

I Introduction of WIMP dark matter.

II WIMP nucleus/nucleon NR int.

III Direct detection experiments

IV Evaluation of effective int of WIMP with nucleus.

V Toward Next-Leading order of d_s .

I Introduction of WIMP dark matter.

Dark matter (DM) in the universe

i) Galaxy rotation curve measurements

ii) CMB measurements

$$\Omega_x \equiv \rho_x / \rho_{\text{critical}} \quad (\rho_{\text{critical}} \approx 10^{-7} \text{ GeV} / \text{cm}^3)$$

→

$$\Omega_{\text{DM}} \approx 27\%$$

$$\Omega_{\text{baryon}} \approx 5\%$$

$$\Omega_{\Lambda} \approx 68\%$$

iii) X-rays & Gravitational lensing of galactic clusters

iv) N-body simulation of structure formation

;

Nature of particle dark matter.

i) Electrically neutral.

ii) Massive

iii) Stable or longer life time than the age of the universe.

iv) "cold"

(free streaming length \ll protogalaxies.)

Candidates for particle dark matter,

$$(10^{-23} \text{ eV} \lesssim M_x \lesssim 10^{18} \text{ GeV})$$

i) Weakly Interacting Massive particles (WIMPs)

- Produced from thermal bath in the early universe.
- $M_x \simeq 10^{2-4} \text{ GeV}$.
- Stability: symmetry.

(example)

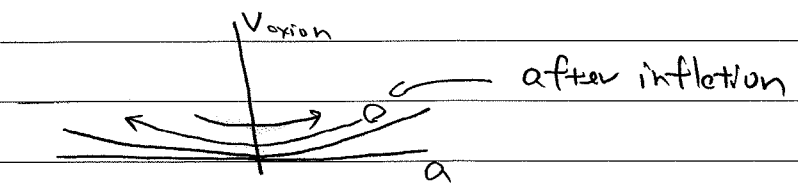
Lightest SUSY particles (LSP) in SUSY SM (R parity)

" Kalzo-Klein " (KKP) in Extradim (KK ")

(linked with the naturalness problem in the SM)

ii) Axion or ALP

- Pseudo NG boson in PQ mechanism for strong CP problem.
- Produced by misalignment mechanism.



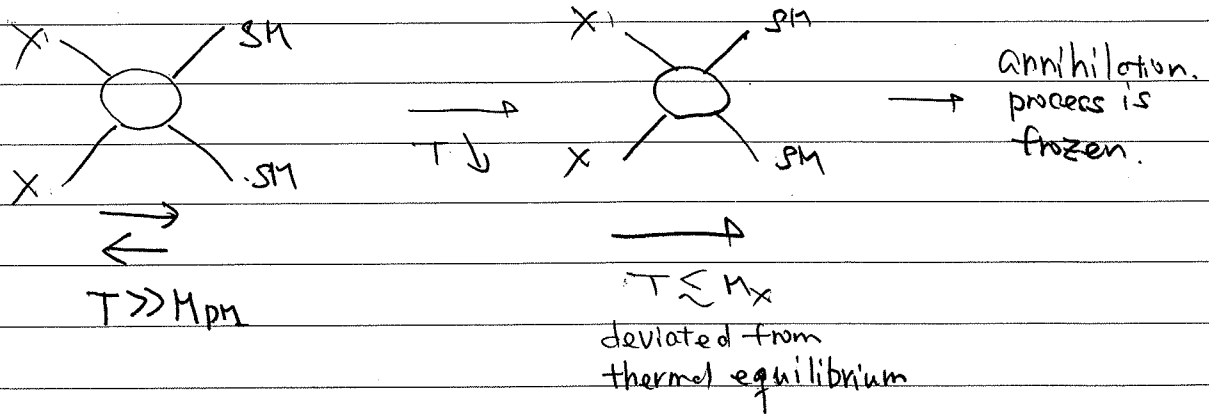
$$M_{\text{axion}} \simeq 10^{-(3-5)} \text{ eV}$$

iii) WIMPzillas

$$M_x \simeq 10^{12-16} \text{ GeV}$$

iv) sterile Neutrinos

Thermal production mechanism of WIMPs



Boltzmann eq of DM particles (n_X : # density of X)

$$\frac{dn_X}{dt} + 3H n_X = -\langle \sigma v \rangle [n_X^2 - (n_X^{EQ})^2]$$

H : Hubble parameter.

