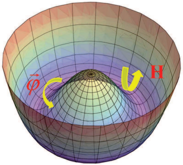
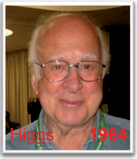
4. Electroweak Effective Theory

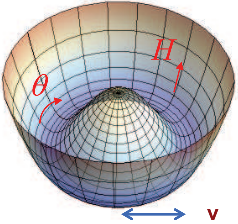


- **Higgs Mechanism**
- **Custodial Symmetry**
- **Equivalence Theorem**
- **EW Effective Theory**

Great success of the Standard Model

BEGHHK (≡ Higgs) Mechanism





$$SU(2)_L \otimes U(1)_Y$$

v = 246 GeV

Fundación Príncipe de Asturias

$$M_Z \cos\theta_W = M_W = \frac{1}{2} v\, g$$

Possible Scenarios of EWSB

① **SM Higgs:** Favoured by EW precision tests

② **Alternative perturbative EWSB:**

Scalar Doublets and singlets $\rho_{\text{tree}} = \frac{M_W^2}{M_Z^2 c_W^2} = \frac{\sum_i v_i^2 [T_i(T_i+1) - Y_i^2]}{2\sum_i v_i^2 Y_i^2}$

③ **Dynamical (non-perturbative) EWSB:**

Pseudo-Goldstone Higgs

Scalar Resonance

Possible Scenarios of EWSB

① **SM Higgs:** Favoured by EW precision tests

② **Alternative perturbative EWSB:**

Scalar Doublets and singlets $$\rho_{\text{tree}} = \frac{M_W^2}{M_Z^2 c_W^2} = \frac{\sum_i v_i^2 [T_i (T_i + 1) - Y_i^2]}{2 \sum_i v_i^2 Y_i^2}$$

