DEPARTMENT OF PHYSICS
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DATE

NO.

I Introduction of WIMP dark matter.

II WINP nucleus/nucleon NR int.

III Direct detection experiments

IV Evaluation of effective int of WINP with nucleon.

V Toward Next-Leading order of α_s.

I Introduction of WIMP darkmatter.

Dark matter (DM) in the universe

i) Galaxy rotation curve measurements.

ii) CMB measurements.

$\Omega_x \equiv \rho_x/\rho_{critical}$ $\quad (\rho_{critical} \simeq 10^{-5} \mathrm{GeV}/\mathrm{cm}^3)$

$\longrightarrow$

$\Omega_{DM} \simeq 27\%$

Ω_{baryon} 5%

$\Omega_\Lambda \simeq 68\%$

iii) Xrays & Gravitational lensing of galactic clusters.

iv) N-bodies simulation of structure formation

:

Nature of particle darkmatter.

i) Electorically neutral.

ii) Massive

iii) Stable or longer lifetime than the age of the universe.

iv) "cold"

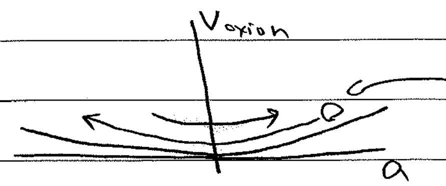
(free streeming length ≪ protogalaxies.)

Candidates for particle dark matter.
(10^{-23} eV $\lesssim M_X \lesssim 10^{18}$ GeV)

i) Weakly Interacting Massive particles (WIMPs)
- Produced from thermal bath in the early universe.
- $M_X \simeq 10^{2-4}$ GeV.
- Stability ; symmetry.

(example)
Lightest SUSY particles (LSP) in SUSY SM (R parity)
" Kaluza-Klein " (LKP) in Extradim (KK ")
⋮
(linked with the naturalness problem in the SM)

ii) Axion or ALP
- Pseudo NG boson in PQ mechanism for strong CP problem.
- Produced by misalignment mechanism.

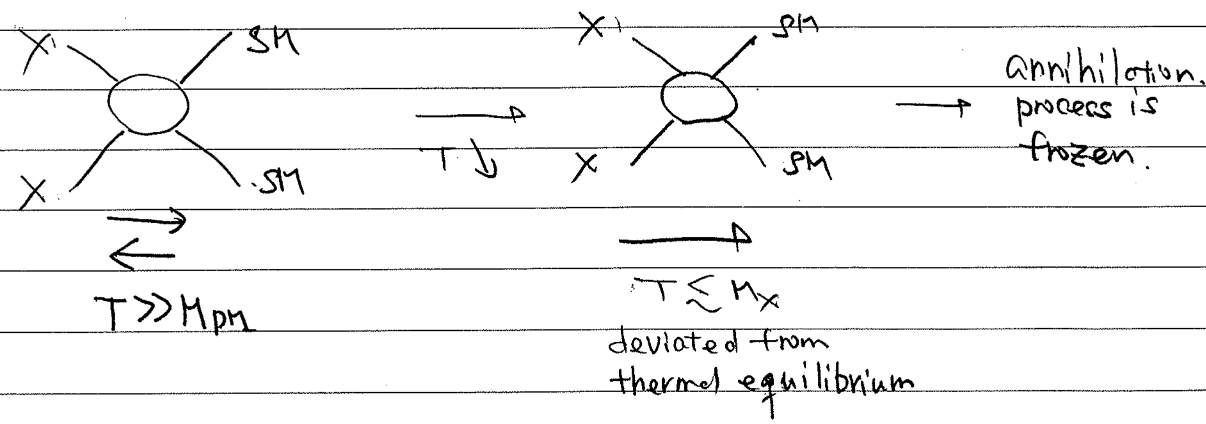


$M_{axion} \simeq 10^{-(3-5)}$ eV.

iii) WIMPzillas
$M_X \simeq 10^{12-16}$ GeV

iv) sterile neutrinos.
⋮

Thermal production mechanism of WIMPs.



Boltzmann eq of DM particles (n_X : # density of X)

$$\frac{dn_X}{dt} + 3Hn_X = -\langle \sigma |v| \rangle [n_X^2 - (n_X^{EQ})^2]$$

H : Hubble parameter.

$$H = \sqrt{\frac{8\pi}{3M_{Pl}^2}\rho} \simeq \sqrt{\frac{4\pi^2}{45}}\, g_*^{1/2} \frac{T^2}{M_{Pl}}$$

$$\left(g_* = \sum_{boson} 1 + \sum_{fermion} \frac{7}{8} \right)$$

$\langle \sigma |v| \rangle$: Thermal averaged total annihilation X section $(XX \to SM)$

n_X^{EQ} : # density of X in thermal equilibrium.

Define

$Y \equiv n_X / s$ $\quad$ (s: Entropy density ($s = \frac{2\pi^2}{45} g_{*s} T^3$ $\quad g_{*s} = g_*$ (massless))

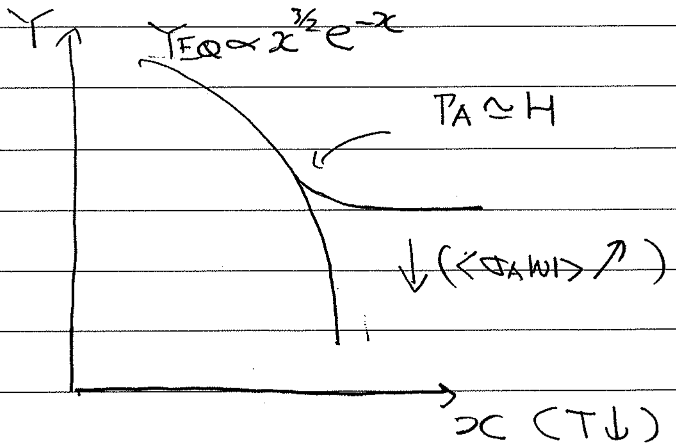
$x \equiv M_X / T$

$\rightarrow$

$$\frac{x}{Y_{EQ}} \frac{dY}{dx} = -\frac{\Gamma_A}{H}\left[\left(\frac{Y}{Y_{EQ}}\right)^2 - 1 \right]$$

where

$\Gamma_A \equiv n_X^{EQ} \langle \sigma_A |v| \rangle$: Probability of annihilation per time



$\Gamma_X \ll H \to X$ decoupled from thermal bath.

Decoupling temperature T_D $(\simeq \frac{1}{20} M_X)$

$$\Gamma_A = n_X^D \langle \sigma_A |v| \rangle \simeq H_D = \sqrt{\frac{4\pi^3}{45}}\, g_*^{1/2} \frac{T_D^2}{M_{PL}}$$

$\therefore$

$$Y_X^{now} \simeq \frac{n_X^D}{s_D} = \frac{45}{2\pi^2} \frac{1}{g_{*s} T_D^3} \frac{1}{\langle \sigma_A |v| \rangle} \sqrt{\frac{4\pi^3}{45}}\, g_*^{1/2} \frac{T_D^2}{M_{PL}}$$

$$= \sqrt{\frac{45}{\pi}} \frac{g_*^{1/2}}{g_{*s}} \frac{1}{(T_D M_{PL})} \frac{1}{\langle \sigma_A |v| \rangle}$$

$$\Omega_X = \frac{1}{\rho_c} \rho_X = \frac{s^{now}}{\rho_c} Y_X^{now} M_X = 3 \times 10^{8}\,\mathrm{GeV}^{-1}\, h^{-2}\, Y_X^{now} M_X$$

$$\simeq 0.4 \times \left(\frac{x_D = M_X / T_D}{20} \right) \left(\frac{\langle \sigma_A |v| \rangle}{10^{-9}\,\mathrm{GeV}^{-2}} \right)^{-1}$$

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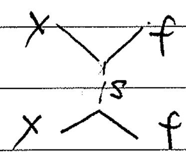
Typical cases.

i) $SU(2)_L$ singlet

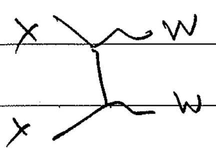


$$\sigma v \simeq \frac{\sum \alpha_Y^2}{M_S^4} M_\chi^2 \simeq 3\times 10^{-9}\,\mathrm{GeV}^{-2}\left(\frac{M_\chi}{300\,\mathrm{GeV}}\right)^{-2}\Bigg|_{M_\chi = M_S}$$

If χ is Majorana, (bino) p-wave suppression of x section

$$v^2 \simeq \frac{T_D}{M} \simeq \frac{1}{20}$$

ii) $SU(2)_L$ multiplet

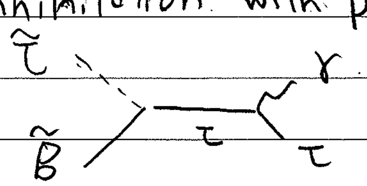


$$\sigma v \simeq \frac{\sum \alpha_2^2}{M_\chi^2} \simeq 3.5\times 10^{-9}\,\mathrm{GeV}^{-2}\left(\frac{M_\chi}{1\,\mathrm{TeV}}\right)^{-2}$$

SU(2) doublet fermion (Higgsino) $\sim 1\,\mathrm{TeV}$

triplet " (wino) $\sim 3\,\mathrm{TeV}$

(cf). Coannihilation with particles degenerate with χ in mass



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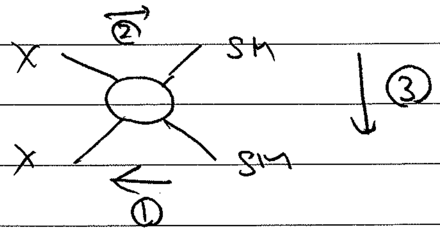
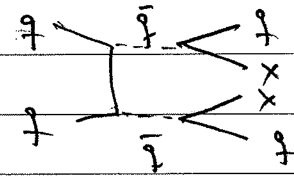
Searches for WIMP DM



① WIMP Production @ colliders (LHC)



missing transverse energy (MET)

② Indirect detection

DM particles are gravitationally accumulated in astrophysical objects (stars, galaxies, ...)

