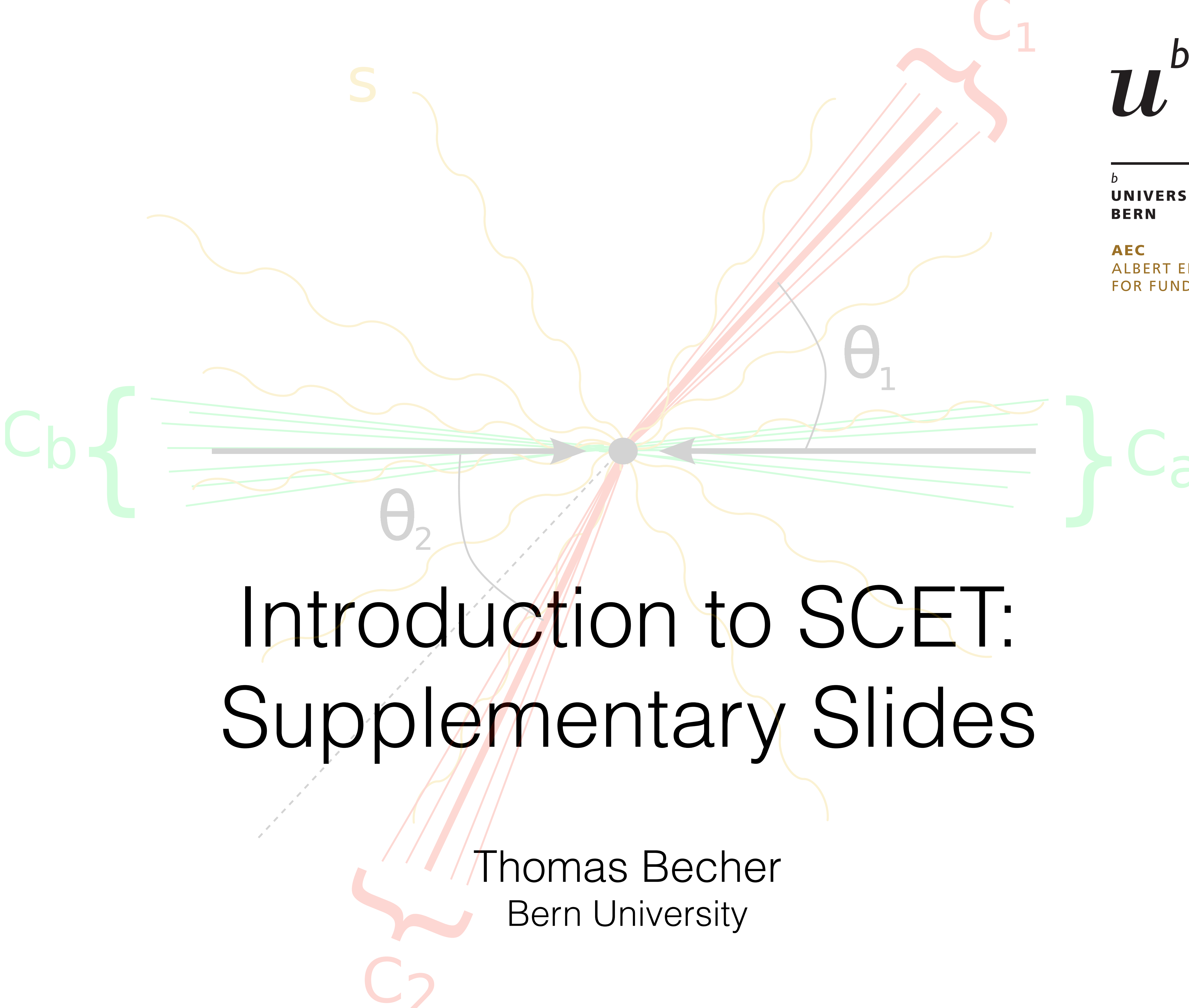

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Introduction to SCET: Supplementary Slides

