

* $\frac{J(x) - J(x_j)}{x_i - x_j}$ Trick was correct independent of GIM, mass-independent forms in J cancel in the difference but this has nothing to do with GIM; indeed, $A(x_i, x_j)$ has a smooth ~~limit~~ limit as $x_j \rightarrow x_i$.

* GIM suppression arises as a consequence of $\sum \lambda_i \lambda_j A(x_i, x_j)$. However, a priori one cannot guess the kind of GIM (power, log) without looking at the details of the amplitude. Indeed, suppression is not quadratic in x_i as one could naively expect from the double sum, but linear due to the IR sensitivity of the diagrams. Writing $\lambda_u = -\lambda_c - \lambda_t$ makes it explicit.

So far we have made ~~an~~ what might look as an academic exercise: what does K - T mixing do with a matrix element among zero-momentum quarks with no strong interactions? ~~Really interesting!~~ For vanishing momentum ~~the~~ a local operator is generated by construction. But this is a reasonable approximation only if q^2 dependence is negligible. Let's check that.

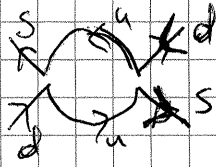
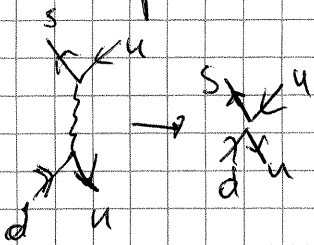
We have neglected external momenta in the evaluation of the diagrams. Is this justified?

- in diagrams containing the top quark, we are neglecting terms of $O\left(\frac{p^2}{m_t^2}, \frac{p^2}{M_W^2}\right) \Rightarrow$ OK

- in diagrams containing the charm quark, we are neglecting terms of $O\left(\frac{p^2}{m_c^2}\right) \sim O\left(\frac{M_W^2}{m_c^2}\right) \sim 15\% \Rightarrow$ relevant

- in diagrams containing the up quark, p^2 cannot be neglected - this is a ~~non~~ non-local contribution!

\Rightarrow ~~le~~ M_{12} gets large non-local contributions - need non-perturbative method to estimate them.



non-local contribution of two $\Delta S=1$ effective Hamiltonians

\Rightarrow $\ln M_{12}$ is however safe: since $\ln \lambda_t = -\ln \lambda_c$, we can write

$$\ln \lambda_t^2 = 2 \ln \lambda_t \operatorname{Re} \lambda_t$$

$$\ln \lambda_c^2 = -2 \ln \lambda_t \operatorname{Re} \lambda_t$$

$$\ln \lambda_c \lambda_t = \ln \lambda_t (\operatorname{Re} \lambda_c - \operatorname{Re} \lambda_t)$$

actually $\frac{\ln \lambda_t}{\lambda_t} = -\frac{\ln \lambda_c}{\lambda_c}$; overall $\arg \lambda_u$ irrelevant, so choose $\arg \lambda_u = 0$

$$\Rightarrow \frac{G_F^2 M_W^2}{2 \alpha^2} \ln \lambda_t (\operatorname{Re} \lambda_t (S_0(x_t) - S_0(x_t, x_c)) - \operatorname{Re} \lambda_c (S_0(x_c) - S_0(x_c, x_t)))$$

$$S_0(x_t) - S_0(x_t, x_c) = A(x_c, x_t) + \bar{A}(x_u, x_u) - 2\bar{A}(x_t, x_u) - \bar{A}(x_t, x_c) - \bar{A}(x_t, x_u)$$

$$+ \bar{A}(x_c, x_u) + \bar{A}(x_t, x_u) = \bar{A}(x_t, x_t) - \bar{A}(x_c, x_u) + \bar{A}(x_t, x_t) + \bar{A}(x_c, x_u)$$

$$S_0(x_c) - S_0(x_t, x_c) = A(x_c, x_c) + A(x_u, x_u) - 2A(x_c, x_u) - \bar{A}(x_t, x_t) - \bar{A}(x_t, x_c) + \bar{A}(x_c, x_c) + \bar{A}(x_c, x_u) - \bar{A}(x_t, x_u)$$

(negative to $O\left(\frac{p^2}{m_c^2}\right)$)