$\chi\chi \to \underline{\gamma\ e^+\ \bar{p}\ \nu}$ contribute to CRs.

Fermi satellite) γ rays from GC, dwarf galaxies . .
HESS (→ CTA)
AMS satellite) e^+, $\bar{p}$
ICECUBE SK) ν from sun.

③ Direct detection.

detection of elastic scattering of nuclei with WIMP DM
$\chi N \to \chi N$.

typical recoil energy

$E \simeq m_r^2/m_T\, v^2$ (m_T : target nuclei mass
m_r : $m_T M_\chi/(m_T+M_\chi)$
v : DM velocity ($\sim 10^{-3}c$))

$= (1\sim 100)$ keV

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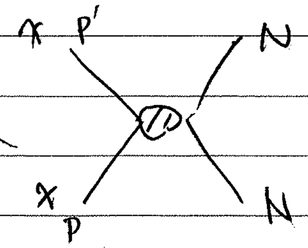
II WIMP nucleus / nucleon NR interaction

WIMP - nucleus NR operators

fermionic WIMP

- momentum transfer $\vec{q}$ $\quad |\vec{q}| \simeq M_\chi v$
- DM incoming velocity relative to target $\vec{v}$
- nucleus spin $\vec{S}$
- WIMP spin $\vec{S}$



$$q^\mu = (p - p')^\mu$$
$$P = (p + p')^\mu = (2p + q)^\mu$$
$$\Rightarrow \quad \vec{P} = 2 m_r \vec{v} + \vec{q}$$

Ref.
~~Fan, Reece~~, Wang
JHEP 1011 (10) 042
arxiv. 1008.1591

Spin Indep (SI) operator
(nucleus)

$$O_1^{(++)} = 1$$
$$O_2^{(-+)} = i \vec{S}_\chi \cdot \vec{q}$$
$$O_3^{(--)} = \vec{S}_\chi \cdot \vec{P}$$
$$O_4^{(++)} = i \vec{S}_\chi \cdot (\vec{P} \times \vec{q})$$

	P	T
$\vec{S}$	+	−
$\vec{q}$	−	−
$\vec{P}$	−	+

Spin-dependent (SD) operator

$$O_5^{(++)} = \vec{S}_\chi \cdot \vec{S}_N$$
$$O_6^{(-+)} = i \vec{S}_N \cdot \vec{q}$$
$$O_7^{(--)} = \vec{S}_N \cdot \vec{P}$$
$$O_8^{(--)} = i (\vec{S}_\chi \times \vec{S}_N) \cdot \vec{q}$$
$$O_9^{(-+)} = (\vec{S}_\chi \times \vec{S}_N) \cdot \vec{P}$$
$$O_{10}^{(++)} = i \vec{S}_N \cdot (\vec{P} \times \vec{q})$$
$$O_{11}^{(++)} = (\vec{S}_N \cdot \vec{q})(\vec{S}_\chi \cdot \vec{q})$$
$$O_{12}^{(++)} = (\vec{S}_N \cdot \vec{P})(\vec{S}_\chi \cdot \vec{P})$$
$$O_{13}^{(+-)} = i((\vec{S}_N \cdot \vec{q})(\vec{S}_\chi \cdot \vec{P}) + (\vec{S}_N \cdot \vec{P})(\vec{S}_\chi \cdot \vec{q}))$$

$$O_{14}^{(+-)} = i((\vec{S}_N \cdot \vec{q})(\vec{S}_\chi \cdot \vec{P}) - (\vec{S}_N \cdot \vec{P})(\vec{S}_\chi \cdot \vec{q}))$$
$$O_{15}^{(-+)} = [\vec{S}_N \cdot (\vec{P} \times \vec{q})](\vec{S}_\chi \cdot \vec{q}) + (\vec{S}_\chi \cdot (\vec{P} \times \vec{q}))(\vec{S}_N \cdot \vec{q})$$
$$O_{16}^{(--)} = i((\vec{S}_N \cdot (\vec{P} \times \vec{q}))(\vec{S}_\chi \cdot \vec{P}) + i[\vec{S}_N \cdot (\vec{P} \times \vec{q})](\vec{S}_N \cdot \vec{P}))$$

Ⓑ Nucleus Spin-dependent (SD) operators.

$$\mathcal{O}_5^{(++)} = \vec{S}_\chi \cdot \vec{S}_N$$

$$\mathcal{O}_6^{(-+)} = i\vec{S}_N \cdot \vec{q}$$

$$\mathcal{O}_7^{(--)} = \vec{S}_N \cdot \vec{P}$$

$$\mathcal{O}_8^{(--)} = i(\vec{S}_\chi \times \vec{S}_N) \cdot \vec{q}$$

$$\mathcal{O}_9^{(-+)} = (\vec{S}_N \times \vec{S}_N) \cdot \vec{P}$$

$$\mathcal{O}_{10}^{(++)} = i\vec{S}_N \cdot (\vec{P} \times \vec{q})$$

$$\mathcal{O}_{11}^{(++)} = (\vec{S}_N \cdot \vec{q})(\vec{S}_\chi \cdot \vec{q})$$

$$\mathcal{O}_{12}^{(++)} = (\vec{S}_N \cdot \vec{P})(\vec{S}_\chi \cdot \vec{P})$$

$$\mathcal{O}_{13}^{(+-)} = i((\vec{S}_N \cdot \vec{q})(\vec{S}_\chi \vec{P}) + (\vec{S}_N \vec{P})(\vec{S}_\chi \vec{q}))$$

$$\mathcal{O}_{14}^{(+-)} = i((\vec{S}_N \cdot \vec{q})(\vec{S}_\chi \vec{P}) - (\vec{S}_N \vec{P})(\vec{S}_\chi \cdot \vec{q}))$$

$$\mathcal{O}_{15}^{(-+)} = [\vec{S}_N \cdot (\vec{P} \times \vec{q})](\vec{S}_\chi \cdot \vec{q}) + (\vec{S}_\chi (\vec{P} \times \vec{q})][\vec{S}_N \vec{q}]$$

$$\mathcal{O}_{16}^{(--)} = i[\vec{S}_N \cdot (\vec{P} \times \vec{q})](\vec{S}_\chi \cdot \vec{P}) + i[\vec{S}_\chi \cdot (\vec{P} \times \vec{q})](\vec{S}_N \cdot \vec{P})$$

Wilson coefficients are suppressed by mediator mass if they are hevier than ~GeV. In the case following two are dominant

$$\begin{cases} \mathcal{O}_1^{(++)} = \mathbb{1} & (SI) \\ \mathcal{O}_5^{(++)} = \vec{S}_\chi \cdot \vec{S}_N & (SD) \end{cases}$$

The other terms are suppressed by v or $|\vec{q}|$.

WIMP-nucleon operators

χ : Majorana fermion

$$\mathcal{L} = \sum_{N=n,p} \Big(\underbrace{f_N \bar{\chi}\chi\bar{N}N}_{SI} + \underbrace{a_N \bar{\chi}\gamma_\mu\gamma_5\chi\bar{N}\gamma_\mu\gamma_5 N}_{SD \to \vec{S}_\chi\cdot\vec{S}_N} \Big)$$

※ "1/2" is not multiplied due to historical reason.

$\sigma = \sigma_{SI} + \sigma_{SD}$

Neglecting nucleus form factor $\left(m_r = M_T m_N / (M_T + m_N) \right)$

$$\sigma_{SI} = \frac{4}{\pi} m_r^2 \left| Z f_p + (A-Z) f_n \right|^2$$

- Amplitude is proportional to Z or $(A-Z)$.
- Large nucleus is more sensitive to DM scattering

$$\sigma_{SD} = \frac{16}{\pi} m_r^2 \frac{J+1}{J} \left| a_p \langle S_p \rangle + a_n \langle S_n \rangle \right|^2$$

- $\langle S_N \rangle$: expectation value of nucleon spin in nucleus.
- J : total spin of nucleus.

- Form factors are not negligible

momentum transfer $q = (2 m_T E_R)^{1/2}$

nucleus radius $r_n = A^{1/3}$ fm.

$\rightarrow r_n q \simeq 7 \times 10^{-3} A^{5/6} (E_R (\mathrm{keV}))^{1/2}$ $m_T \simeq A m_p$

σ_{SI} is suppressed for large A and large E_R.

- Future sensitivites and bounds on SI interaction is represented by σ_p^{SI}.

