

# Miroirs et cavités temporelles

**Emmanuel Fort**

*Institut Langevin, ESPCI*  
[emmanuel.fort@espci.fr](mailto:emmanuel.fort@espci.fr)



**Institut Langevin**  
ONDES ET IMAGES



**AXA**  
Research Fund

# Authors



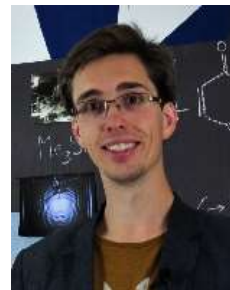
**Stéphane Perrard**



**Vincent Bacot**



**Matthieu Labousse**



**Guillaume Durey**



**Mathias Fink**



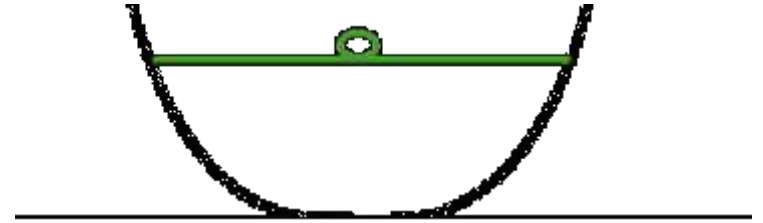
**Yves Couder**



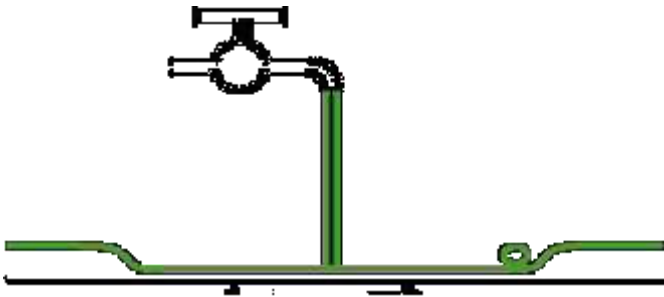
**Antonin Eddi**

# Coalescence

- Drop deposited on the surface of the same fluid coalesce within 0.1 seconde



- It is possible to prevent coalescence



For instance, in hydraulic jumps

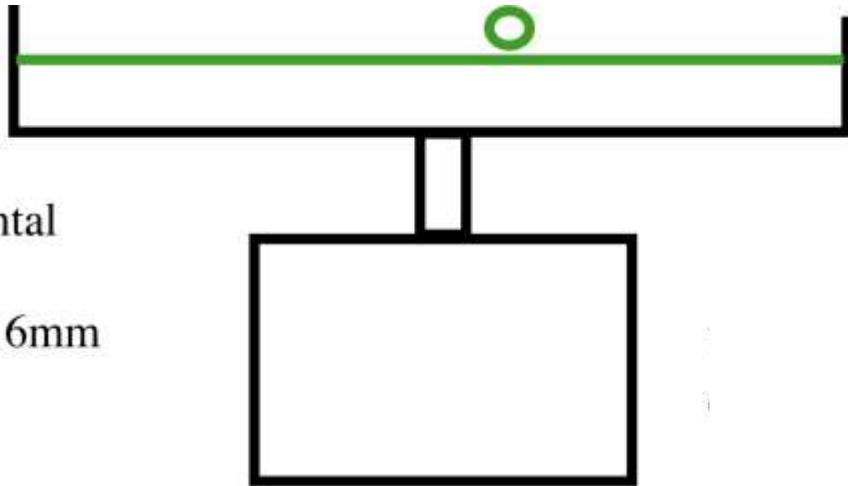


*By courtesy of L. Limat*

# Basic experimental setup

Fluids: Silicon oils with viscosities

$$5 \cdot 10^{-3} \text{ Pa}\cdot\text{s} < \mu < 500 \cdot 10^{-3} \text{ Pa}\cdot\text{s}$$



Experimental  
cell  
130x130x 6mm

Vertical acceleration

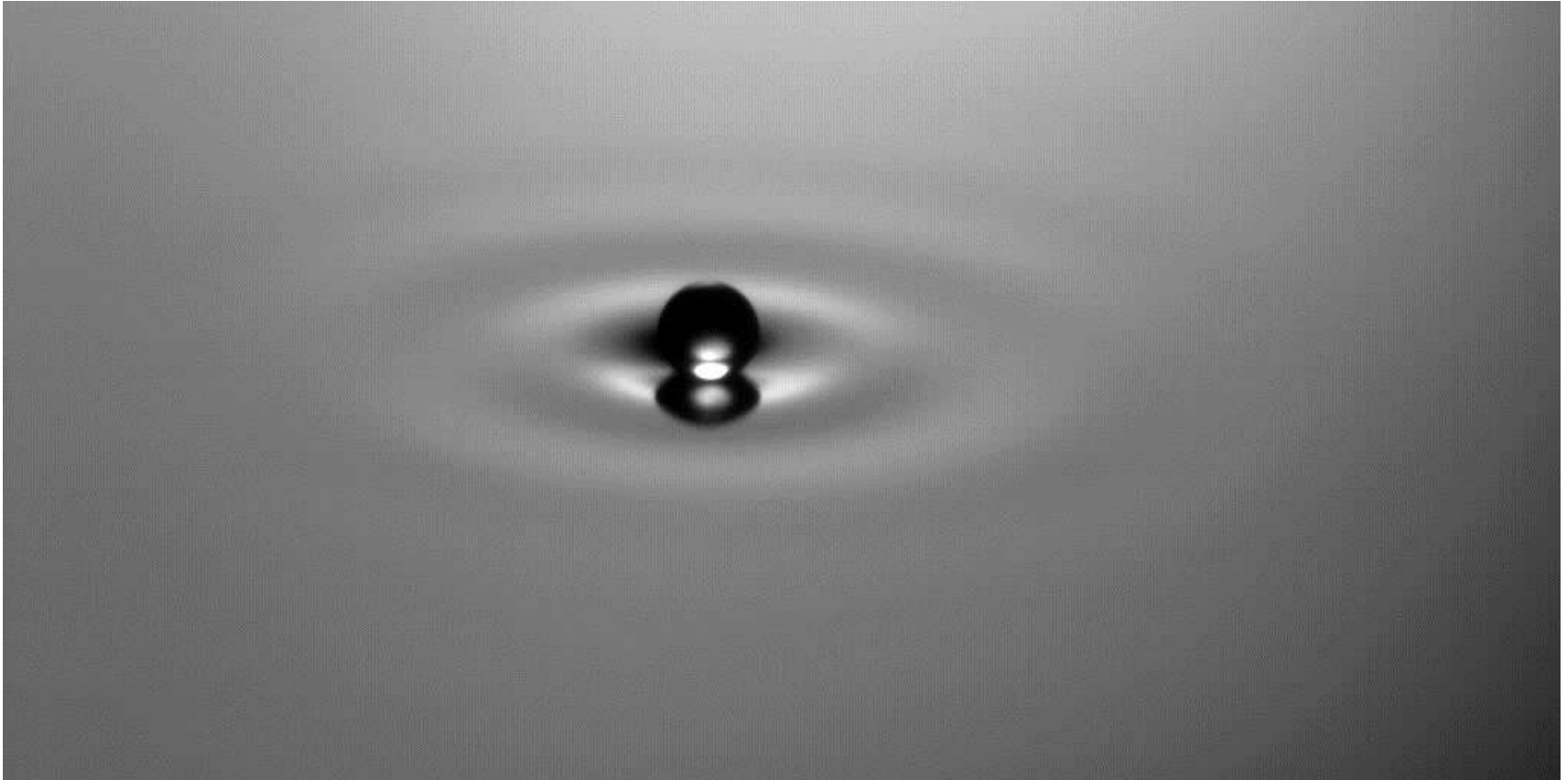
$$\gamma = \gamma_m \cos(\omega t)$$

with  $\gamma_m$  ranging from 0 to 5g

Forcing frequency range:

$$10 < \omega/2\pi < 300 \text{ Hz}$$

# Bouncing droplet

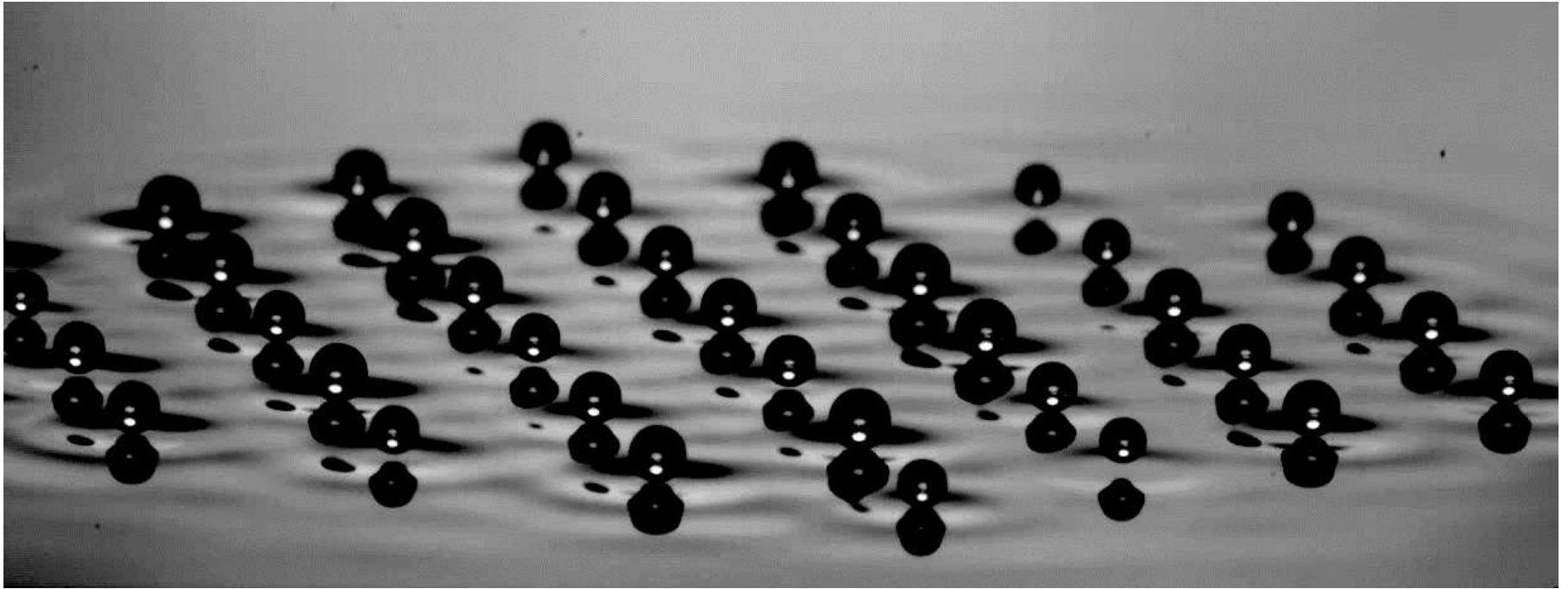


Fast camera

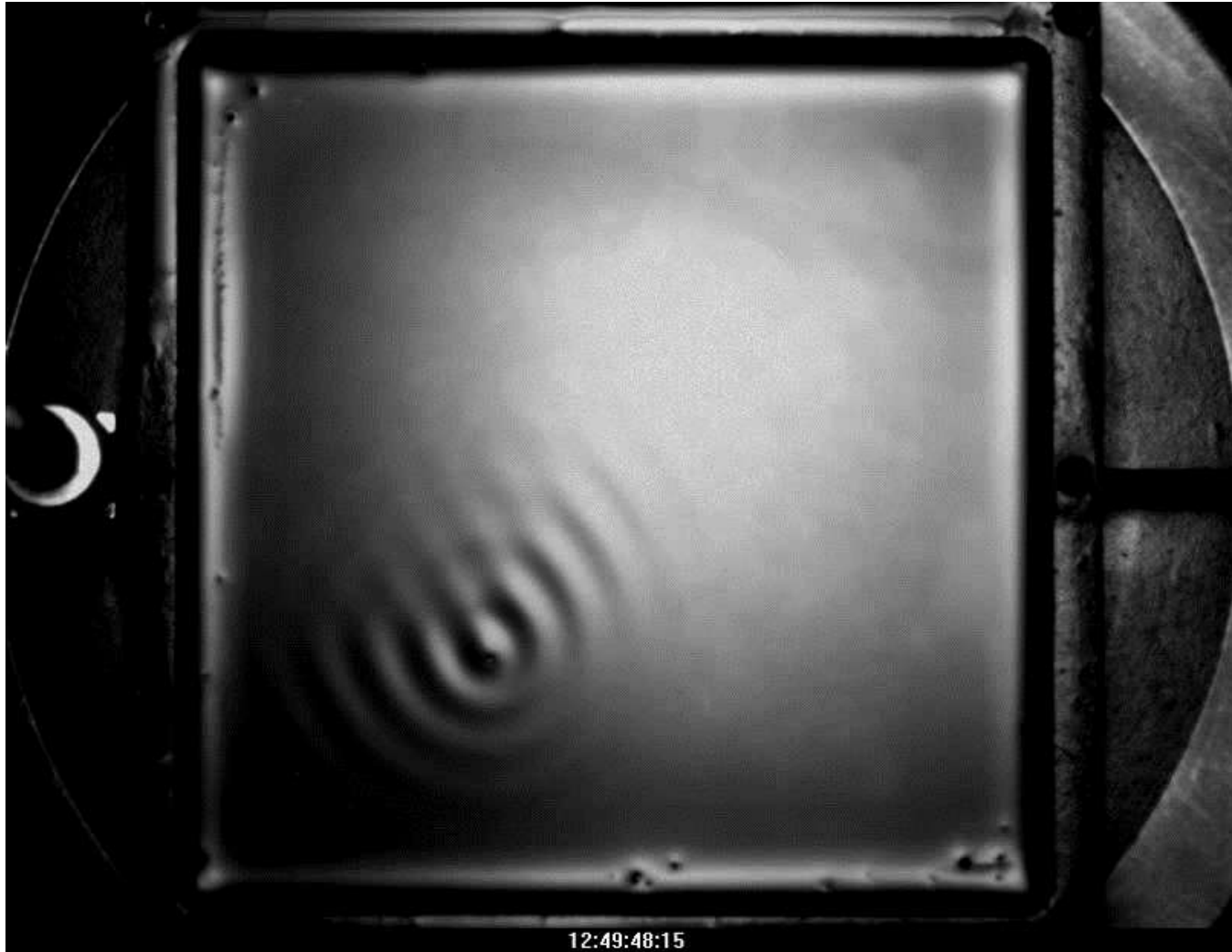
# How to make a droplet?