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Lectures on Soft-Collinear Effective Field Theory
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Momentum Regions in the Sudakov Problem

Method of regions

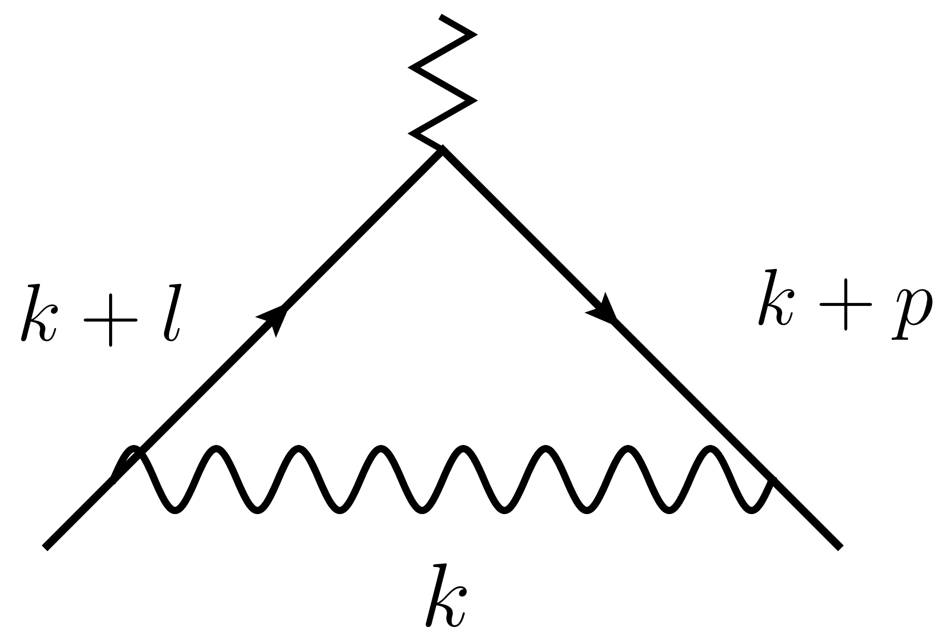
For a review: V.A. Smirnov Springer, Tracts Mod. Phys.177:1-262, 2002
Steps towards a proof: B. Jantzen, JHEP 1112 (2011) 076

In general, the expansion of integral in dim. reg. is obtained as follows:

- Identify all regions of the integration which lead to singularities in the limit under consideration.
- Expand the integrand in each region and integrate over the *full* phase space.
- Summing the contribution from the different regions gives the expansion of the original integral.

Application to the Sudakov problem

Let us now perform the expansion in a situation, where particles have large energies, but small invariant masses. Simplest example is the integral



$$L^2 \equiv -l^2 - i0\,, \qquad P^2 \equiv -p^2 - i0\,, \qquad Q^2 \equiv -(l-p)^2 - i0$$

We consider the limit $L^2 \sim P^2 \ll Q^2$.

$$I = i\pi^{-d/2}\mu^{4-d}\int d^dk\,\frac{1}{(k^2+i0)\,[(k+l)^2+i0]\,[(k+p)^2+i0]}$$

We consider the scalar integral *I*, but the same momentum regions appear in tensor integrals.

To obtain the expansion introduce light-like reference vectors in the directions of p and l

$$n_\mu = (1,0,0,1) \qquad \bar{n}_\mu = (1,0,0,-1)$$

$$n^2 = \bar{n}^2 = 0 \qquad n\cdot\bar{n} = 2$$

Any vector can be decomposed as

$$p^\mu = (n\cdot p)\,\frac{\bar{n}^\mu}{2} + (\bar{n}\cdot p)\,\frac{n^\mu}{2} + p_\perp^\mu \equiv p_+^\mu + p_-^\mu + p_\perp^\mu\,,$$