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χ = Dirac fermion

$$\mathcal{L}_V = \sum_{N=n,p} f_N^{(V)} \bar{\chi}\gamma_\mu\chi \bar{N}\gamma_\mu N$$

$$\mathcal{L}_S = \sum_{N=n,p} f_N^{(S)} \bar{\chi}\chi \bar{N}N$$

$$\mathcal{L}_\gamma = \underbrace{\frac{\mu_\chi}{2}\bar{\chi}\sigma^{\mu\nu}\chi F_{\mu\nu}}_{\text{MDM}} + \underbrace{b_\chi \bar{\chi}\gamma^\mu\chi \partial^\nu F_{\mu\nu}}_{\text{charge radius}} + \underbrace{\frac{d_\chi}{2}\bar{\chi} i\sigma^{\mu\nu}\gamma_5\chi F_{\mu\nu}}_{\text{EDM}}$$

WIMP - Nuclei$_{(A)}$ - SI cross section.

$$\frac{d\sigma}{dE_R} = \alpha \mu_\chi^2 Z^2 \left(\frac{1}{E_R} - \frac{M}{2m_p^2 v^2} \right) |F_{SI}(E_R)|^2$$

$$+ \alpha d_\chi^2 Z^2 \frac{1}{v^2 E_R} |F_{SI}(E_R)|^2$$

$$+ \frac{m_A}{2\pi v^2} (f)^2 \ |F_{SI}(E_R)|^2$$

$$f = Z\left(f_p^{(S)} + f_p^{(V)} - e b_\chi - e\frac{\mu_\chi}{2m_\chi}\right)$$

$$+ (A-Z)\left(f_n^{(S)} + f_n^{(V)}\right)$$

$$E_{th} \le E_R \le E_{max} \qquad \left(E_{max} = 2\frac{m_r^2}{m_A} v^2\right)$$

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When Dirac DM has weak charge, it is severely constrained.
$(Q = T_{3\chi} + Y_\chi = 0)$

$$\mathcal{L} = + \frac{g_Z^2}{m_Z^2} Y_\chi \bar{\chi}\gamma_\mu \chi \left[(\underbrace{\tfrac{1}{4} - s_W^2}_{\simeq 0}) \bar{p}\gamma_\mu p + (-\tfrac{1}{4}) \bar{n}\gamma_\mu n \right]$$

$\rightarrow$

$$\sigma_n^{SI} \simeq \frac{1}{4\pi} m_n^2 \frac{g_Z^4}{m_Z^4} Y_\chi^2 \simeq 7 \times 10^{-40}\,\mathrm{cm}^2$$

XENON1T $\sigma_n \lesssim 10^{-45}\,\mathrm{cm}^2$ $(M_\chi \simeq 1\,\mathrm{TeV})$

$\longrightarrow M_\chi \gtrsim 10^5\,\mathrm{TeV}$.

Pseudo Dirac DM

$$\mathcal{L} = -M\bar{\chi}\chi - \frac{1}{2}\mu(\bar{\chi^c}\chi + h.c.)$$
$$= -\frac{1}{2}\left[\overline{(\chi_L)^c}, \bar{\chi}_R\right] \begin{bmatrix} \mu & M \\ M & \mu \end{bmatrix} \begin{bmatrix} \chi_L \\ (\chi_R)^c \end{bmatrix} + h.c.$$

$\rightarrow$

$$\left.\begin{aligned} M_1 &= M + \mu \\ M_2 &= \mu - M \end{aligned}\right\} \Delta M = 2\mu.$$

When $\Delta M \gtrsim m_N v^2 \simeq 10^2\,\mathrm{KeV}$, elinelastic scattering $(\chi_1 N \to \chi_2 N)$ is suppressed.

III, Direct detection experiments.

Dark matter around us.

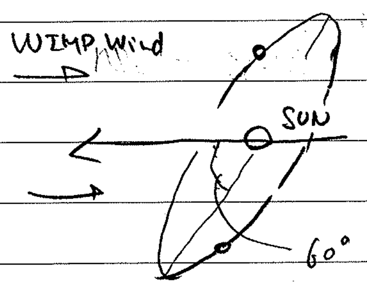
· Density : $\rho_X \simeq (0.3 \sim 0.7)$ GeV/cm^3.

· Velocity distribution.
(assume Maxwellian.)

$$f(\vec{v}) \simeq \frac{1}{(\pi v_0^2)^{3/2}} e^{-(\vec{v}+\vec{v}_E)^2/v_0^2}$$

$v_0 \simeq 230$ km/s

· Velocity of earth.



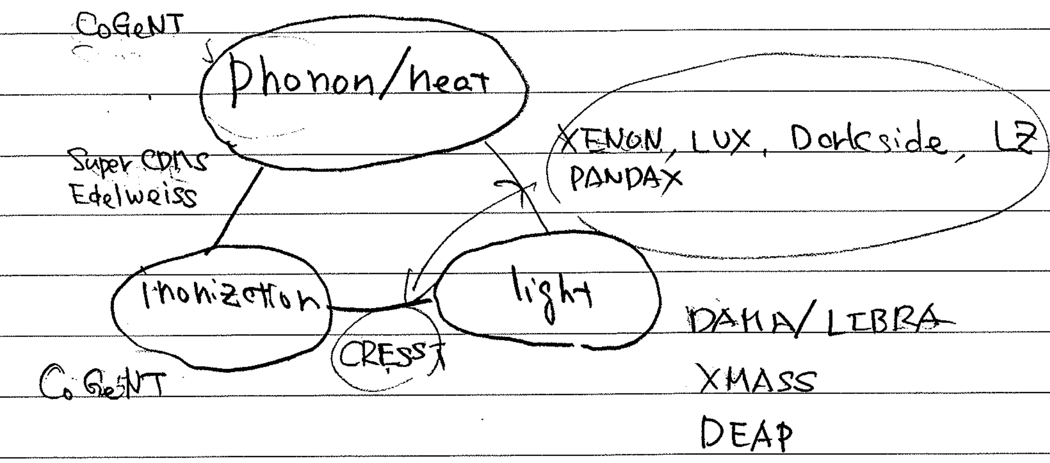
$|\vec{v}_E| \simeq 244 + 15 \sin(2\pi y)$ (km/s)

y: elapsed time from 2nd, March.

$\Rightarrow$

· Annual modulation of event rate $\sim 3\%$
· Directional detection is still challenging.

Detectional technique



Low BG: underground
coincidence

Event rate

$$dR = \frac{N_0}{A}\sigma v \, dn_x \quad : \text{event rate/unit target mass}$$

Avogadro #

atomic #

Assuming zero-momentum transfer cross section $\sigma \simeq \text{const} = \sigma_0$

$$R_0 = \frac{2}{\pi}\frac{N_0}{A}\frac{\rho_x}{M_x}\sigma_0 v_0 \qquad (\text{escape velocity } v_E \to 0)$$

$$\simeq \frac{540}{A M_x}\left(\frac{\sigma_0}{10^{-36}\,\mathrm{cm}^2}\right)\left(\frac{\rho_0}{0.4\,\mathrm{GeV/cm^3}}\right)\left(\frac{v_0}{230\,\mathrm{km/s}}\right)\ \text{event/kg/day}$$

GeV unit.

· Event spectrum. (E_R: recoil energy)

$$\frac{dR}{dE_R} \simeq \frac{R_0}{E_0} e^{-E_R/E_0}$$

where

$$E_0 = \frac{1}{2}M_x v^2 \cdot 4\frac{M_x m_T}{(M_x + m_T)^2}$$

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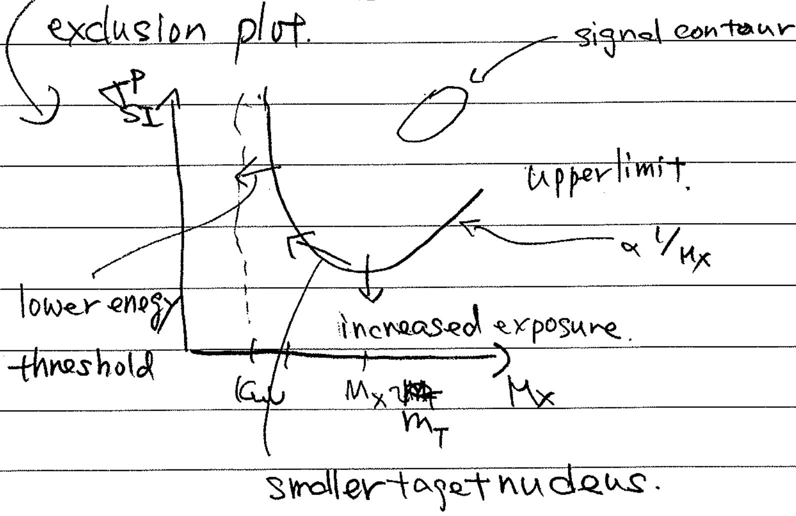
- assuming isosinglet SI interaction

exclusion plot.



Background from ν coherent scattering.

- Solar ν.

$$\sigma^p_{SI} \lesssim 10^{-44}\,\mathrm{cm}^2 \quad (M_X \lesssim \text{several GeV})$$

- Atmospheric ν.

$$\sigma^p_{SI} \lesssim 10^{-43}\,\mathrm{cm}^2 \quad (M_X \simeq 100\,\mathrm{GeV})$$

$$10^{-48}\,\mathrm{cm}^2 \quad (M_X \simeq 1\,\mathrm{TeV})$$

If directional detection is available, we may detect WIMP with smaller cross section than BG limit.

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Current limit.

PandaX. LUX XENON-1T

$\sigma_{SI}^{p} \sim 10^{-46}$ cm ($M_\chi \simeq 50$ GeV)

Future experiments.

