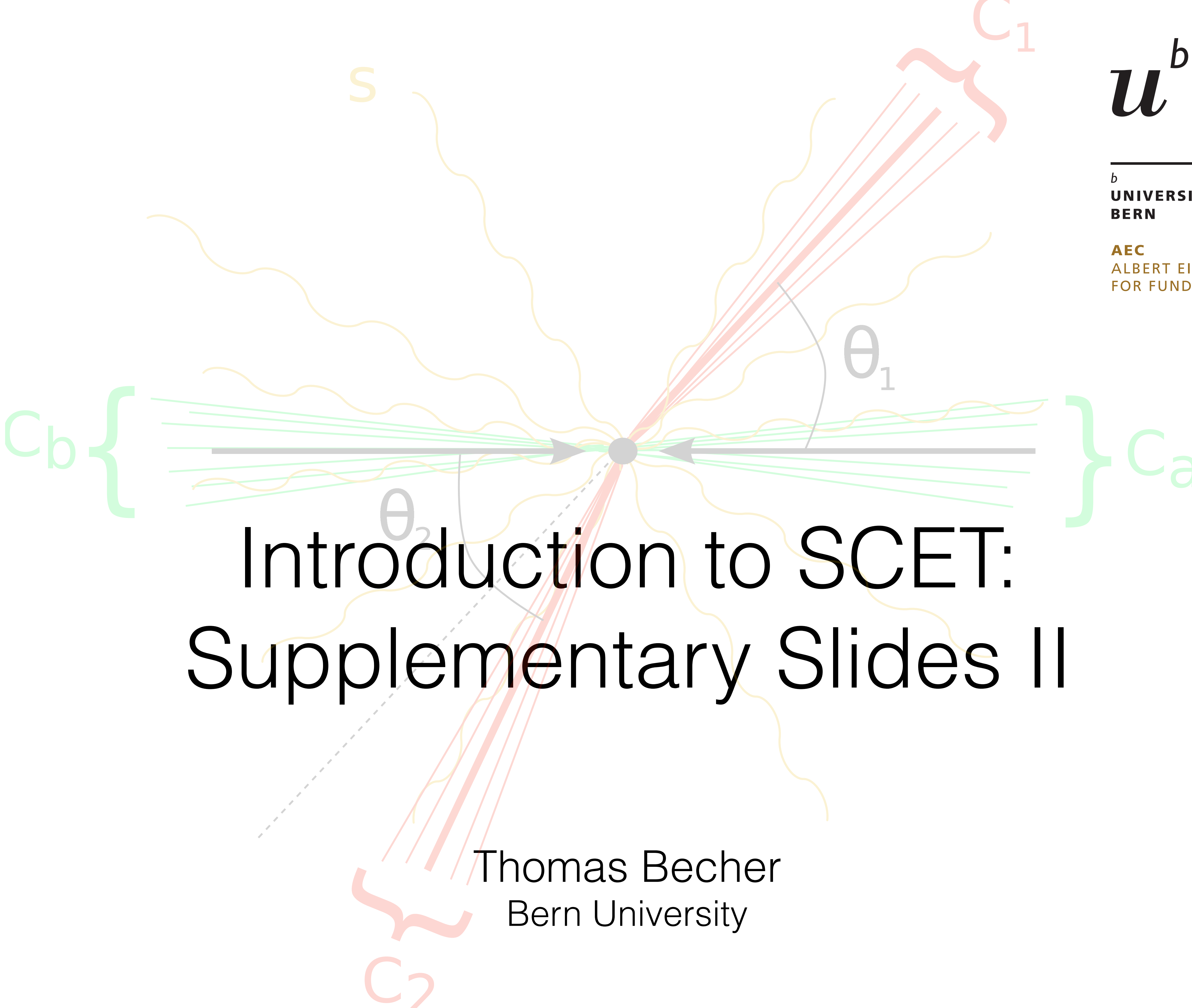

UNIVERSITÄT BERN
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FOR FUNDAMENTAL PHYSICS

Introduction to SCET: Supplementary Slides II

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Lectures on Soft-Collinear Effective Field Theory
Les Houches Summer School, July 2017

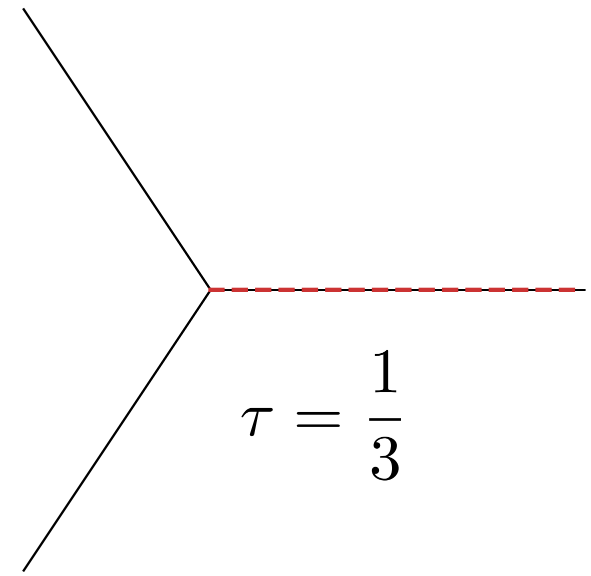
Thrust

Thrust and thrust axis

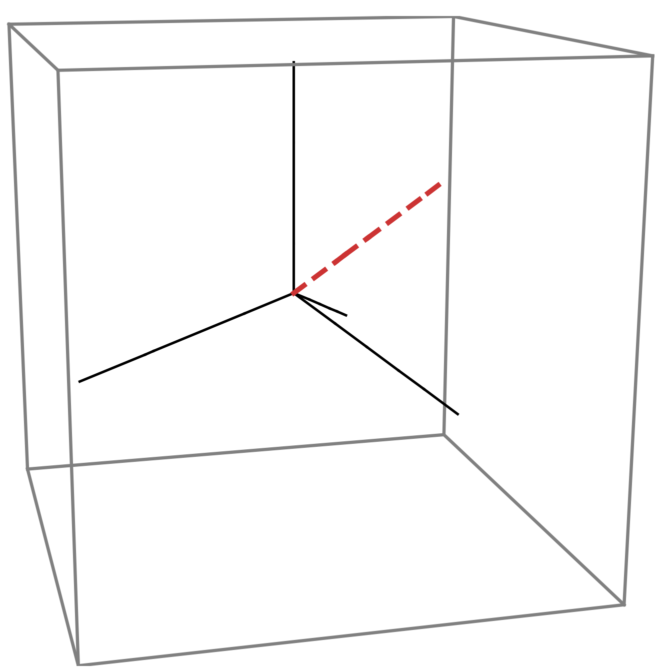
$$T = \frac{1}{Q} \max_{\vec{n}} \sum_i |\vec{n} \cdot \vec{p_i}|$$