③ **Dynamical (non-perturbative) EWSB:**

Pseudo-Goldstone Higgs

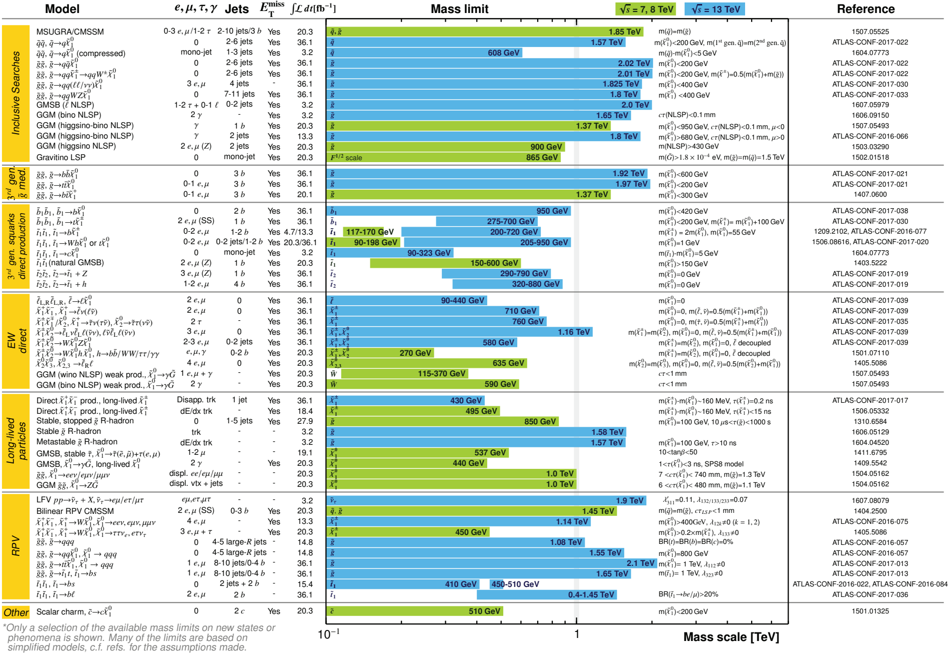
Scalar Resonance

ATLAS SUSY Searches* - 95% CL Lower Limits

May 2017

ATLAS Preliminary

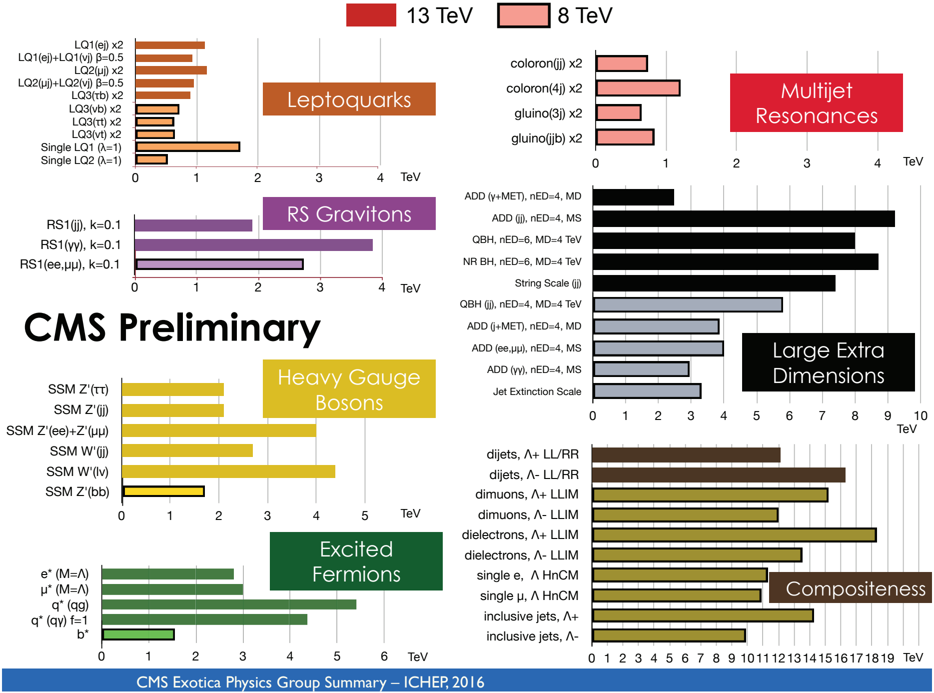
$\sqrt{s}$ = 7, 8, 13 TeV



🟩 $\sqrt{s}$ = 7, 8 TeV 🟦 $\sqrt{s}$ = 13 TeV

	Model	e,μ,τ,γ	Jets	E_T^{miss}	$\int\mathcal{L}\,dt[\mathrm{fb}^{-1}]$	Mass limit		Reference
Inclusive Searches	MSUGRA/CMSSM	0-3 e,μ/1-2 τ	2-10 jets/3 b	Yes	20.3	🟩 $\tilde{q},\tilde{g}$ 1.85 TeV	m($\tilde{q}$)=m($\tilde{g}$)	1507.05525
	$\tilde{q}\tilde{q}$, $\tilde{q}\to q\tilde{\chi}_1^0$	0	2-6 jets	Yes	36.1	🟦 $\tilde{q}$ 1.57 TeV	m($\tilde{\chi}_1^0$)<200 GeV, m(1$^{\mathrm{st}}$ gen. $\tilde{\mathrm{q}}$)=m(2$^{\mathrm{nd}}$ gen. $\tilde{\mathrm{q}}$)	ATLAS-CONF-2017-022
	$\tilde{q}\tilde{q}$, $\tilde{q}\to q\tilde{\chi}_1^0$ (compressed)	mono-jet	1-3 jets	Yes	3.2	🟦 $\tilde{q}$ 608 GeV	m($\tilde{q}$)-m($\tilde{\chi}_1^0$)<5 GeV	1604.07773
	$\tilde{g}\tilde{g}$, $\tilde{g}\to q\bar{q}\tilde{\chi}_1^0$	0	2-6 jets	Yes	36.1	🟦 $\tilde{g}$ 2.02 TeV	m($\tilde{\chi}_1^0$)<200 GeV	ATLAS-CONF-2017-022
	$\tilde{g}\tilde{g}$, $\tilde{g}\to qq\tilde{\chi}_1^\pm\to qqW^\pm\tilde{\chi}_1^0$	0	2-6 jets	Yes	36.1	🟦 $\tilde{g}$ 2.01 TeV	m($\tilde{\chi}_1^0$)<200 GeV, m($\tilde{\chi}^\pm$)=0.5(m($\tilde{\chi}_1^0$)+m($\tilde{g}$))	ATLAS-CONF-2017-022
	$\tilde{g}\tilde{g}$, $\tilde{g}\to qq(\ell\ell/\nu\nu)\tilde{\chi}_1^0$	3 e,μ	4 jets	-	36.1	🟦 $\tilde{g}$ 1.825 TeV	m($\tilde{\chi}_1^0$)<400 GeV	ATLAS-CONF-2017-030
	$\tilde{g}\tilde{g}$, $\tilde{g}\to qqWZ\tilde{\chi}_1^0$	0	7-11 jets	Yes	36.1	🟦 $\tilde{g}$ 1.8 TeV	m($\tilde{\chi}_1^0$) <400 GeV	ATLAS-CONF-2017-033
	GMSB ($\tilde{\ell}$ NLSP)	1-2 τ + 0-1 ℓ	0-2 jets	Yes	3.2	🟦 $\tilde{g}$ 2.0 TeV		1607.05979
	GGM (bino NLSP)	2 γ	-	Yes	3.2	🟦 $\tilde{g}$ 1.65 TeV	$c\tau$(NLSP)<0.1 mm	1606.09150
	GGM (higgsino-bino NLSP)	γ	1 b	Yes	20.3	🟩 $\tilde{g}$ 1.37 TeV	m($\tilde{\chi}_1^0$)<950 GeV, $c\tau$(NLSP)<0.1 mm, μ<0	1507.05493
	GGM (higgsino-bino NLSP)	γ	2 jets	Yes	13.3	🟦 $\tilde{g}$ 1.8 TeV	m($\tilde{\chi}_1^0$)>680 GeV, $c\tau$(NLSP)<0.1 mm, μ>0	ATLAS-CONF-2016-066
	GGM (higgsino NLSP)	2 e,μ (Z)	2 jets	Yes	20.3	🟩 $\tilde{g}$ 900 GeV	m(NLSP)>430 GeV	1503.03290
	Gravitino LSP	0	mono-jet	Yes	20.3	🟩 $F^{1/2}$ scale 865 GeV	m($\tilde{G}$)>1.8×10^{-4} eV, m($\tilde{g}$)=m($\tilde{q}$)=1.5 TeV	1502.01518
3rd gen. $\tilde{g}$ med.	$\tilde{g}\tilde{g}$, $\tilde{g}\to b\bar{b}\tilde{\chi}_1^0$	0	3 b	Yes	36.1	🟦 $\tilde{g}$ 1.92 TeV	m($\tilde{\chi}_1^0$)<600 GeV	ATLAS-CONF-2017-021
	$\tilde{g}\tilde{g}$, $\tilde{g}\to t\bar{t}\tilde{\chi}_1^0$	0-1 e,μ	3 b	Yes	36.1	🟦 $\tilde{g}$ 1.97 TeV	m($\tilde{\chi}_1^0$)<200 GeV	ATLAS-CONF-2017-021
	$\tilde{g}\tilde{g}$, $\tilde{g}\to b\bar{t}\tilde{\chi}_1^+$	0-1 e,μ	3 b	Yes	20.1	🟩 $\tilde{g}$ 1.37 TeV	m($\tilde{\chi}_1^0$)<300 GeV	1407.0600
3rd gen. squarks direct production	$\tilde{b}_1\tilde{b}_1$, $\tilde{b}_1\to b\tilde{\chi}_1^0$	0	2 b	Yes	36.1	🟦 $\tilde{b}_1$ 950 GeV	m($\tilde{\chi}_1^0$)<420 GeV	ATLAS-CONF-2017-038
	$\tilde{b}_1\tilde{b}_1$, $\tilde{b}_1\to t\tilde{\chi}_1^\pm$	2 e,μ (SS)	1 b	Yes	36.1	$\tilde{b}_1$ 🟦 275-700 GeV	m($\tilde{\chi}_1^0$)<200 GeV, m($\tilde{\chi}_1^\pm$)= m($\tilde{\chi}_1^0$)+100 GeV	ATLAS-CONF-2017-030
	$\tilde{t}_1\tilde{t}_1$, $\tilde{t}_1\to b\tilde{\chi}_1^\pm$	0-2 e,μ	1-2 b	Yes	4.7/13.3	$\tilde{t}_1$ 🟩 117-170 GeV 🟦 200-720 GeV	m($\tilde{\chi}_1^\pm$) = 2m($\tilde{\chi}_1^0$), m($\tilde{\chi}_1^0$)=55 GeV	1209.2102, ATLAS-CONF-2016-077
	$\tilde{t}_1\tilde{t}_1$, $\tilde{t}_1\to Wb\tilde{\chi}_1^0$ or $t\tilde{\chi}_1^0$	0-2 e,μ	0-2 jets/1-2 b	Yes	20.3/36.1	🟩 $\tilde{t}_1$ 90-198 GeV 🟦 205-950 GeV	m($\tilde{\chi}_1^0$)=1 GeV	1506.08616, ATLAS-CONF-2017-020
	$\tilde{t}_1\tilde{t}_1$, $\tilde{t}_1\to c\tilde{\chi}_1^0$	0	mono-jet	Yes	3.2	🟦 $\tilde{t}_1$ 90-323 GeV	m($\tilde{t}_1$)-m($\tilde{\chi}_1^0$)=5 GeV	1604.07773
	$\tilde{t}_1\tilde{t}_1$(natural GMSB)	2 e,μ (Z)	1 b	Yes	20.3	$\tilde{t}_1$ 🟩 150-600 GeV	m($\tilde{\chi}_1^0$)>150 GeV	1403.5222
	$\tilde{t}_2\tilde{t}_2$, $\tilde{t}_2\to\tilde{t}_1+Z$	3 e,μ (Z)	1 b	Yes	36.1	$\tilde{t}_2$ 🟦 290-790 GeV	m($\tilde{\chi}_1^0$)=0 GeV	ATLAS-CONF-2017-019
	$\tilde{t}_2\tilde{t}_2$, $\tilde{t}_2\to\tilde{t}_1+h$	1-2 e,μ	4 b	Yes	36.1	$\tilde{t}_2$ 🟦 320-880 GeV	m($\tilde{\chi}_1^0$)=0 GeV	ATLAS-CONF-2017-019
EW direct	$\tilde{\ell}_{\mathrm{L,R}}\tilde{\ell}_{\mathrm{L,R}}$, $\tilde{\ell}\to\ell\tilde{\chi}_1^0$	2 e,μ	0	Yes	36.1	🟦 $\tilde{\ell}$ 90-440 GeV	m($\tilde{\chi}_1^0$)=0	ATLAS-CONF-2017-039
	$\tilde{\chi}_1^+\tilde{\chi}_1^-$, $\tilde{\chi}_1^+\to\tilde{\ell}\nu(\ell\tilde{\nu})$	2 e,μ	0	Yes	36.1	🟦 $\tilde{\chi}_1^\pm$ 710 GeV	m($\tilde{\chi}_1^0$)=0, m($\tilde{\ell},\tilde{\nu}$)=0.5(m($\tilde{\chi}_1^\pm$)+m($\tilde{\chi}_1^0$))	ATLAS-CONF-2017-039
	$\tilde{\chi}_1^\pm\tilde{\chi}_1^\mp/\tilde{\chi}_2^0$, $\tilde{\chi}_1^+\to\tilde{\tau}\nu(\tau\tilde{\nu})$, $\tilde{\chi}_2^0\to\tilde{\tau}\tau(\nu\tilde{\nu})$	2 τ	-	Yes	36.1	🟦 $\tilde{\chi}_1^\pm$ 760 GeV	m($\tilde{\chi}_1^0$)=0, m($\tilde{\tau},\tilde{\nu}$)=0.5(m($\tilde{\chi}_1^\pm$)+m($\tilde{\chi}_1^0$))	ATLAS-CONF-2017-035