· XENON1T (16-18) → XENONnT (19-)
$\sigma_{SI}^{p} \sim 10^{-47} cm^2$ $\sigma_{SI}^{p} \sim 10^{-48} cm^2$.

· LUX (~16) → LZ (19-)

· PandaX → PandaX-xT

ν_R

· DARWIN (24?)

· Darkside20T (?)

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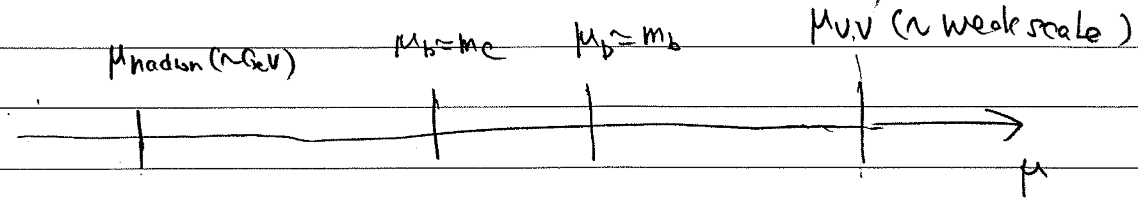
IV Evaluation of effective interaction of WIMP with nucleons.

IV-I Strategy



Standard procedure

(i) Construct effective theory of WIMP and quark/gluon. (relevant to the direct detection)

$$\mathcal{L} = C_i \bar{X} \Gamma_i X \bar{q} \Gamma_i q.$$

(ii) Evaluate Wilson coefficients for the effective operator $C_i^{UV}(\mu_{UV})$ by integrating out massive particles in UV theory.

(iii) Evaluate $C_i(\mu_{hadron})$ by RG equations.

~~Include~~ threshold correction @ μ_b or μ_c ~~if necessary from viewpoint of accuracy.~~

(iv) Evaluate effective coupling in nucleon level using matrix elements.

$$\langle p | O_i(\mu_{hadron}) | p \rangle .$$

If we get to know matrix element @ μ_{UV}, we can skip step (iii).

$$\langle p | \sum_i C_i(\mu_{UV}) O_i(\mu_{UV}) | p \rangle = \langle p | \sum_i C_i(\mu_{had}) O_i(\mu_{hadron}) | p \rangle$$

IV-II effective interaction @ parton Level

χ : Majorana fermion

- up to eq of motion
- up to $D=4$ for quark/gluon operators
- Neglect terms suppressed DM velocity or momentum transfer.

$$\mathcal{L}_{eff} = \sum_{p=q,g} C_S^p O_S^p + \sum_{i=1,2} \sum_{p=q,g} C_{T_i}^p O_{T_i}^p + \sum_q C_{AV}^q O_{AV}^q$$

$$O_S^q = \bar{\chi}\chi \, \underline{\underline{m_q}} \bar{q} q \qquad (D=7)$$

$$O_S^g = \underline{\underline{\frac{\alpha_s}{\pi}}} \bar{\chi}\chi \, G_{\mu\nu}^A G_{\mu\nu}^A \qquad (D=7)$$

$$O_{T_1}^p = \frac{1}{M_\chi} \bar{\chi} i\partial_\mu \gamma_\nu \chi \, O_{\mu\nu}^p \qquad (D=8)$$

$$O_{T_2}^p = \frac{1}{M_\chi^2} \bar{\chi} i\partial_\mu i\partial_\nu \chi \, O_{\mu\nu}^p \qquad (D=9)$$

(SI for the above four)

$$O_{AV}^q = \bar{\chi}\gamma_\nu\gamma_5\chi \, \bar{q}\gamma_\nu\gamma_5 q \qquad (D=6)$$

(SD)

where Spin-2 twist-2 operator

$$\left\{ \begin{array}{l} O_{\mu\nu}^q = \frac{1}{2} \bar{q} i \left(D_\mu \gamma_\nu + D_\nu \gamma_\mu - \frac{1}{2} g_{\mu\nu} \not{D} \right) q \\ O_{\mu\nu}^g = G_\mu^{A\rho} G_{\nu\rho}^A - \frac{1}{4} g_{\mu\nu} G_{\mu\nu}^A G^{A\mu\nu}. \end{array} \right.$$

twist = dim − spin.

① In NR limit of χ

$$O^P_{T_1} \simeq \chi^\dagger \chi \, O^P_{00} \qquad O^P_{T_2} = \bar{\chi}\chi \, O^P_{00}.$$

② Trace anomaly in QCD

$$\Theta^\mu_\mu = \frac{\beta(\alpha_s)}{4\alpha_s} G^A_{\mu\nu} G^{A\mu\nu} + (1-\gamma_m)\sum_q m_q \bar{q} q$$

where $\beta(\alpha_s) \equiv \mu \frac{\partial}{\partial\mu}\alpha_s \simeq 2b_1 \frac{\alpha_s^2}{4\pi} + 2b_2 \frac{\alpha_s^3}{(4\pi)^2}$

$$b_1 = -\frac{11}{3}N_c + \frac{2}{3}N_f$$

$$b_2 = -\frac{34}{3}N_c^2 + \frac{10}{3}N_c N_f + 2C_F N_f$$

$$\gamma_m m_q \equiv \mu \frac{\partial}{\partial\mu} m_q = -6C_F \frac{\alpha_s}{4\pi}$$

$$\left[\begin{array}{lll} O^q_S = m_q \bar{q} q & \text{RG inv } (\overline{MS}) & \\ O^g_S = \frac{\alpha_s}{\pi} G^2. & \text{〃} & @ \ O(\alpha_s) \end{array}\right.$$

∵ $\mu \frac{\partial}{\partial\mu} \Theta^\mu_\mu = 0$

∴

$$\mu \frac{\partial}{\partial\mu} \begin{pmatrix} O^q_S \\ O^g_S \end{pmatrix} = -\begin{bmatrix} 0 & 0 \\ \underbrace{-\frac{4\alpha_s^2}{\pi} \frac{\partial\gamma_m}{\partial\alpha_s}}_{\sim O(\alpha_s^2)} & \underbrace{\alpha_s^2 \frac{\partial}{\partial\alpha_s}\left(\frac{\beta(\alpha_s)}{\alpha_s^2}\right)}_{O(\alpha_s^2)} \end{bmatrix} \begin{bmatrix} O^q_S \\ O^g_S \end{bmatrix}$$

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IV-III Matrix elements.

mass factor

① $\langle N | m_q \bar{q} q | N \rangle \equiv f_{Tq}^{(N)} m_N$.

QCD lattice ($N_f = 2+1+1$, ETM collab. 1601.01624)

$f_u^{(p)} = 0.0149(17)\binom{21}{16}$ $\quad f_u^{(n)} = 0.0117(15)\binom{18}{12}$

$f_d^{(p)} = 0.0234(23)\binom{27}{16}$ $\quad f_d^{(n)} = 0.0298(23)\binom{30}{16}$

$f_s^{(p)} = 0.0440(88)\binom{72}{51}$

$f_c^{(p)} = 0.085(22)\binom{11}{7}$

statistical $\quad$ systematical

$(r = m_u/m_d = 0.50(4))$

② $\langle N | \frac{\alpha_s}{\pi} G^2 | N \rangle$

trace anomaly $\quad m_N = \langle N | \Theta^\mu_\mu | N \rangle$

$$\langle N | \frac{\alpha_s}{\pi} G^2 | N \rangle = m_N \frac{4\alpha_s^2}{\pi \beta(\alpha_s)} \left[1 - (1-\gamma_m) \sum_q f_{Tq}^{(N)} \right]$$

$$\left(\simeq \frac{8}{b_1} m_N \left(1 - \sum_q f_{Tq}^{(N)}\right) \right) \quad O(\alpha_s^0)$$

$$= -\frac{8}{9} m_N \Big(1 - \underbrace{\sum_q f_{Tq}^{(N)}}_{\lesssim 0.1}\Big)$$

$\therefore \langle N | \frac{\alpha_s}{\pi} G^2 | N \rangle$ is $O(1)$

$\langle N | m_Q \bar{Q} Q | N \rangle$ Q : heavy quark

$\langle N | \Theta^\mu_\mu | N \rangle |_{N_f} = \langle N | \Theta^\mu_\mu | N \rangle |_{N_f - 1}$

$\therefore$

$\langle N | m_Q \bar{Q} Q | N \rangle \simeq -\frac{1}{12}(1 + 11 \frac{\alpha_s}{4\pi}) \langle N | \frac{\alpha_s}{\pi} G^2 | N \rangle$

@ $\mu \simeq m_Q$

$\simeq \frac{2}{27} m_N$

~ 0.074 (consistent with lattice)

Integrating out Q is ok ?