$$\tau = 1 - T$$

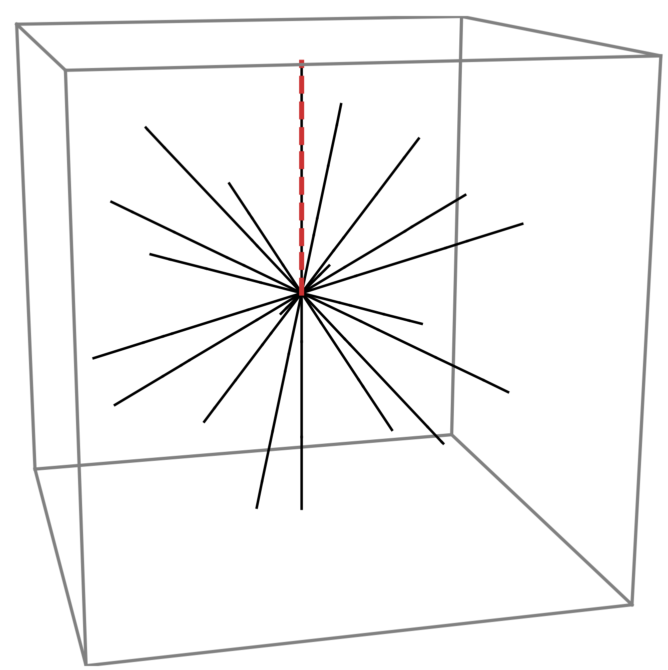
$\tau = 0$



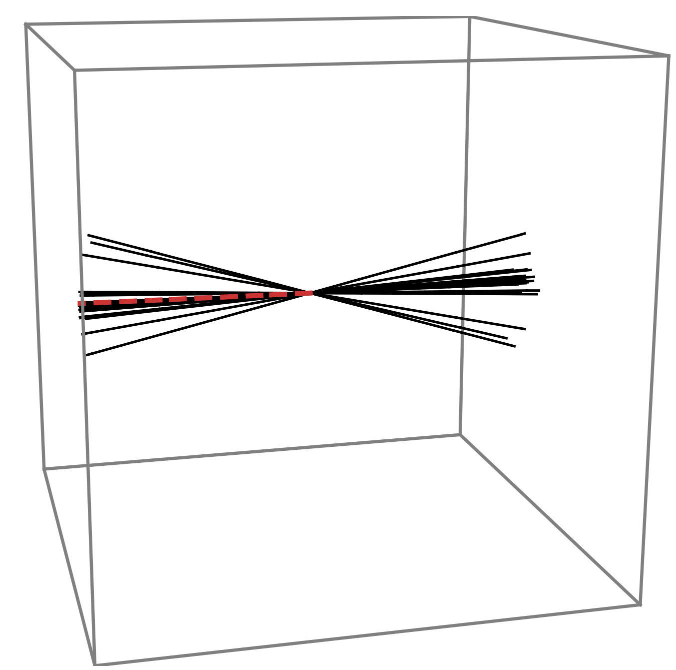
$\tau = \frac{1}{3}$



$\tau = 1 - \frac{1}{\sqrt{3}} = 0.42$



$\tau = 0.48$



$\tau = \frac{M_1^2 + M_2^2}{Q^2}$

Logarithmically enhanced contributions

- The LO thrust distribution has the form

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \frac{2\alpha_s}{3\pi}\left[-\frac{3}{\tau}+6+9\tau+\frac{(6\tau^2-6\tau+4)}{(1-\tau)\tau}\ln\frac{1-2\tau}{\tau}\right]$$

$$= \frac{2\alpha_s}{3\pi}\left[\frac{-4\ln\tau-3}{\tau}+d_{\mathrm{regular}}(\tau)\right]$$