# Droplet crystals



**For specific excitation conditions...  
the droplet becomes selfpropelled**

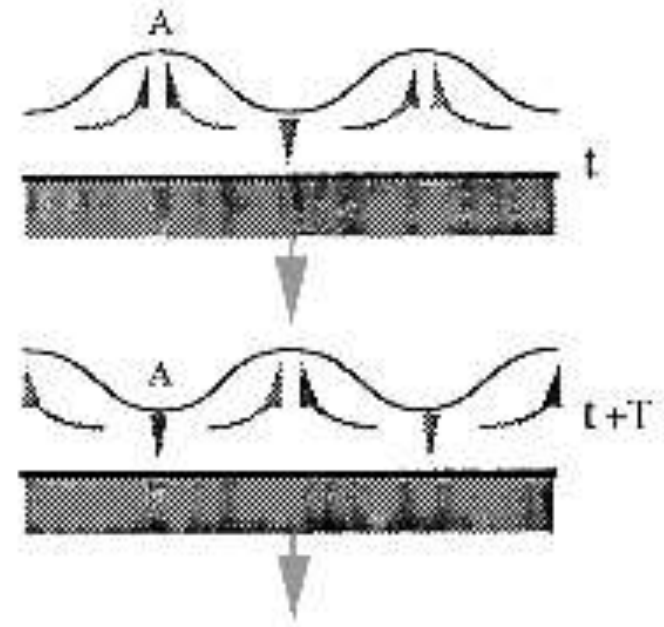
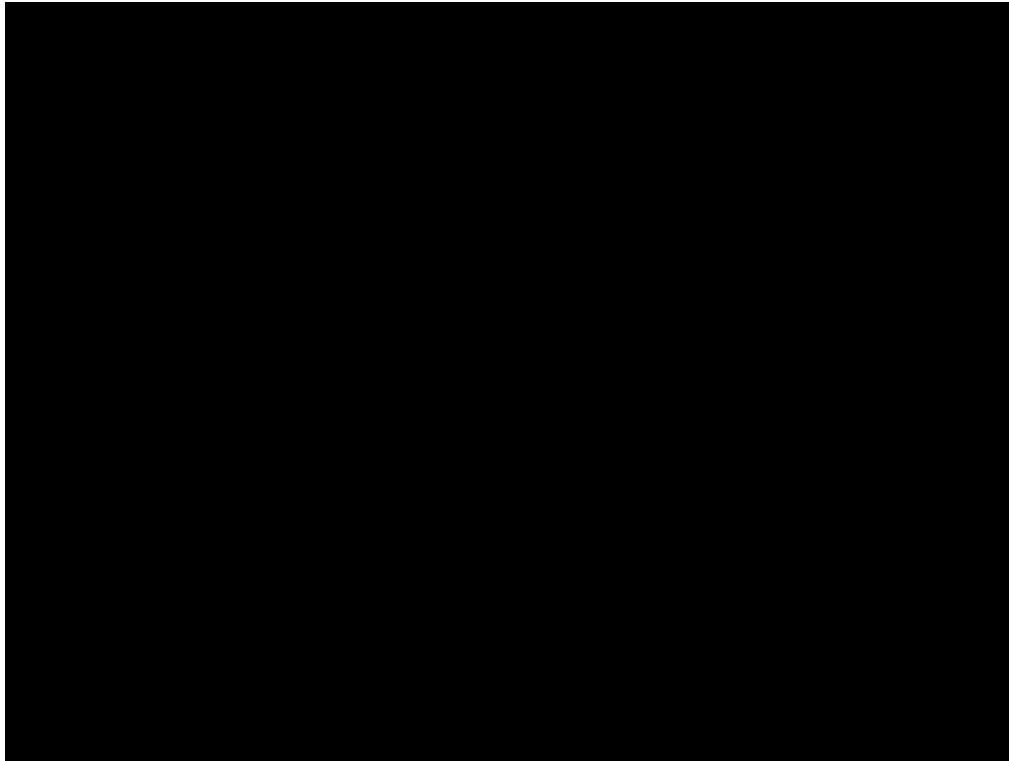


**Top view, real time**



# Faraday instability

# Faraday instability



# Specialist of Faraday instability

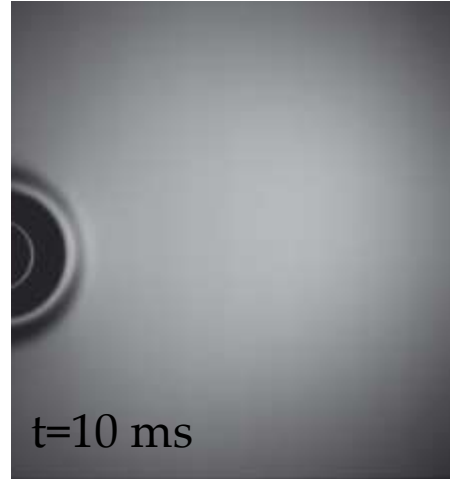


# **Path memory dynamics**

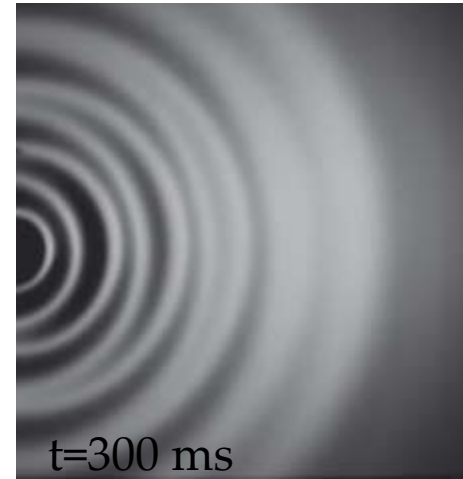
# Wavefield produced by a single collision

Experiment performed with a 2 mm steel ball

Without periodic forcing

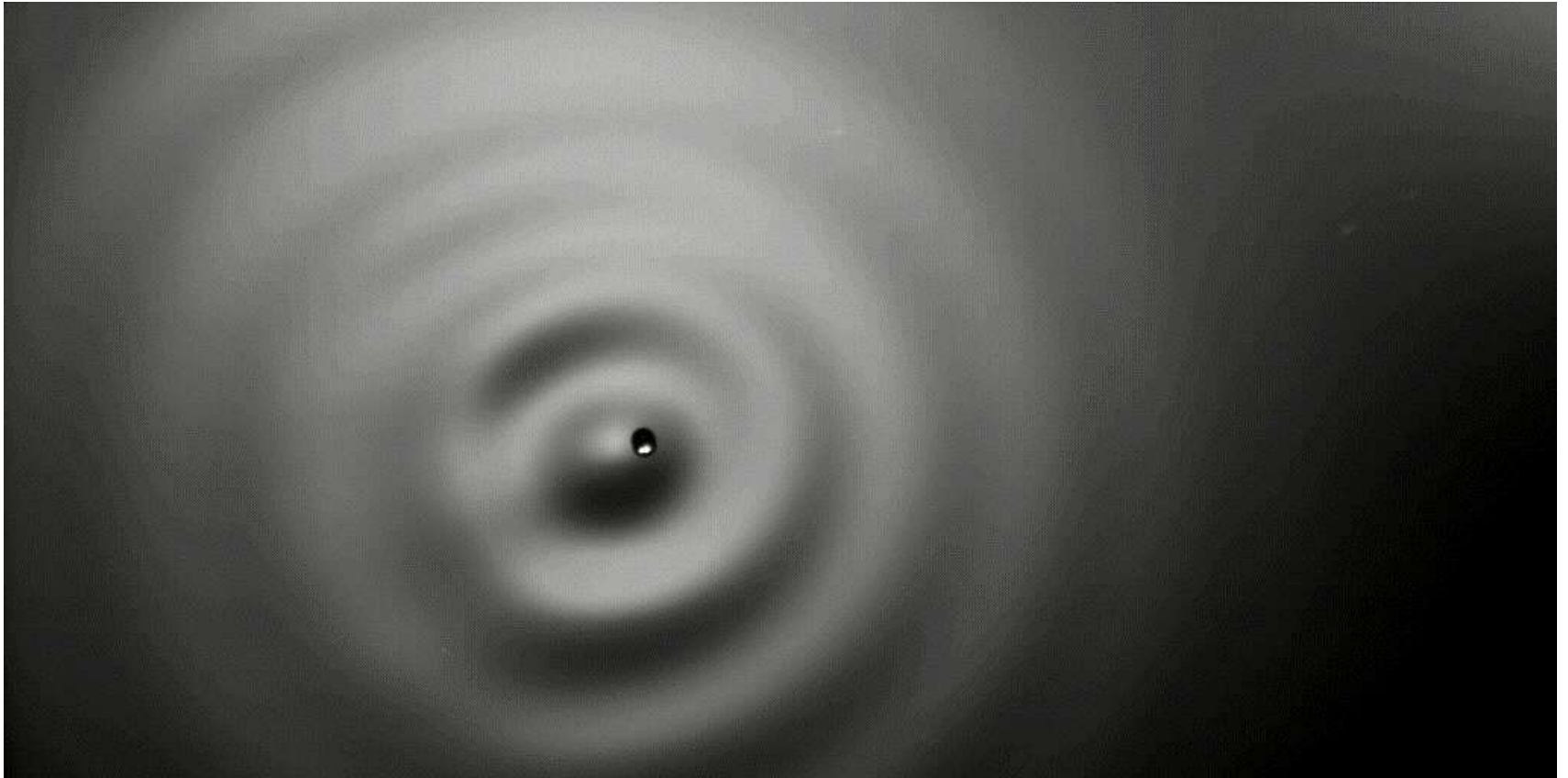


With a periodic forcing near the Faraday instability threshold



**Near the Faraday threshold, the propagating capillary wave triggers  
Stationary Faraday waves around the droplet**

# A Walker: oscillatory motion



**Fast camera, top view**

The droplet falls on the front side of the bump created by the previous bounces  
It gives the droplet a « kick » at each bounce to maintain self propulsion  
The trajectory can be deduced iteratively

# The path-memory model

The relative surface height

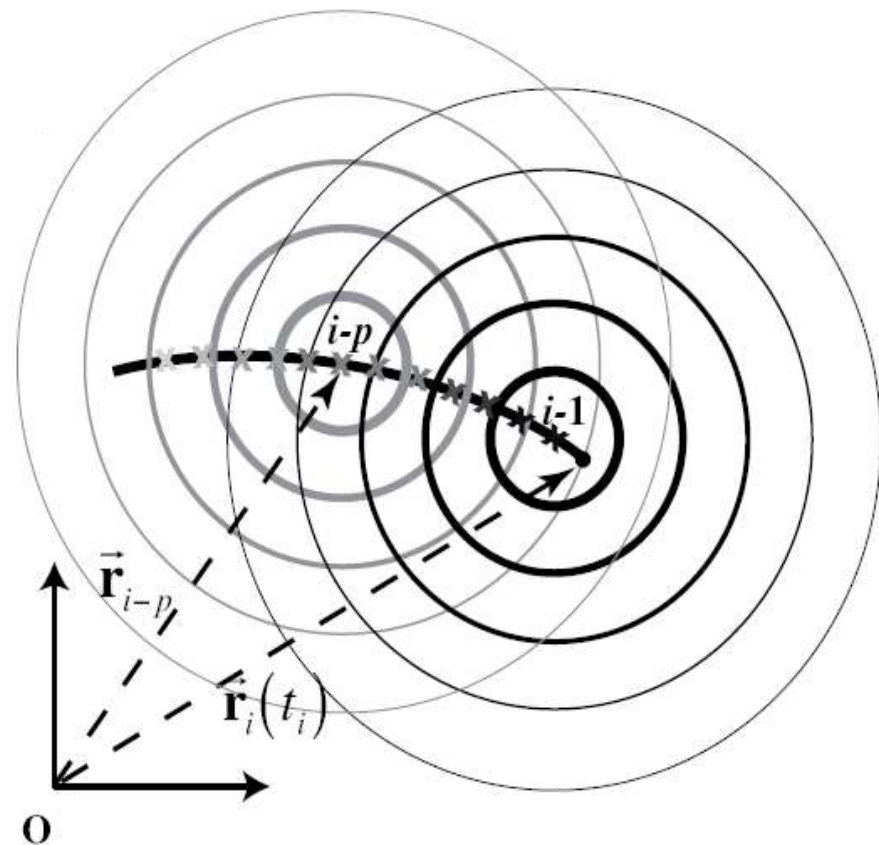
$h(\vec{r}, t_i)$  position  $\vec{r}$  and time  $t_i$  is given by:

$$h(\vec{r}, t_i) = \sum_{p=i-1}^{-\infty} \exp\left(-\frac{t_i - t_p}{\tau}\right) J_0\left(\frac{2\pi|\vec{r} - \vec{r}_p|}{\lambda_F}\right)$$

where  $\vec{r}_p$  is the position of a previous impact which occurred at time  $t_p = t_i - (i - p)T_F$ .

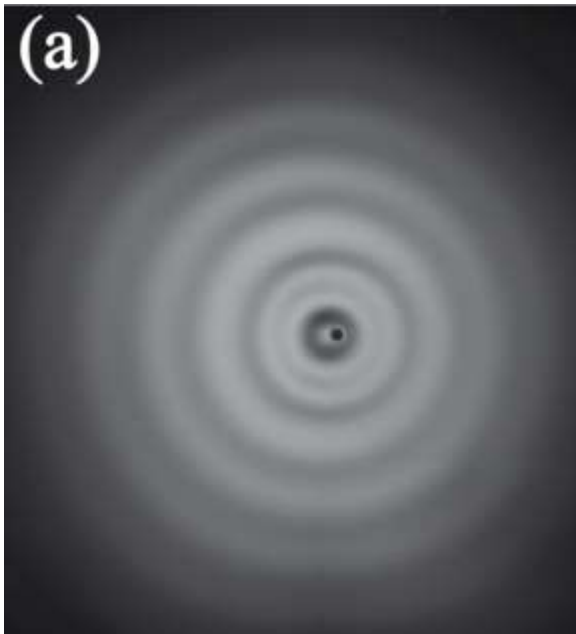
**The damping time,  $\tau$**  is the path memory parameter.

$\tau$  is tunable, being determined by the distance to the Faraday instability threshold.

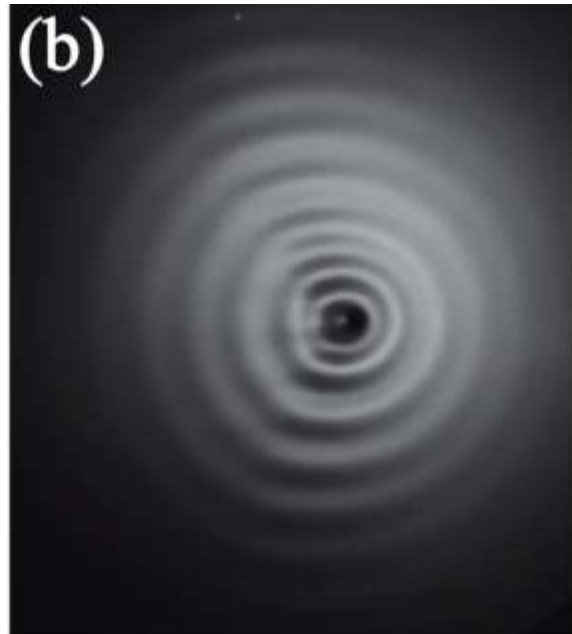


# The path-memory concept

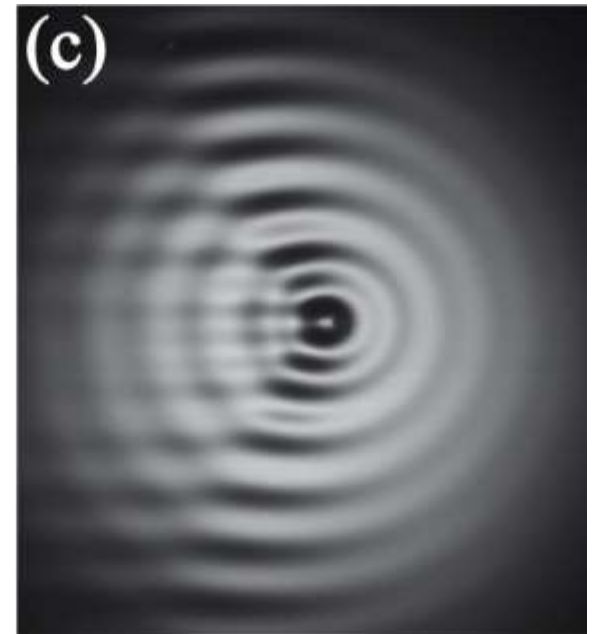
- At each bounce, a circular localized mode of standing waves is generated
- The localized Faraday instability is damped on a typical time :  $\tau \propto |\gamma_m - \gamma_m^F|^{-1}$
- The waves generated during the previous bounces along the trajectory form a "path-memory" of the walker wavefield.
- **Memory parameter**  $M = \tau/T_F$  is the number of bounces that contribute to the wave field.
- **M** can be tuned easily by changing the excitation amplitude



