Les Houches Lectures: EFT of Large-Scale Structure Exercises

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July 23, 2017

1 - Clustering of Fixed Height Subsamples Consider a Gaussian random field δ described by power spectrum P. The corresponding real space correlation function is ξ and the variance $\sigma^2 = \xi(0)$. Consider the PDF of fluctuations

$$\mathbb{P}_{\mathrm{2pt}}(\delta_1, \delta_2 | r) = \frac{1}{\sqrt{(2\pi)^2 |C(r)|}} \exp\left[-\frac{1}{2} \boldsymbol{Y}^{\mathsf{T}} C^{-1}(r) \boldsymbol{Y} \right],$$

where $\boldsymbol{Y} = (\delta_1, \delta_2) = \sigma(\nu_1, \nu_2)$ is the state vector and $C_{ij}(r) = \langle \delta_i \delta_j \rangle = \sigma^2 \delta^{(\mathrm{K})} + \left(1 - \delta^{(\mathrm{K})}\right) \xi(r)$ is the covariance matrix. The correlation function of the field can be recovered as

$$\xi(r) = \langle \delta(\boldsymbol{x}) \delta(\boldsymbol{x} + \boldsymbol{r}) \rangle = \sigma^4 \int \mathrm{d}\nu_1 \int \mathrm{d}\nu_2 \; \nu_1 \nu_2 \mathbb{P}_{\mathrm{2pt}}(\sigma\nu_1, \sigma\nu_2 | r).$$

Consider the subset of fluctuations of fixed amplitude ν_{c} and calculate their correlation function

$$\xi_{\mathrm{c}}(r) = \frac{\mathbb{P}_{\mathrm{2pt}}(\sigma\nu_{\mathrm{c}}, \sigma\nu_{\mathrm{c}}; r)}{\mathbb{P}_{\mathrm{1pt}}(\sigma\nu_{\mathrm{c}}) \mathbb{P}_{\mathrm{1pt}}(\sigma\nu_{\mathrm{c}})} - 1.$$

For large separations this allows an expansion in the small quantity ξ/σ^2. Write down this expansion to second order. The prefactors of this expansion are called bias parameters. Fourier transform the expression to k-space and consider the low-k limit. Can you identify contributions that would require a counterterm? *Remark:* The above model can be used to model regions that will eventually form dark matter haloes, i.e., formation sites of galaxies. In this context the field is smoothed on a scale R to consider fluctuations of a given mass $M \propto \bar{\rho} R^3$

$$\delta_R(\boldsymbol{x}) = \int \mathrm{d}^3 x W_R(|\boldsymbol{x} - \boldsymbol{x}'|) \delta(\boldsymbol{x}'),$$

where $W_R(r)$ is a Gaussian or top hat filter.

2 - Equality Scale Integrating the Bose-Einstein distribution, we get that the radiation energy density is related to the temperature T_{CMB} of the CMB photons by $\rho_{\mathrm{rad}} = \pi^2/15 \; T_{\mathrm{CMB}}^4$. Use the measured values of the CMB temperature of $T_{\mathrm{CMB}} = 2.725\mathrm{K}$ and matter density $\Omega_{\mathrm{m},0} = 0.28$ to calculate the scale factor of matter-radiation equality a_{eq}. Calculate the size of the horizon at a_{eq} and the wavenumber k_{eq} of fluctuations entering at matter-radiation equality. This is the characteristic scale, at which the transfer function transitions from the large scale k^0 behaviour to the small scale $\ln(k)/k^2$ behaviour.

3 - Fluid Equations Using the definitions of density, mean streaming velocity and velocity dispersion in terms of the distribution function $f(\boldsymbol{x}, \boldsymbol{p}, \eta)$ and the conservation of phase space density $\mathrm{d}f/\mathrm{d}\eta = 0$, derive the continuity and Euler equations for collisonless dark matter. You will need to use the energy momentum conservation of the homogeneous background Universe $\bar{\rho}' + 3\mathcal{H}\bar{\rho} = 0$.

4 - Recursion Relations Starting from the k-space version of the continuity and Euler equations in a matter-only EdS Universe, and the ansatz

$$\delta(\boldsymbol{k},\eta) = \sum_{i=1}^{\infty} a^i(\eta)\delta^{(i)}(\boldsymbol{k}) \qquad \theta(\boldsymbol{k},\eta) = -\mathcal{H}(\eta)\sum_{i=1}^{\infty} a^i(\eta)\,\tilde{\theta}^{(i)}(\boldsymbol{k}),$$

derive the recursion relations for the gravitational coupling kernels F_n and G_n relating the n-th order fields to the linear density fields

$$\delta^{(n)}(\boldsymbol{k}) = \prod_{m=1}^{n}\left\{\int \frac{\mathrm{d}^3 q_m}{(2\pi)^3}\delta^{(1)}(\boldsymbol{q}_m)\right\} F_n(\boldsymbol{q}_1,\ldots,\boldsymbol{q}_n)\delta^{(\mathrm{D})}(\boldsymbol{k}-\boldsymbol{q}|_1^n)$$
$$\tilde{\theta}^{(n)}(\boldsymbol{k}) = \prod_{m=1}^{n}\left\{\int \frac{\mathrm{d}^3 q_m}{(2\pi)^3}\delta^{(1)}(\boldsymbol{q}_m)\right\} G_n(\boldsymbol{q}_1,\ldots,\boldsymbol{q}_n)\delta^{(\mathrm{D})}(\boldsymbol{k}-\boldsymbol{q}|_1^n)\,.$$

5 - Two-loop power spectrum Write down the the diagrams and integrals contributing to the two-loop matter power spectrum in terms of the gravitational coupling kernels F_n. Try to identify the diagrams with the strongest UV-sensitivity.