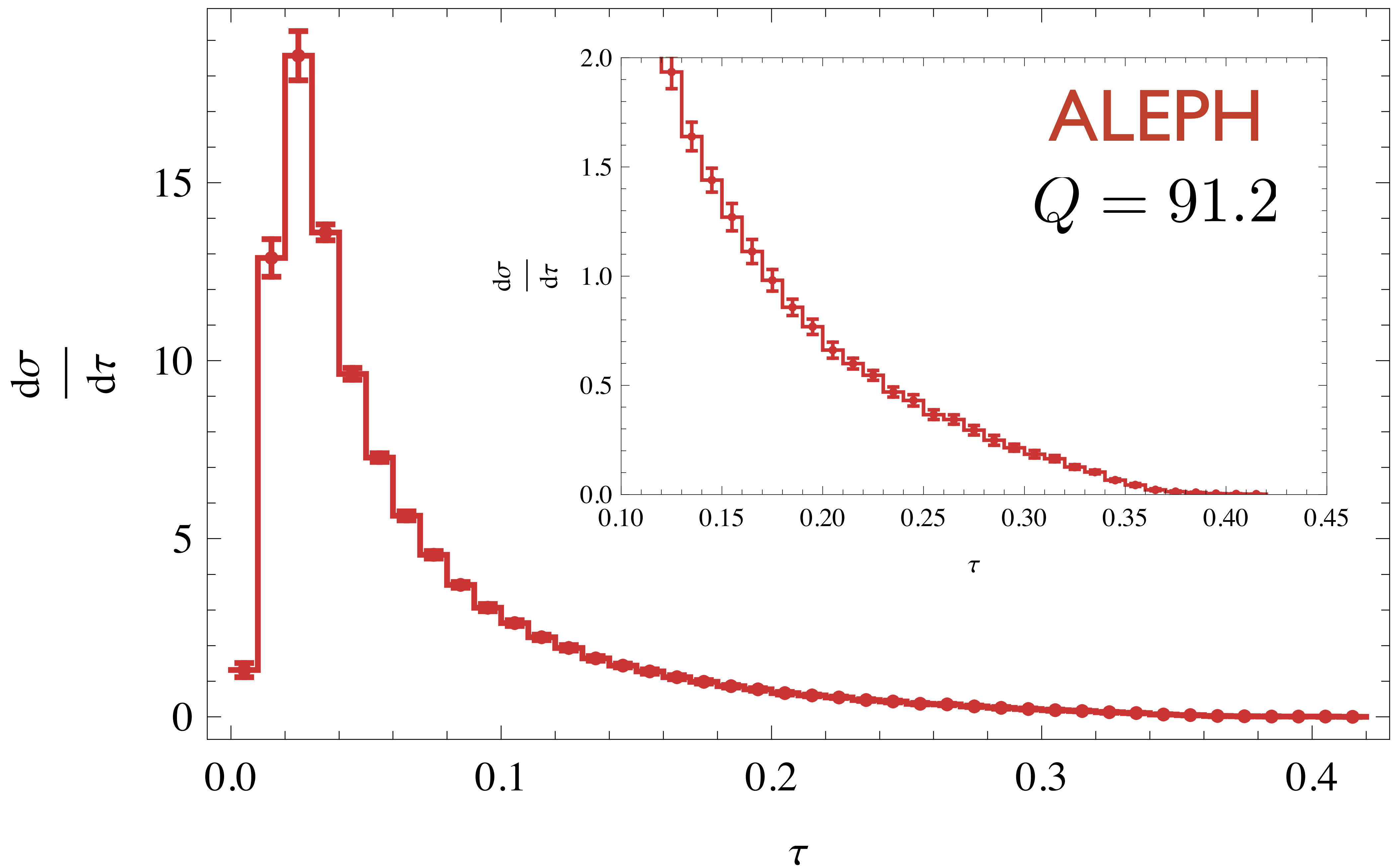
singular terms

- Integral over the end-point is

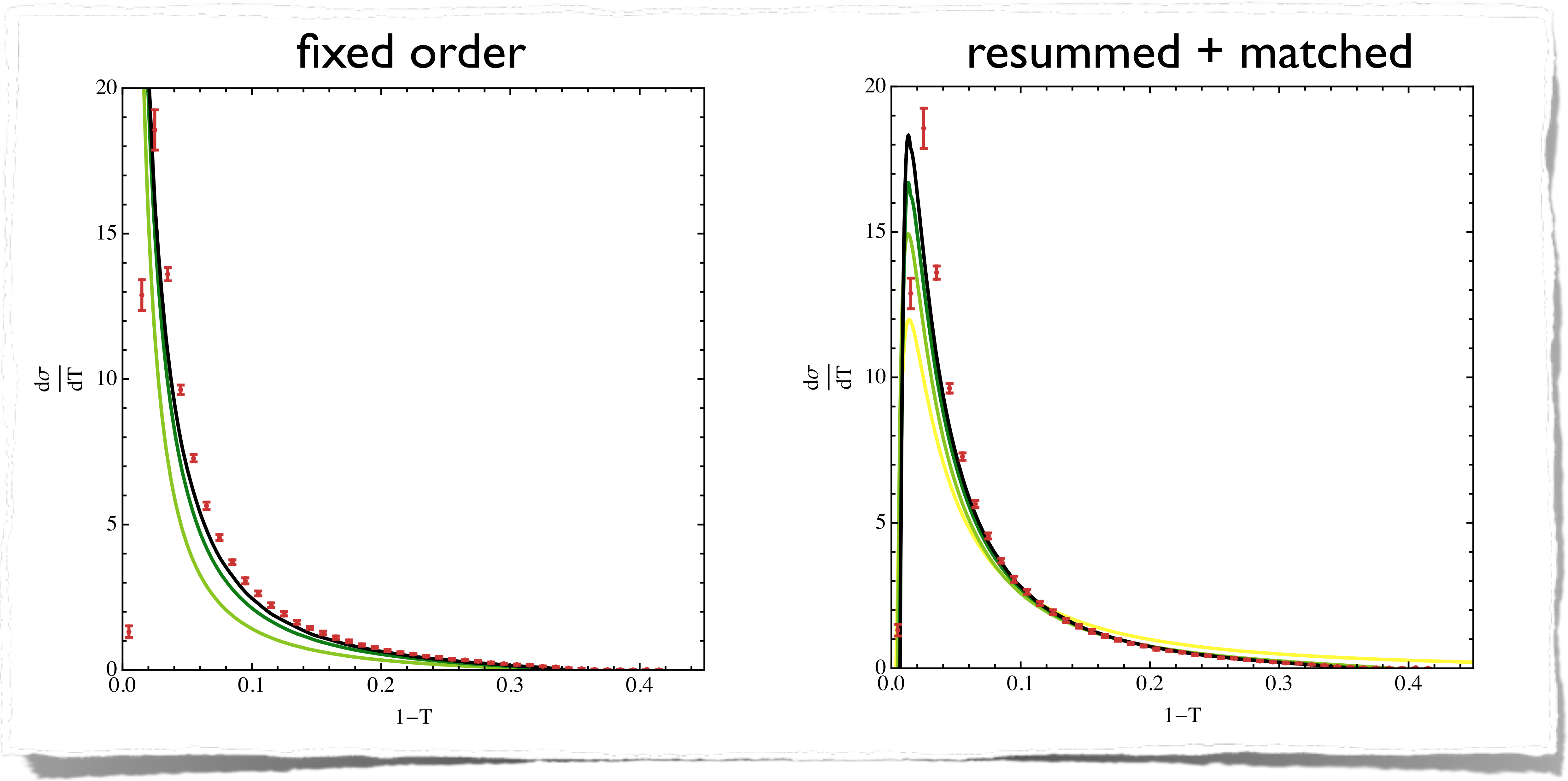
$$R(\tau)=\int_0^\tau d\tau'\,\frac{1}{\sigma_0}\frac{d\sigma}{d\tau'}=\frac{2\alpha_s}{3\pi}\left[-2\ln^2\tau-3\ln\tau+\ldots\right]$$