**M=5**



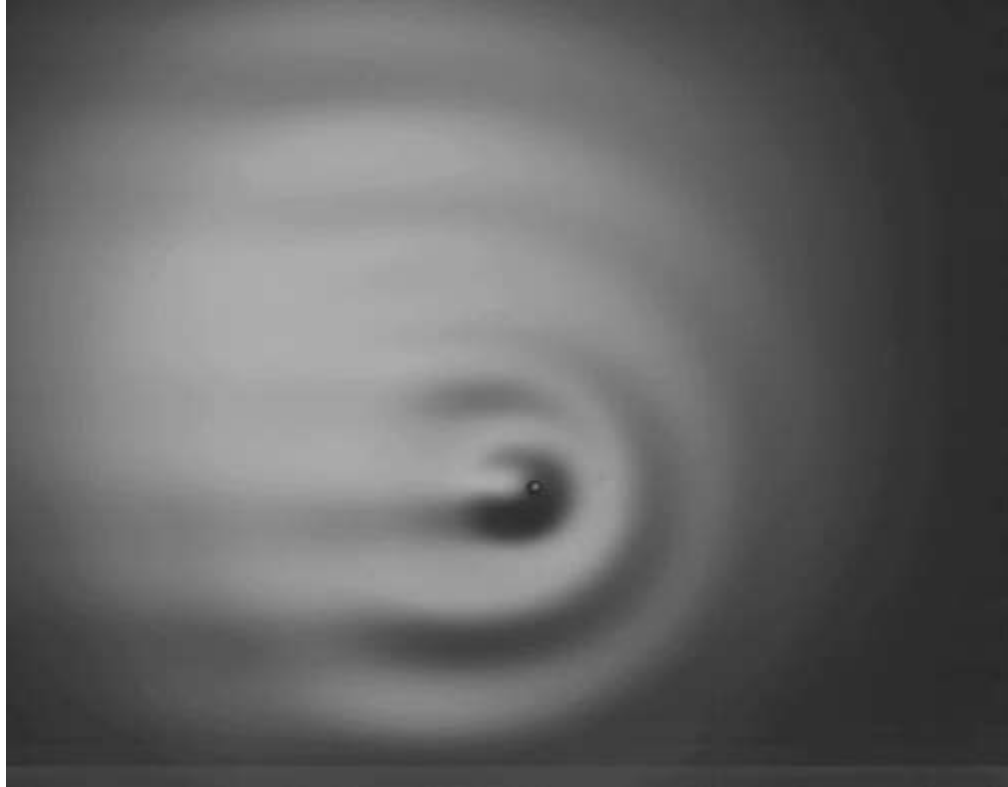
**M=10**



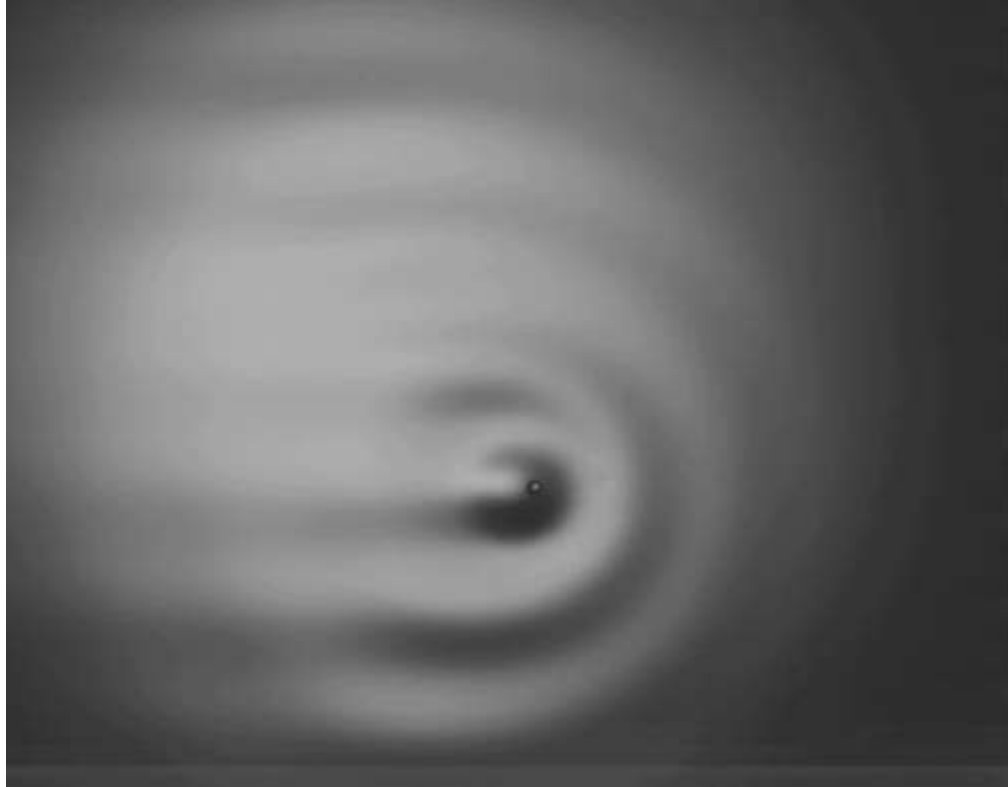
**M=40**



# The soul of a walker



# The soul of a walker



**A walker is a spatio-temporal non-local object**

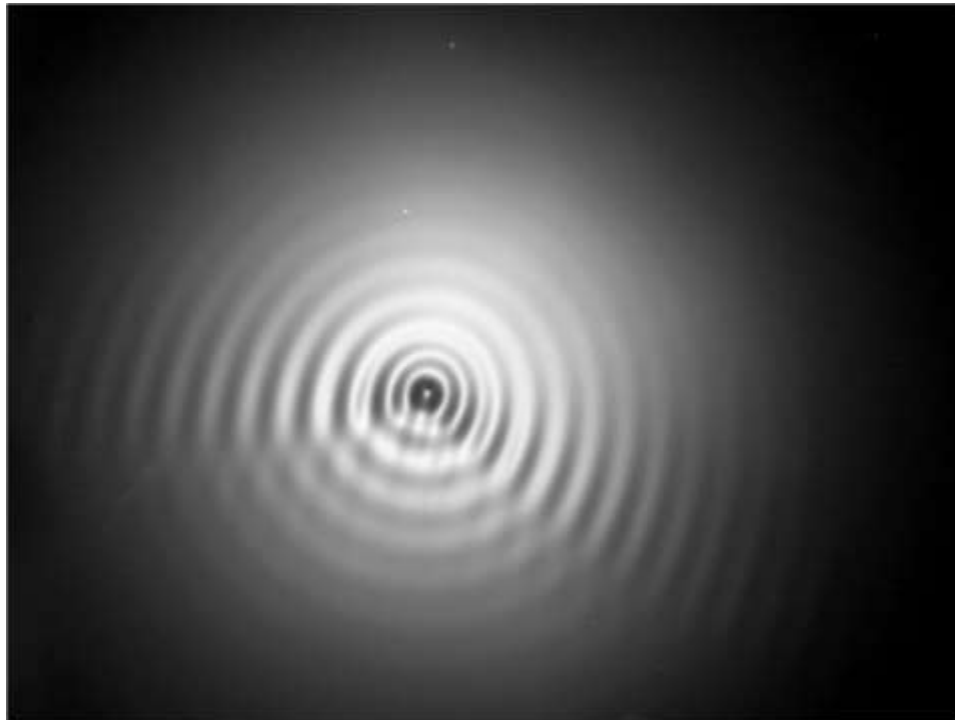
## A “classical” dual object

A walker is a “symbiotic” association of a particle with a wave:

If the particle vanishes, so does the wave

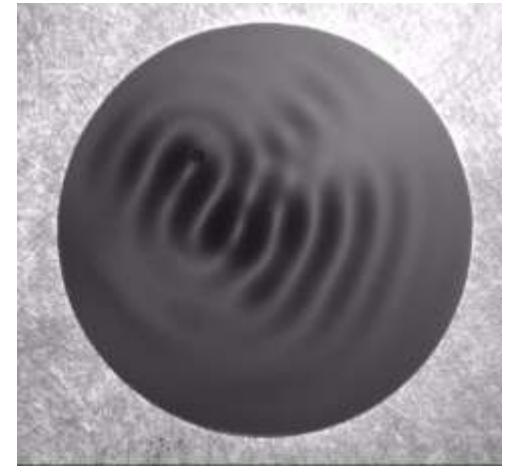
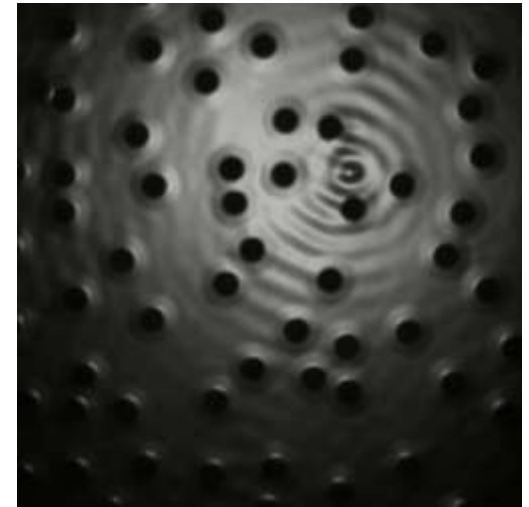
If the wave is damped, the droplet stops

**A walker is a non-quantum macroscopic dual object!**



# Quantum-like Experiments

- Interacting walkers
- Landau-like quantization
- Zeeman-like experiments
- Crystal and Phonons modes
- Central force
- Single particle diffraction and interferences
- Tunneling effect
- Cavity confinement
- Anderson-like localization
- ...



# Collision



13:49:25:11

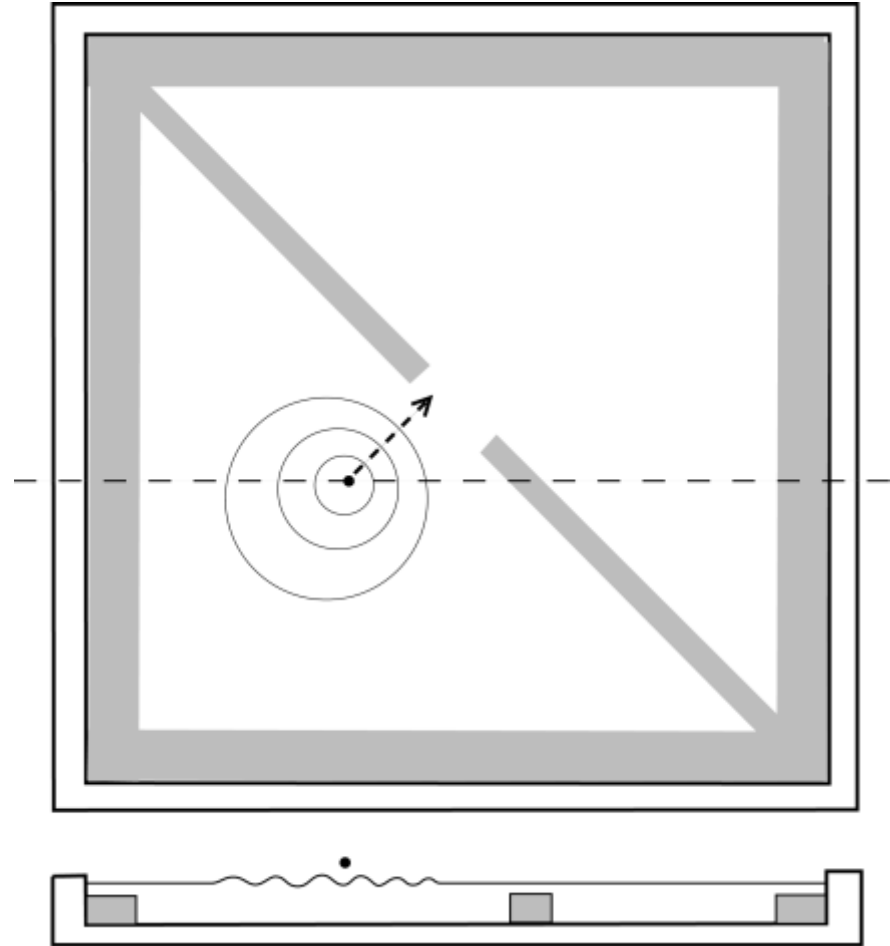
# Collision



# The experimental setup for diffraction and interference experiments

In the grey regions the fluid layer thickness is reduced to  $h_1=1\text{mm}$  ( $h_0=4\text{mm}$  elsewhere)

**In these regions the Faraday threshold being shifted, the walkers do not propagate**



Cross section

# Measurements on the droplet's trajectory

The relevant parameters:

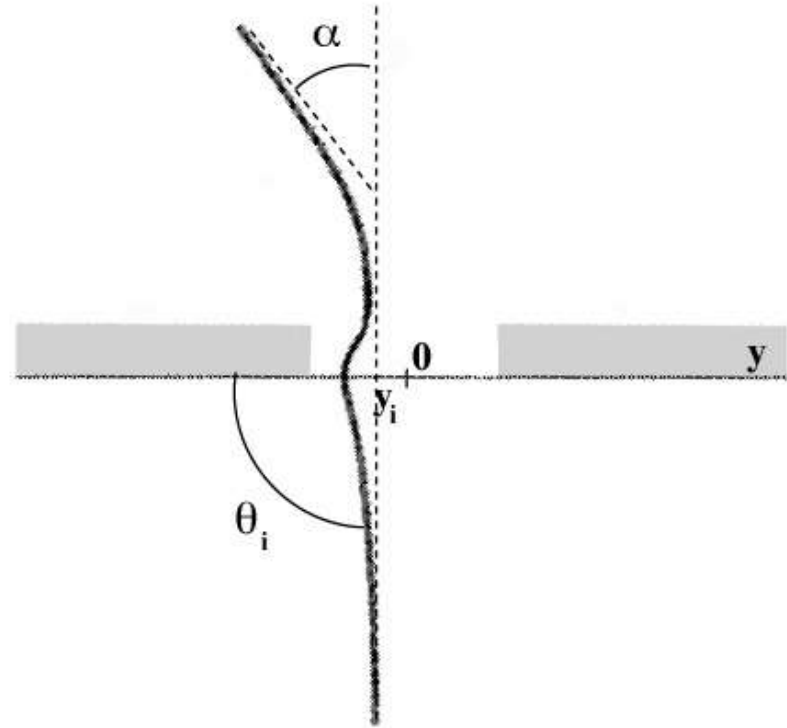
$L$  : the width of the slit,

$q_i$  : angle of incidence (chosen  $\theta_i = \pi/2$ )