$-\frac{\alpha_s}{12\pi} G^2 + \frac{\alpha_s}{64\pi m_Q^2} (D^\nu G^A_{\nu\mu})(D^\rho G^A_{\rho\mu}) - \frac{g_s \alpha_s(m_Q)}{720 m_Q^2} f_{ABC} G^A_{\mu\nu} G^{B\nu\rho} G^C_{\nu\rho}$

$\frac{\Lambda^2_{QCD}}{m_c^2} \simeq O(10\%) \longrightarrow$ ~~o(few%)~~

parametrically

③ Spin-2 twist-2 operators

$$\langle N(p)|O^{q(\mu)}_{\mu\nu}|N(p)\rangle = m_N\left(\frac{p_\mu p_\nu}{m_N^2}-\frac{1}{4}g_{\mu\nu}\right)\left(q^{(N)}(2,\mu)+\bar{q}^{(N)}(2,\mu)\right)$$

$$\langle N(p)|O^{g(\mu)}_{\mu\nu}|N(p)\rangle = -m_N\left(\quad \text{〃} \quad\right) g^{(N)}(2,\mu)$$

where

$$q^{(N)}(n,\mu) = \int_0^1 dx\, x^{n-1}\, q^{(N)}(x,\mu)$$

$$\bar{q}^{(N)}(n,\mu) = \int_0^1 dx\, x^{n-1}\, \bar{q}^{(N)}(x,\mu) \qquad \text{(n-th moments of PDF)}$$

$$g^{(N)}(n,\mu) = \int_0^1 dx\, x^{n-1}\, g^{(N)}(x,\mu)$$

$q^{(N)}(x,\mu)$ $\bar{q}^{(N)}(x,\mu)$, $g(x,\mu)$: PDF of μ.

Proof)

- OPE for DIS, optical theorem ($\gamma^* N \to \gamma^* N$) and, contour integral $(\gamma_q N \to X)$
  - See Peskin & Schroeder Ch 18.
  - Assume integral for $|w| = |\frac{1}{x}| \to \infty$ is zero.
  - RGeq of twist-2 ops are consistent with Altarelli-Parisi evolution of PDF.

$$\{p^{\mu_1}p^{\mu_2}\cdots p^{\mu_J}\}_{TS}\langle N|O_J^{(q)}|N\rangle = \frac{1}{2}\langle N|\bar{q}_J(\gamma^{\mu_1}iD^{\mu_2}\cdots iD^{\mu_J})_{TS}\, q_J|N\rangle$$

$\uparrow$ traceless sym. $\quad$ $\leftarrow$ ~~spin~~ twist 2 op

$$\langle N|O_J^{(q)}|N\rangle = \int_0^1 dx\, x^{J-1}\left(f_J^{(N)}(x,\mu)+(-)^J \bar{f}_J^{(N)}(x,\mu)\right)\frac{1}{2m_N}$$

total x-sec for DIS $\left(\sum \sigma(\gamma_q N \to X)\right)$

2nd moments of PPF of Proton ($\mu = M_Z$; $N_f = 5$)

CTEQ-Jefferson Lab. collaboration

$g(2) = 0.464(2)$

$u(2) = 0.223(3)$ $\bar{u}(2) = 0.036(2)$

$d(2) = 0.118(3)$ $\bar{d}(2) = 0.037(3)$

$s(2) = 0.0258(4)$ $\bar{s}(2) = s(2)$

$c(2) = 0.0187(2)$ $\bar{c}(2) = c(2)$

$b(2) = 0.0117(1)$ $\bar{b}(2) = b(2)$

Valence quarks and gluon are sizable.

RGEs of twist op

$$\mu \frac{d}{d\mu} \left(O^q_{\mu\nu} \ O^g_{\mu\nu} \right) = - \begin{bmatrix} \gamma_{qq} & \gamma_{qg} \\ \gamma_{gq} & \gamma_{gg} \end{bmatrix} \begin{bmatrix} O^q_{\mu\nu} \\ O^g_{\mu\nu} \end{bmatrix}$$

$$\gamma_{qq} = \frac{16}{3} C_F \frac{\alpha_s}{4\pi} + \left(-\frac{208}{27} C_F N_f - \frac{224}{27} C_F^2 + \frac{752}{27} C_F N_c \right) \left(\frac{\alpha_s}{4\pi} \right)^2$$

$$\gamma_{qg} = \frac{4}{3} \frac{\alpha_s}{4\pi} + \left(\frac{148}{27} C_F + \frac{70}{27} N_c \right) \left(\frac{\alpha_s}{4\pi} \right)^2$$

$$\gamma_{gq} = \frac{16}{3} C_F \frac{\alpha_s}{4\pi} + \left(-\frac{208}{27} C_F N_f - \frac{224}{27} C_F^2 + \frac{752}{27} C_F N_c \right) \left(\frac{\alpha_s}{4\pi} \right)^2$$

$$\gamma_{gg} = \frac{4}{3} N_f \frac{\alpha_s}{4\pi} + \left(\frac{148}{27} C_F N_f + \frac{70}{27} N_c N_f \right) \left(\frac{\alpha_s}{4\pi} \right)^2$$

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④ Axial vector coupling

nucleon spin

$$\langle N|\bar{q}\gamma_\mu\gamma_5 q|N\rangle \equiv 2 S_\mu \Delta q_N$$

spin fraction.

$\Delta u_p = 0.77$
$\Delta d_p = -0.47$
$\Delta s_p = -0.15$

Summary

$$\mathcal{L}_{SI}^{(N)} = \sum_N f_N \bar{X} X \bar{N} N$$

$$f_N/m_N = \sum_{q=uds} C_S^q(\mu_{had}) f_{Tq}^{(N)} - \frac{8}{9} C_S^g(\mu_{hadron}) (1 - \sum_q f_{Tq}^{(N)})$$

$$+ \frac{3}{4} \sum_q^{N_f} \sum_{i=1,2} C_{T_i}^q (q(2;\mu) + \bar{q}(2;\mu))$$

$$- \frac{3}{4} \sum_{i=1,2} C_{T_i}^g(\mu)\, g(2;\mu)$$

$$\mathcal{L}_{SD}^{(N)} = \sum_N a_N \bar{X} \gamma_\mu \gamma_5 X \bar{N} \gamma_\mu \gamma_5 N$$

$$a_N = \sum_{q=uds} C_{AV}^q(\mu_{hadron}) \Delta q$$

At leading order of $O(\alpha_s)$, we can evaluate f_N and a_N at UV scale μ directly.

$\longrightarrow$

$$\sigma = \frac{4}{\pi} m_r^2 [| N_p f_p + N_n f_n |^2 + 4 \frac{J+1}{J} | a_p \langle S_p \rangle + a_n \langle S_n \rangle |^2$$

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$D_\mu \equiv \partial_\mu + i g A_\mu^A T_A$

IV-IV. Fock-Schwinger gauge

convenient gauge fixing for calcution of operetors with $G^{\mu\nu}$.

$$x_\mu A_\mu^A(x) = 0 \leftarrow$$

$\rightarrow$

$$A_\mu^A(x) = \int_0^1 d\alpha\, G_{\rho\mu}^A(\alpha x)\, \alpha x_\rho$$

$$(\because)\quad A_\mu^A(y) = \frac{\partial}{\partial y_\mu}(\cancel{A_\rho^A(y) y_\rho}) - y_\rho \frac{\partial A_\rho^A}{\partial y_\mu}$$

$\swarrow\ y_\rho A_\rho^C = 0$

$$= -y_\rho \left(\frac{\partial A_\rho^A}{\partial y_\mu} - \frac{\partial A_\mu^A}{\partial y_\rho} - g f^{ABC} A_\mu^B A_\rho^C \right)$$

$$- y_\rho \frac{\partial A_\mu^A}{\partial y_\rho}$$

$$= -y_\rho G_{\mu\rho}^A - y_\rho \frac{\partial A_\mu^A}{\partial y_\rho}$$

$$A_\mu^A(y) + y_\rho \frac{\partial A_\mu^A(y)}{\partial y_\rho} = y_\rho G_{\rho\mu}^A(y) \leftarrow$$

$y = \alpha x$ $\leftarrow$

$$A_\mu^A(\alpha x) + x_\rho \frac{\partial}{\partial x_\rho} A_\mu^A(\alpha x) = \frac{d}{d\alpha}\left(\alpha A_\mu^A(\alpha x)\right) = \alpha x_\rho G_{\rho\mu}^A(\alpha x)$$

$$A_\mu^A(x) = \int_0^1 d\alpha\, G_{\rho\mu}(\alpha x)\, \alpha x_\rho. \leftarrow$$

Expansion with local operetor at $x \simeq 0$

$$A_\mu^A(x) = \frac{1}{2\cdot 0!} x_\rho G_{\rho\mu}^A(0) + \frac{1}{3\times 1!} x_\alpha x_\rho (D_\alpha G_{\rho\mu}(0))^A$$

$$+ \frac{1}{4\cdot 2!} x_\alpha x_\beta x_\rho (D_\alpha D_\beta G_{\rho\mu}(0))^A + \cdots$$

From gauge fixing

$$x_\mu\left[A^A_\mu(0)+x_{\alpha_1}\partial_{\alpha_1}A^A_\mu(0)+\frac{1}{2}x_{\alpha_1}x_{\alpha_2}\partial_{\alpha_1}\partial_{\alpha_2}A^A_\mu(0)\cdots\right]=0$$

$\rightarrow$

$$x_\mu A^A_\mu(0)=0$$
$$x_\mu x_{\alpha_1}\partial_{\alpha_1}A^A_\mu(0)=0$$
$$x_\mu x_{\alpha_1}x_{\alpha_2}\partial_{\alpha_1}\partial_{\alpha_2}A^A_\mu(0)=0$$

$\therefore$

$$x_{\alpha_1}\partial_{\alpha_1}G^A_{\rho\mu}(0)=x_{\alpha_1}(D_{\alpha_1}G_{\rho\mu}(0))^A$$
$$x_{\alpha_1}x_{\alpha_2}\partial_{\alpha_1}\partial_{\alpha_2}G^A_{\rho\mu}(0)=x_{\alpha_1}x_{\alpha_2}(\underbrace{D_{\alpha_1}D_{\alpha_2}}_{(\partial_{\alpha_1}+\cancel{A_{\alpha_1}})(\partial_{\alpha_1}+A_{\alpha_1})}G_{\rho\mu}(0))^A$$