Sudakov double logarithm

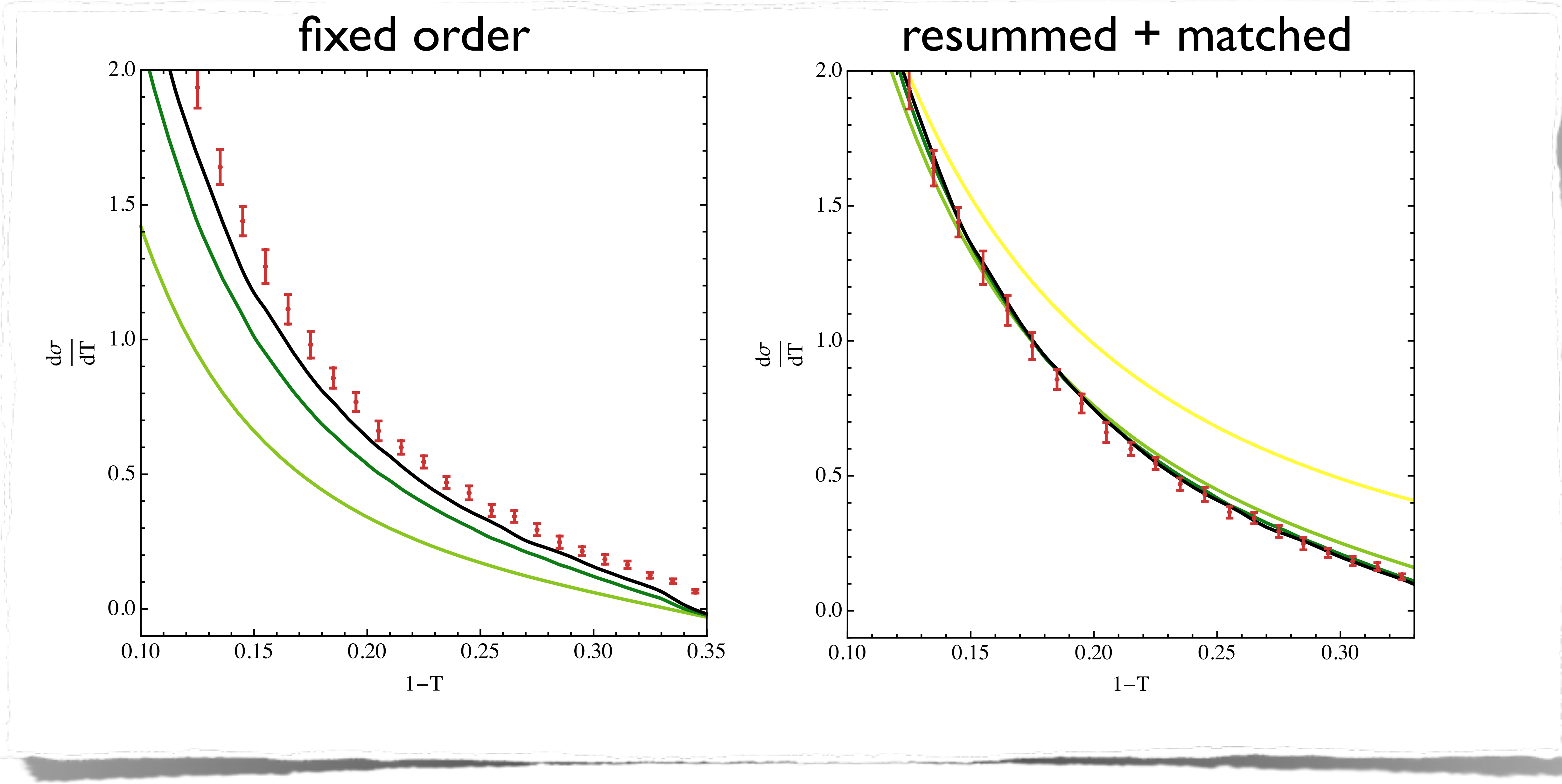
Thrust measurement by ALEPH



Resummed vs fixed order

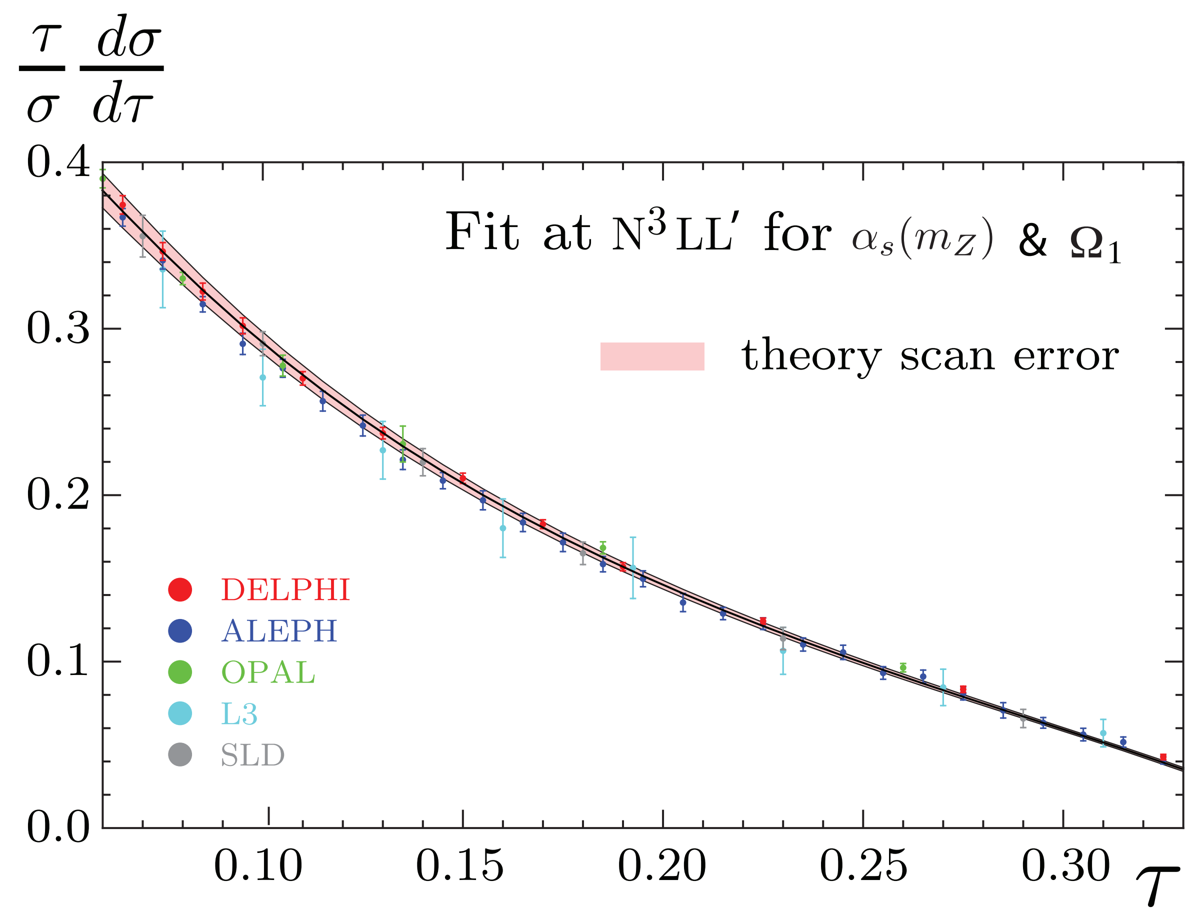


τ=1-T



- For $\alpha_s(M_Z)$=0.118
- This is the region relevant for α_s determination

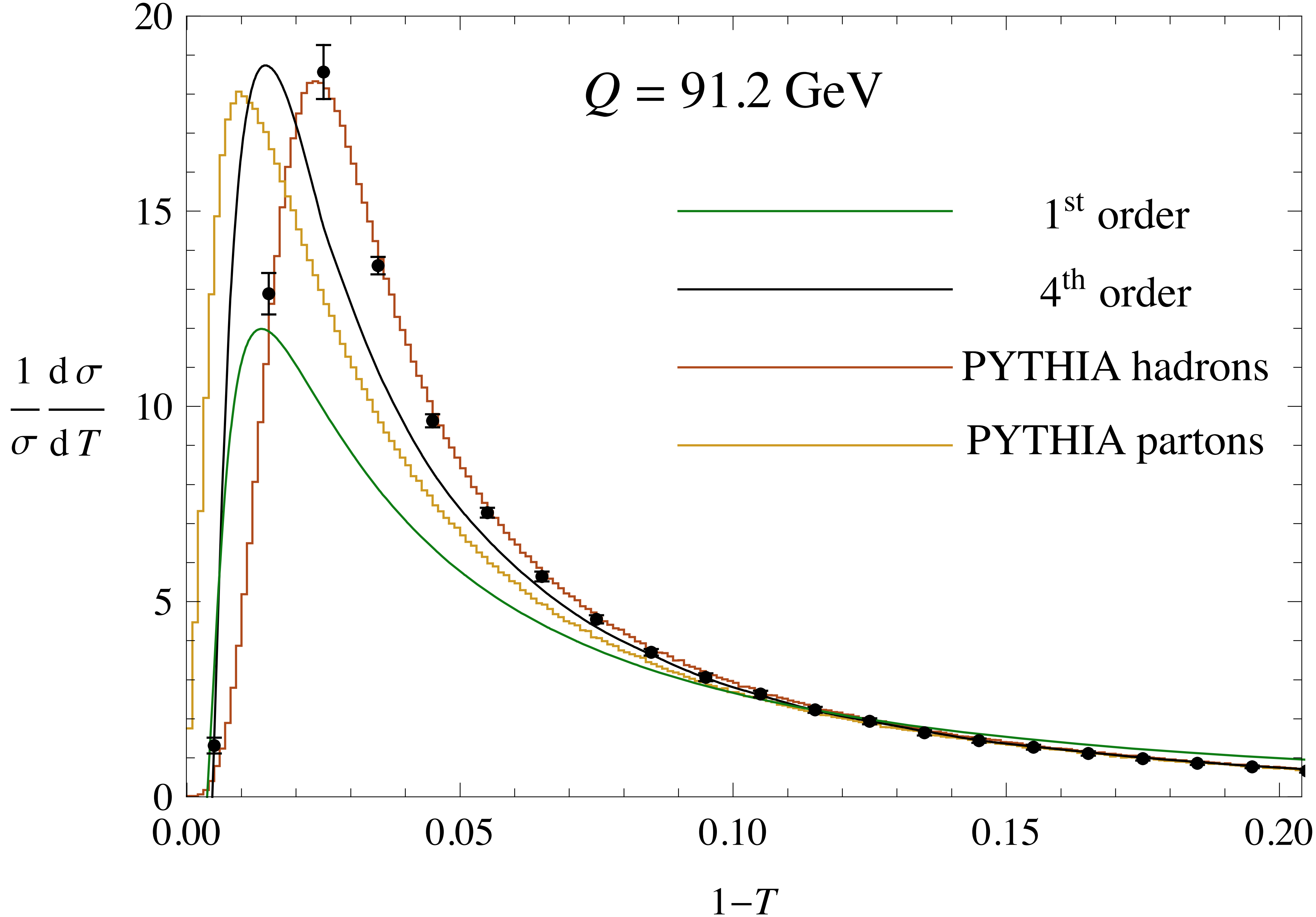
Precision determination of α_s