	$\tilde{\chi}_1^\pm\tilde{\chi}_2^0\to\tilde{\ell}_{\mathrm{L}}\nu\tilde{\ell}_{\mathrm{L}}\ell(\tilde{\nu}\nu)$, $\ell\tilde{\nu}\tilde{\ell}_{\mathrm{L}}\ell(\tilde{\nu}\nu)$	3 e,μ	0	Yes	36.1	🟦 $\tilde{\chi}_1^\pm,\tilde{\chi}_2^0$ 1.16 TeV	m($\tilde{\chi}_1^\pm$)=m($\tilde{\chi}_2^0$), m($\tilde{\chi}_1^0$)=0, m($\tilde{\ell},\tilde{\nu}$)=0.5(m($\tilde{\chi}_1^\pm$)+m($\tilde{\chi}_1^0$))	ATLAS-CONF-2017-039
	$\tilde{\chi}_1^\pm\tilde{\chi}_2^0\to W\tilde{\chi}_1^0Z\tilde{\chi}_1^0$	2-3 e,μ	0-2 jets	Yes	36.1	🟦 $\tilde{\chi}_1^\pm,\tilde{\chi}_2^0$ 580 GeV	m($\tilde{\chi}_1^\pm$)=m($\tilde{\chi}_2^0$), m($\tilde{\chi}_1^0$)=0, $\tilde{\ell}$ decoupled	ATLAS-CONF-2017-039
	$\tilde{\chi}_1^\pm\tilde{\chi}_2^0\to W\tilde{\chi}_1^0h\tilde{\chi}_1^0$, $h\to b\bar{b}/WW/\tau\tau/\gamma\gamma$	e,μ,γ	0-2 b	Yes	20.3	🟩 $\tilde{\chi}_1^\pm,\tilde{\chi}_2^0$ 270 GeV	m($\tilde{\chi}_1^\pm$)=m($\tilde{\chi}_2^0$), m($\tilde{\chi}_1^0$)=0, $\tilde{\ell}$ decoupled	1501.07110
	$\tilde{\chi}_2^0\tilde{\chi}_3^0$, $\tilde{\chi}_{2,3}^0\to\tilde{\ell}_{\mathrm{R}}\ell$	4 e,μ	0	Yes	20.3	🟩 $\tilde{\chi}_{2,3}^0$ 635 GeV	m($\tilde{\chi}_2^0$)=m($\tilde{\chi}_3^0$), m($\tilde{\chi}_1^0$)=0, m($\tilde{\ell},\tilde{\nu}$)=0.5(m($\tilde{\chi}_2^0$)+m($\tilde{\chi}_1^0$))	1405.5086
	GGM (wino NLSP) weak prod., $\tilde{\chi}_1^0\to\gamma\tilde{G}$	1 e,μ + γ	-	Yes	20.3	$\tilde{W}$ 🟩 115-370 GeV	$c\tau$<1 mm	1507.05493
	GGM (bino NLSP) weak prod., $\tilde{\chi}_1^0\to\gamma\tilde{G}$	2 γ	-	Yes	20.3	🟩 $\tilde{W}$ 590 GeV	$c\tau$<1 mm	1507.05493
Long-lived particles	Direct $\tilde{\chi}_1^+\tilde{\chi}_1^-$ prod., long-lived $\tilde{\chi}_1^\pm$	Disapp. trk	1 jet	Yes	36.1	🟦 $\tilde{\chi}_1^\pm$ 430 GeV	m($\tilde{\chi}_1^\pm$)-m($\tilde{\chi}_1^0$)~160 MeV, τ($\tilde{\chi}_1^\pm$)=0.2 ns	ATLAS-CONF-2017-017
	Direct $\tilde{\chi}_1^+\tilde{\chi}_1^-$ prod., long-lived $\tilde{\chi}_1^\pm$	dE/dx trk	-	Yes	18.4	🟩 $\tilde{\chi}_1^\pm$ 495 GeV	m($\tilde{\chi}_1^\pm$)-m($\tilde{\chi}_1^0$)~160 MeV, τ($\tilde{\chi}_1^\pm$)<15 ns	1506.05332
	Stable, stopped $\tilde{g}$ R-hadron	0	1-5 jets	Yes	27.9	🟩 $\tilde{g}$ 850 GeV	m($\tilde{\chi}_1^0$)=100 GeV, 10 μs<τ($\tilde{g}$)<1000 s	1310.6584
	Stable $\tilde{g}$ R-hadron	trk	-	-	3.2	🟦 $\tilde{g}$ 1.58 TeV		1606.05129
	Metastable $\tilde{g}$ R-hadron	dE/dx trk	-	-	3.2	🟦 $\tilde{g}$ 1.57 TeV	m($\tilde{\chi}_1^0$)=100 GeV, τ>10 ns	1604.04520
	GMSB, stable $\tilde{\tau}$, $\tilde{\chi}_1^0\to\tilde{\tau}(\tilde{e},\tilde{\mu})+\tau(e,\mu)$	1-2 μ	-	-	19.1	🟩 $\tilde{\chi}_1^0$ 537 GeV	10<tanβ<50	1411.6795
	GMSB, $\tilde{\chi}_1^0\to\gamma\tilde{G}$, long-lived $\tilde{\chi}_1^0$	2 γ	-	Yes	20.3	🟩 $\tilde{\chi}_1^0$ 440 GeV	1<τ($\tilde{\chi}_1^0$)<3 ns, SPS8 model	1409.5542
	$\tilde{g}\tilde{g}$, $\tilde{\chi}_1^0\to ee\nu/e\mu\nu/\mu\mu\nu$	displ. $ee/e\mu/\mu\mu$	-	-	20.3	🟩 $\tilde{\chi}_1^0$ 1.0 TeV	7 <$c\tau$($\tilde{\chi}_1^0$)< 740 mm, m($\tilde{g}$)=1.3 TeV	1504.05162
	GGM $\tilde{g}\tilde{g}$, $\tilde{\chi}_1^0\to Z\tilde{G}$	displ. vtx + jets	-	-	20.3	🟩 $\tilde{\chi}_1^0$ 1.0 TeV	6 <$c\tau$($\tilde{\chi}_1^0$)< 480 mm, m($\tilde{g}$)=1.1 TeV	1504.05162
RPV	LFV $pp\to\tilde{\nu}_\tau+X$, $\tilde{\nu}_\tau\to e\mu/e\tau/\mu\tau$	$e\mu$,$e\tau$,$\mu\tau$	-	-	3.2	🟦 $\tilde{\nu}_\tau$ 1.9 TeV	λ'_{311}=0.11, $\lambda_{132/133/233}$=0.07	1607.08079
	Bilinear RPV CMSSM	2 e,μ (SS)	0-3 b	Yes	20.3	🟩 $\tilde{q},\tilde{g}$ 1.45 TeV	m($\tilde{q}$)=m($\tilde{g}$), $c\tau_{LSP}$<1 mm	1404.2500
	$\tilde{\chi}_1^+\tilde{\chi}_1^-$, $\tilde{\chi}_1^+\to W\tilde{\chi}_1^0$, $\tilde{\chi}_1^0\to ee\nu, e\mu\nu, \mu\mu\nu$	4 e,μ	-	Yes	13.3	🟦 $\tilde{\chi}_1^\pm$ 1.14 TeV	m($\tilde{\chi}_1^0$)>400GeV, $\lambda_{12k}\neq0$ ($k = 1, 2$)	ATLAS-CONF-2016-075
	$\tilde{\chi}_1^+\tilde{\chi}_1^-$, $\tilde{\chi}_1^+\to W\tilde{\chi}_1^0$, $\tilde{\chi}_1^0\to\tau\tau\nu_e, e\tau\nu_\tau$	3 e,μ + τ	-	Yes	20.3	🟩 $\tilde{\chi}_1^\pm$ 450 GeV	m($\tilde{\chi}_1^0$)>0.2×m($\tilde{\chi}_1^\pm$), $\lambda_{133}\neq0$	1405.5086
	$\tilde{g}\tilde{g}$, $\tilde{g}\to qqq$	0	4-5 large-R jets	-	14.8	🟦 $\tilde{g}$ 1.08 TeV	BR(t)=BR(b)=BR(c)=0%	ATLAS-CONF-2016-057
	$\tilde{g}\tilde{g}$, $\tilde{g}\to qq\tilde{\chi}_1^0$, $\tilde{\chi}_1^0\to qqq$	0	4-5 large-R jets	-	14.8	🟦 $\tilde{g}$ 1.55 TeV	m($\tilde{\chi}_1^0$)=800 GeV	ATLAS-CONF-2016-057
	$\tilde{g}\tilde{g}$, $\tilde{g}\to t\bar{t}\tilde{\chi}_1^0$, $\tilde{\chi}_1^0\to qqq$	1 e,μ	8-10 jets/0-4 b	-	36.1	🟦 $\tilde{g}$ 2.1 TeV	m($\tilde{\chi}_1^0$)= 1 TeV, $\lambda_{112}\neq0$	ATLAS-CONF-2017-013
	$\tilde{g}\tilde{g}$, $\tilde{g}\to\tilde{t}_1t$, $\tilde{t}_1\to bs$	1 e,μ	8-10 jets/0-4 b	-	36.1	🟦 $\tilde{g}$ 1.65 TeV	m($\tilde{t}_1$)= 1 TeV, $\lambda_{323}\neq0$	ATLAS-CONF-2017-013
	$\tilde{t}_1\tilde{t}_1$, $\tilde{t}_1\to bs$	0	2 jets + 2 b	-	15.4	🟦 $\tilde{t}_1$ 410 GeV 🟦 450-510 GeV		ATLAS-CONF-2016-022, ATLAS-CONF-2016-084
	$\tilde{t}_1\tilde{t}_1$, $\tilde{t}_1\to b\ell$	2 e,μ	2 b	-	36.1	$\tilde{t}_1$ 🟦 0.4-1.45 TeV	BR($\tilde{t}_1\to be/\mu$)>20%	ATLAS-CONF-2017-036
Other	Scalar charm, $\tilde{c}\to c\tilde{\chi}_1^0$	0	2 c	Yes	20.3	🟩 $\tilde{c}$ 510 GeV	m($\tilde{\chi}_1^0$)<200 GeV	1501.01325