$$H = \sqrt{\frac{8\pi}{3M_{pl}^2} \rho} \approx \sqrt{\frac{4\pi^2}{45} g_*} \frac{T^2}{M_{pl}}$$

$$(g_* = \sum_{\text{boson}} 1 + \sum_{\text{fermion}} \frac{7}{8})$$

$\langle \sigma v \rangle$: Thermal averaged total annihilation σ section ($XX \rightarrow SM$)

n_X^{EQ} : # density of X in thermal equilibrium.

Define

$$Y \equiv n_X / s \quad \left(\begin{array}{l} s: \text{Entropy density} \\ s = \frac{2\pi^2}{45} g_* T^3 \quad g_* = g \text{ (massless)} \end{array} \right)$$

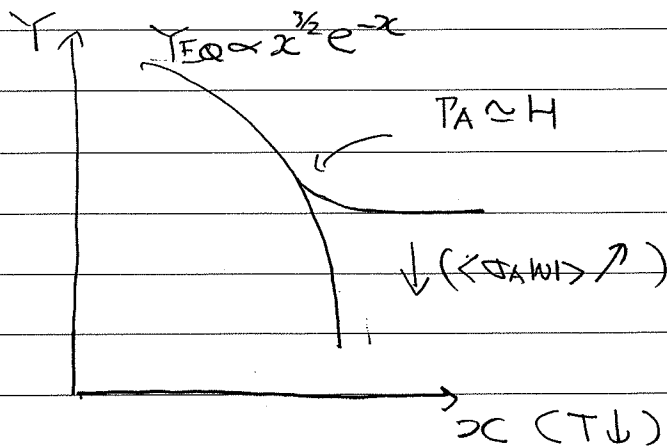
$$x \equiv M_X / T$$

→

$$\frac{x}{Y_{EQ}} \frac{dY}{dx} = - \frac{\Gamma_A}{H} \left[\left(\frac{Y}{Y_{EQ}} \right)^2 - 1 \right]$$

where

$$\Gamma_A \equiv n_X^{EQ} \langle \sigma v \rangle: \text{Probability of annihilation per time}$$



$R_x \ll H \rightarrow x$ decoupled from thermal bath.

Decoupling temperature $T_D (\simeq \frac{1}{20} M_x)$

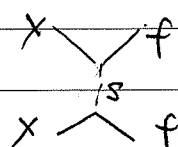
$$R_A = n_x^p \langle \sigma_{ANN} \rangle \simeq H_D = \sqrt{\frac{42^3}{45}} g_x^{1/2} \frac{T_D^2}{M_{pl}}$$

$$\begin{aligned} Y_x^{now} &\simeq \frac{n_x^p}{s_D} = \frac{45}{2\pi^2} \frac{1}{g_p T_D^3} \langle \sigma_{ANN} \rangle \sqrt{\frac{42^3}{45}} g_x^{1/2} \frac{T_D^2}{M_{pl}} \\ &= \sqrt{\frac{45}{\pi}} \frac{g_x^{1/2}}{g_p} \frac{1}{(T_D M_{pl})} \langle \sigma_{ANN} \rangle \end{aligned}$$

$$\begin{aligned} \Omega_x &= \frac{1}{\rho_c} R_x = \frac{s^{now}}{\rho_c} Y_x^{now} M_x = 3 \times 10^{-8} \text{GeV}^{-1} \text{h}^{-2} Y_x^{now} M_x \\ &\simeq 0.4 \times \left(\frac{x_D = M_x/T_D}{20} \right) \left(\frac{\langle \sigma_{ANN} \rangle}{10^{-9} \text{GeV}^{-2}} \right)^{-1} \end{aligned}$$

Typical cases.