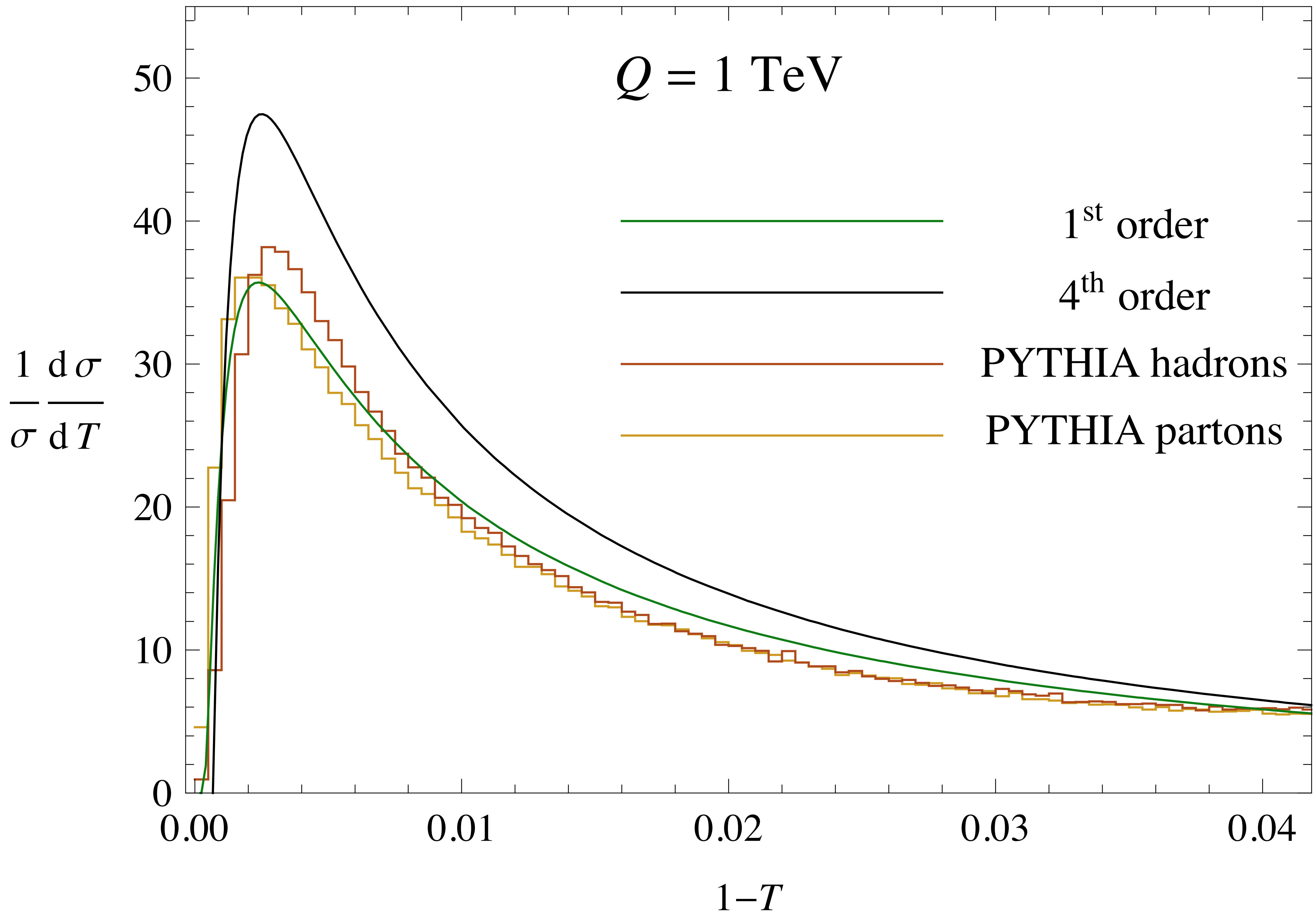
$$\alpha_s(m_Z) = 0.1135 \pm (0.0002)_{\text{expt}} \pm (0.0005)_{\Omega_1} \pm (0.0009)_{\text{pert}}$$

(hadronisation)

Abbate, Fickinger, Hoang, Mateu and Stewart 1004.4894



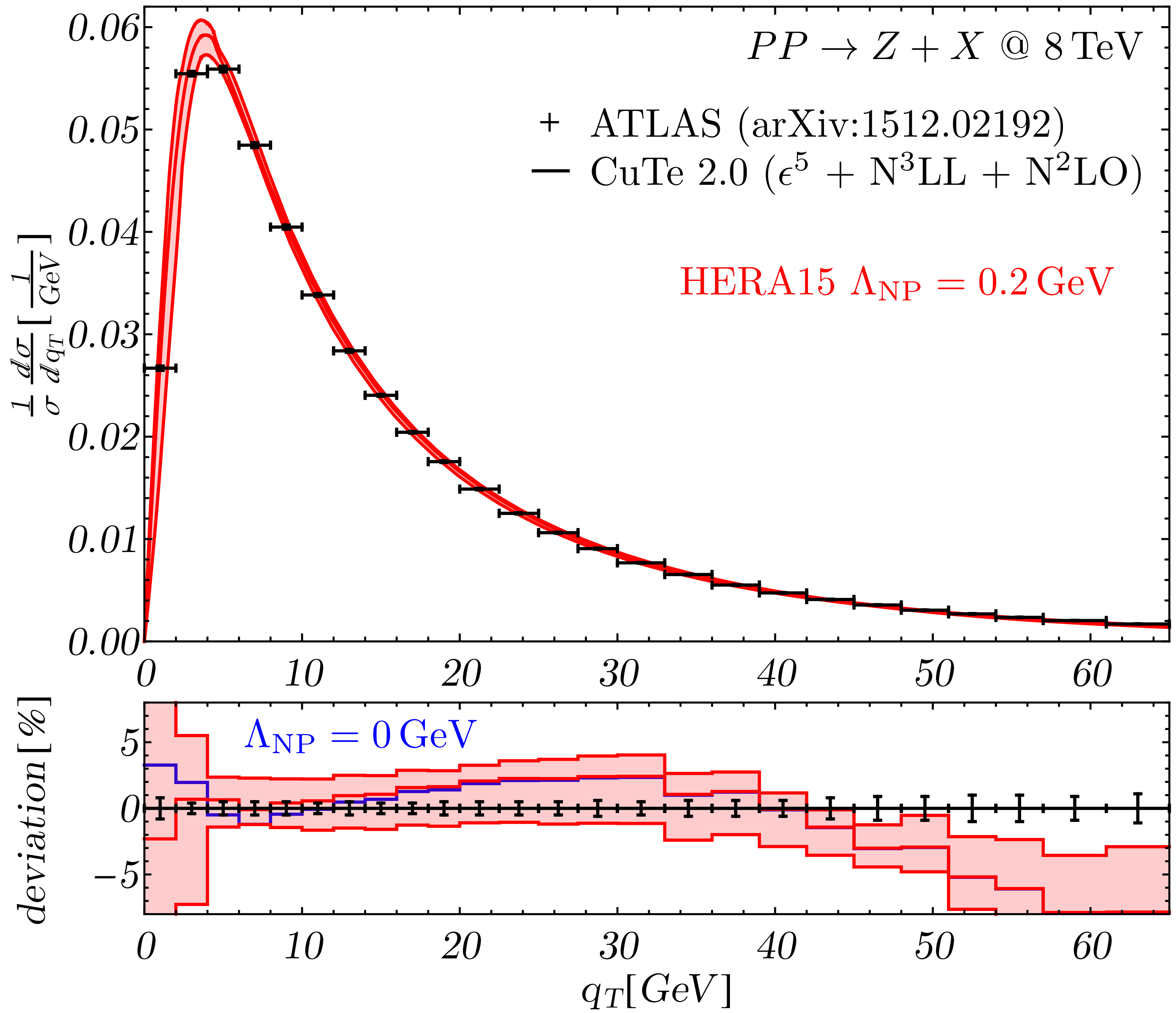
$Q = 91.2$ GeV
1st order
4th order
PYTHIA hadrons
PYTHIA partons
$\frac{1}{\sigma}\frac{d\sigma}{dT}$
20
15
10
5
0
0.00
0.05
0.10
0.15
0.20
$1-T$



$Q = 1$ TeV
1^{st} order
4^{th} order
PYTHIA hadrons
PYTHIA partons
$\frac{1}{\sigma}\frac{d\sigma}{dT}$
50
40
30
20
10
0
0.00
0.01
0.02
0.03
0.04
$1-T$

Another example: q_T resummation at LHC

CuTe 2.0 TB, Lübbert, Neubert, Wilhelm



Cannot use fixed-order computation in peak region.