Introduce expansion parameter $\lambda^2 \sim P^2/Q^2 \sim L^2/Q^2$

The different components of p^μ scale differently. Since

$$p^2 = n \cdot p \; \bar{n} \cdot p + p_\perp^2 \sim \lambda^2 Q^2$$

and $p^\mu \approx \frac{1}{2} Q \, n^\mu$, we must have

$$(n \cdot p \, , \, \bar{n} \cdot p \, , \, p_\perp)$$

$$p^\mu \sim \left(\lambda^2 \; , \; 1 \; , \; \lambda\right) Q$$

$$l^\mu \sim \left(1 \; , \; \lambda^2 \; , \; \lambda\right) Q$$

Regions in the Sudakov problem

The following momentum regions contribute to the expansion of the integral

$$(n \cdot k \,,\, \bar{n} \cdot k \,,\, k_\perp)$$

- Hard (h) $\quad k^\mu \sim (1 \;,\; 1 \;,\; 1\;)\, Q$
- Collinear to p (c1) $\quad k^\mu \sim (\lambda^2 \;,\; 1 \;,\; \lambda\;)\, Q$
- Collinear to l (c2) $\quad k^\mu \sim (1 \;,\; \lambda^2 \;,\; \lambda\;)\, Q$
- Soft (s) $\quad k^\mu \sim (\lambda^2 \;,\; \lambda^2 \;,\; \lambda^2)\, Q$

All other possible scalings $(\lambda^a, \lambda^b, \lambda^c)$ lead to scaleless integrals upon expanding. →Exercise

Have expanded away small momentum components

$$I_{\rm h} = i\pi^{-d/2}\mu^{4-d}\int d^dk\,\frac{1}{(k^2+i0)\,(k^2+2k_-\cdot l_+ +i0)\,(k^2+2k_+\cdot p_- +i0)}$$

$$= \frac{\Gamma(1+\epsilon)}{2l_+\cdot p_-}\,\frac{\Gamma^2(-\epsilon)}{\Gamma(1-2\epsilon)}\left(\frac{\mu^2}{2l_+\cdot p_-}\right)^{\epsilon}$$

$$= \frac{\Gamma(1+\epsilon)}{Q^2}\left(\frac{1}{\epsilon^2}+\frac{1}{\epsilon}\ln\frac{\mu^2}{Q^2}+\frac{1}{2}\ln^2\frac{\mu^2}{Q^2}-\frac{\pi^2}{6}\right)+O(\epsilon)\,,$$

IR divergences!

The hard region is given by the on-shell form factor integral.

$$p^\mu \longrightarrow (\bar n\cdot p)\,\frac{n^\mu}{2}\equiv p_-^\mu\,, \qquad l^\mu \longrightarrow (n\cdot l)\,\frac{\bar n^\mu}{2}\equiv l_+^\mu$$

Collinear contribution

$$
\begin{aligned}
I_{\rm c} &= i\pi^{-d/2}\mu^{4-d}\int d^dk\,\frac{1}{(k^2+i0)\,(2k_-\cdot l_+ + i0)\,[(k+p)^2+i0]} \\
&= -\frac{\Gamma(1+\epsilon)}{2l_+\cdot p_-}\,\frac{\Gamma^2(-\epsilon)}{\Gamma(1-2\epsilon)}\left(\frac{\mu^2}{P^2}\right)^{\epsilon} \\
&= \frac{\Gamma(1+\epsilon)}{Q^2}\left(-\frac{1}{\epsilon^2}-\frac{1}{\epsilon}\ln\frac{\mu^2}{P^2}-\frac{1}{2}\ln^2\frac{\mu^2}{P^2}+\frac{\pi^2}{6}\right)+O(\epsilon)\,.
\end{aligned}
$$

The other collinear contribution I_{c2} is obtained from exchanging $l \leftrightarrow p$.