*Only a selection of the available mass limits on new states or phenomena is shown. Many of the limits are based on simplified models, c.f. refs. for the assumptions made.


13 TeV
8 TeV
Leptoquarks
LQ1(ej) x2
LQ1(ej)+LQ1(νj) β=0.5
LQ2(μj) x2
LQ2(μj)+LQ2(νj) β=0.5
LQ3(τb) x2
LQ3(νb) x2
LQ3(τt) x2
LQ3(νt) x2
Single LQ1 (λ=1)
Single LQ2 (λ=1)
0 1 2 3 4 TeV
RS Gravitons
RS1(jj), k=0.1
RS1(γγ), k=0.1
RS1(ee,μμ), k=0.1
0 1 2 3 4 TeV
CMS Preliminary
Heavy Gauge Bosons
SSM Z'(ττ)
SSM Z'(jj)
SSM Z'(ee)+Z'(μμ)
SSM W'(jj)
SSM W'(lν)
SSM Z'(bb)
0 1 2 3 4 5 TeV
Excited Fermions
e* (M=Λ)
μ* (M=Λ)
q* (qg)
q* (qγ) f=1
b*
0 1 2 3 4 5 6 TeV
Multijet Resonances
coloron(jj) x2
coloron(4j) x2
gluino(3j) x2
gluino(jjb) x2
0 1 2 3 4 TeV
Large Extra Dimensions
ADD (γ+MET), nED=4, MD
ADD (jj), nED=4, MS
QBH, nED=6, MD=4 TeV
NR BH, nED=6, MD=4 TeV
String Scale (jj)
QBH (jj), nED=4, MD=4 TeV
ADD (j+MET), nED=4, MD
ADD (ee,μμ), nED=4, MS
ADD (γγ), nED=4, MS
Jet Extinction Scale
0 1 2 3 4 5 6 7 8 9 10 TeV
Compositeness
dijets, Λ+ LL/RR
dijets, Λ- LL/RR
dimuons, Λ+ LLIM
dimuons, Λ- LLIM
dielectrons, Λ+ LLIM
dielectrons, Λ- LLIM
single e, Λ HnCM
single μ, Λ HnCM
inclusive jets, Λ+
inclusive jets, Λ-
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 TeV
CMS Exotica Physics Group Summary – ICHEP, 2016

Energy Scale	Fields	Effective Theory
$\Lambda_{NP} \sim$ TeV	S_n, P_n, V_n, A_n, F_n H, W, Z, γ, g τ, μ, e, ν_i t, b, c, s, d, u	Underlying Dynamics
M_W	H, W, Z, γ, g τ, μ, e, ν_i t, b, c, s, d, u	Standard Model

······ **Energy Gap** ······

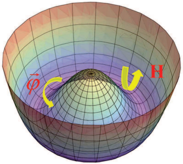
Effective Field Theory

$$\mathcal{L}_{\rm eff} \; = \; \mathcal{L}^{(4)} \, + \, \sum_{D>4} \sum_{i} \, \frac{c_i^{(D)}}{\Lambda^{D-4}} \, \mathcal{O}_i^{(D)}$$

- **Most general Lagrangian with the SM gauge symmetries**
- **Light $(m \ll \Lambda_{NP})$ fields only**
- **The SM Lagrangian corresponds to $D = 4$**
- **$c_i^{(D)}$ contain information on the underlying dynamics:**

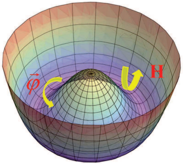
$$\mathcal{L}_{\rm NP} \doteq g_x \, (\bar{q}_L \gamma^\mu q_L) \, X_\mu \qquad \rightarrow \qquad \frac{g_x^2}{M_X^2} \, (\bar{q}_L \gamma^\mu q_L) \, (\bar{q}_L \gamma_\mu q_L)$$

- **Options for H(126):**
  - **$SU(2)_L$ doublet (SM)**
  - **Scalar singlet**
  - **Additional light scalars**



$$\mathcal{L}_\Phi = (D_\mu \Phi)^\dagger D^\mu \Phi - \lambda \left(|\Phi|^2 - \frac{v^2}{2} \right)^2$$

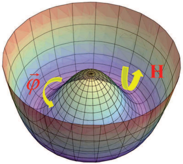
$$\Sigma \equiv (\Phi^c, \Phi) = \begin{pmatrix} \Phi^{0*} & \Phi^+ \\ -\Phi^- & \Phi^0 \end{pmatrix}$$



$$\begin{aligned} \mathcal{L}_\Phi &= (D_\mu\Phi)^\dagger D^\mu\Phi - \lambda \left(|\Phi|^2 - \frac{v^2}{2} \right)^2 \\ &= \frac{1}{2}\,\mathrm{Tr}\left[(D^\mu\Sigma)^\dagger D_\mu\Sigma\right] - \frac{\lambda}{4}\left(\mathrm{Tr}\left[\Sigma^\dagger\Sigma\right] - v^2\right)^2 \end{aligned}$$

Custodial Symmetry

$$\Sigma \equiv (\Phi^c, \Phi) = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^- & \phi^0 \end{pmatrix}$$



$$\mathcal{L}_\Phi = (D_\mu\Phi)^\dagger D^\mu\Phi - \lambda \left(|\Phi|^2 - \frac{v^2}{2}\right)^2$$

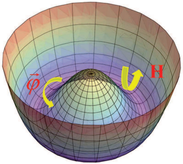
$$= \frac{1}{2}\,\mathrm{Tr}\left[(D^\mu\Sigma)^\dagger D_\mu\Sigma\right] - \frac{\lambda}{4}\left(\mathrm{Tr}\left[\Sigma^\dagger\Sigma\right] - v^2\right)^2$$

$SU(2)_L \otimes SU(2)_R \to SU(2)_{L+R}$ **Symmetry:** $\Sigma \to g_L\, \Sigma\, g_R^\dagger$

Custodial Symmetry

$$\Sigma \equiv (\Phi^c, \Phi) = \begin{pmatrix} \Phi^{0*} & \Phi^+ \\ -\Phi^- & \Phi^0 \end{pmatrix} \equiv \frac{1}{\sqrt{2}}\,(v+H)\; U(\vec{\varphi}\,)$$

$$U(\vec{\varphi}\,) \equiv \exp\left\{ i\,\vec{\sigma}\cdot\frac{\vec{\varphi}}{v}\right\}$$



$$\boxed{\begin{aligned} \mathcal{L}_\Phi &= (D_\mu\Phi)^\dagger D^\mu\Phi - \lambda\left(|\Phi|^2 - \frac{v^2}{2}\right)^2 \\ &= \frac{1}{2}\,\mathrm{Tr}\left[(D^\mu\Sigma)^\dagger D_\mu\Sigma\right] - \frac{\lambda}{4}\left(\mathrm{Tr}\left[\Sigma^\dagger\Sigma\right] - v^2\right)^2 \\ &= \frac{v^2}{4}\,\mathrm{Tr}\left[(D^\mu U)^\dagger D_\mu U\right] + O(H/v) \end{aligned}}$$

$SU(2)_L \otimes SU(2)_R \to SU(2)_{L+R}$ **Symmetry:** $\Sigma \to g_L\,\Sigma\,g_R^\dagger$

Custodial Symmetry

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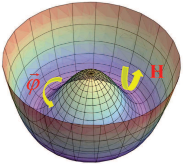

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Same Goldstone Lagrangian as QCD pions:

$$f_\pi \to v \qquad , \qquad \vec{\pi} \to \vec{\varphi} \to W_L^\pm, Z_L$$

EFFECTIVE LAGRANGIAN:

$$\mathcal{L}(U) = \sum_n \mathcal{L}_{2n}$$

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- **Goldstone Bosons**

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Parity $\longrightarrow$ even dimension ; $U\,U^\dagger = 1$ $\longrightarrow$ $2n \geq 2$

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Derivative Coupling

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Derivative Coupling

Goldstones become free at zero momenta

Goldstone Electroweak Effective Theory

$$\boxed{\mathcal{L}_{\mathrm{EW}}^{(2)} \; = \; -\frac{1}{2g^2}\,\langle \hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\rangle \;-\; \frac{1}{2g'^{\,2}}\,\langle \hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\rangle + \frac{v^2}{4}\,\langle D^\mu U^\dagger D_\mu U\rangle}$$

$$U(\varphi) \;=\; \exp\left\{\frac{i\sqrt{2}}{v}\,\Phi\right\} \qquad , \qquad \Phi \;\equiv\; \frac{1}{\sqrt{2}}\,\vec{\sigma}\cdot\vec{\varphi} \;=\; \begin{pmatrix} \frac{1}{\sqrt{2}}\,\varphi^0 & \varphi^+ \\ \varphi^- & -\frac{1}{\sqrt{2}}\,\varphi^0 \end{pmatrix}$$

$$D^\mu U \;=\; \partial^\mu U - i\,\hat{W}^\mu U + i\,U\,\hat{B}^\mu \qquad , \qquad D^\mu U^\dagger \;=\; \partial^\mu U^\dagger + i\,U^\dagger\,\hat{W}^\mu - i\,\hat{B}^\mu\,U^\dagger \qquad , \qquad \langle A\rangle \;\equiv\; \mathrm{Tr}(A)$$

$$\hat{W}^{\mu\nu} \;=\; \partial^\mu\hat{W}^\nu - \partial^\nu\hat{W}^\mu - i\,[\hat{W}^\mu,\hat{W}^\nu] \qquad , \qquad \hat{B}^{\mu\nu} \;=\; \partial^\mu\hat{B}^\nu - \partial^\nu\hat{B}^\mu - i\,[\hat{B}^\mu,\hat{B}^\nu]$$