Cone jets & NGLs

Factorization theorem

TB, Neubert, Rothen, Shao ’15 ’16, see also Caron-Huot ‘15

Hard function.
m hard partons along
fixed directions $\{n_1, \ldots, n_m\}$

Soft function
with *m* Wilson lines

$$\sigma(\beta) = \sum_{m=2}^{\infty} \big\langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, Q, \mu) \otimes \boldsymbol{\mathcal{S}}_m(\{\underline{n}\}, Q\beta, \mu) \big\rangle \,;$$

color trace

integration over the *m* directions

First all-order factorization theorem for non-global observable. Achieves scale separation!

$$\sigma(\beta) = \sum_{m=2}^{\infty} \langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, Q, \mu) \otimes \boldsymbol{\mathcal{S}}_m(\{\underline{n}\}, Q\beta, \mu) \rangle \,;$$

High-*E* physics
Wilson coefficients

Low-*E* physics
EFT Operator

- Renormalization of hard Wilson coefficients

$$\boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, Q, \delta, \epsilon) = \sum_{l=2}^{m} \boldsymbol{\mathcal{H}}_l(\{\underline{n}\}, Q, \delta, \mu)\, \boldsymbol{Z}^H_{lm}(\{\underline{n}\}, Q, \delta, \epsilon, \mu)$$

- Same *Z*-factor must render $\mathcal{S}_m$ finite!
- Associated anomalous dimension $\boldsymbol{\Gamma}^H$

$$\frac{d}{d\ln\mu}\, \boldsymbol{Z}^H_{km}(\{\underline{n}\}, Q, \delta, \epsilon, \mu) = \sum_{l=k}^{m} \boldsymbol{Z}^H_{kl}(\{\underline{n}\}, Q, \delta, \epsilon, \mu)\, \hat{\otimes}\, \boldsymbol{\Gamma}^H_{lm}(\{\underline{n}\}, Q, \delta, \mu)$$

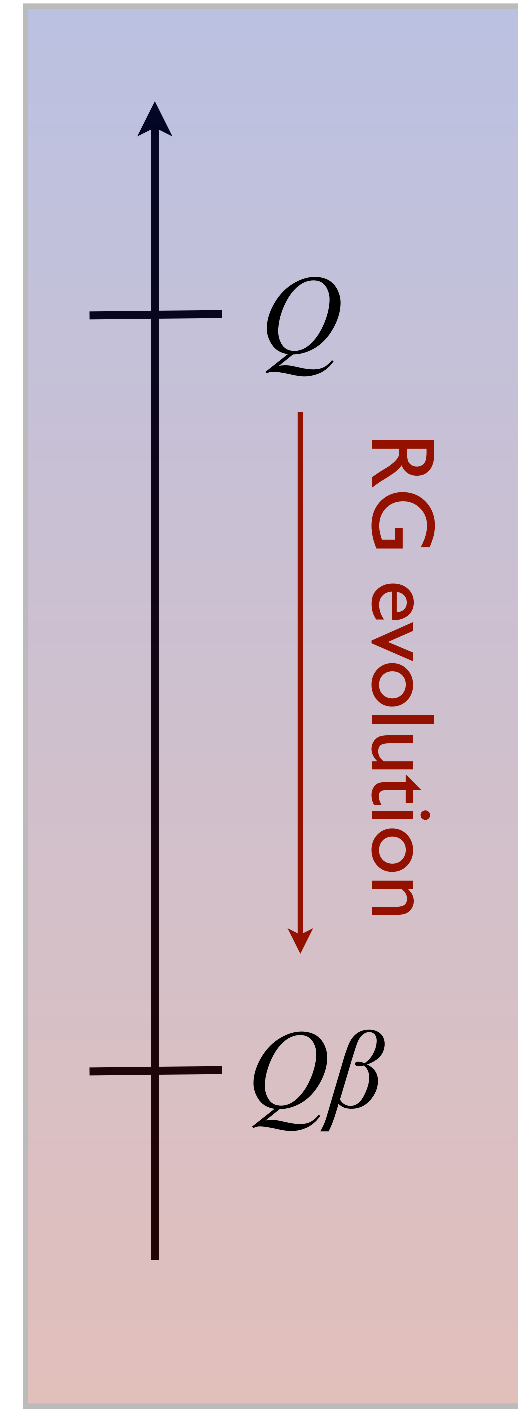
Resummation by RG evolution

Wilson coefficients fulfill renormalization group (RG) equations

$$\frac{d}{d\ln\mu}\boldsymbol{\mathcal{H}}_m(Q,\mu) = -\sum_{l=2}^{m}\boldsymbol{\mathcal{H}}_l(Q,\mu)\,\boldsymbol{\Gamma}^H_{lm}(Q,\mu)$$

1. Compute $\mathcal{H}_m$ at a characteristic high scale $\mu_h \sim Q$
2. Evolve $\mathcal{H}_m$ to the scale of low energy physics $\mu_l \sim Q\beta$

Avoids large logarithms $\alpha_s{}^n \ln^n(\beta)$ of scale ratios which can spoil convergence of perturbation theory.



RG = Parton Shower

- Ingredients for LL

$$\boldsymbol{\mathcal{H}}_2(\mu = Q) = \sigma_0$$
$$\boldsymbol{\mathcal{H}}_m(\mu = Q) = 0 \text{ for } m > 2$$
$$\boldsymbol{\mathcal{S}}_m(\mu = \beta Q) = 1$$

$$\boldsymbol{\Gamma}^{(1)} = \begin{pmatrix} \boldsymbol{V}_2 & \boldsymbol{R}_2 & 0 & 0 & \dots \\ 0 & \boldsymbol{V}_3 & \boldsymbol{R}_3 & 0 & \dots \\ 0 & 0 & \boldsymbol{V}_4 & \boldsymbol{R}_4 & \dots \\ 0 & 0 & 0 & \boldsymbol{V}_5 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

- RG

$$\frac{d}{dt}\boldsymbol{\mathcal{H}}_m(t) = \boldsymbol{\mathcal{H}}_m(t)\boldsymbol{V}_m + \boldsymbol{\mathcal{H}}_{m-1}(t)\boldsymbol{R}_{m-1}\,.$$

$$t = \int_{\alpha(\mu)}^{\alpha(Q)} \frac{d\alpha}{\beta(\alpha)}\,\frac{\alpha}{4\pi}$$