$a$  : the angle of deviation

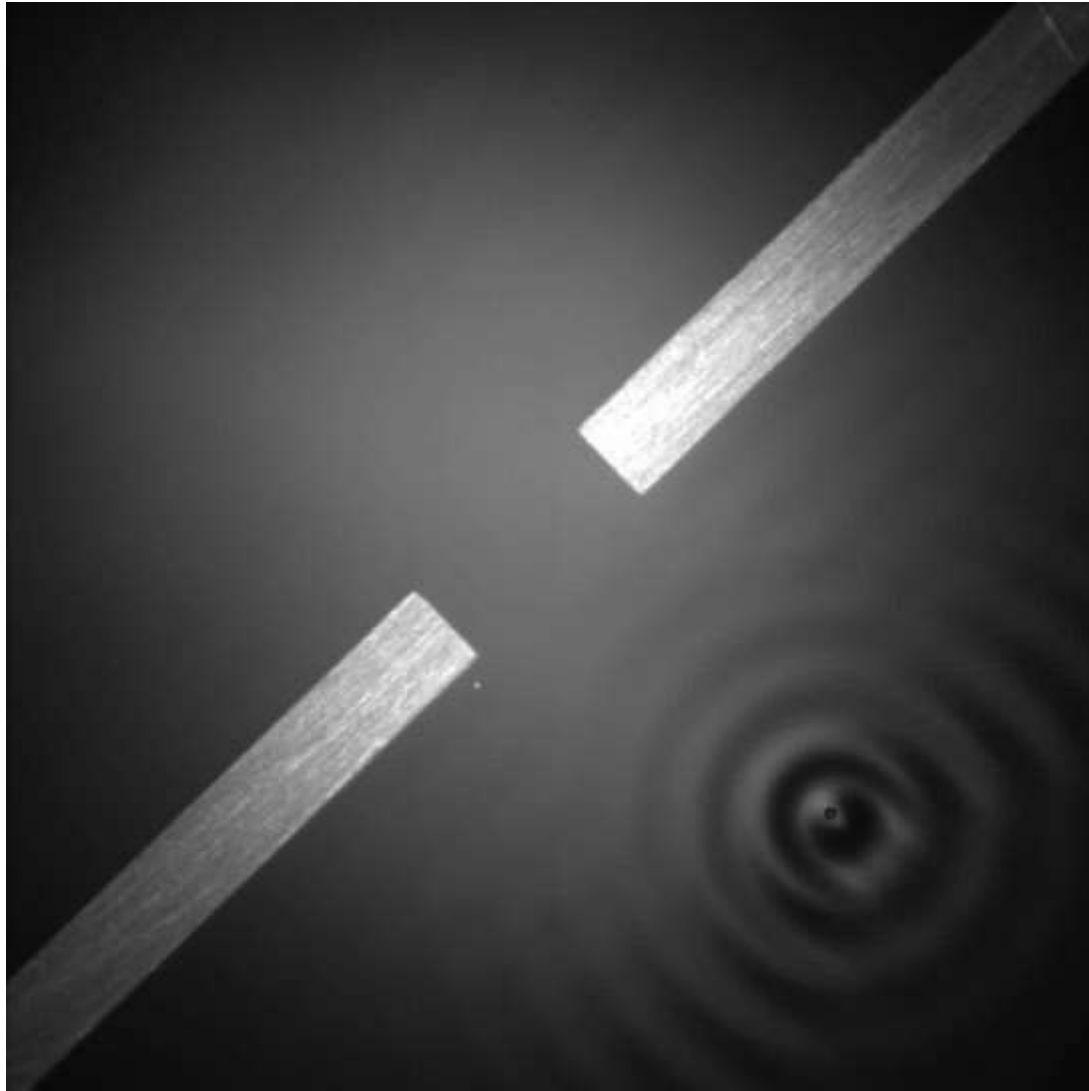
$Y_i = y_i/L$  : the impact parameter

(With  $-0.5 < Y_i < 0.5$ )





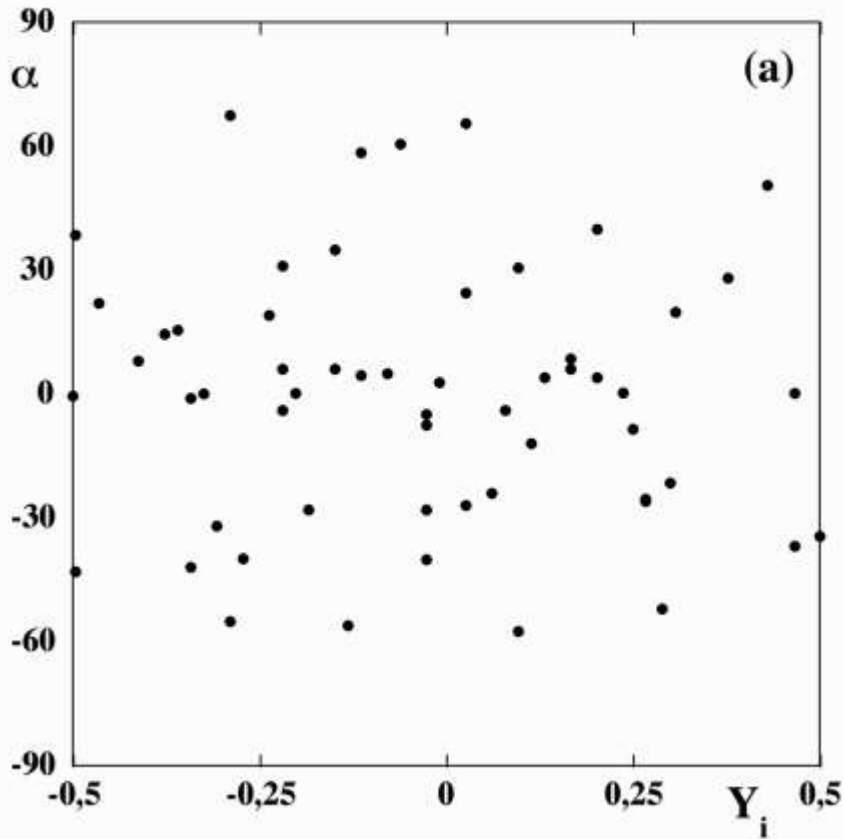
# A single "diffracting" droplet



# Deviation angle $\alpha$ vs the impact parameter $Y_i$ ?

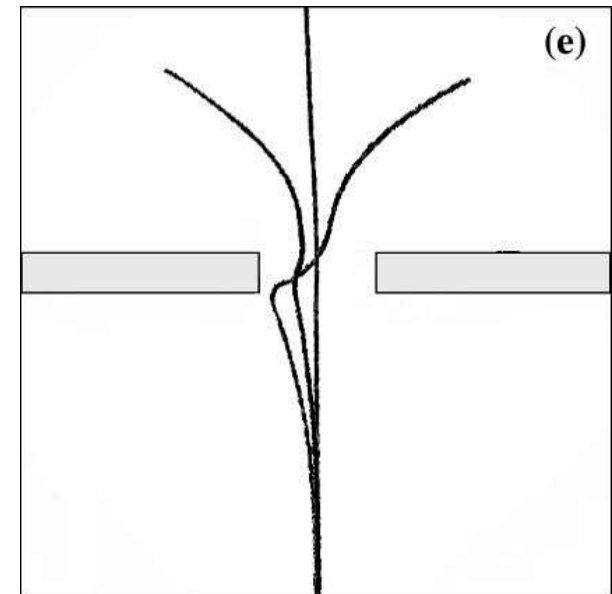
*The measured deviation in experiments performed with the same walker, the same angle of incidence, but various impact parameters*

$L/\lambda_F=3.1$  ( $L=14.7\text{mm}$  and  $\lambda_F=4.75\text{mm}$ ).



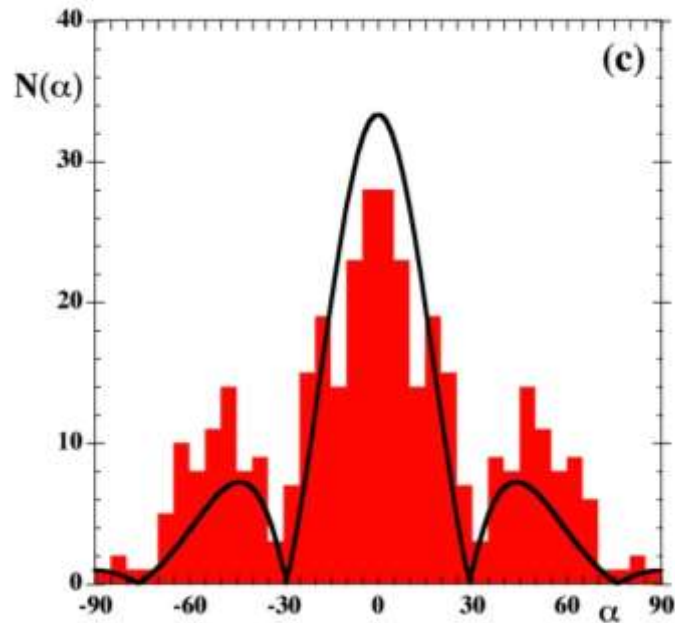
Impact parameter

Three independent trajectories with the same initial conditions (within experimental accuracy)

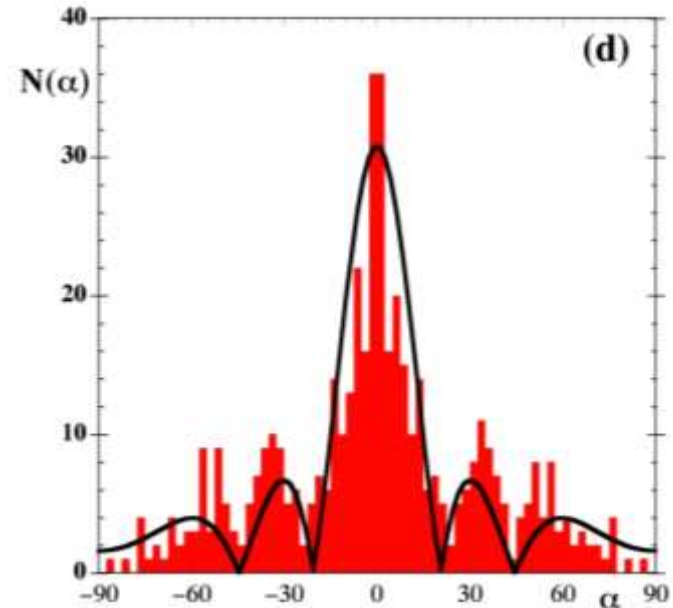


# Deviation histograms

Slit width/wavelength=2.11



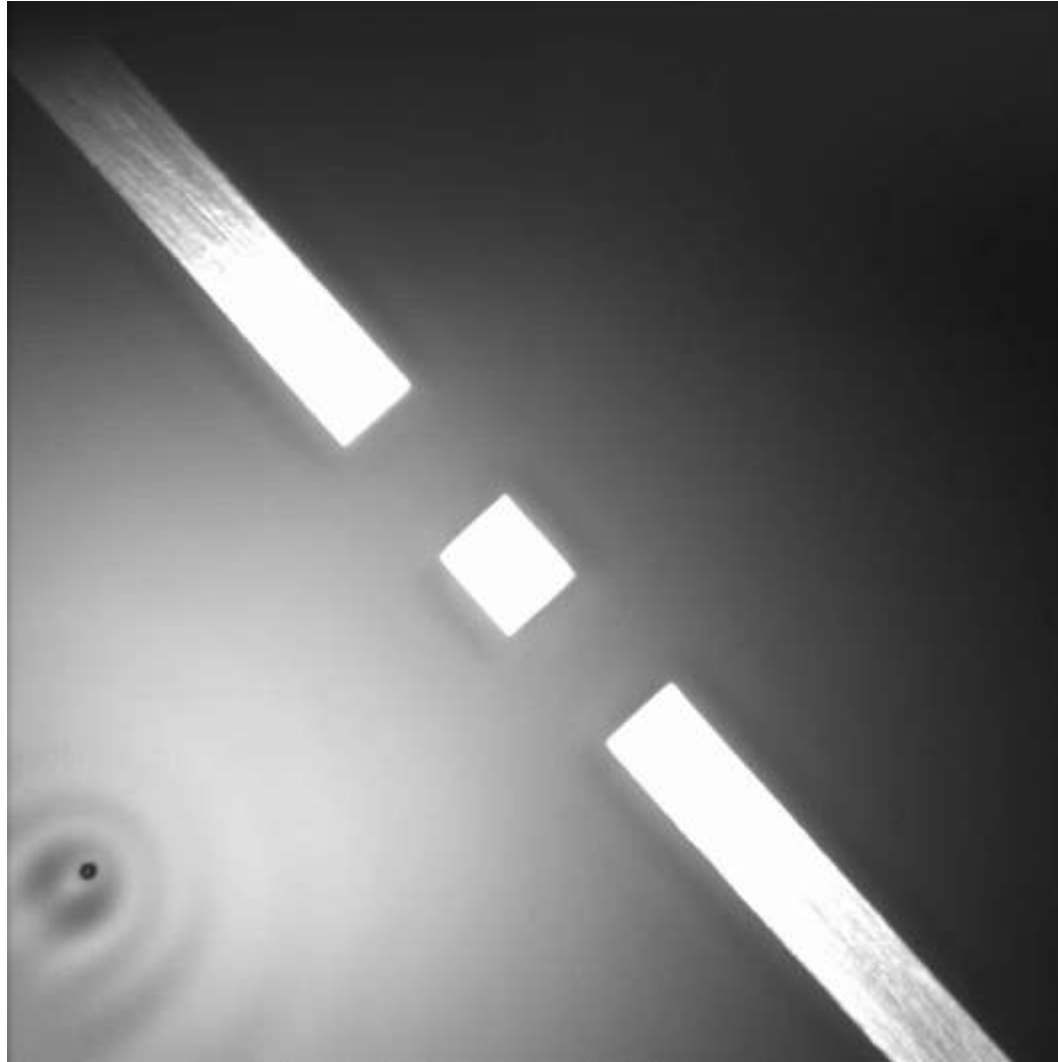
Slit width/wavelength=3.1



The curve is the modulus of the amplitude of diffraction of a plane wave with  $L/\lambda_F = 1.96$ .

$$f(\alpha) = A \left| \frac{\sin(\pi L \sin \alpha / \lambda_F)}{\pi L \sin \alpha / \lambda_F} \right|$$

# Young's double slit experiment



# Young's two slits experiment with walkers

## Interference Fringes with feeble light

G.I. Taylor, *Proc. Camb. Phil. Soc.*, 15, 114-115, (1909)

The phenomena of ionisation by light and by Röntgen rays have led to a theory according to which energy is distributed unevenly over the wave-front (J.J. Thomson, *Proc. Camb. Phil. Soc.* XIV, p.417, 1907). There are regions of maximum energy widely separated by large undisturbed areas. When the intensity of light is reduced these regions become more widely separated, but the amount of energy in any one of them does not change, that is, they are indivisible units.

So far all the evidence brought forward in support of the theory has been of an indirect nature; for all ordinary optical phenomena are average effects, and are therefore incapable of differentiating between the usual electromagnetic theory and the modification of it that we are considering. Sir J.J. Thomson however suggested that if the intensity of light in a diffraction pattern were so greatly reduced that only a few of these indivisible units of energy should occur on a Huygens zone at once the ordinary phenomena of diffraction would be modified. Photographs were taken of the shadow of a needle, the source of light being a narrow slit placed in front of a gas flame. The intensity of the light was reduced by means of smoked glass screens.

Before making any exposures it was necessary to find out what proportion of the light was cut off by these screens. A plate was exposed to direct gas light for a certain time. The gas flame was then shaded by the various screens that were to be used, and other plates of the same kind were exposed till they came out as black as the first plate on being completely developed. The times of exposure necessary to produce this result were taken as inversely proportional to the intensities. Experiments made to test the truth of this assumption showed it to be true if the light was not very feeble.

