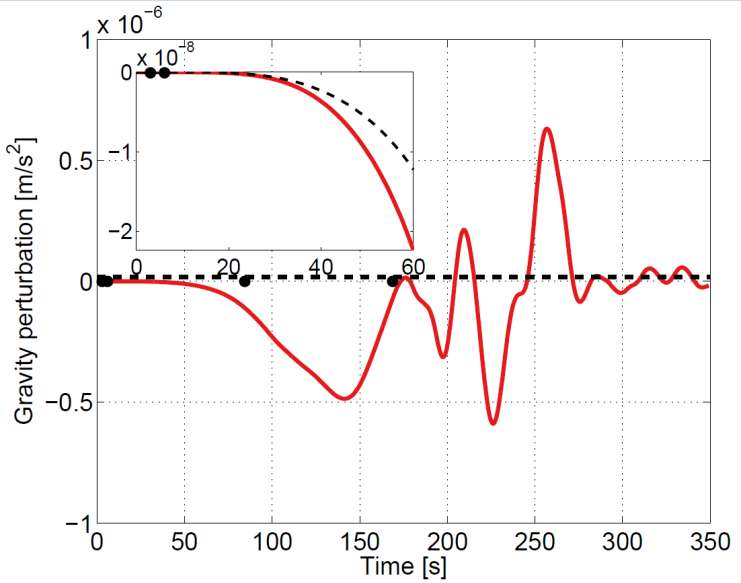


Gravity perturbation along the vertical at 100km distance to epicenter at a direction perpendicular to the fault line

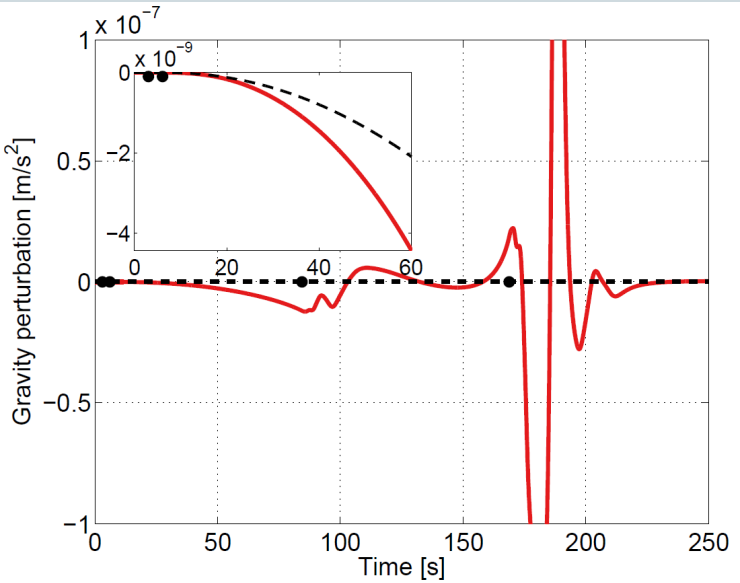


Dependence on Magnitude

Tohoku-Oki (M=9.1 according to moment function)



Same focal mechanism, same depth, but: rupture duration and moment rate divided by 10 (M=7.8)



Both plots for Kamioka mine

Fullspace / Halfspace