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SM Symmetry Breaking: $\quad \hat{W}^\mu \;=\; -\frac{g}{2}\,\vec{\sigma}\cdot\vec{W}^\mu \quad , \quad \hat{B}^\mu \;=\; -\frac{g'}{2}\,\sigma_3\,B^\mu$

Electroweak Symmetry Breaking

$$\mathcal{L}_2 = \frac{v^2}{4}\,\mathrm{Tr}\left(D_\mu U^\dagger D^\mu U\right) \xrightarrow{U=1} \mathcal{L}_2 = M_W^2\, W_\mu^\dagger W^\mu + \frac{1}{2} M_Z^2\, Z_\mu Z^\mu$$

$$\mathbf{M_W = M_Z \cos\theta_W = \tfrac{1}{2}\, g\, v}$$

- **EW Goldstones are responsible for $M_{W,Z}$** **(not the Higgs!)**

- **QCD pions also generate small W, Z masses:** $\delta_\pi M_W = \frac{1}{2}\, g\, f_\pi$

Goldstone interactions are determined by the underlying symmetry

$$
\begin{aligned}
\frac{v^2}{4}\,\langle\partial_\mu U^\dagger\partial^\mu U\rangle \quad &= \quad \partial_\mu\varphi^-\partial^\mu\varphi^+ + \frac{1}{2}\,\partial_\mu\varphi^0\partial^\mu\varphi^0 \\
&+ \quad \frac{1}{6v^2}\,\left\{\left(\varphi^+\overleftrightarrow{\partial}_\mu\,\varphi^-\right)\left(\varphi^+\overleftrightarrow{\partial}^\mu\varphi^-\right) + 2\left(\varphi^0\overleftrightarrow{\partial}_\mu\,\varphi^+\right)\left(\varphi^-\overleftrightarrow{\partial}^\mu\varphi^0\right)\right\} \\
&+ \quad O\left(\varphi^6/v^4\right)
\end{aligned}
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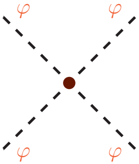
$$



$$T\left(\varphi^+\varphi^-\to\varphi^+\varphi^-\right) \;=\; \frac{s+t}{v^2}$$

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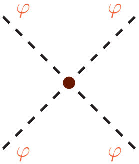
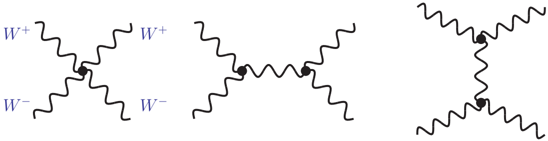


$$T\left(\varphi^+\varphi^- \to \varphi^+\varphi^-\right) \; = \; \frac{s+t}{v^2}$$

Non-Linear Lagrangian: $2\varphi \to 2\varphi,\ 4\varphi\ \cdots$ related

Equivalence Theorem



Cornwall–Levin–Tiktopoulos

Vayonakis

Lee–Quigg–Thacker

$$
\boxed{\begin{aligned}
T(W_L^+ W_L^- \to W_L^+ W_L^-) &= \frac{s+t}{v^2} + O\left(\frac{M_W}{\sqrt{s}}\right) \\
&= T(\varphi^+ \varphi^- \to \varphi^+ \varphi^-) + O\left(\frac{M_W}{\sqrt{s}}\right)
\end{aligned}}
$$

The scattering amplitude grows with energy

Goldstone dynamics ⟷ **derivative interactions**

Tree-level violation of unitarity

Longitudinal Polarizations

$k^\mu = \left(k^0, 0, 0, |\vec{k}| \right)$ ➡ $\epsilon_L^\mu(\vec{k}) = \frac{1}{M_W} \left(|\vec{k}|, 0, 0, k^0 \right) \; = \; \frac{k^\mu}{M_W} + O\!\left(\frac{M_W}{|\vec{k}|} \right)$

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One naively expects

$$T(W_L^+ W_L^- \to W_L^+ W_L^-) \;\sim\; g^2\,\frac{|\vec{k}|^4}{M_W^4}$$

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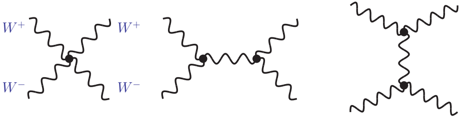
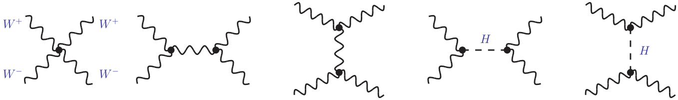


Gauge Cancelation

$$\begin{aligned} T(W_L^+ W_L^- \to W_L^+ W_L^-) \;&=\; \frac{s+t}{v^2} + O\left(\frac{M_W}{\sqrt{s}}\right) \\ &=\; T(\varphi^+\varphi^- \to \varphi^+\varphi^-) + O\left(\frac{M_W}{\sqrt{s}}\right) \end{aligned}$$

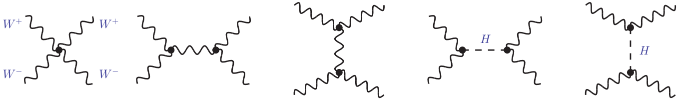
$W_L^+ W_L^- \to W_L^+ W_L^-$:



$$T_{\mathrm{SM}} = \frac{1}{v^2}\left\{s + t - \frac{s^2}{s - M_H^2} - \frac{t^2}{t - M_H^2}\right\} = -\frac{M_H^2}{v^2}\left\{\frac{s}{s - M_H^2} + \frac{t}{t - M_H^2}\right\}$$

Higgs-exchange exactly cancels the $O(s,t)$ terms in the SM

$W_L^+ W_L^- \to W_L^+ W_L^-$:



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Higgs-exchange exactly cancels the $O(s,t)$ terms in the SM

When $s \gg M_H^2$, $\quad T_{\mathrm{SM}} \approx -\frac{2M_H^2}{v^2}$, $\quad a_0 \equiv \frac{1}{32\pi}\int_{-1}^{1} d\cos\theta \; T_{\mathrm{SM}} \approx -\frac{M_H^2}{8\pi v^2}$

Unitarity:

Lee–Quigg–Thacker

$|a_0| \leq 1$ ➡ $\mathbf{M_H} < \sqrt{\mathbf{8}\pi}\,\mathbf{v}\; \underbrace{\sqrt{2/3}}_{\mathrm{W^+W^-,\, ZZ,\, HH}} \approx \mathbf{1\ TeV}$

What happens in QCD?

- QCD satisfies unitarity (it is a renormalizable theory)
- Pion scattering unitarized by exchanges of resonances (composite objects):
  - P-wave $(\mathsf{J} = 1)$ unitarized by ρ exchange
  - S-wave $(\mathsf{J} = 0)$ unitarized by σ exchange
- The σ meson is the QCD equivalent of the SM Higgs
- **BUT**, the σ is an 'effective' object generated through π rescattering (summation of pion loops)

Does not seem to work this way in the EW case, but . . .

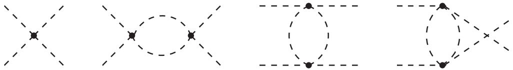
$\varphi^a \varphi^b \to \varphi^c \varphi^d$:

$$\mathcal{A}(\varphi^a \varphi^b \to \varphi^c \varphi^d) \;=\; A(s,t,u)\,\delta_{ab}\,\delta_{cd} + A(t,s,u)\,\delta_{ac}\,\delta_{bd} + A(u,t,s)\,\delta_{ad}\,\delta_{bc}$$

$$\boxed{\begin{aligned} A(s,t,u) \;=\; & \frac{s}{v^2} + \frac{4}{v^2}\left[a_4^r(\mu)\,(t^2+u^2) + 2\,a_5^r(\mu)\,s^2\right] \\ & + \frac{1}{16\pi^2 v^2}\left\{\frac{5}{9}\,s^2 + \frac{13}{18}\,(t^2+u^2) + \frac{1}{12}\,(s^2-3t^2-u^2)\,\log\left(\frac{-t}{\mu^2}\right)\right. \\ & \qquad \left. + \frac{1}{12}\,(s^2-t^2-3u^2)\,\log\left(\frac{-u}{\mu^2}\right) - \frac{1}{2}\,s^2\,\log\left(\frac{-s}{\mu^2}\right)\right\} \end{aligned}}$$

$$a_i \;=\; a_i^r(\mu) + \frac{\gamma_i}{16\pi^2}\left[\frac{2\,\mu^{D-4}}{4-D} + \log\,(4\pi) - \gamma_E\right] \qquad , \qquad \gamma_4 \;=\; -\frac{1}{12} \qquad , \qquad \gamma_5 \;=\; -\frac{1}{24}$$

$$\varphi^a \varphi^b \to \varphi^c \varphi^d$$



+ Higgs (tree + 1-loop) contributions

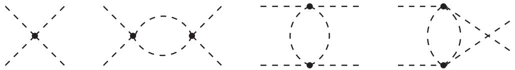
$$\mathcal{L} = \frac{v^2}{4} \langle D^\mu U^\dagger D_\mu U \rangle \left[1 + 2\, a\, \frac{H}{v} + b\, \frac{H^2}{v^2} \right]$$