i) $SU(2)_L$ singlet

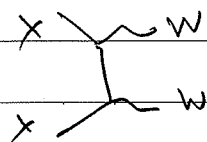


$$\sigma N \approx \frac{\pi d_f^2}{M^2} M_X^2 \approx 3 \times 10^{-9} \text{ GeV}^{-2} \left(\frac{M_X}{300 \text{ GeV}} \right)^{-2} \quad | M_X = M_f$$

If X is Majorana, (bino) p-wave suppression of X section

$$N^2 \approx \frac{T_D}{M} \approx \frac{1}{20}$$

ii) $SU(2)_L$ multiplet



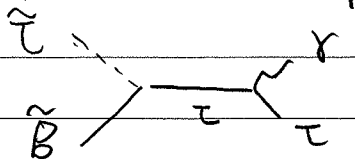
$$\sigma N \approx \frac{\pi d_f^2}{M_X^2} = 3.5 \times 10^{-9} \text{ GeV}^{-2} \left(\frac{M_X}{1 \text{ TeV}} \right)^{-2}$$

(Higgs)

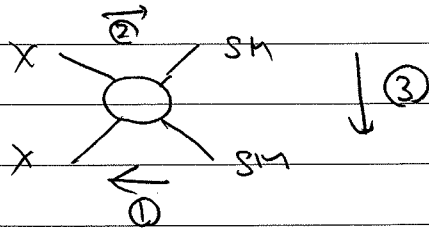
$SU(2)$ doublet fermion $\sim 1 \text{ TeV}$

triplet " $\sim 3 \text{ TeV}$
(wins)

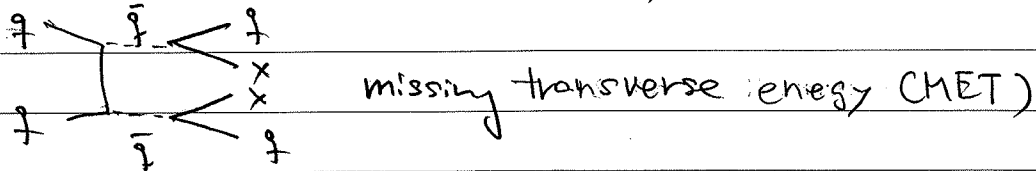
cf). Coannihilation with particles degenerate with X in mass



Searches for WIMP DM

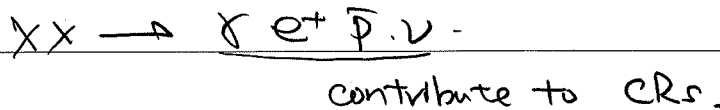


① WIMP Production @ colliders (LHC)



② Indirect detection

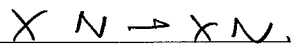
DM particles are gravitationally accumulated in astrophysical objects
(stars, galaxies, ...)



- Fermi satellite) γ rays from GC, dwarf galaxies.
- HESS (→ CTA)
- AMS satellite) e^+ , \bar{p}
- ICECUBE SK) ν from sun.

③ Direct detection

detection of elastic scattering of nuclei with WIMP DM



typical recoil energy

$$E \approx \frac{m_r^2}{m_T} v^2 \quad \left(\begin{array}{l} m_T : \text{target nuclei mass} \\ m_r : \frac{m_T m_X}{m_T + m_X} \\ v : \text{DM velocity } (\sim 10^{-3} c) \end{array} \right)$$

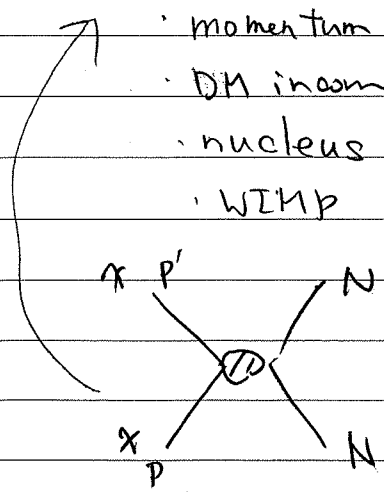
$= (1-100) \text{ keV}$

II WIMP, nucleon/nucleus NR interaction.