Five diffraction photographs were then taken, the first with direct light and the others with the various screens inserted between the gas flame and the slit. The time of exposure for the first photograph was obtained by trial, a certain standard of blackness being attained by the plate when fully developed. The remaining times of exposure were taken from the first in the inverse ratio of the corresponding intensities. The longest time was 2000 hours or about 3 months. In no case was there any diminution in the sharpness of the pattern although the plates did not all reach the standard black-

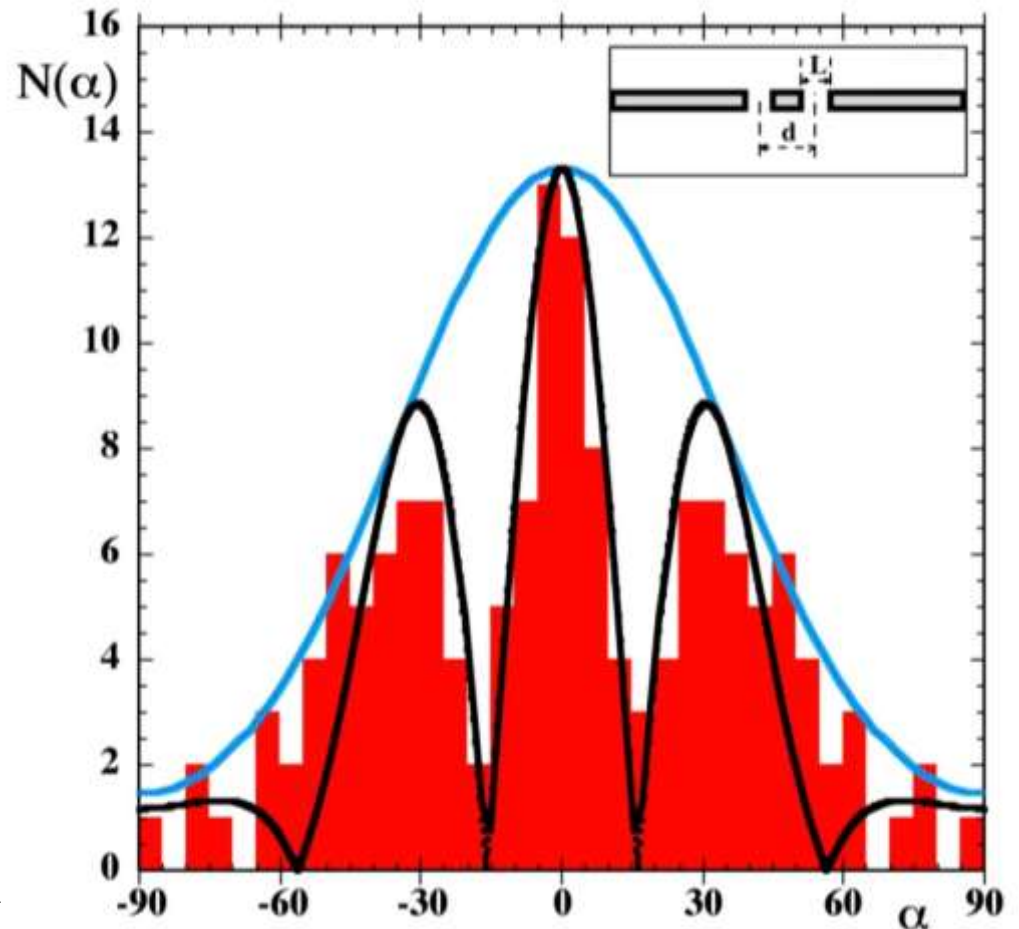
ness of the first photograph.

In order to get some idea of the energy of the light falling on the plates in these experiments a plate of the same kind was exposed at a distance of two metres from a standard candle till complete development brought it up to the standard of blackness. Ten seconds sufficed for this. A simple calculation will show that the amount of energy falling on the plate during the longest exposure was the same as that due to a standard candle burning at a distance slightly exceeding a mile. Taking the value given by Drude for the energy in the visible part of the spectrum of a standard candle, the amount of energy falling on 1 square centimetre of the plate is  $5 \times 10^{-6}$  ergs per sec, and the amount of energy per cubic centimetre of this radiation is  $1.6 \times 10^{-16}$  ergs.

According to Sir J.J. Thomson this value sets an upper limit to the amount of energy contained in one of the indivisible units mentioned above.

# Young's double slit experiment

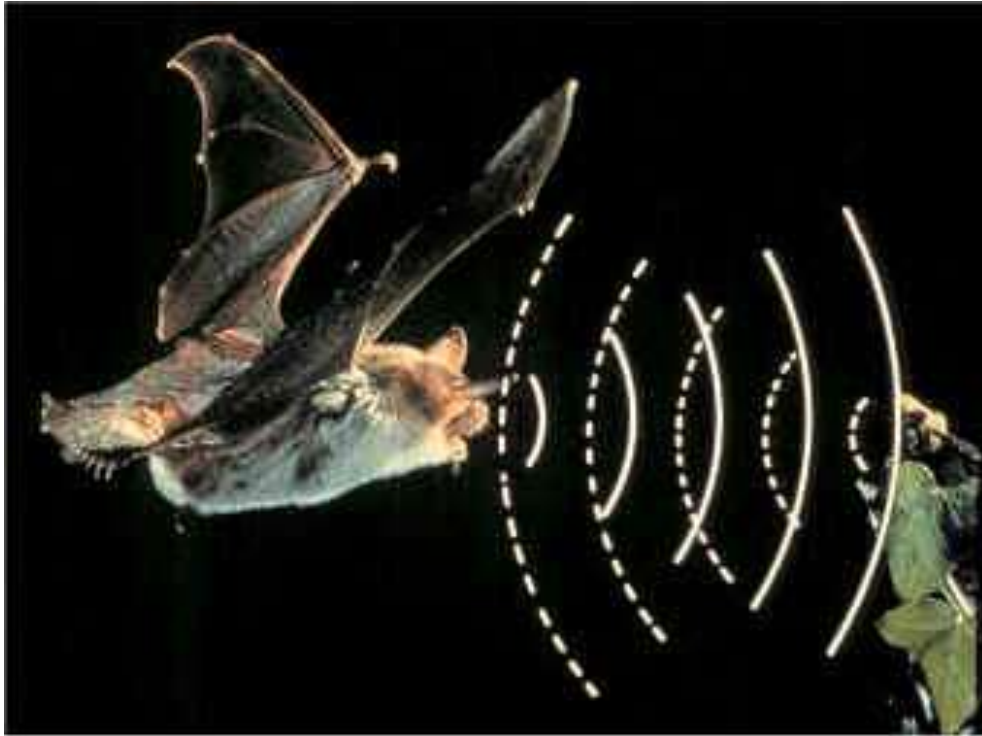
## Deviation histogram



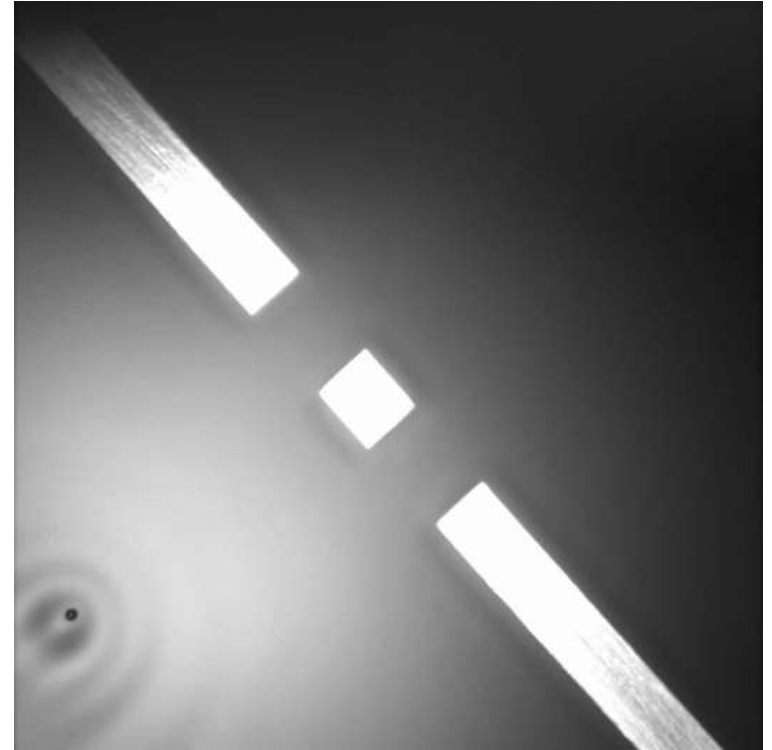
The curve is the modulus of the amplitude of the interference of a plane wave through two slits with  $L/\lambda_F=0.9$  and  $d/\lambda_F=1.7$ .

$$f(\alpha) = A \left| \frac{\sin(\pi L \sin \alpha / \lambda_F)}{\pi L \sin \alpha / \lambda_F} \cos(\pi d \sin \alpha / \lambda_F) \right|$$

# Echolocation



**Echolocation incohérente animale**



**Echolocation cohérente self-walker**

# Central force experiments

(with Stéphane Perrard, Matthieu Labousse, Yves Couder)

Perrard S. *et al.*, *Nature Com* (2014)

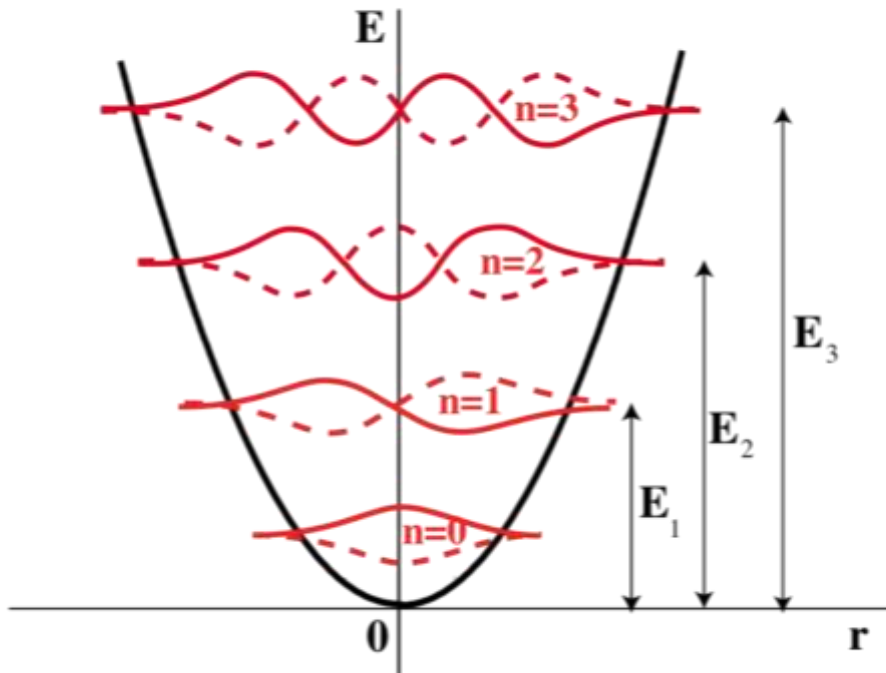
Perrard S. *et al.*, *PRL* (2014)



# Tuning the central force... energy tuning

Instead of varying the particle energy, we vary the width of the potential well

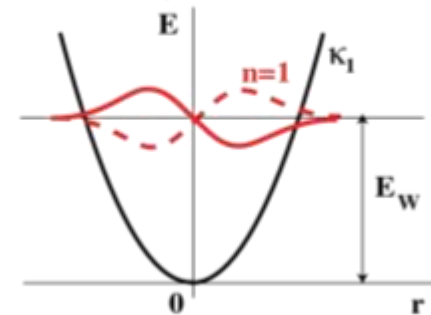
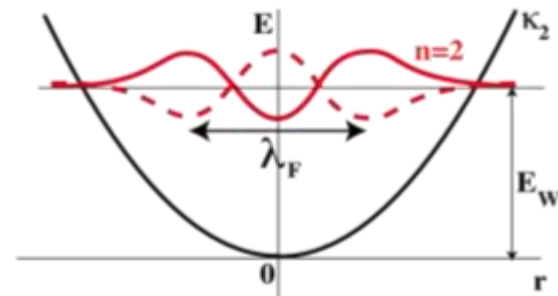
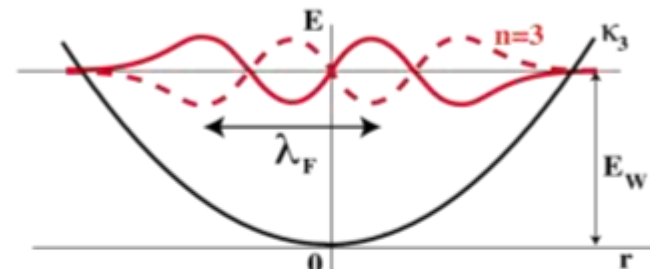
Since  $\kappa$  can be tuned continuously by varying  $d$ , we should obtain the successive eigenmodes one by one



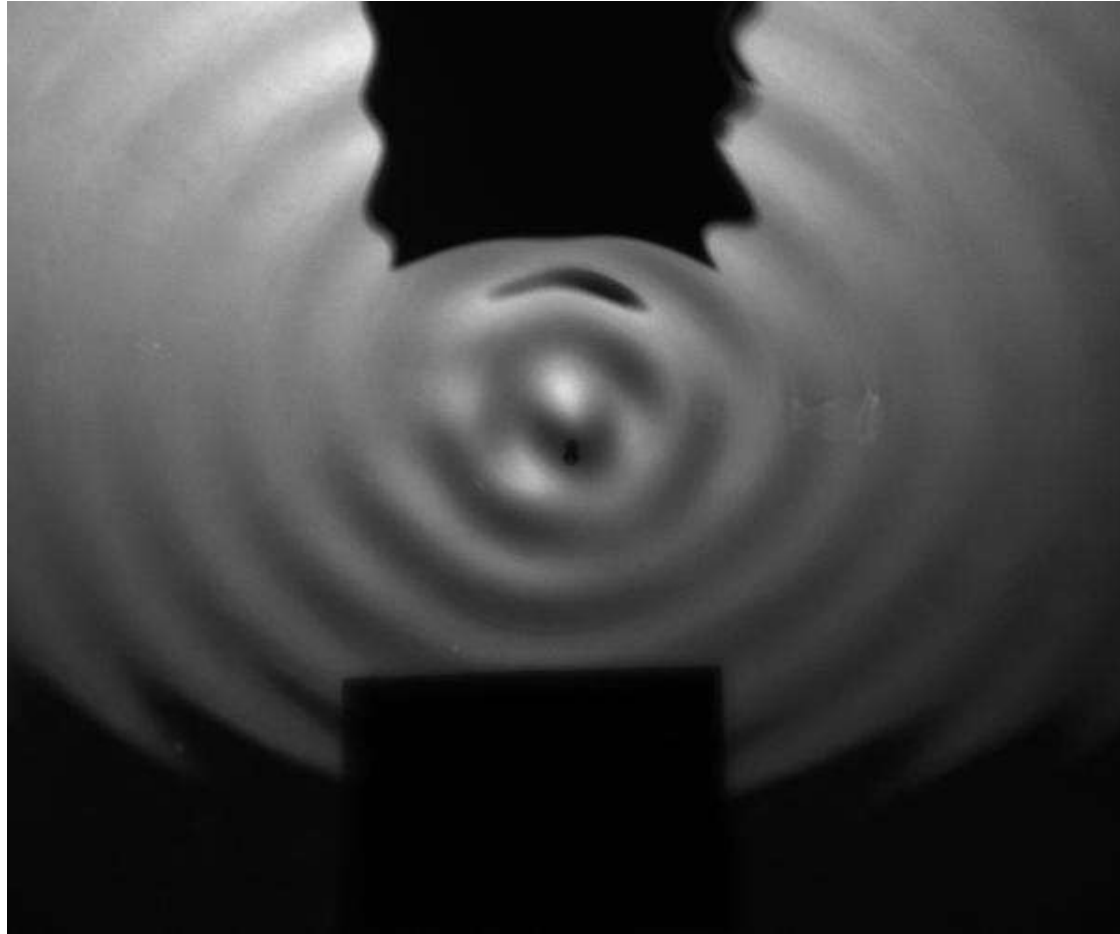
$$\vec{F}_m = -\kappa(d) r \vec{u}_r$$

**Confinement parameter**

$$\Lambda = \frac{V}{\omega \lambda_F} \quad \text{with} \quad \omega = \sqrt{\frac{\kappa(d)}{m}}$$