Espriu–Mescia–Yencho, Delgado–Dobado–Llanes-Estrada

$$\begin{aligned} A(s,t,u) \; = \; & \frac{s}{v^2}\,(1-a^2) + \frac{4}{v^2}\left[a_4^r(\mu)\,(t^2+u^2) + 2\, a_5^r(\mu)\; s^2 \right] \\ & + \frac{1}{16\pi^2 v^2} \left\{ \frac{1}{9}\,(14\,a^4 - 10\,a^2 - 18\,a^2 b + 9\,b^2 + 5)\; s^2 + \frac{13}{18}\,(1-a^2)^2\,(t^2+u^2) \right. \\ & \qquad - \frac{1}{2}\,(2\,a^4 - 2\,a^2 - 2\,a^2 b + b^2 + 1)\; s^2\, \log\left(\frac{-s}{\mu^2}\right) \\ & \qquad \left. + \frac{1}{12}\,(1-a^2)^2 \left[(s^2 - 3t^2 - u^2)\, \log\left(\frac{-t}{\mu^2}\right) + (s^2 - t^2 - 3u^2)\, \log\left(\frac{-u}{\mu^2}\right) \right] \right\} \end{aligned}$$

$$\gamma_4 = -\tfrac{1}{12}\,(1-a^2)^2 \qquad , \qquad \gamma_5 = -\tfrac{1}{48}\,(2 + 5\,a^4 - 4\,a^2 - 6\,a^2 b + 3\,b^2)$$

$$\varphi^a \varphi^b \to \varphi^c \varphi^d$$



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SM: $a = b = 1$, $a_4 = a_5 = 0$ ➡ $A(s,t,u) \sim \mathcal{O}(M_H^2/v^2)$

Low-Energy Effective Theory ➡ Power Counting

- **Momentum expansion:** $\Lambda \sim 4\pi v\,,\, M_X$

$$\mathcal{A} = \sum_n \mathcal{A}_n \left(\frac{p}{\Lambda}\right)^n$$

- $U(\varphi)\,,\ \varphi\,,\ h \ \sim\ O(p^0)$

  $D_\mu U\,,\ \hat{W}_\mu\,,\ \hat{B}_\mu \ \sim\ O(p^1) \qquad , \qquad \hat{W}_{\mu\nu}\,,\ \hat{B}_{\mu\nu} \ \sim\ O(p^2)$

- **A general connected diagram with N_d vertices of $O(p^d)$ and L Goldstone loops has a power dimension:** Weinberg

$$D = 2L + 2 + \sum_d N_d\,(d-2)$$

➡ **Finite number of divergences / counterterms**

Electroweak Effective Theory

$$\mathcal{L}_{\mathrm{EWET}} = \underbrace{\mathcal{L}_{YM} + i \sum_f \bar{f}\gamma^\mu D_\mu f + \Delta\mathcal{L}_2}_{\mathcal{L}_{\mathrm{EW}}^{(2)}} + \mathcal{L}_{\mathrm{EW}}^{(4)} + \cdots$$

$$\Delta\mathcal{L}_2^{\mathrm{Bosonic}} = \frac{1}{2}\,\partial_\mu h\,\partial^\mu h - \frac{1}{2}\,m_h^2\,h^2 - V(h/v) + \frac{v^2}{4}\,\mathcal{F}_u(h/v)\,\langle u_\mu u^\mu\rangle$$

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$$V(h/v) \;=\; v^4\,\sum_{n=3} c_n^{(V)}\left(\frac{h}{v}\right)^n \qquad , \qquad \mathcal{F}_u(h/v) \;=\; 1 + \sum_{n=1} c_n^{(u)}\left(\frac{h}{v}\right)^n$$

Electroweak Effective Theory

$$\mathcal{L}_{\rm EWET} \;=\; \underbrace{\mathcal{L}_{YM} + i\sum_f \bar f\gamma^\mu D_\mu f + \Delta\mathcal{L}_2}_{\mathcal{L}_{\rm EW}^{(2)}} + \mathcal{L}_{\rm EW}^{(4)} + \cdots$$

$$\Delta\mathcal{L}_2^{\rm Bosonic} \;=\; \frac{1}{2}\,\partial_\mu h\,\partial^\mu h \;-\; \frac{1}{2}\,m_h^2\,h^2 \;-\; V(h/v) \;+\; \frac{v^2}{4}\,\mathcal{F}_u(h/v)\,\langle u_\mu u^\mu\rangle$$

$$V(h/v) \;=\; v^4\,\sum_{n=3} c_n^{(V)}\left(\frac{h}{v}\right)^n \qquad , \qquad \mathcal{F}_u(h/v) \;=\; 1 + \sum_{n=1} c_n^{(u)}\left(\frac{h}{v}\right)^n$$

SM: $c_3^{(V)} = \frac{m_h^2}{2v^2}\;,\;\; c_4^{(V)} = \frac{m_h^2}{8v^2}\;,\;\; c_{n>4}^{(V)} = 0 \quad ; \quad c_1^{(u)} = 2\;,\;\; c_2^{(u)} = 1\;,\;\; c_{n>2}^{(u)} = 0$

Yukawa Couplings

$$\boxed{\Delta\mathcal{L}_2^{\mathrm{Ferm.}} = -v\left\{\bar{Q}_L\, U(\varphi)\left[\hat{\mathbf{Y}}_{\mathbf{u}}\,\mathcal{P}_+ + \hat{\mathbf{Y}}_{\mathbf{d}}\,\mathcal{P}_-\right] Q_R + \bar{L}_L\, U(\varphi)\,\hat{\mathbf{Y}}_{\boldsymbol{\ell}}\,\mathcal{P}_+\, L_R + \mathrm{h.c.}\right\}}$$

$$Q = \begin{pmatrix} u \\ d \end{pmatrix} \qquad , \qquad L = \begin{pmatrix} \nu_\ell \\ \ell \end{pmatrix}$$

$$U(\varphi) \to g_L\, U(\varphi)\, g_R^\dagger \quad , \quad Q_L \to g_L\, Q_L \quad , \quad Q_R \to g_R\, Q_R \quad , \quad \mathcal{P}_\pm \to g_R\, \mathcal{P}_\pm\, g_R^\dagger$$

Yukawa Couplings

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$$Q = \left(\begin{array}{c} u \\ d \end{array}\right) \qquad , \qquad L = \left(\begin{array}{c} \nu_\ell \\ \ell \end{array}\right)$$

$$U(\varphi) \to g_L\,U(\varphi)\,g_R^\dagger \quad , \quad Q_L \to g_L\,Q_L \quad , \quad Q_R \to g_R\,Q_R \quad , \quad \mathcal{P}_\pm \to g_R\,\mathcal{P}_\pm\,g_R^\dagger$$

- **Symmetry Breaking:** $\qquad \mathcal{P}_\pm = \frac{1}{2}\,(\mathrm{I}_2 \pm \sigma_3)$

Yukawa Couplings

$$\boxed{\Delta\mathcal{L}_2^{\mathrm{Ferm.}} = -v\,\left\{\bar{Q}_L\, U(\varphi)\left[\hat{\mathbf{Y}}_{\mathbf{u}}\,\mathcal{P}_+ + \hat{\mathbf{Y}}_{\mathbf{d}}\,\mathcal{P}_-\right] Q_R \;+\; \bar{L}_L\, U(\varphi)\,\hat{\mathbf{Y}}_{\boldsymbol{\ell}}\,\mathcal{P}_+\, L_R \;+\; \mathrm{h.c.}\right\}}$$

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- **Symmetry Breaking:** $\mathcal{P}_\pm = \frac{1}{2}\,(\mathrm{I}_2 \pm \sigma_3)$
- **Flavour Structure:** $\hat{\mathbf{Y}}_{\mathbf{u},\mathbf{d},\boldsymbol{\ell}}$ **3×3 matrices in flavour space**

Yukawa Couplings

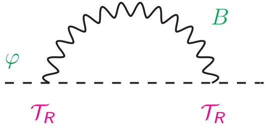
$$\boxed{\Delta\mathcal{L}_2^{\mathrm{Ferm.}} = -v \left\{ \bar{Q}_L\, U(\varphi) \left[\hat{\mathbf{Y}}_{\mathbf{u}}\, \mathcal{P}_+ + \hat{\mathbf{Y}}_{\mathbf{d}}\, \mathcal{P}_- \right] Q_R \; + \; \bar{L}_L\, U(\varphi)\, \hat{\mathbf{Y}}_{\ell}\, \mathcal{P}_+\, L_R \; + \; \mathrm{h.c.} \right\}}$$

$$Q = \begin{pmatrix} u \\ d \end{pmatrix} \quad , \quad L = \begin{pmatrix} \nu_\ell \\ \ell \end{pmatrix}$$

$$U(\varphi) \to g_L\, U(\varphi)\, g_R^\dagger \quad , \quad Q_L \to g_L\, Q_L \quad , \quad Q_R \to g_R\, Q_R \quad , \quad \mathcal{P}_\pm \to g_R\, \mathcal{P}_\pm\, g_R^\dagger$$