∂_{α_1}

$\therefore$

$$x_{\alpha_1}\cdots x_{\alpha_n}(\partial_{\alpha_1}\cdots\partial_{\alpha_n}G^A_{\rho\mu}(0))=x_{\alpha_1}\cdots x_{\alpha_n}(D_{\alpha_1}\cdots D_{\alpha_n}G_{\rho\mu}(0))^A$$

$$A^a_\mu(x)=\int_0^1 d\alpha\, G^A_{\rho\mu}(\alpha x)\, \alpha x_\rho$$

$$=\int_0^1 d\alpha\sum_n\frac{1}{n!}\partial_{\alpha_1}\cdots\partial_{\alpha_n}G^A_{\rho\mu}(0)\cdot\alpha x_{\alpha_1}\cdots\alpha x_{\alpha_n}\alpha x_\rho$$

$$=\sum_n\frac{1}{n+2}\frac{1}{n!}x_{\alpha_1}\cdots x_{\alpha_n}x_\rho\,\partial_{\alpha_1}\cdots\partial_{\alpha_n}G^A_{\rho\mu}(0)$$

$$=\sum_n\frac{1}{n+2}\frac{1}{n!}x_{\alpha_1}\cdots x_{\alpha_n}x_\rho(D_{\alpha_1}\cdots D_{\alpha_n}G^A_{\rho\mu}(0))$$

Similarly

$$\psi(x)=\psi(0)+x_{\alpha_1}D_{\alpha_1}\psi(0)+\frac{1}{2}x_{\alpha_1}x_{\alpha_2}D_{\alpha_1}D_{\alpha_2}\psi(0)\cdots$$

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Quark propagator under gluon BG.

$$(i\partial_{x\mu}\gamma_\mu - g A^A_\mu T_A \gamma_\mu - m)\, iS(x,y) = i\delta^{(4)}(x-y)$$

$$iS(x,y) = iS^{(0)}(x-y) + \int d^4z\, iS^{(0)}(x-z)(-ig\not{A}(z))\, iS^{(0)}(z-y)$$

$$+ \int d^4z\, d^4z'\, iS^{(0)}(x-z)(-ig\not{A}(z))\, iS^{(0)}(z-z')(-ig\not{A}(z'))\, iS^{(0)}(z'-y)$$

$$+ \cdots$$

$$A^A_\mu(x) = +\frac{1}{2} x_\rho G^A_{\rho\mu}(0) + \cdots$$ $D^n G$'s are ignored

$$A^A_\mu(k) = \int A^a_\mu(x)\, e^{ikx} d^4x + \cdots$$

$$= -\frac{1}{2}(2\pi)^4 G^a_{\rho\mu}(0) \frac{\partial}{\partial k^\rho} \delta^{(4)}(k) + \cdots$$

$$iS(p) \equiv \int d^4x\, iS(x,0)\, e^{ipx}$$

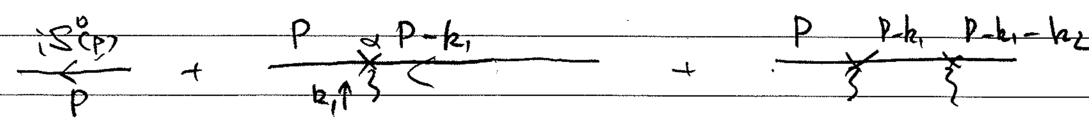
$$= iS^{(0)}(p)$$

$$+ iS^{(0)}(p)\left(\frac{1}{2} g\gamma_\alpha G_{\alpha\rho}(0) \frac{\partial}{\partial k_1^\rho}\delta^{(4)}(k_1)\right) iS^{(0)}(p-k_1)\, d^4k_1$$

$$+ iS^{(0)}(p)\left(\frac{1}{2} g\gamma_\alpha G_{\alpha\rho}(0) \frac{\partial}{\partial k_1^\rho}\delta^{(4)}(k_1)\right) iS^{(0)}(p-k_1)$$

$$\times \left(\frac{1}{2} g\gamma_\beta G_{\beta\sigma} \frac{\partial}{\partial k_2^\sigma}\delta^{(4)}(k_2)\right) iS^{(0)}(p-k_1-k_2)\, d^4k_1\, d^4k_2$$

$\longrightarrow$ =

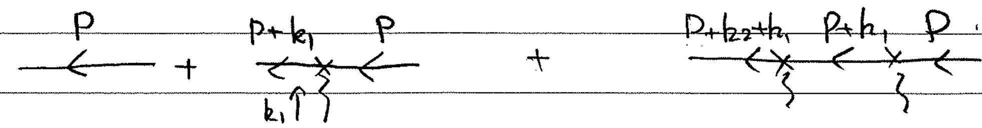


$$\frac{1}{2} g\gamma_\alpha G_{\alpha\beta} \frac{\partial}{\partial k_1^\beta}\delta^{(4)}(k_1)$$

$$d^4k_1$$

Notice translation inv. is broken

$$i\tilde{S}(p) \equiv \int iS(0,x)\, e^{-ipx}\, d^4x$$

$=$ 

$$\frac{1}{2} g\, \gamma_\alpha G_{\alpha\rho}(0) \frac{\partial}{\partial k_1^\rho} \delta^{(4)}(k_1)\, d^4k_1$$

We are interested in bilinear ops of G.

$$G^A_{\alpha\mu} G^A_{\beta\nu} = \frac{1}{12} G^A_{\rho\sigma} G^A_{\rho\sigma} \left(g_{\alpha\beta} g_{\mu\nu} - g_{\alpha\nu} g_{\beta\mu}\right)$$

$$+ \frac{1}{2} g_{\alpha\beta} O^g_{\mu\nu} + \frac{1}{2} g_{\mu\nu} O^g_{\alpha\beta} + \frac{1}{2} g_{\alpha\nu} O^g_{\beta\mu} + \frac{1}{2} g_{\beta\mu} O^g_{\alpha\nu}$$

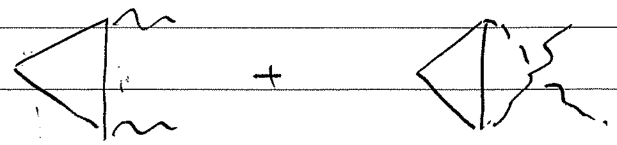
$$+ O^g_{\mu\nu\rho\sigma}$$

$$O^g_{\alpha\mu\beta\nu} = G^A_{\alpha\mu} G^A_{\beta\nu}$$
$$- \frac{1}{2} g_{\alpha\beta} G^{A\rho}{}_{\mu} G^A{}_{\rho\nu} - \frac{1}{2} g_{\mu\nu} G^{A\rho}{}_{\alpha} G^A{}_{\rho\beta}$$
$$+ \frac{1}{2} g_{\alpha\nu} G^{A\rho}{}_{\rho} G^A{}_{\rho\mu} + \frac{1}{2} g_{\beta\mu} G^{A\rho}{}_{\alpha} G^A{}_{\rho\nu}$$
$$+ \frac{1}{6} G^A_{\rho\sigma} G^{A\rho\sigma} \left(g_{\alpha\beta} g_{\mu\nu} - g_{\alpha\nu} g_{\beta\mu}\right)$$

$$\therefore\quad O^g_{\alpha\mu\beta\nu} = - O^g_{\mu\alpha\beta\nu} = - O^g_{\alpha\mu\nu\beta}$$

Example.

$$\mathcal{L} = -m\bar{Q}Q.$$



$$iM = (-im)\,\mathrm{Tr}_{c+L}[\,i S(p)\,]$$

$$= (-im)\,\mathrm{Tr}_{c+L}\,\frac{i}{\not p - m}\left(\frac{1}{2} g \gamma_\alpha G_{\alpha\rho}\frac{\partial}{\partial k_1^\rho}\delta^{(4)}(k_1)\right)\frac{i}{\not p - \not k_1 - m}$$

$$\left(\frac{1}{2} g \gamma_\beta G_{\beta\sigma}\frac{\partial}{\partial k_2^\sigma}\delta^{(4)}(k_2)\right)\frac{i}{\not p - \not k_1 - \not k_2 - m}\quad d^4k_1\, d^4k_2\,\frac{d^4p}{(2\pi)^4}$$

$$= +im\frac{1}{8}g^2\,\underline{G^A_{\alpha\rho}G^A_{\beta\sigma}}\int\frac{d^4p}{(2\pi)^4}\frac{\partial}{\partial k_1^\rho}\frac{\partial}{\partial k_2^\sigma}\mathrm{Tr}\left[\frac{1}{\not p - m}\gamma_\alpha\frac{1}{\not p - \not k_1 - m}\gamma_\beta\frac{1}{\not p - \not k_1 - \not k_2 - m}\right]\Bigg|_{k_1=k_2=0}$$

$$\to \frac{1}{12}G^A_{\mu\nu}G^A_{\mu\nu}(\delta_{\alpha\beta}\delta_{\rho\sigma} - \delta_{\alpha\sigma}\delta_{\rho\beta})$$

$$= im\frac{1}{96}g^2G^2\int\frac{d^4p}{(2\pi)^4}\Bigg[\frac{\partial}{\partial k_1^\rho}\frac{\partial}{\partial k_2^\rho}\mathrm{Tr}\left[\frac{1}{\not p - m}\gamma_\alpha\frac{1}{\not p - \not k_1 - m}\gamma_\alpha\frac{1}{\not p - \not k_1 - \not k_2 - m}\right]$$

$$-\frac{\partial}{\partial k_1^\rho}\frac{\partial}{\partial k_2^\sigma}\mathrm{Tr}\left[\frac{1}{\not p - m}\gamma_\alpha\frac{1}{\not p - \not k_1 - m}\gamma_\rho\frac{1}{\not p - \not k_1 - \not k_2 - m}\right]\Bigg]\Bigg|_{k_1=k_2}$$

$$= \vdots$$

$$= +i\frac{\alpha_s}{12\pi}GG$$

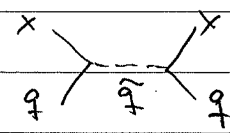
IV-V Evaluation of Wilson coefficients @ UV scale
at leading order of α_s.

