Proposed Problems on Chiral Perturbation Theory

A. Pich **(Les Houches 2017)**

Problem 1

The quadratic mass term of the $\mathcal{O}(p^2)$ χPT Lagrangian generates a small mixing between the π_3 and η_8 fields, proportional to the quark mass difference $\Delta m \equiv m_d - m_u$.

a) Diagonalize the neutral meson mass matrix and find out the correct mass eigenstates and their masses.

b) When isospin is conserved, Bose symmetry forbids the decay $\eta \to \pi^0\pi^+\pi^-$ (why?). Compute the decay amplitude to first-order in Δm.

Problem 2

a) Compute the axial current at $\mathcal{O}(p^2)$ in χPT and check that $f_\pi = f$ at this order.

b) Expand the $\mathcal{O}(p^2)$ axial current to $\mathcal{O}(\Phi^3)$ and compute the 1-loop corrections to f_π. Remember to include the pion wave-function renormalization.

c) Find the tree-level contribution of the $\mathcal{O}(p^4)$ χPT Lagrangian to the axial current. Renormalize the UV loop divergences with the $\mathcal{O}(p^4)$ LECs.

Problem 3

Assume the existence of a hypothetical light Higgs which couples to quarks with the Yukawa interaction

$$\mathcal{L}_{h^0\bar{q}q} \;=\; -\frac{h^0}{v}\,\sum_q k_q\, m_q\, \bar{q} q\,.$$

a) Determine at lowest-order in the χPT expansion the effective Lagrangian describing the Higgs coupling to pseudoscalar mesons induced by the light-quark Yukawas.

b) Determine the effective $h^0 G^{\mu\nu}_a G^a_{\mu\nu}$ coupling induced by heavy quark loops.

c) The $G^{\mu\nu}_a G^a_{\mu\nu}$ operator can be related to the trace of the energy-momentum tensor, in the 3-flavour QCD theory:

$$\Theta^\mu_\mu \;=\; \frac{\beta_1\alpha_s}{4\pi}\, G^{\mu\nu}_a G^a_{\mu\nu} + \bar{q}\mathcal{M} q\,,$$

where $\beta_1 = -\frac{9}{2}$ is the first coefficient of the β function. Using this relation, determine the lowest-order χPT Lagrangian incorporating the Higgs coupling to pseudoscalar mesons induced by the heavy-quark Yukawas.

d) Compute the decay amplitudes $h^0 \to 2\pi$ and $\eta \to h^0\pi^0$.