- **Symmetry Breaking:** $\mathcal{P}_\pm = \frac{1}{2}\left(\mathrm{I}_2 \pm \sigma_3\right)$
- **Flavour Structure:** $\hat{\mathbf{Y}}_{\mathbf{u},\mathbf{d},\ell}$ **3×3 matrices in flavour space**
- **Higgs field:** $\hat{\mathbf{Y}}_{\mathbf{u},\mathbf{d},\ell}(h/v) = \sum_{n=0} \hat{\mathbf{Y}}^{(\mathbf{n})}_{\mathbf{u},\mathbf{d},\ell} \left(\frac{h}{v}\right)^n$

Custodial Symmetry Breaking: $\hat{B}_\mu \equiv -g' \frac{\sigma_3}{2} B_\mu$



$$\longrightarrow \quad \langle U^\dagger D^\mu U \, \mathcal{T}_R \, U^\dagger D_\mu U \, \mathcal{T}_R \rangle$$

$$U^\dagger D_\mu U = i \, \frac{\sqrt{2}}{v} \, D_\mu \Phi + \cdots \qquad , \qquad \mathcal{T}_R \to g_R \, \mathcal{T}_R \, g_R^\dagger \qquad , \qquad \mathcal{T}_R = -g' \, \frac{\sigma_3}{2}$$

$$\langle U^\dagger D^\mu U \, \mathcal{T}_R \, U^\dagger D_\mu U \, \mathcal{T}_R \rangle = \langle U^\dagger D^\mu U \, \mathcal{T}_R \rangle \, \langle U^\dagger D_\mu U \, \mathcal{T}_R \rangle + \frac{1}{2} \, \langle (D_\mu U)^\dagger D_\mu U \rangle \, \langle \mathcal{T}_R \mathcal{T}_R \rangle$$

Power-Counting Rules:

A.P., Rosell, Santos, Sanz-Cillero, 1609.06659

$$v\,,\ \frac{\varphi}{v}\,,\ u(\varphi)\,,\ U(\varphi)\,,\ \frac{h}{v}\,,\ \frac{\vec{W}_\mu}{v}\,,\ \frac{B_\mu}{v} \quad\sim\quad \mathcal{O}\left(p^0\right)$$

$$\frac{\psi}{v}\,,\ \frac{\bar{\psi}}{v} \quad\sim\quad \mathcal{O}\left(p^{1/2}\right)$$

$$D_\mu U\,,\ u_\mu\,,\ \partial_\mu\,,\ \hat{W}_\mu\,,\ \hat{B}_\mu\,,\ m_h\,,\ m_W\,,\ m_Z\,,\ m_\psi\,,\ g\,,\ g'\,,\ \mathcal{Y}\,,\ \mathcal{T}_R \quad\sim\quad \mathcal{O}\left(p\right)$$

$$\hat{W}_{\mu\nu}\,,\ \hat{B}_{\mu\nu}\,,\ f_{\pm\,\mu\nu}\,,\ c_n^{(V)} \quad\sim\quad \mathcal{O}\left(p^2\right)$$

$$\partial_{\mu_1}\partial_{\mu_2}\ldots\partial_{\mu_n}\,\mathcal{F}(h/v) \quad\sim\quad \mathcal{O}\left(p^n\right)$$

$$\left(\frac{\bar{\psi}'\,\Gamma\,\psi}{v^2}\right)^n \quad\sim\quad \mathcal{O}\left(p^{2n}\right)$$

$$\Gamma \sim p^{\hat{d}_\Gamma} \qquad , \qquad \hat{d}_\Gamma = 2 + 2L + \sum_{\hat{d}} (\hat{d} - 2)\, N_{\hat{d}}$$

CP-Invariant Bosonic Operators

A.P., Rosell, Santos,
Sanz-Cillero
1609.06659

i	$\mathcal{O}_i$	$\widetilde{\mathcal{O}}_i$
1	$\frac{1}{4}\langle\, f_+^{\mu\nu} f_{+\,\mu\nu} - f_-^{\mu\nu} f_{-\,\mu\nu}\,\rangle$	$\frac{i}{2}\langle\, f_-^{\mu\nu}[u_\mu, u_\nu]\,\rangle$
2	$\frac{1}{2}\langle\, f_+^{\mu\nu} f_{+\,\mu\nu} + f_-^{\mu\nu} f_{-\,\mu\nu}\,\rangle$	$\langle\, f_+^{\mu\nu} f_{-\,\mu\nu}\,\rangle$
3	$\frac{i}{2}\langle\, f_+^{\mu\nu}[u_\mu, u_\nu]\,\rangle$	$\frac{1}{v}\,(\partial_\mu h)\,\langle\, f_+^{\mu\nu} u_\nu\,\rangle$
4	$\langle\, u_\mu u_\nu\,\rangle\,\langle\, u^\mu u^\nu\,\rangle$	—
5	$\langle\, u_\mu u^\mu\,\rangle^2$	—
6	$\frac{1}{v^2}\,(\partial_\mu h)(\partial^\mu h)\,\langle\, u_\nu u^\nu\,\rangle$	—
7	$\frac{1}{v^2}\,(\partial_\mu h)(\partial_\nu h)\,\langle\, u^\mu u^\nu\,\rangle$	—
8	$\frac{1}{v^4}\,(\partial_\mu h)(\partial^\mu h)(\partial_\nu h)(\partial^\nu h)$	—
9	$\frac{1}{v}\,(\partial_\mu h)\,\langle\, f_-^{\mu\nu} u_\nu\,\rangle$	—
10	$\langle\mathcal{T} u_\mu\rangle^2$	—
11	$\hat{X}_{\mu\nu}\hat{X}^{\mu\nu}$	—

$$\mathcal{L}_4^{\mathrm{Bosonic}} = \sum_{i=1}^{11}\mathcal{F}_i(h/v)\;\mathcal{O}_i \;+\; \sum_{i=1}^{3}\widetilde{\mathcal{F}}_i(h/v)\;\widetilde{\mathcal{O}}_i$$

A.P., Rosell, Santos, Sanz-Cillero, 1609.06659

$$J_\Gamma = \begin{cases} \bar\psi_L \,\Gamma \psi_L + \bar\psi_R \,\Gamma \psi_R & (\Gamma = \gamma^\mu, \gamma^\mu\gamma_5) \\ \bar\psi_L \,\Gamma\, U \,\psi_R + \bar\psi_R \,\Gamma\, U^\dagger \psi_L & (\Gamma = I, i\gamma_5, \sigma^{\mu\nu}) \end{cases}$$

CP-Invariant Fermionic Operators

i	$\mathcal{O}_i^{\psi^2}$	$\widetilde{\mathcal{O}}_i^{\psi^2}$	$\mathcal{O}_i^{\psi^4}$	$\widetilde{\mathcal{O}}_i^{\psi^4}$
1	$\langle\, J_S \,\rangle \langle\, u_\mu u^\mu \,\rangle$	$\langle\, J_T^{\mu\nu} f_{-\,\mu\nu} \,\rangle$	$\langle\, J_S J_S \,\rangle$	$\langle\, J_V^\mu J_{A,\mu} \,\rangle$
2	$i\,\langle\, J_T^{\mu\nu}\,[u_\mu, u_\nu] \,\rangle$	$\frac{1}{v}\,(\partial_\mu h)\,\langle\, u_\nu J_T^{\mu\nu} \,\rangle$	$\langle\, J_P J_P \,\rangle$	$\langle\, J_V^\mu \,\rangle \langle\, J_{A,\mu} \,\rangle$
3	$\langle\, J_T^{\mu\nu} f_{+\,\mu\nu} \,\rangle$	$\langle\, J_V^\mu \,\rangle \langle\, u_\mu \mathcal{T} \,\rangle$	$\langle\, J_S \,\rangle \langle\, J_S \,\rangle$	—
4	$\hat{X}_{\mu\nu}\,\langle\, J_T^{\mu\nu} \,\rangle$	—	$\langle\, J_P \,\rangle \langle\, J_P \,\rangle$	—
5	$\frac{1}{v}\,(\partial_\mu h)\,\langle\, u^\mu J_P \,\rangle$	—	$\langle\, J_V^\mu J_{V,\mu} \,\rangle$	—
6	$\langle\, J_A^\mu \,\rangle \langle\, u_\mu \mathcal{T} \,\rangle$	—	$\langle\, J_A^\mu J_{A,\mu} \,\rangle$	—
7	$\frac{1}{v^2}\,(\partial_\mu h)(\partial^\mu h)\,\langle\, J_S \,\rangle$	—	$\langle\, J_V^\mu \,\rangle \langle\, J_{V,\mu} \,\rangle$	—
8	—	—	$\langle\, J_A^\mu \,\rangle \langle\, J_{A,\mu} \,\rangle$	—
9	—	—	$\langle\, J_T^{\mu\nu} J_{T\,\mu\nu} \,\rangle$	—
10	—	—	$\langle\, J_T^{\mu\nu} \,\rangle \langle\, J_{T\,\mu\nu} \,\rangle$	—

$$\mathcal{L}_4^{\text{Ferm.}} = \sum_{i=1}^{7} \mathcal{F}_i^{\psi^2}(h/v)\; \mathcal{O}_i^{\psi^2} + \sum_{i=1}^{3} \widetilde{\mathcal{F}}_i^{\psi^2}(h/v)\; \widetilde{\mathcal{O}}_i^{\psi^2} + \sum_{i=1}^{10} \mathcal{F}_i^{\psi^4}(h/v)\; \mathcal{O}_i^{\psi^4} + \sum_{i=1}^{2} \widetilde{\mathcal{F}}_i^{\psi^4}(h/v)\; \widetilde{\mathcal{O}}_i^{\psi^4}$$