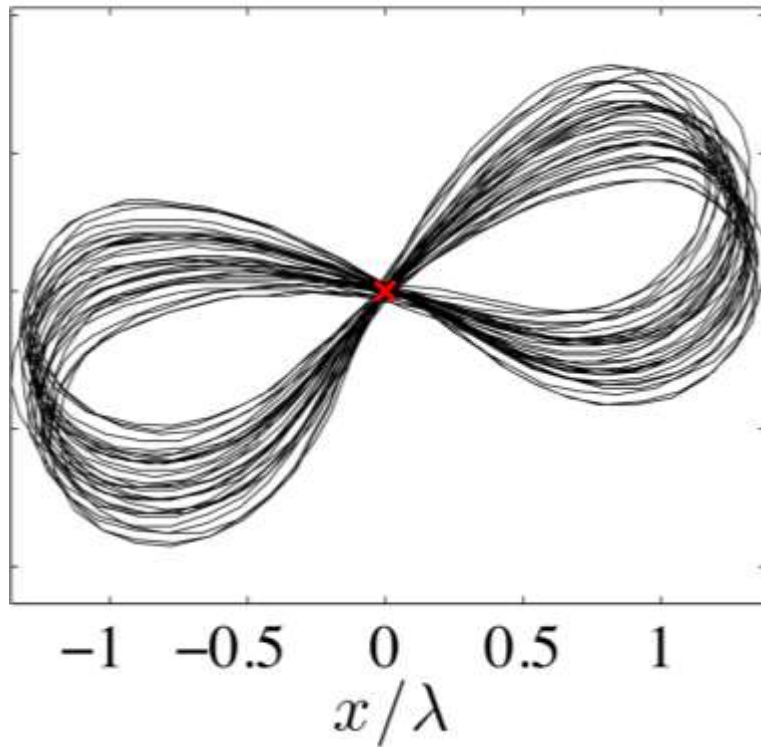
# Trapped walker



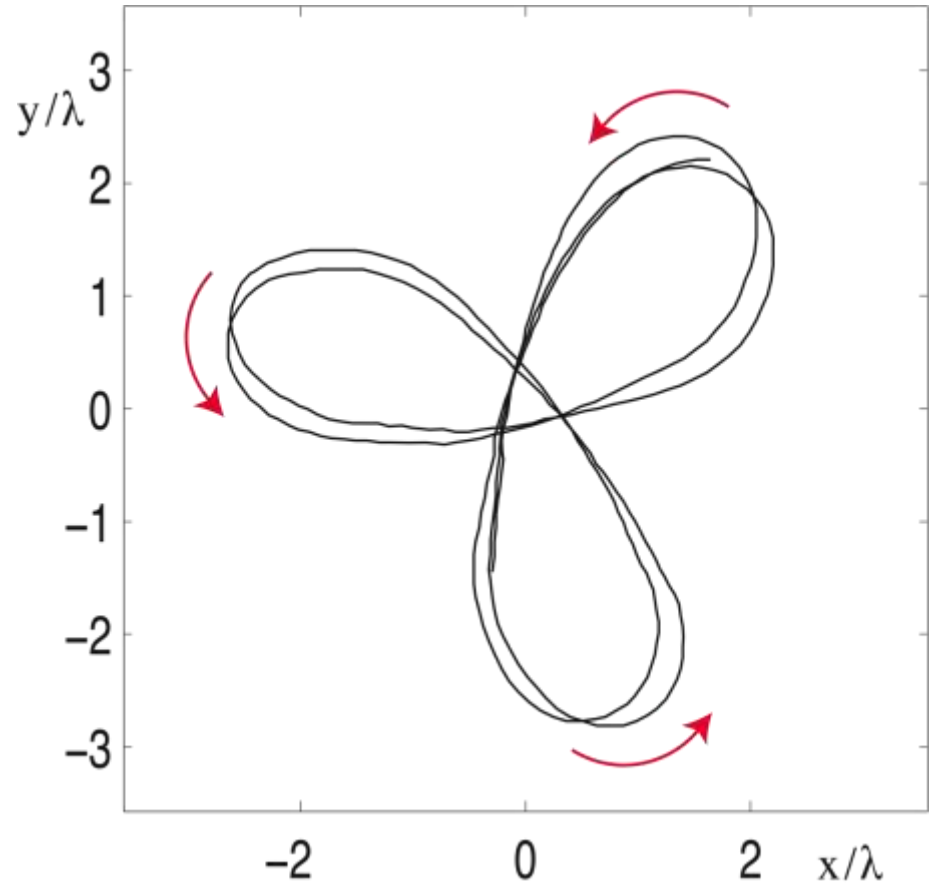
© Stéphane Perrard

# Eigenstates

Examples of stable quantized trajectories

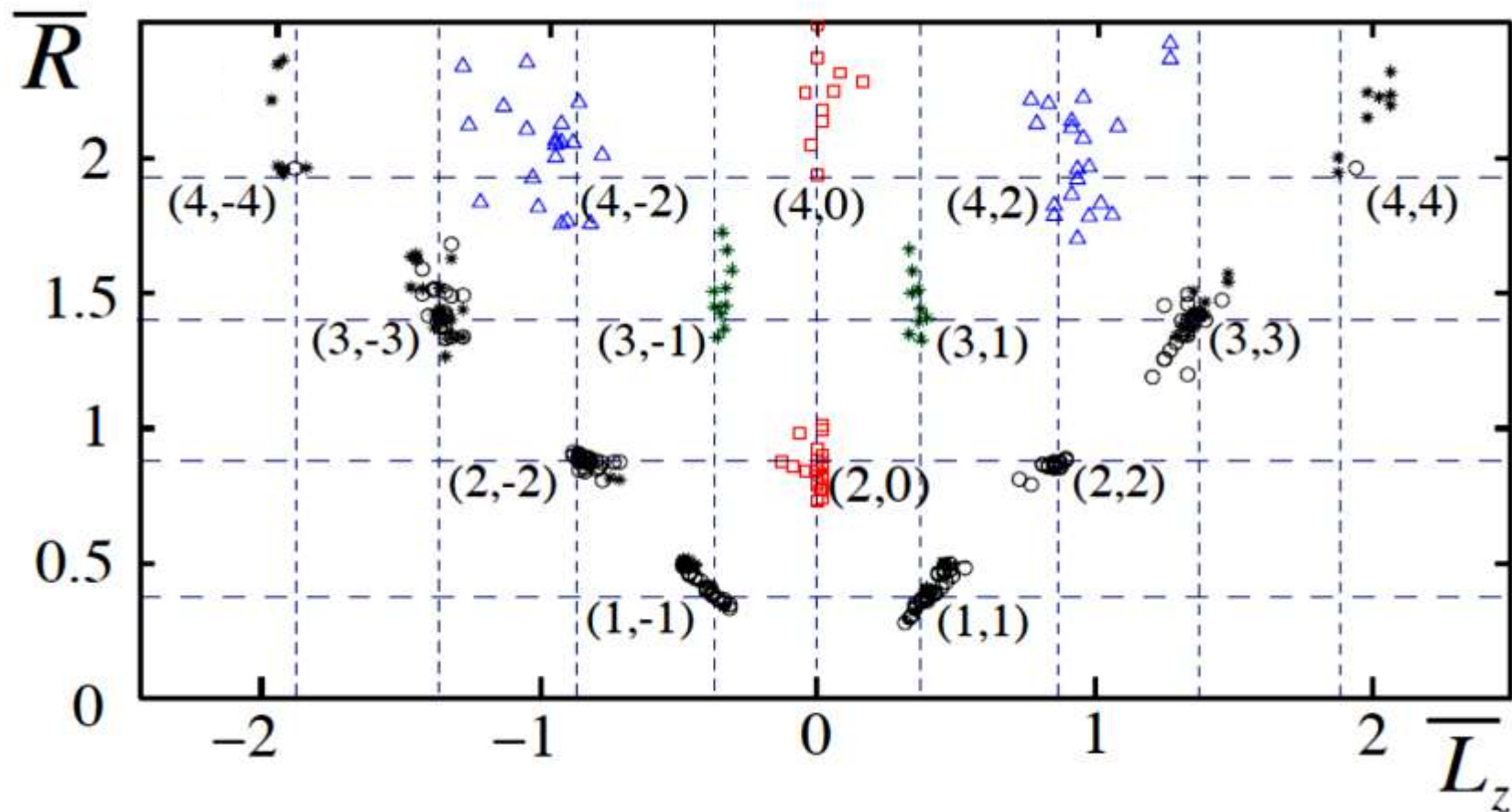


**Lemniscates**

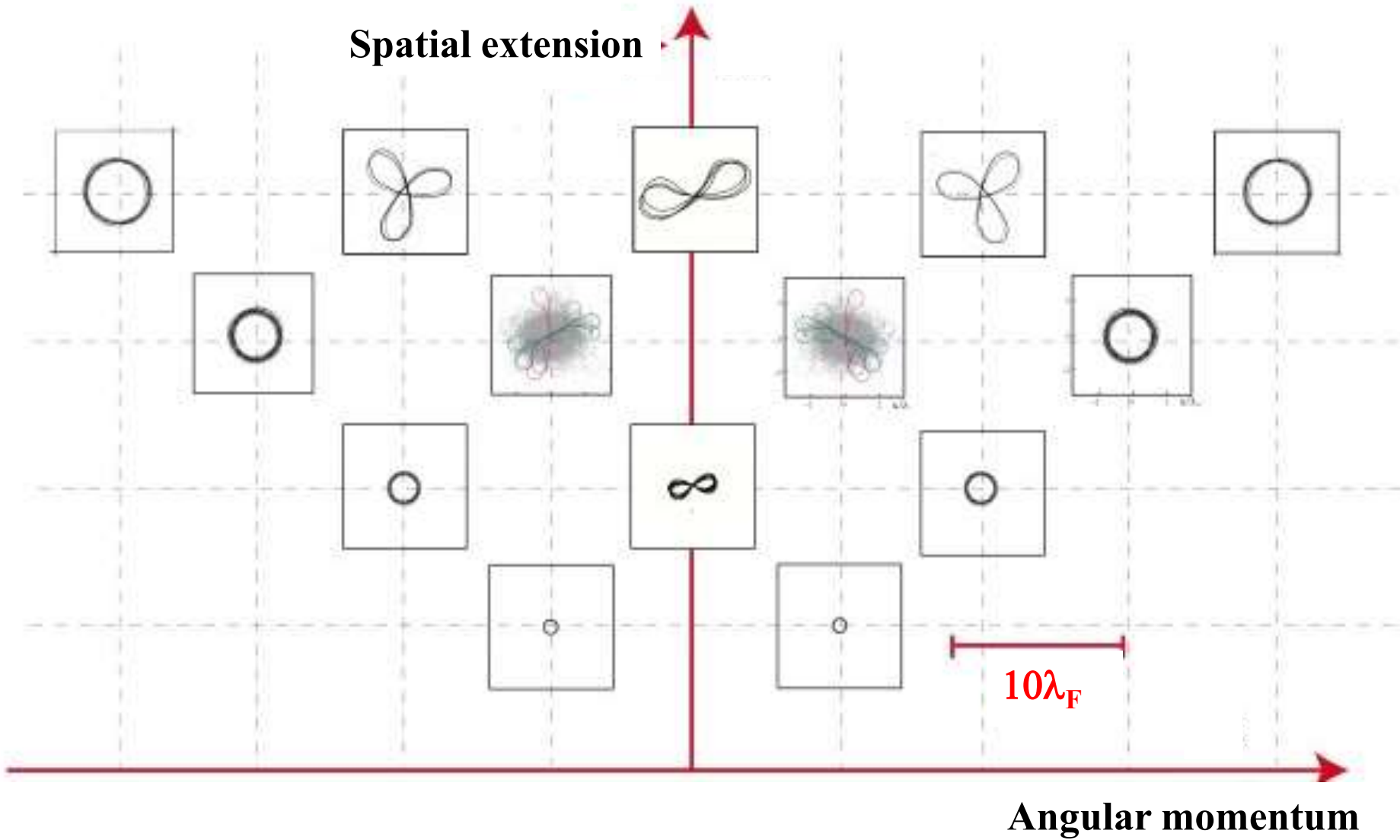


**Trefoils**

# Double quantization of experimental states (M=30)

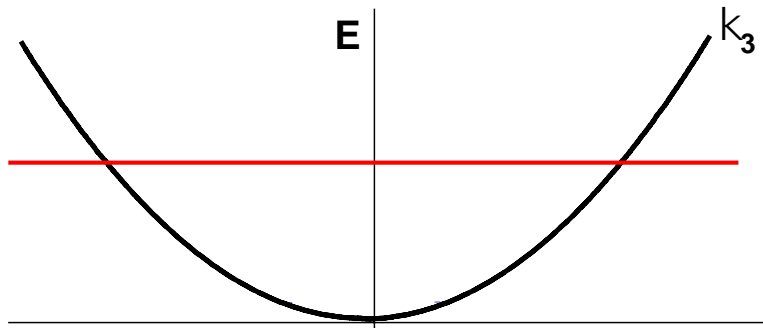


# Double quantization of experimental states

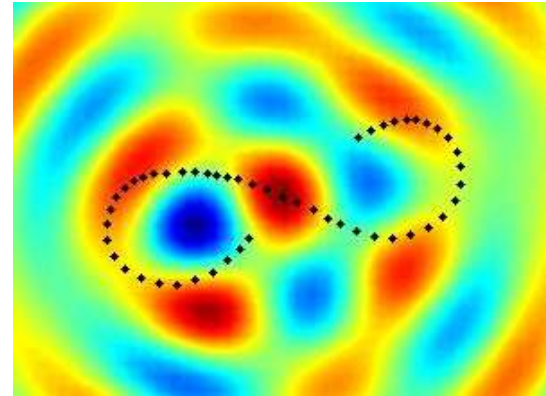


# Self-organization

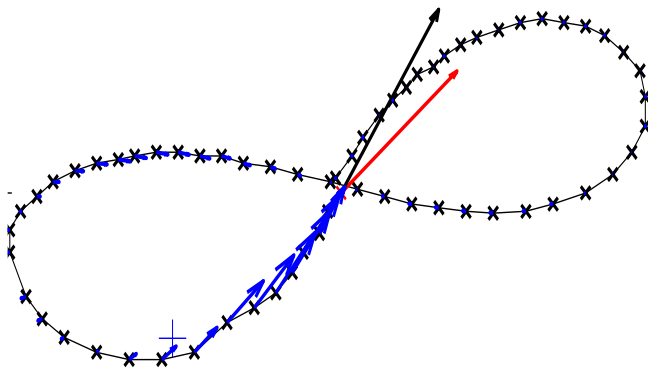
*Droplet in an energy potential*



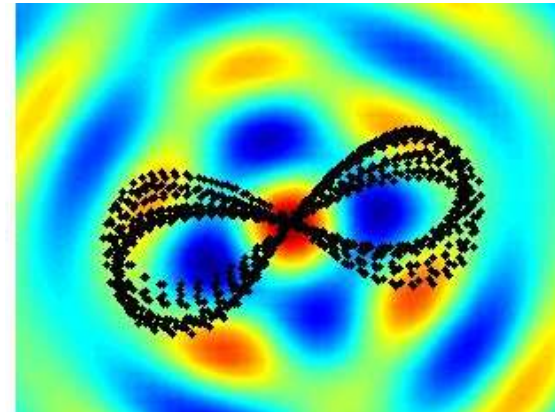
*... wave field construction*



*... dynamical modification of its trajectory*



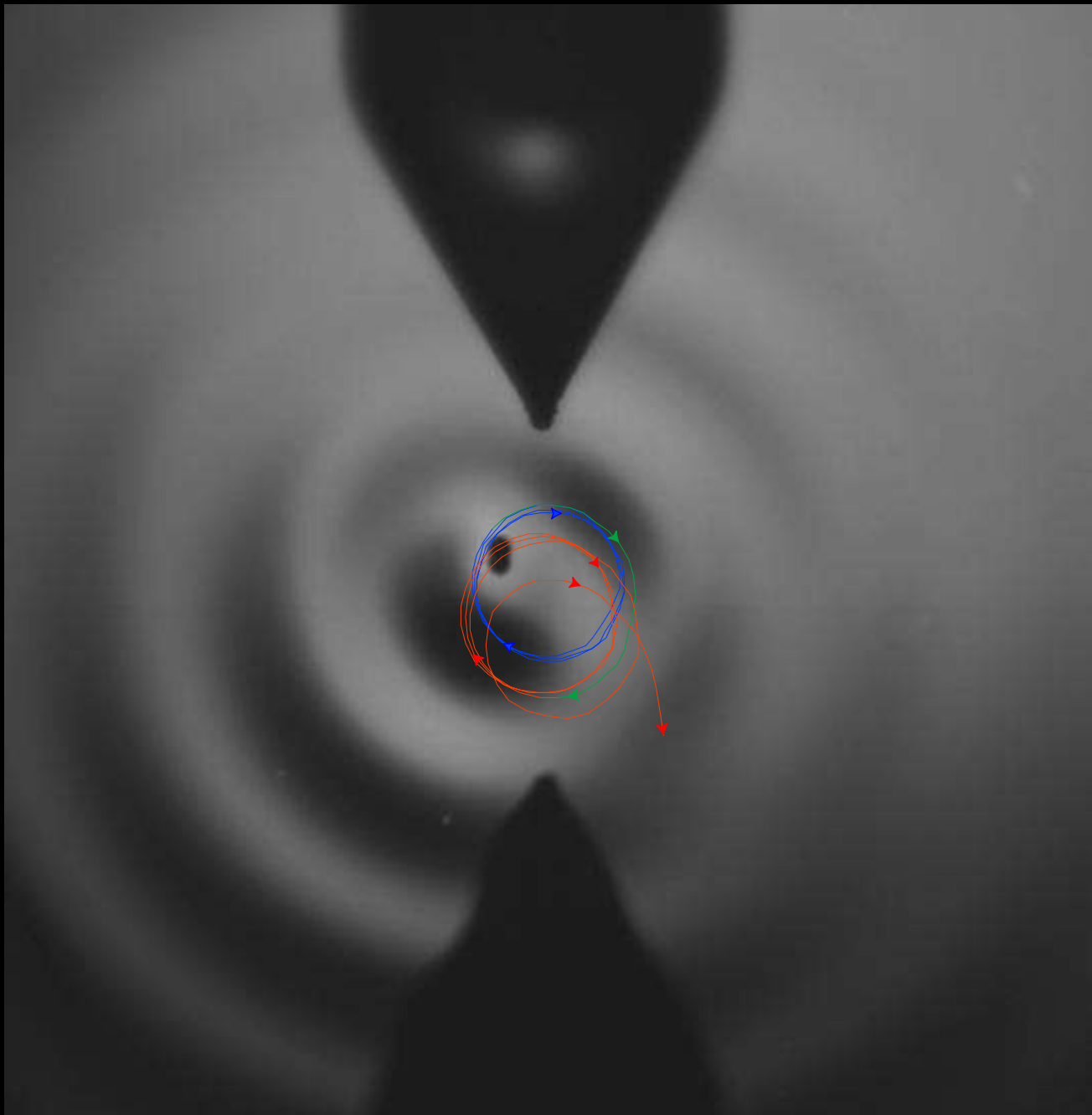
*...Guiding along wave field modes*



**Self-organization induced by the interplay between the wave field and the droplet**

# Self-attraction...

M. Labousse et al., PRE (2016)





# **Time mirrors**

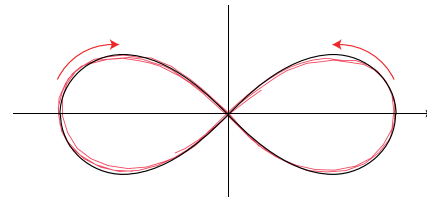
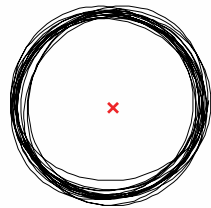
(with Vincent, Matthieu Labousse, Antonin Eddi, Mathias Fink)

# Definition of mean parameters



**Spatial extension:**

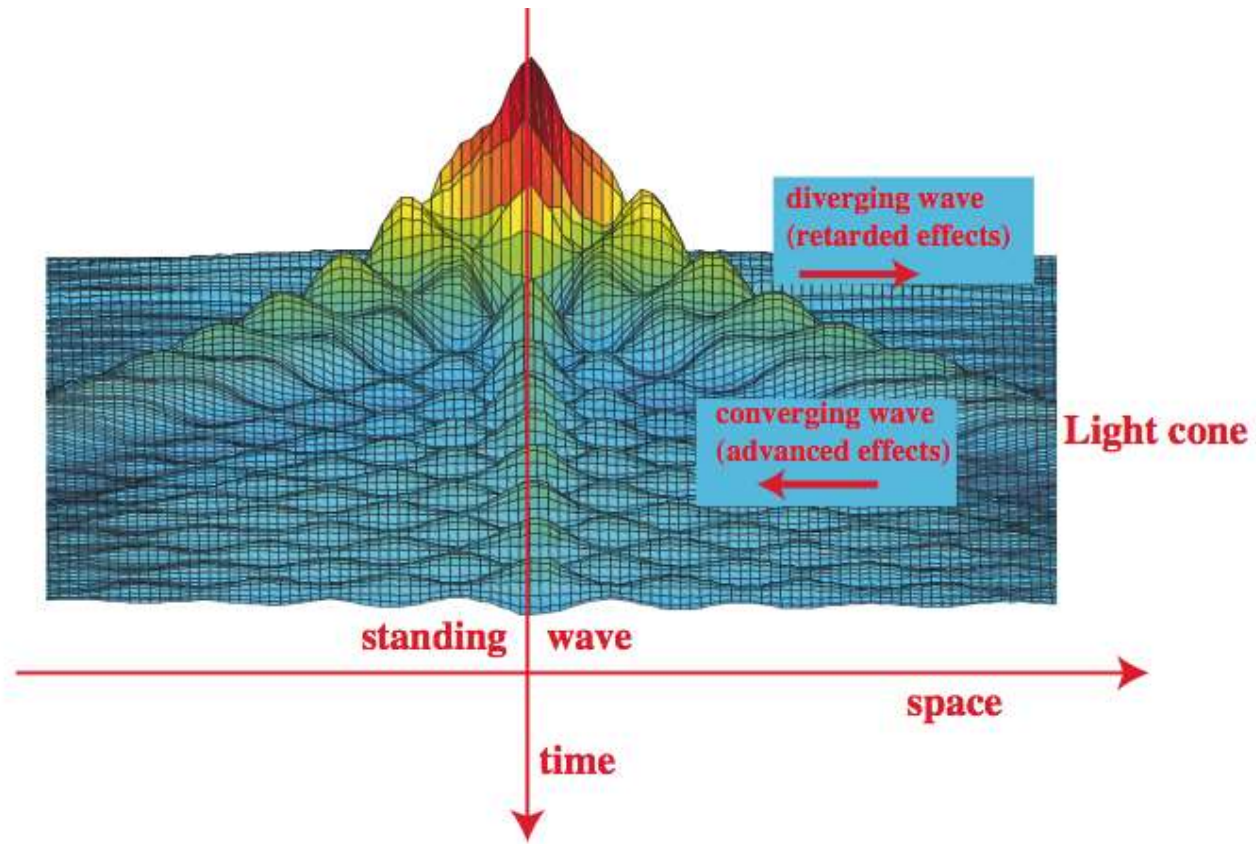
$$\bar{R}^2 = \frac{1}{T} \int_0^T \frac{r^2(t)}{\lambda_F^2} dt$$



**Angular momentum:**

$$\bar{L}_z = \frac{1}{T} \int_0^T \frac{(\vec{r}(t) \wedge \vec{V}(t)) \cdot \vec{e}_z}{\lambda_F V_0} dt$$

# Single bounce on an oscillating bath



Convergent waves are retro-propagating waves triggered by the propagation of the divergent wavefront.

**Each point of the wave front emits a wave moving backwards**

« Advanced effects » but with no problem with causality!

# The pre-Schrödinger de Broglie model (1926)

de Broglie assumes that there are well defined particles that he considers as point sources or singularities.