Have expanded $(k+l)^2 = 2k_-\cdot l_+ + \mathcal{O}(\lambda^2)$

Scales as $(P^2)^{-\varepsilon}$

Soft contribution

$$I_{\mathrm{s}} = i\pi^{-d/2}\mu^{4-d}\int d^dk\,\frac{1}{(k^2+i0)\,(2k_-\cdot l_+ + l^2 + i0)\,(2k_+\cdot p_- + p^2 + i0)}$$

$$= -\frac{\Gamma(1+\epsilon)}{2l_+\cdot p_-}\,\Gamma(\epsilon)\,\Gamma(-\epsilon)\,\left(\frac{2\mu^2 l_+\cdot p_-}{L^2P^2}\right)^{\epsilon}$$

$$= \frac{\Gamma(1+\epsilon)}{Q^2}\left(\frac{1}{\epsilon^2}+\frac{1}{\epsilon}\ln\frac{\mu^2Q^2}{L^2P^2}+\frac{1}{2}\ln^2\frac{\mu^2Q^2}{L^2P^2}+\frac{\pi^2}{6}\right)+O(\epsilon)\,.$$

UV divergences!

Scales as $(\Lambda_{\mathrm{soft}}{}^2)^{-\varepsilon} \sim (P^2\, L^2/Q^2)^{-\varepsilon}$.

Expand $$(k+p)^2 = 2k_+\cdot p_- + p^2 + \mathcal{O}(\lambda^3)$$

Grand total

$$I_{\mathrm{h}} = \frac{\Gamma(1+\epsilon)}{Q^2}\left(\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\ln\frac{\mu^2}{Q^2} + \frac{1}{2}\ln^2\frac{\mu^2}{Q^2} - \frac{\pi^2}{6}\right)$$

$$I_{\mathrm{c}} = \frac{\Gamma(1+\epsilon)}{Q^2}\left(-\frac{1}{\epsilon^2} - \frac{1}{\epsilon}\ln\frac{\mu^2}{P^2} - \frac{1}{2}\ln^2\frac{\mu^2}{P^2} + \frac{\pi^2}{6}\right)$$

$$I_{\bar{\mathrm{c}}} = \frac{\Gamma(1+\epsilon)}{Q^2}\left(-\frac{1}{\epsilon^2} - \frac{1}{\epsilon}\ln\frac{\mu^2}{L^2} - \frac{1}{2}\ln^2\frac{\mu^2}{L^2} + \frac{\pi^2}{6}\right)$$

$$I_{s} = \frac{\Gamma(1+\epsilon)}{Q^2}\left(\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\ln\frac{\mu^2 Q^2}{L^2 P^2} + \frac{1}{2}\ln^2\frac{\mu^2 Q^2}{L^2 P^2} + \frac{\pi^2}{6}\right)$$

$$I = I_{\mathrm{h}} + I_{\mathrm{c}} + I_{\bar{\mathrm{c}}} + I_s = \frac{1}{Q^2}\left(\ln\frac{Q^2}{L^2}\ln\frac{Q^2}{P^2} + \frac{\pi^2}{3} + O(\lambda)\right)$$

Finite (and correct!)

Cancellations

IR divergences of the hard part are in one-to-one correspondence to UV divergences of the low-energy regions

- True in general: IR divergences of on-shell amplitudes are equal to UV divergences of soft+collinear contributions

The cancellation of divergences involves a nontrivial interplay of soft and collinear log's

$$-\frac{1}{\epsilon}\ln\frac{\mu^2}{P^2}-\frac{1}{\epsilon}\ln\frac{\mu^2}{L^2}+\frac{1}{\epsilon}\ln\frac{\mu^2 Q^2}{L^2 P^2}=-\frac{1}{\epsilon}\ln\frac{\mu^2}{Q^2}$$

- Leads to interesting constraints on IR structure of on-shell amplitudes. TB Neubert, '09, Gardi Magnea '09

Soft region

Note that the soft region has

$$p_s^2 \sim \lambda^4 Q^2 \sim \frac{L^2 P^2}{Q^2}$$

Interestingly, loop diagrams involve a lower scale than what is present on the external lines. Sometimes scaling $p_s^2 \sim \lambda^4 Q^2$ is called *ultra*-soft.

Implies e.g. that jet-production processes can involve non-perturbative physics, even when the masses of the jets are perturbative.