WIMP - nucleus NR operators.

fermionic WIMP

- momentum transfer \vec{q} $|\vec{q}| \approx M \times v$.
- DM incoming velocity relative to target \vec{v}
- nucleus spin \vec{S}
- WIMP spin \vec{s}



$$\vec{q} = (p - p')$$

$$P = (p + p') = (2p + \vec{q})$$

$$\Rightarrow \vec{P} = 2M \vec{v} + \vec{q}$$

Ref.
Fan, Reese, Wang
JHEP 1011 (10) 024
arXiv: 1008.1391

Spin-Indep (PT) operator

Operator	P	T
$O_1^{(++)} = 1$	S	+
$O_2^{(-+)} = i \vec{S}_x \cdot \vec{q}$	P	-
$O_3^{(-)} = \vec{S}_x \cdot \vec{P}$	P	+
$O_4^{(++)} = i \vec{S}_x \cdot (P \times \vec{q})$		

Spin-dependent (SD) operator.

$O_5^{(++)} = \vec{S}_x \cdot \vec{S}_N$	$O_4^{(++)} = i (\vec{S}_N \cdot \vec{q}) (\vec{S}_x \cdot P) - (\vec{S}_N \cdot \vec{P}) (\vec{S}_x \cdot \vec{q})$
$O_6^{(-+)} = i \vec{S}_N \cdot \vec{q}$	$O_{10}^{(+)} = [(\vec{S}_x \cdot (P \times \vec{q}))] (\vec{S}_x \cdot \vec{q}) + (\vec{S}_x \cdot (P \times \vec{q})) (\vec{S}_N \cdot \vec{q})$
$O_7^{(-)} = \vec{S}_N \cdot \vec{P}$	$O_{16}^{(-)} = i (\vec{S}_N \cdot (P \times \vec{q})) (\vec{S}_x \cdot \vec{P}) + i [\vec{S}_N \cdot (P \times \vec{q})] (\vec{S}_N \cdot \vec{P})$
$O_8^{(-+)} = i (\vec{S}_x \times \vec{S}_N) \cdot \vec{q}$	
$O_9^{(-)} = (\vec{S}_x \times \vec{S}_N) \cdot \vec{P}$	
$O_{10}^{(++)} = i \vec{S}_N \cdot (P \times \vec{q})$	
$O_{11}^{(++)} = (\vec{S}_N \cdot \vec{q}) (\vec{S}_x \cdot \vec{q})$	
$O_{12}^{(++)} = (\vec{S}_N \cdot \vec{P}) (\vec{S}_x \cdot \vec{P})$	
$O_{13}^{(+-)} = i ((\vec{S}_N \cdot \vec{q}) (\vec{S}_x \cdot \vec{P}) + (\vec{S}_N \cdot \vec{P}) (\vec{S}_x \cdot \vec{q}))$	

③ Nucleus Spin-dependent (SD) operators.

$$O_5^{(++)} = \vec{S}_X \cdot \vec{S}_N$$

$$O_6^{(+-)} = i \vec{S}_N \cdot \vec{q}$$

$$O_7^{(-)} = \vec{S}_N \cdot \vec{P}$$

$$O_8^{(-)} = i (\vec{S}_X \times \vec{S}_N) \cdot \vec{q}$$

$$O_9^{(-+)} = (\vec{S}_N \times \vec{S}_N) \cdot \vec{P}$$

$$O_{10}^{(++)} = i \vec{S}_N \cdot (\vec{P} \times \vec{q})$$

$$O_{11}^{(++)} = (\vec{S}_N \cdot \vec{q}) (\vec{S}_X \cdot \vec{q})$$

$$O_{12}^{(++)} = (\vec{S}_N \cdot \vec{P}) (\vec{S}_X \cdot \vec{P})$$

$$O_{13}^{(+-)} = i ((\vec{S}_N \cdot \vec{q}) (\vec{S}_X \cdot \vec{P}) + (\vec{S}_N \cdot \vec{P}) (\vec{S}_X \cdot \vec{q}))$$