This material point has an internal oscillation (*zitterbewegung*) and emitting in the surrounding medium a wave of frequency :

$$\nu_0 = \frac{1}{h} m_0 c^2$$

The particle is surrounded by a stationary spherical wave, the superposition of a divergent and a convergent wave.

$$\varphi(r_0, t_0) = \frac{A}{2r_0} \left\{ \cos \left[ 2\pi\nu_0 \left( t_0 - \frac{r_0}{c} \right) \right] - \cos \left[ 2\pi\nu_0 \left( t_0 + \frac{r_0}{c} \right) \right] \right\}$$

**Non causal!**

de Broglie writes :

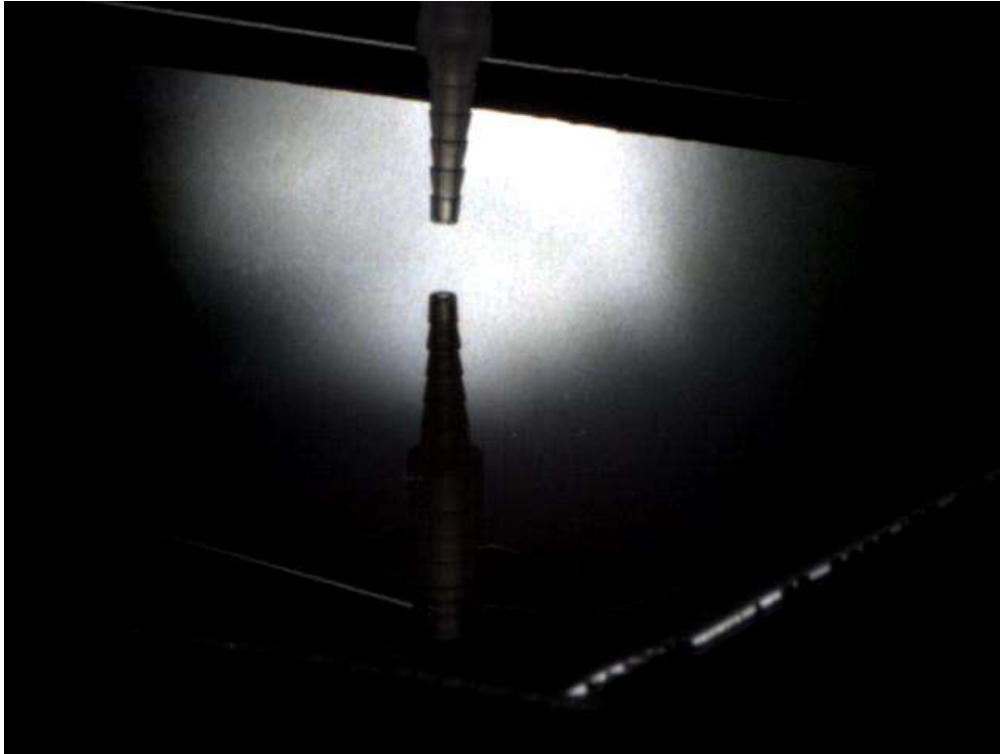
*« But there is also the convergent wave, the interpretation of which could raise interesting philosophical issues, but that appears necessary to insure the stability of the material point »*

# Instantaneous time mirror



**Top view**

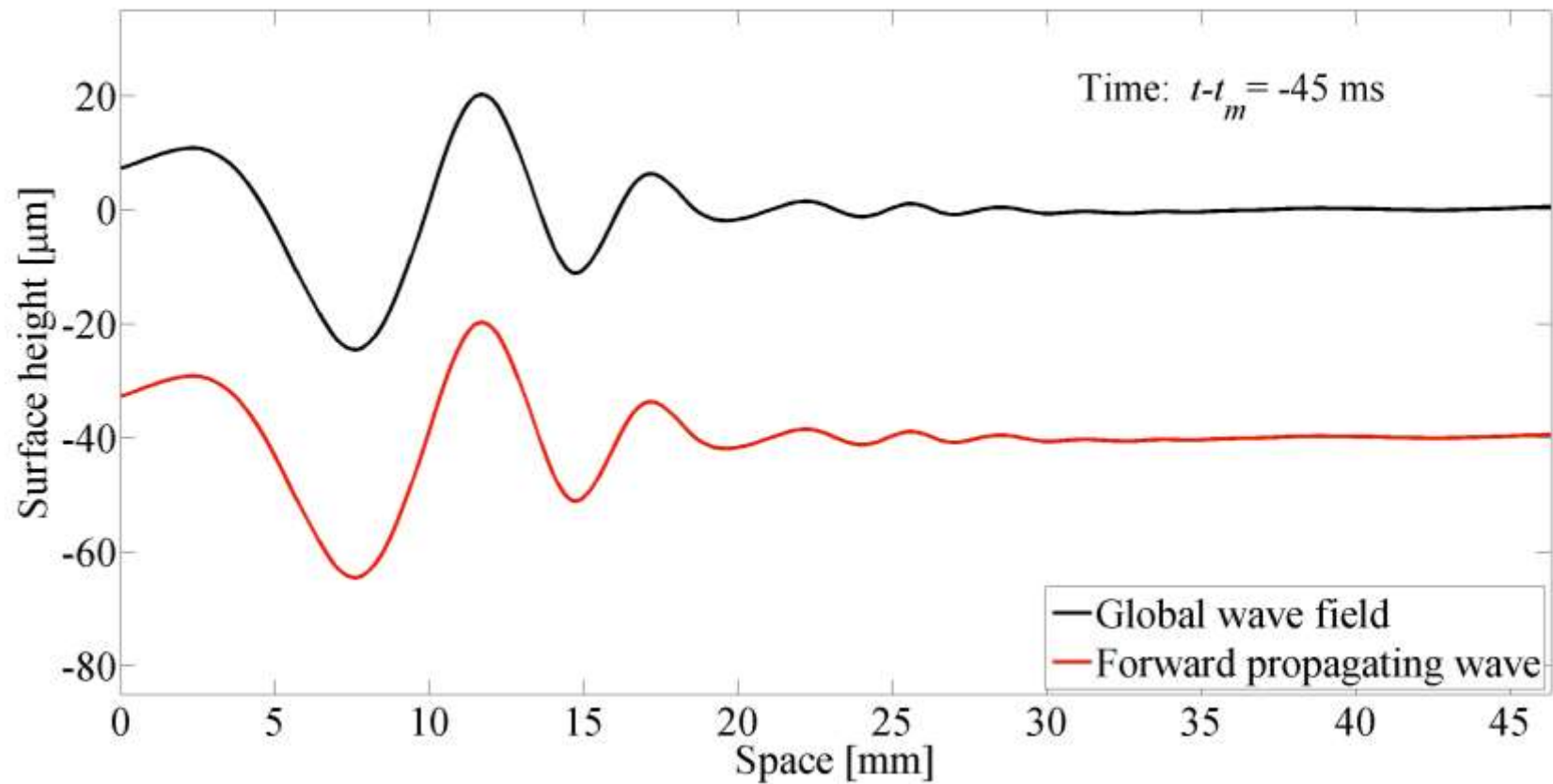
# Instantaneous Time Mirror



**Time reversed wave  
at the jolt instant**

**From the side, slowed down 50 times**

# Measurement of the wave field



# Loschmidt's Daemon

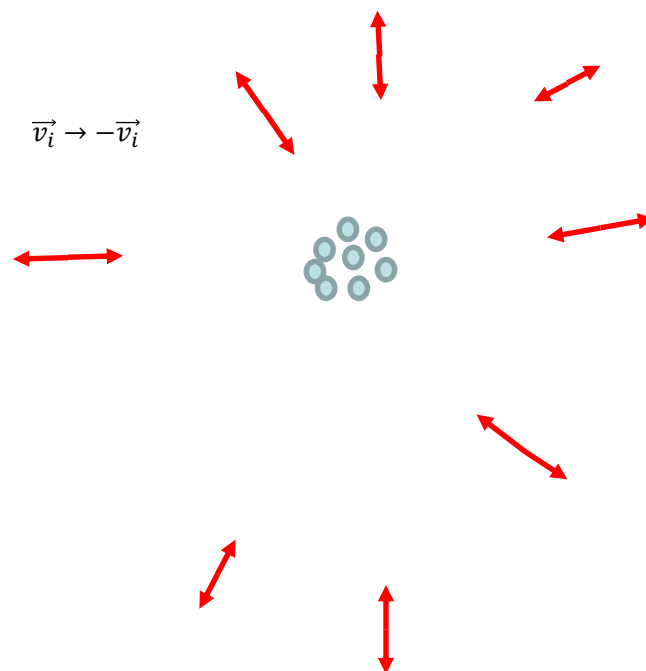


**Johann Loschmidt**  
(1821 – 1895)



**Ludwig Boltzmann**  
(1844-1906)

**Instantaneous reversal of  
velocities of all particles**





# Loschmidt's wave Daemon?

- **Wave equation (time symmetric and *linear*):**  $\Delta\varphi - \frac{1}{c^2} \frac{\partial^2 \varphi}{\partial t^2} = 0$
- **Cauchy's theorem (Initial Conditions):**  $\begin{pmatrix} \varphi(t_0) \\ \frac{\partial \varphi}{\partial t}(t_0) \end{pmatrix} \Rightarrow t > t_0, \varphi(t)$
- **Loschmidt's daemon:**  $\begin{pmatrix} \varphi(t_0) \\ \frac{\partial \varphi}{\partial t}(t_0) \end{pmatrix} \rightarrow \begin{pmatrix} \varphi(t_0) \\ -\frac{\partial \varphi}{\partial t}(t_0) \end{pmatrix} \Rightarrow t > t_0, \varphi(2t_0 - t)$