- Equivalent to parton shower equation

$$\boldsymbol{\mathcal{H}}_m(t) = \boldsymbol{\mathcal{H}}_m(t_1)e^{(t-t_1)V_n} + \int_{t_1}^{t} dt'\boldsymbol{\mathcal{H}}_{m-1}(t')\boldsymbol{R}_{m-1}e^{(t-t')\boldsymbol{V}_n}$$

1-loop anomalous dimension

$$\boldsymbol{V}_m = \boldsymbol{\Gamma}^{(1)}_{m,m} = 2 \sum_{(ij)} (\boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,L} + \boldsymbol{T}_{i,R} \cdot \boldsymbol{T}_{j,R}) \int \frac{d\Omega(n_k)}{4\pi} W^k_{ij},$$

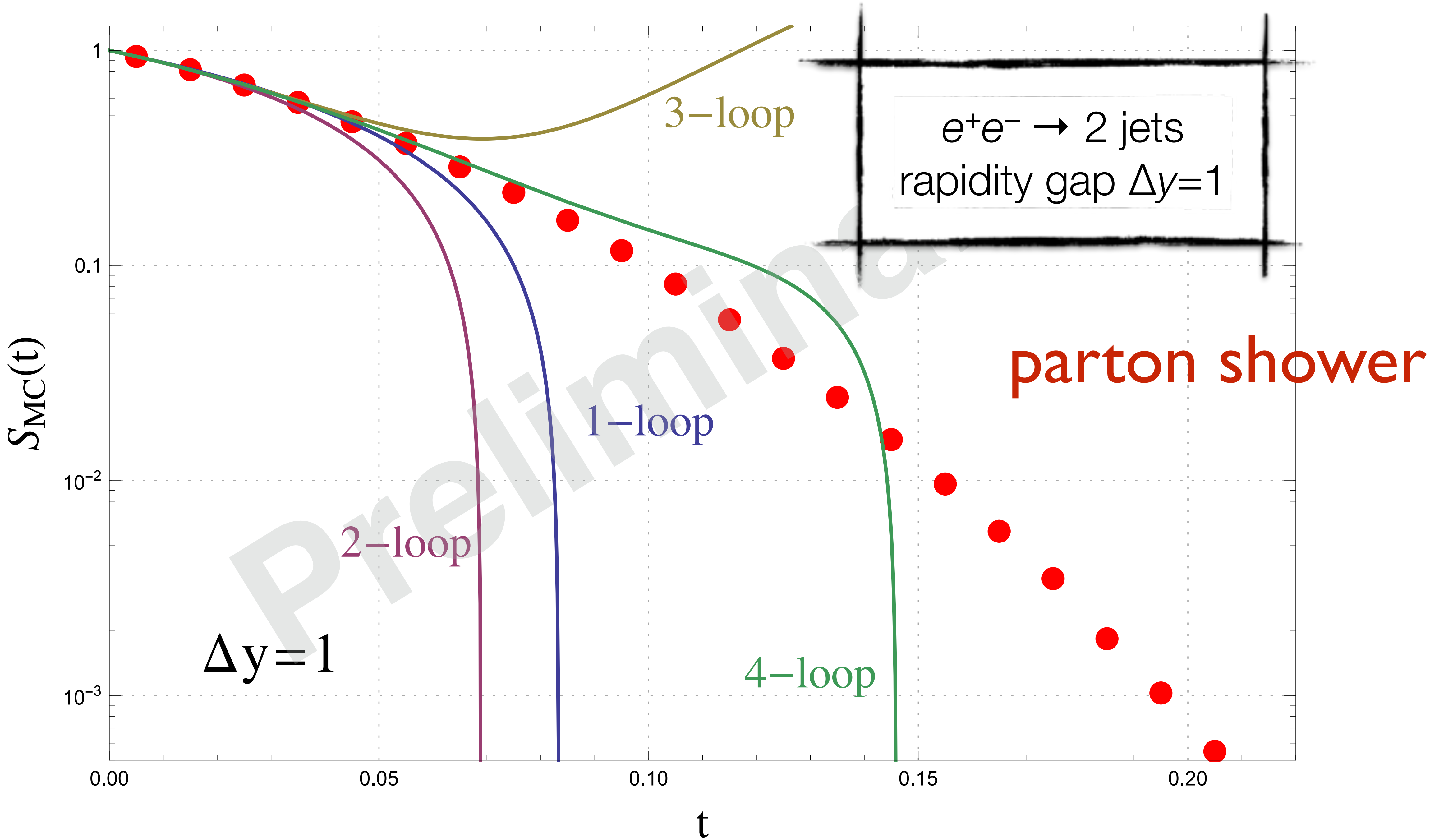
$$\boldsymbol{R}_m = \boldsymbol{\Gamma}^{(1)}_{m,m+1} = -4 \sum_{(ij)} \boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,R} \, W^{m+1}_{ij} \, \Theta^{n\bar{n}}_{\text{in}}(n_{m+1}) \, .$$

- Contain dipoles → dipole shower

$$W^k_{ij} = \frac{n_i \cdot n_j}{n_i \cdot n_k \, n_j \cdot n_k}$$

- Trivial color structure at large N_c :

$$\boldsymbol{T}_i \cdot \boldsymbol{T}_j \to -\frac{N_c}{2} \, \delta_{j,i\pm 1}$$



- Equivalent to the dipole shower used by Dasgupta and Salam '02.

Work in progress

- Finite N_c
  - nontrivial color structure, interference
  - MC over colors? Expand in 1/N_c ? Plätzer, Sjödahl ’12, Plätzer ‘13
- Subleading logarithms

$$\boldsymbol{\Gamma}^{(2)} = \begin{pmatrix} \boldsymbol{v}_2 & \boldsymbol{r}_2 & \boldsymbol{d}_2 & 0 & \dots \\ 0 & \boldsymbol{v}_3 & \boldsymbol{r}_3 & \boldsymbol{d}_2 & \dots \\ 0 & 0 & \boldsymbol{v}_4 & \boldsymbol{r}_4 & \dots \\ 0 & 0 & 0 & \boldsymbol{v}_5 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

v_m: two-loop virtual

r_m: real-virtual

d_m: double real

see Caron-Huot ‘15

- Hadronic collisions and super-leading logs