X: Majorana (Bino-like WIMP)

$\mathcal{L}_{int} = \bar{q}(a_q + b_q \gamma_5) X \tilde{q} + h.c.$

<u>leading order calculation of α_s</u>

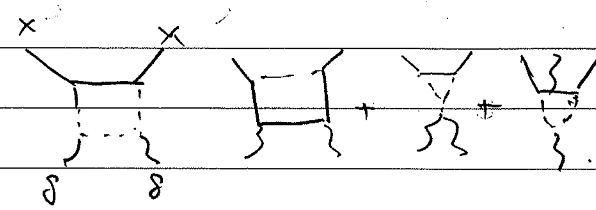
Quark scaler op — tree



" twist-2 op — " — "

" AV op — "

Gluon scaler op — 1 loop



" twist2 op — No

Quark mass threshold $N_f = 5 \to 4$ ($\mu_b \simeq m_b$)

$C_S^q(\mu_b)\big|_{N_f=4} = C_S^q(\mu_b)\big|_{N_f=5}$

$C_S^g(\mu_b)\big|_{N_f=4} = C_S^g(\mu_b)\big|_{N_f=5} - \frac{1}{12} C_S^b(\mu_b)\big|_{N_f=5}$

$C_{T_i}^{q/g}(\mu_b)\big|_{N_f=4} = C_{T_i}^{q/g}(\mu_b)\big|_{N_f=5}$

$C_{AV}^q(\mu_b)\big|_{N_f=4} = C_{AV}^q(\mu_b)\big|_{N_f=5}$

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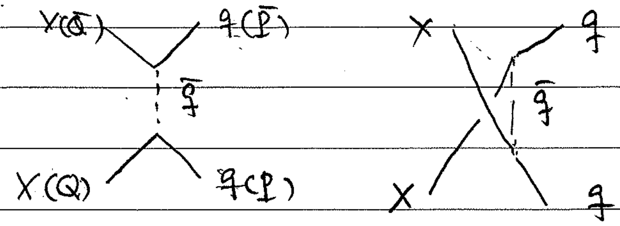
① quark operators



$$iM = \bar{U}_q\, i\,(a_q + b_q \gamma_5)\, V_X\ \bar{V}_X\, i\,(a_q - b_q\gamma_5)\, U_q \ \frac{i}{(Q+P)^2 - m^2}$$

$$- \bar{U}_q\, i\,(a_q + b_q\gamma_5)\, U_X\ \bar{U}_X\, i\,(a_q - b_q\gamma_5)\, U_q\ \frac{i}{(Q-\bar{P}) - m^2}$$

$$= \ \text{〃} \qquad \left[\frac{1}{M_X^2 - m^2} - \frac{2(QP)}{(M_X^2-m^2)^2}\right]$$

$$- \ \text{〃} \qquad \left[\frac{1}{M_X^2 - m^2} + \frac{2(Q\bar{P})}{(M_X^2-m^2)^2}\right]$$

Firtz tr.

$$= \frac{i}{M_X^2 - m^2}\ \frac{a_q^2 + b_q^2}{2}\ \bar{U}_q U_q\ \bar{U}_X U_X$$

$$- \frac{i}{M_X^2 - m^2}\ \frac{a_q^2 + b_q^2}{2}\ \bar{U}_q \gamma_\mu \gamma_5 U_q\ \bar{U}_X \gamma_\mu \gamma_5 U_X$$

$$+ i\ \frac{(Q(P+\bar{P}))}{(M_X^2 - m^2)^2}\ \frac{a_q^2 + b_q^2}{2}\ \bar{U}_q \gamma_\mu U_q\ \bar{U}_X \gamma_\mu U_X$$

$$C_S^q(M_{UV}) = \frac{a_q^2 - b_q^2}{4 m_q}\ \frac{1}{M_X^2 - m^2} + \frac{a_q^2 + b_q^2}{8}\ \frac{M_X}{(M_X^2 - m^2)^2}$$

$$C_{T1}^q(M_{UV}) = \frac{a_q^2 + b_q^2}{2}\ \frac{M_X}{(M_X^2 - m^2)^2}.$$

$$C_{T2}^q(M_{UV}) = 0$$

$$C_{AV}^q(M_{UV}) = -\ \frac{a_q^2 + b_q^2}{4}\ \frac{1}{M_X^2 - m^2}$$

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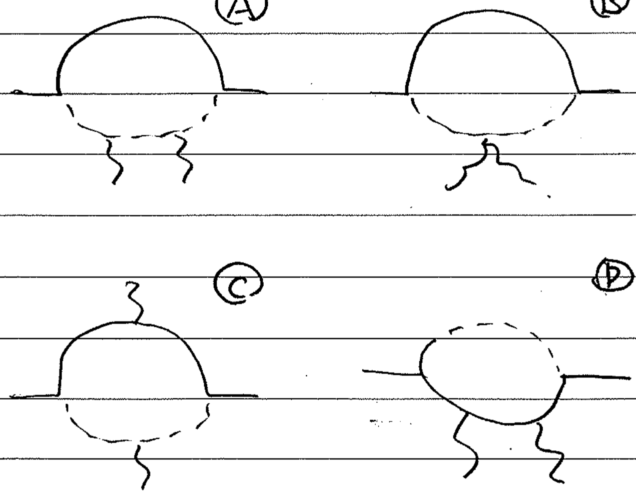
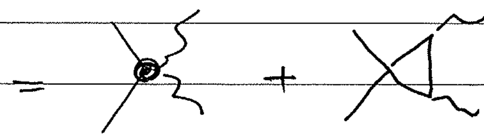
② gluon operators.

Ⓐ Ⓑ



"matching"



$C_s^g \qquad\qquad -\frac{1}{12} C_s^q .$

Ⓒ Ⓓ

FS gauge Ⓐ = Ⓒ = 0

$m_q \ll m,\ M_x$

$$C_s^g\Big|_{Ⓑ} = \frac{a_q^2+b_q^2}{16}\left[+\frac{M_x}{6m^2(M_x^2-m^2)}\right] + \frac{a_q^2-b_q^2}{16}\cancel{\left[-\frac{m_q(2m^2+M_x^2)}{6m^2(M^2-m^2)^2}\right]}$$

$$C_s^g\Big|_{Ⓓ} = \frac{a_q^2+b_q^2}{16}\left[-\frac{M_x}{6(M_x^2-m^2)^2}\right] + \frac{a_q^2-b_q^2}{16}\left[-\frac{1}{3m_q(M_x^2-m^2)}\right]$$

$$= -\frac{1}{12} C_s^q$$

After matching,

$$C_s^g(\mu_{UV}) = \sum_{q=\mathrm{all}} C_s^g\Big|_{Ⓑ} + \sum_{q(m_q>\mu_{UV})} C_s^g\Big|_{Ⓓ}$$

V. Toward Next-leading order of α_s.

V-I Previous model

$$\mathcal{L}_{int} = \bar{q}(a_q + b_q \gamma_5) \chi \tilde{q} + h.c.$$

- Wilson coefficient @ UV scale.

Quark	scaler	op	tree → 1 loop
〃	twist-2	〃	〃
〃	AV	〃	〃
Gluon	scaler	op	1 loop → 2 loop
〃	twist 2		No → 1 loop.

- RG evolution @ two-loop.

- Quark mass threshold. ($N_f = 5 \to 4$ @ $\mu_b \simeq m_b$)

$$C_S^q(\mu_b)\big|_{N_f=4} = C_S^q(\mu_b)\big|_{N_f=5}$$

$$C_S^g(\mu_b)\big|_{N_f=4} = \left[1 + \frac{\alpha_s(\mu_b)}{4\pi}\frac{2}{3}\log\frac{m_b^2}{\mu_b^2}\right] C_S^g(\mu_b)\big|_{N_f=5}$$

$$-\frac{1}{12}\left[1 + \frac{\alpha_s(\mu_b)}{4\pi}\left(11 + \frac{2}{3}\log\frac{m_b^2}{\mu_b^2}\right)\right] C_S^b(\mu_b)\big|_{N_f=5}$$

$$C_{T_i}^q(\mu_b)\big|_{N_f=4} = C_{T_i}^q(\mu_b)\big|_{N_f=5} \qquad (i=1,2)$$

$$C_{T_i}^g(\mu_b)\big|_{N_f=4} = \left(1 + \frac{\alpha_s(\mu_b)}{4\pi}\frac{2}{3}\log\frac{m_b^2}{\mu_b^2}\right) C_{T_i}^g(\mu_b)\big|_{N_f=5}$$

$$+ \frac{\alpha_s(\mu_b)}{4\pi}\frac{2}{3}\log\frac{m_b^2}{\mu_b^2}\, C_{T_i}^b(\mu_b)\big|_{N_f=5}$$

DEPARTMENT OF PHYSICS
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V-II WIMP from SU(2) multiplets.