Higher-Order Goldstone Interactions

$$\mathcal{L}_{\mathrm{EW}}^{(4)}\Big|_{\text{Bosonic}} = \sum_i \mathcal{F}_i(h/v)\,\mathcal{O}_i \qquad\qquad \mathcal{F}_i(h/v) = \sum_{n=0} \mathcal{F}_{i,n}\left(\frac{h}{v}\right)^n$$

Appelquist-Bernard, Longhitano, Buchalla et al, Alonso et al, Pich et al...

$\mathcal{O}(\mathbf{p}^4)$ $\mathcal{P}$-even bosonic operators

A.P., Rosell, Santos, Sanz-Cillero

$\mathcal{O}_1 = \frac{1}{4}\langle\, f_+^{\mu\nu} f_{\mu\nu}^+ - f_-^{\mu\nu} f_{\mu\nu}^- \,\rangle$	$\mathcal{O}_6 = \frac{1}{v^2}\,(\partial_\mu h)(\partial^\mu h)\,\langle\, u_\nu u^\nu \,\rangle$
$\mathcal{O}_2 = \frac{1}{2}\langle\, f_+^{\mu\nu} f_{\mu\nu}^+ + f_-^{\mu\nu} f_{\mu\nu}^- \,\rangle$	$\mathcal{O}_7 = \frac{1}{v^2}\,(\partial_\mu h)(\partial_\nu h)\,\langle\, u^\mu u^\nu \,\rangle$
$\mathcal{O}_3 = \frac{i}{2}\langle\, f_+^{\mu\nu} [u_\mu, u_\nu] \,\rangle$	$\mathcal{O}_8 = \frac{1}{v^4}\,(\partial_\mu h)(\partial^\mu h)(\partial_\nu h)(\partial^\nu h)$
$\mathcal{O}_4 = \langle\, u_\mu u_\nu \,\rangle\langle\, u^\mu u^\nu \,\rangle$	$\mathcal{O}_9 = \frac{1}{v}\,(\partial_\mu h)\,\langle\, f_-^{\mu\nu} u_\nu \,\rangle$
$\mathcal{O}_5 = \langle\, u_\mu u^\mu \,\rangle^2$	

$$U = u^2 = \exp\left\{\tfrac{i}{v}\,\vec{\sigma}\,\vec{\varphi}\right\} \quad , \quad u_\mu \equiv i\,u\,(D_\mu U)^\dagger u = u_\mu^\dagger \quad , \quad f_\pm^{\mu\nu} = u^\dagger \hat{W}^{\mu\nu} u \pm u\,\hat{B}^{\mu\nu} u^\dagger$$

Custodial symmetry assumed

Unitary Gauge: $\mathbf{U = u = 1}$

All invariants reduce to polynomials of h and gauge fields

- **Bilinear gauge terms:** $\mathcal{O}_1, \mathcal{O}_2$

  ➡ **Oblique corrections** $(\Delta r, \Delta\rho, \Delta k \longleftrightarrow S, T, U)$

- **Trilinear gauge couplings:** $\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3$
- **Quartic gauge couplings:** $\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3, \mathcal{O}_4, \mathcal{O}_5$
- **Higgs interactions:** $\mathcal{O}_{1\text{–}9}$

Backup Slides

Higher-Order Goldstone Interactions

$$\mathcal{L}_{\mathrm{EW}}^{(4)}\Big|_{\mathrm{CP-even}} \; = \; \sum_{i=0}^{14} a_i\, \mathcal{O}_i$$ (Appelquist, Longhitano)

$\mathcal{O}_0 = v^2 \langle T_L V_\mu \rangle^2$

$\mathcal{O}_1 = \langle U \hat{B}_{\mu\nu} U^\dagger \hat{W}^{\mu\nu} \rangle$ $\qquad$ $\mathcal{O}_2 = i \langle U \hat{B}_{\mu\nu} U^\dagger [V^\mu, V^\nu] \rangle$

$\mathcal{O}_3 = i \langle \hat{W}_{\mu\nu} [V^\mu, V^\nu] \rangle$ $\qquad$ $\mathcal{O}_4 = \langle V_\mu V_\nu \rangle \langle V^\mu V^\nu \rangle$

$\mathcal{O}_5 = \langle V_\mu V^\mu \rangle^2$ $\qquad$ $\mathcal{O}_6 = 4 \langle V_\mu V_\nu \rangle \langle T_L V^\mu \rangle \langle T_L V^\nu \rangle$

$\mathcal{O}_7 = 4 \langle V_\mu V^\mu \rangle \langle T_L V_\nu \rangle^2$ $\qquad$ $\mathcal{O}_8 = \langle T_L \hat{W}_{\mu\nu} \rangle^2$

$\mathcal{O}_9 = -2 \langle T_L \hat{W}_{\mu\nu} \rangle \langle T_L [V^\mu, V^\nu] \rangle$ $\qquad$ $\mathcal{O}_{10} = 16 \{ \langle T_L V_\mu \rangle \langle T_L V_\nu \rangle \}^2$

$\mathcal{O}_{11} = \langle (D_\mu V^\mu)^2 \rangle$ $\qquad$ $\mathcal{O}_{12} = 4 \langle T_L D_\mu D_\nu V^\nu \rangle \langle T_L V^\mu \rangle$

$\mathcal{O}_{13} = 2 \langle T_L D_\mu V_\nu \rangle^2$ $\qquad$ $\mathcal{O}_{14} = -2i\, \varepsilon^{\mu\nu\rho\sigma} \langle \hat{W}_{\mu\nu} V_\rho \rangle \langle T_L V_\sigma \rangle$

$$V_\mu \equiv D_\mu U\, U^\dagger \quad , \quad D_\mu V_\nu \equiv \partial_\mu V_\nu - i\, [\hat{W}_\mu, V_\nu] \quad , \quad (V_\mu, D_\mu V_\nu, T_L) \to g_L\, (V_\mu, D_\mu V_\nu, T_L)\, g_L^\dagger$$

Symmetry breaking: $\quad T_L \equiv U \frac{\sigma_3}{2} U^\dagger \quad , \quad \hat{B}_{\mu\nu} \equiv -g' \frac{\sigma_3}{2} B_{\mu\nu}$

$\varphi^a\varphi^b \to \varphi^c\varphi^d$:

- **Isospin:**

$$\begin{aligned} A_0(s,t,u) &= 3\,A(s,t,u) + A(t,s,u) + A(u,t,s) \\ A_1(s,t,u) &= A(t,s,u) - A(u,t,s) \\ A_2(s,t,u) &= A(t,s,u) + A(u,t,s) \end{aligned}$$

- **Partial Waves:**

$$\begin{aligned} A_{IJ}(s) &= \frac{1}{64\pi}\int_{-1}^{+1} d\cos\theta\; P_J(\cos\theta)\; A_I(s,t,u) \\ \sigma(s) &= \frac{64\pi}{s}\sum_{I,J}(2I+1)\,(2J+1)\;|A_{IJ}|^2 \end{aligned}$$

$$\boxed{\begin{aligned} A_{00}(s) &= \frac{s}{16\pi v^2}\left\{1+\frac{s}{16\pi^2 v^2}\left[\frac{101}{36}+\frac{64\pi^2}{3}\,(7\,a_4^r+11\,a_5^r)-\frac{25}{18}\,\log\left(\frac{s}{\mu^2}\right)+i\,\pi\right]+\cdots\right\} \\ A_{11}(s) &= \frac{s}{96\pi v^2}\left\{1+\frac{s}{16\pi^2 v^2}\left[\frac{1}{9}+64\pi^2\,(a_4^r-2\,a_5^r)+i\,\frac{\pi}{6}\right]+\cdots\right\} \\ A_{20}(s) &= \frac{-s}{32\pi v^2}\left\{1+\frac{s}{16\pi^2 v^2}\left[-\frac{91}{36}-\frac{256\pi^2}{3}\,(2\,a_4^r+a_5^r)+\frac{10}{9}\,\log\left(\frac{s}{\mu^2}\right)-i\,\frac{\pi}{2}\right]+\cdots\right\} \end{aligned}}$$