- « Freezing » the field:

$$\begin{pmatrix} \varphi(\vec{r}, t_0) \\ \frac{\partial \varphi}{\partial t}(\vec{r}, t_0) \end{pmatrix} \rightarrow \begin{pmatrix} \varphi(\vec{r}, t_0) \\ 0 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} \varphi(t_0) \\ \frac{\partial \varphi}{\partial t}(t_0) \end{pmatrix} + \frac{1}{2} \begin{pmatrix} \varphi(t_0) \\ -\frac{\partial \varphi}{\partial t}(t_0) \end{pmatrix}$$

(Superposition of) Initial wave + Time reversed wave

# Cauchy time sources

- Decoupling the wave field and its time derivative

$$\begin{pmatrix} \varphi(\vec{r}, t_0) \\ \frac{\partial \varphi}{\partial t}(\vec{r}, t_0) \end{pmatrix} \longrightarrow \begin{pmatrix} \varphi(\vec{r}, t_0) \\ 0 \end{pmatrix} = \begin{pmatrix} \varphi(\vec{r}, t_0) \\ \frac{\partial \varphi}{\partial t}(\vec{r}, t_0) \end{pmatrix} + \begin{pmatrix} \varphi(\vec{r}, t_0) \\ -\frac{\partial \varphi}{\partial t}(\vec{r}, t_0) \end{pmatrix}$$

Initial wave + **Time reversed wave**

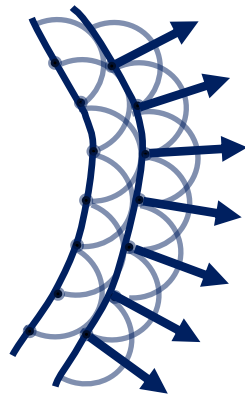
- During time disruption, « **Cauchy sources** » generated:

$$\Delta \varphi - \frac{1}{c^2} \frac{\partial^2 \varphi}{\partial t^2} = s(\vec{r}, t); \quad s(\vec{r}, t) = \frac{\partial \varphi}{\partial t}(\vec{r}, t_m^-) \delta(t - t_m)$$

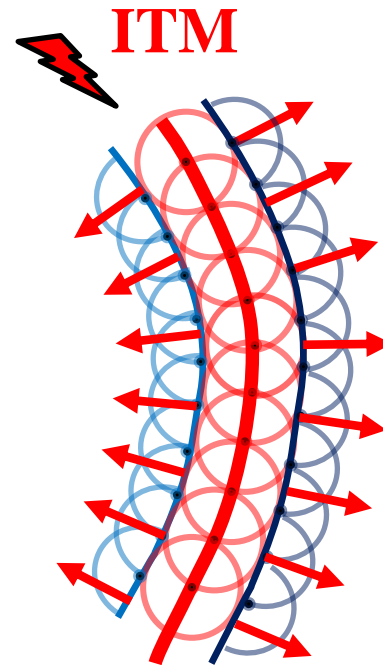
**Space is a distributed memory** (compared with standard Time Reversal Mirrors)

# Revisit Huygens-Fresnel principle

« Cauchy » time sources: generalize Huygens-Fresnel principle on time boundaries



Huygens-Fresnel sources



Time "Cauchy" sources

# Celerity time control

- Time varying celerity 
$$\Delta\varphi - \frac{n(t)^2}{c^2} \frac{\partial^2 \varphi}{\partial t^2} = 0$$

- Source generated during the time disruption:

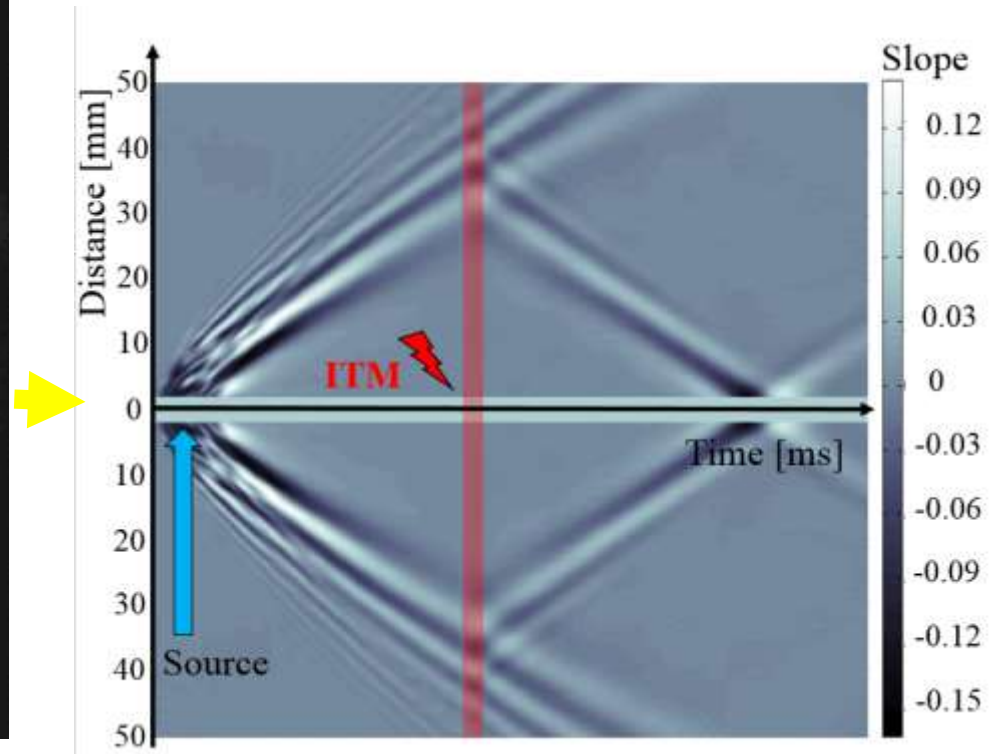
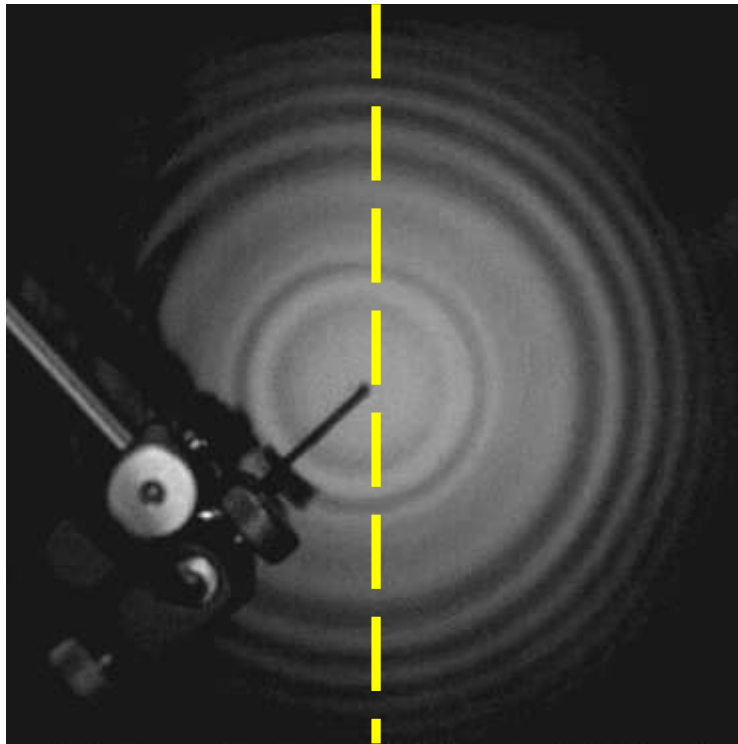
$$\Delta\varphi - \frac{1}{c^2} \frac{\partial^2 \varphi}{\partial t^2} = s(\vec{r}, t); \quad s(\vec{r}, t) = -\frac{\partial^2 \varphi}{\partial t^2}(\vec{r}, t_m^-) \delta(t - t_m)$$

Instantaneous time mirror: time reversal of the wave field time derivative.

In water experiments, the gravity jolt changes the wave velocity

$$v \propto \sqrt{g}$$

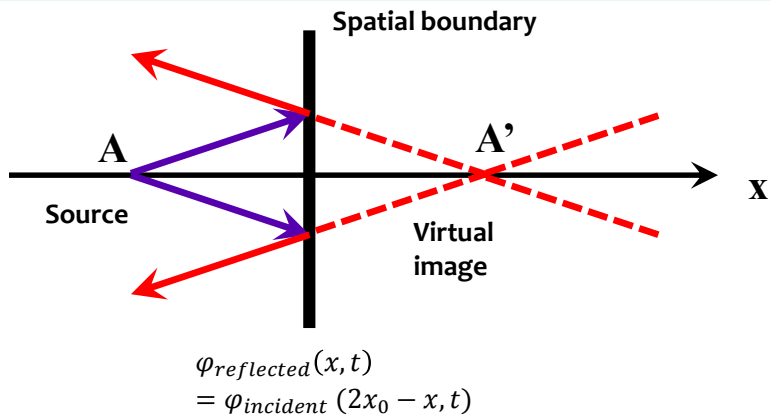
# Spatiotemporal measurement



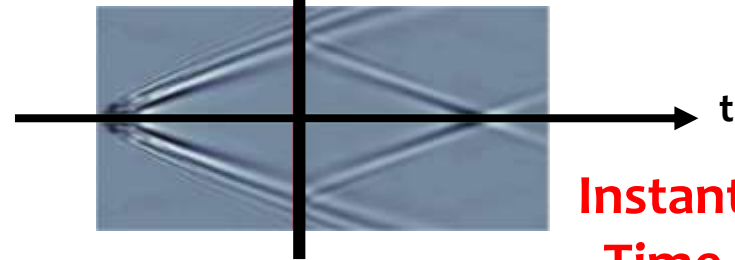
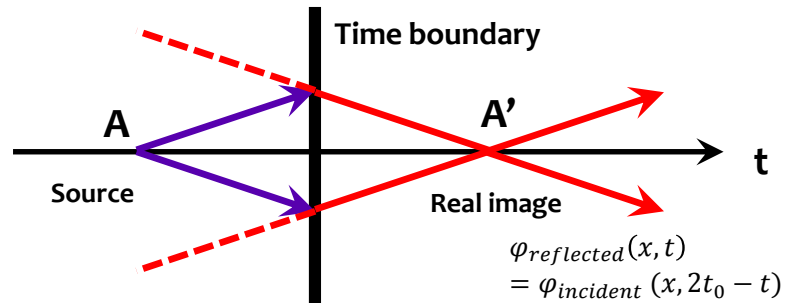
**Celerity shock  
(jolt)**

# Through the (time) mirror...

Standard Mirror :

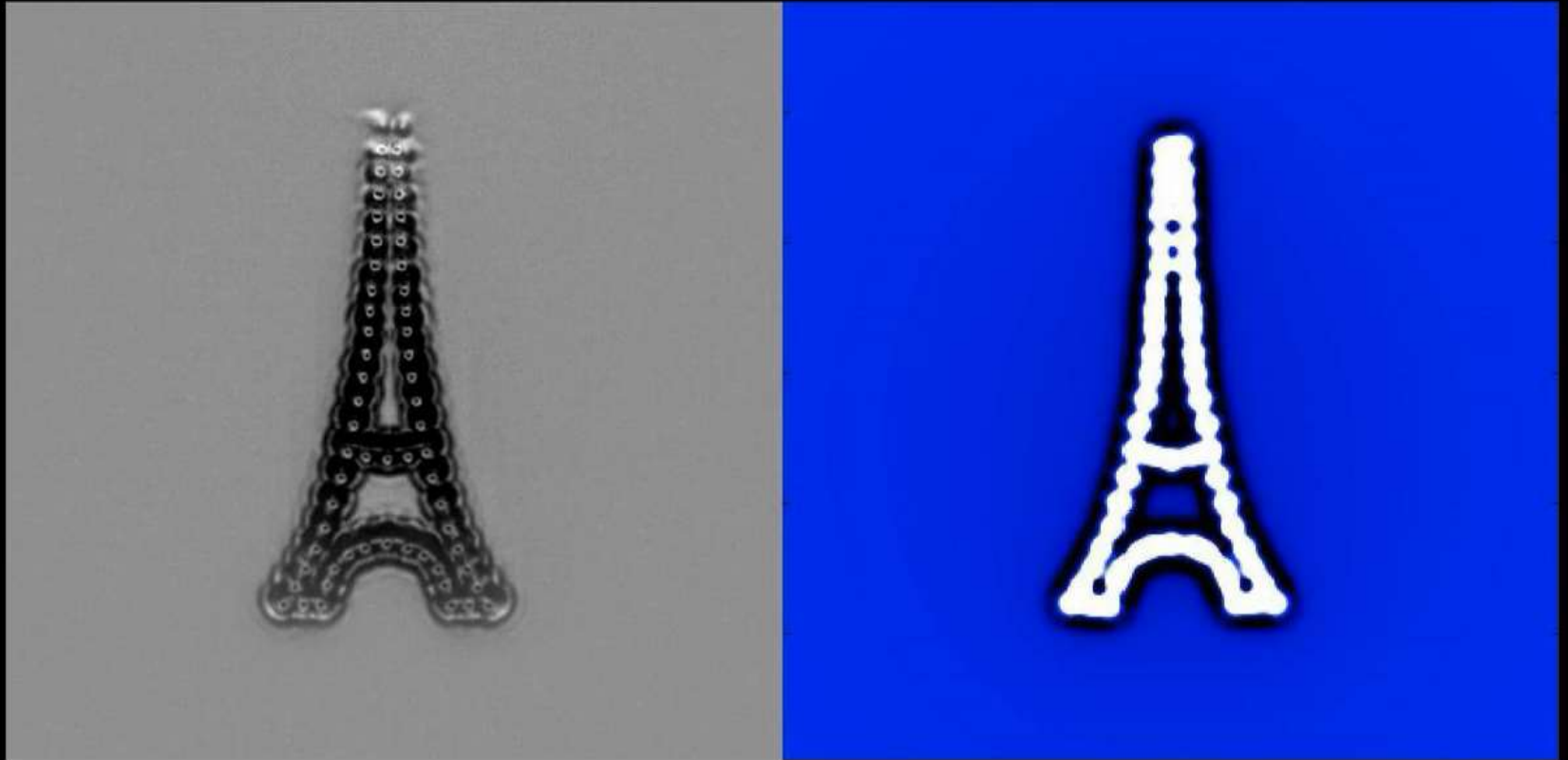


Time Mirror:



**Instantaneous  
Time Mirror**

# Complexe sources



$$t - t_{ITM} = -202 \text{ ms}$$

**Thank you**