Low energy regions

In contrast to expansion problems in Euclidean space, we encounter several low-energy regions. Each one is represented by a field in SCET.

Q	QCD	hard	H
M	SCET	collinear	J
M^2/Q	SET	soft	S

Leading-power SCET Lagrangian

Leading-power SCET Lagrangian

$$\mathcal{L}_{\text{SCET}} = \bar{\xi}\,\frac{\bar{\not{n}}}{2}\left[in\cdot D + i\not{D}_{c\perp}\frac{1}{i\bar{n}\cdot D_c}i\not{D}_{c\perp}\right]\xi - \frac{1}{4}(F^{c\,a}_{\mu\nu})^2 + \bar{q}_s\, i\not{D}_s\, q_s - \frac{1}{4}(F^{s\,a}_{\mu\nu})^2$$

where

$$igF^{s\,a}_{\mu\nu}\;t^a = [iD^\mu_s, iD^\mu_s] \quad \text{and} \quad igF^{c\,a}_{\mu\nu}\;t^a = [iD^\mu, iD^\mu]$$

with

$$iD^\mu_s = i\partial^\mu + g\,A_s(x)^\mu$$

$$iD^\mu_c = i\partial^\mu + g\,A_c(x)^\mu$$

$$iD^\mu = i\partial^\mu + g\,A^\mu_c(x) + g\,n\cdot A_s(x_-)\frac{\bar{n}^\mu}{2}$$

This Lagrangian is exact, i.e. there are no matching corrections!

Gauge invariance

When performing gauge transformations, we must make sure that they respect the scaling of the fields. For example, transforming a soft field under a gauge transformation $\alpha(x)$ with collinear scaling would turn it into collinear field.

We consider two gauge transformations

$$V_s(x) = \exp\left[\, i\alpha_s^a(x)\, t^a \,\right] \qquad V_c(x) = \exp\left[\, i\alpha_c^a(x)\, t^a \,\right]$$

where $\alpha_{\mathrm{s}}(x)$ has soft scaling, i.e. $\partial^\mu \alpha_s(x) \sim \lambda^2 \alpha_s(x)$, and $\alpha_{\mathrm{c}}(x)$ collinear scaling.

Soft gauge transformations

The soft fields transform in the usual way

$$\psi_s(x) \to V_s(x)\,\psi_s(x) \qquad A_s^\mu(x) \to V_s(x) A_s^\mu(x) V_s^\dagger(x) + \frac{i}{g} V_s(x)\left[\partial^\mu, V_s^\dagger\right]$$

while the collinear fields transform as

$$\xi(x) \to V_s(x_-)\,\xi(x) \qquad A_c^\mu(x) \to V_s(x_-) A_c^\mu(x) V_s^\dagger(x_-)$$

The x_- instead of x dependence ensures that the transformation does not induce higher power corrections. Also, the $V_s(x)\left[\partial^\mu, V_s^\dagger\right]$ is a higher power correction for $A_{c\perp}^\mu$ and $\bar{n}\cdot A_c$. The small component $n\cdot A_c$ only appears in the combination

$$in\cdot D \to V_s(x_-)\, in\cdot D\, V_s^\dagger(x_-)$$

Collinear gauge transformation

Under collinear transformations, the soft fields remain unchanged. The collinear fields transform as

$$\xi(x) \to V_c(x)\,\xi(x)\,, \quad A_c^\mu(x) \to V_c(x) A_c^\mu(x) V_c^\dagger(x) + \frac{1}{g} V_c(x) \left[i\partial^\mu + \frac{\bar{n}^\mu}{2} n \cdot A_s(x_-), V_c^\dagger(x) \right]$$

The soft piece in this transformation law ensures that

$$in \cdot D \to V_c(x)\, in \cdot D\, V_c^\dagger(x)$$

Both collinear and soft gauge transformations are homogenous and leave the Lagrangian invariant (*exercise*). A thorough discussion of the gauge transformations and the construction of higher power terms in the Lagrangian is given in Beneke and Feldmann hep-ph/0211358.