SU(2) triplet (3, 0) : "wino"

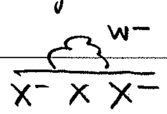
$M_\chi \simeq 3\,\mathrm{TeV}$ (Themal production)

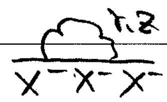
SU(2) doublets $(2, +\frac{1}{2}) \oplus (2, -\frac{1}{2})$ with Majorana mass term.

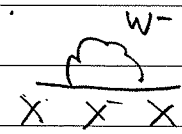
$$\mathcal{L} = -M_\chi \tilde{H}_1 \tilde{H}_2 + \frac{1}{\Lambda}(H\tilde{H}_1)^2 + \frac{1}{\Lambda}(H\tilde{H}_2)^2$$

$\rightarrow M_\chi \simeq 1\,\mathrm{TeV}$ (Thermal ")

⊛ Neutral - charged wino mass splitting.

$\Delta m =$ 

$+$ 

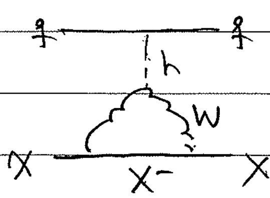
$-$ 

$+$ 2loop

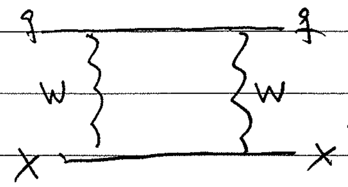
$\simeq 165\,\mathrm{MeV}$ (Ibe et al , 1212, 5989)

⊛ In SUSY SM, if SUSY particles except WIMP and SU(2) partners are decoupled, WIMP interacts with matters via SU(2) × U(1) interactions.

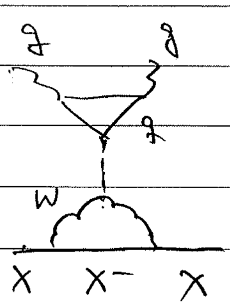
Leading order of wino-nucleon scattering SI cross section.



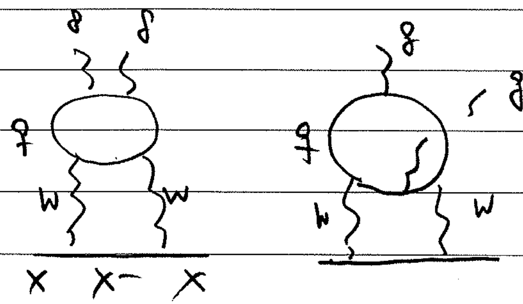
quark scaler op



quark scaler/twist-2 op



gluon scaler op



gluon scaler / twist-2 op.

$$\mathcal{L} = -g_2(\bar{\chi}\gamma_\mu \chi^- W^+_\mu + h.c.)$$

$M_\chi \gg m_t\ m_W$

$$C_S^q = \frac{\alpha_2^2}{4 m_W m_h^2}(-2\pi) \qquad (q = u.d.s)$$

$$C_S^g = -3\underbrace{\frac{1}{12}\frac{\alpha_s^2}{4 m_W m_h^2}(-2\pi)}_{(c.b.t)} + 2\underbrace{\frac{1}{4}\frac{\alpha_2^2}{m_W^3}\frac{\pi}{12}}_{(u.d)\,(c.s)}$$

$$+\underbrace{\frac{1}{4}\frac{\alpha_2^2}{m_W^3}\frac{\pi}{8}}_{(t.b)} x_{wt}^2 \frac{1+\frac{2}{3}x_{wt}}{(1+x_{wt})^3}$$

$x_{wt} = \frac{m_W}{m_t}$ $\to$ 14% $\times \frac{\pi}{12}$.

$C^q_{T_1} = \frac{\alpha_3}{m_W}\frac{\pi}{3}$

$C^q_{T_2} = O\left(\frac{m_W}{M_X}\right) \to 0$

$C^q_{AV} = O\left(\frac{m_W}{M_X}\right) \to 0$

※ SI cross section is insensitive to wino mass

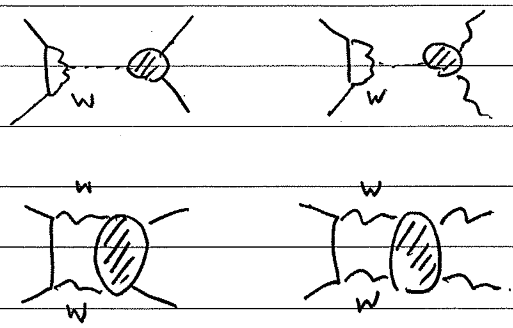
※ Accidental cancellation

$$f_N/m_N = \underbrace{\frac{\alpha_2^2}{m_W^3}(0.4)}_{\text{quark twist}} - \underbrace{\frac{\alpha_2^2}{m_W^3}(0.27)}_{\text{gluon scalar}} - \underbrace{\frac{\alpha_2^2}{m_W^3}(0.03)}_{\text{quark scalar}}$$

$\to$ { Uncertainties are large.

Cross section is larger than ν BG ?

How to go to NLO of α_s (J.H. Ishiwata Nagata 1504.00915)



$\mu \simeq \mu_{weak} < m_t$

Parton		Higgs LO	Higgs NLO	Box LO	Box NLO
1st/2nd generation quark	scaler	1L	2L	1L	2L
	twist2	—	—	1L	2L
bottom	" scaler	1L	2L	1L	2L (neglect)
	twist	—	—	1L	2L (")
gluon	scaler	2L	3L	2L	3L
(1st/2nd gene. quark)	twist	—	—	—	2L
gluon	scalar	2L	3L	2L	3L (")
(3rd ")	twist			—	2L (")

OPE of current-current correlator.

$$\Pi^W_{\mu\nu}(q) \equiv i\int d^4x\, e^{iqx}\, T[J^W_\mu(x), J^W_\nu(0)]$$

where $J^W_\mu \equiv \sum_i \frac{g_2}{\sqrt{2}} \bar{u}_i \gamma_\mu P_L d_i$

$$\Pi^W_{\mu\nu}(q)\Big|_{\rm scalar} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\Pi^W_T(q^2) + \cancel{\frac{q_\mu q_\nu}{q^2}}\,\Pi^W_L(q^2)$$

→ Not contribute to cross section

$$\Pi^W_T(q^2) = \sum_q C^q_{WS}(q^2)\, m_q \bar{q}q + C^g_{WS}(q^2)\frac{\alpha_s}{\pi}G^2 .$$

$$C^q_{WS}(q^2) = -\frac{\alpha_s}{4\pi}\frac{g_2^2}{q^2}$$ (1st/2nd gen quarks)

$$C^q_{WS}(q^2) = -\frac{1}{48}\frac{g_2^2}{q^2}\left[2\Big(1+\underbrace{\frac{7}{6}\frac{\alpha_s}{\pi}}_{\text{Broadhurst et al}}\Big)\right]$$

$$\Pi^W_{\mu\nu}(q)\Big|^q_{\rm twist\text{-}2} = \sum_{q=u,d,c,s}\frac{g_2^2}{2}\Big[-\frac{1}{(q^2)^2}\left[g_{\mu\rho}g_{\nu\sigma}q^2 - g_{\mu\rho}q_\nu q_\sigma - g_{\nu\sigma}q_\mu q_\rho + g_{\mu\nu}q_\rho q_\sigma\right]$$

$$\times C^q_{W2}$$

$$+\left[g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right]\frac{q_\rho q_\sigma}{(q^2)^2}C^q_{WL}\Big]O^{q\rho\sigma}.$$

$$C^q_{W2} = 1 + \frac{\alpha_s}{4\pi}\left[-\frac{1}{2}\left(\frac{64}{9}\right)\log\frac{-q^2}{\mu^2_{UV}} + \frac{4}{9}\right]$$

$$C^q_{WL} = \frac{\alpha_s}{4\pi}\frac{16}{9}$$

$$\Pi^W_{\mu\nu}(q)\Big|^g_{\rm twist\text{-}2} = \left(C^q_{W2}\to C^g_{W2},\ C^q_{WL}\to C^g_{WL},\ O^{q\rho\sigma}\to O^{g\rho\sigma}\right)$$

$$C^g_{W2} = 4\frac{\alpha_s}{4\pi}\left[-\frac{1}{2}\left(\frac{4}{3}\right)\log\frac{-q^2}{\mu^2_{UV}} + \frac{1}{2}\right]$$

$$C^g_{WL} = 4\frac{\alpha_s}{4\pi}\left[-\frac{2}{3}\right]$$

→

$$\sigma^{p}_{SI} = 2.3^{+0.2+0.5}_{-0.3-0.4} \times 10^{-47}\,\mathrm{cm}^2.$$

inputs

perturbation.

Other WIMPs.

- Doublet ("Higgsino") $\sigma^{p}_{SI} \sim 10^{-49}\,\mathrm{cm}^2 < \nu\mathrm{BG}$
- Quintet ("minimal DM") $\sigma^{p}_{SI} \sim 2\times 10^{-47}\,\mathrm{cm}^2$