$$O_{14}^{(+-)} = i ((\vec{S}_N \cdot \vec{q}) (\vec{S}_X \cdot \vec{P}) - (\vec{S}_N \cdot \vec{P}) (\vec{S}_X \cdot \vec{q}))$$

$$O_{15}^{(-+)} = [\vec{S}_N \cdot (\vec{P} \times \vec{q})] (\vec{S}_X \cdot \vec{q}) + (\vec{S}_X \cdot (\vec{P} \times \vec{q})) (\vec{S}_N \cdot \vec{q})$$

$$O_{16}^{(-+)} = i [\vec{S}_N \cdot (\vec{P} \times \vec{q})] (\vec{S}_X \cdot \vec{P}) + i [\vec{S}_X \cdot (\vec{P} \times \vec{q})] (\vec{S}_N \cdot \vec{P})$$

Wilson coefficients are suppressed by mediator mass if they are heavier than $\sim \text{GeV}$. In the case following two are dominant.

$$O_1^{(SI)} = 1 \quad (SI)$$

$$O_5^{(SD)} = \vec{S}_X \cdot \vec{S}_N \quad (SD)$$

The other terms are suppressed by v or $|\vec{q}|$.

WIMP-nucleon operators

χ : Majorana fermion

$$\mathcal{L} = \sum_{N=p,n} (f_N \bar{\chi} \chi \bar{N} N + a_N \bar{\chi} \gamma_\mu \gamma_5 \chi \bar{N} \gamma_\mu \gamma_5 N)$$

SI
 $\rightarrow \vec{S}_\chi \cdot \vec{S}_N$

SD

* $\frac{1}{2}$ is not multiplied due to historical reason.

$$\sigma = \sigma_{SI} + \sigma_{SD}$$

Neglecting nucleus form factor ($m_r = M_T m_N / (M_T + m_N)$)

$$\sigma_{SI} = \frac{4}{\pi} m_r^2 |Z f_p + (A-Z) f_n|^2$$

- Amplitude is proportional to Z or $(A-Z)$
- Large nucleus is more sensitive to DM scattering

$$\sigma_{sp} = \frac{16}{\pi} m_r^2 \frac{J+1}{J} |Q_p \langle S_p \rangle + Q_n \langle S_n \rangle|^2$$

- $\langle S_N \rangle$: expectation value of nucleon spin in nucleus.
- J : total spin of nucleus.

• Form factors are not negligible

$$\left(\begin{array}{l} \text{Momentum transfer } q = (2m_T E_R)^{1/2} \\ \text{nucleus radius } r_n = A^{1/3} \text{ fm} \end{array} \right.$$

$$\rightarrow r_n q \approx 7 \times 10^{-3} A^{1/6} (E_R \text{ (keV)})^{1/2} \quad m_T \approx A m_p$$

σ_{SI} is suppressed for large A and large E_R .

• Future sensitivities and bounds on SI interaction is represented by σ_p^{SI} .

$X =$ Dirac fermion

$$\mathcal{L}_V = \sum_{\nu=HP} (f_N^{(\nu)} \bar{X} \gamma_\mu X \bar{N} \gamma^\mu N)$$

$$\mathcal{L}_S = \sum_{\nu=HP} f_N^{(\nu)} \bar{X} X \bar{N} N$$

$$\mathcal{L}_A = \frac{M_X}{2} \bar{X} \gamma^{\mu\nu} X F_{\mu\nu} + b_X \bar{X} \gamma^\mu X \partial^\nu F_{\mu\nu} + \frac{d_X}{2} \bar{X} i \gamma^{\mu\nu} \gamma_5 X F_{\mu\nu}$$

MDM charge radius EDM

WIMP - Nuclei - SI cross section

$$\frac{d\sigma}{dE_R} = \alpha \mu_X^2 Z^2 \left(\frac{1}{E_R} - \frac{M}{2M_P^2 v^2} \right) |F_{SI}(E_R)|^2$$

$$+ \alpha d_X^2 Z^2 \frac{1}{v^2 E_R} |F_{SI}(E_R)|^2$$

$$+ \frac{M_A}{2\pi v^2} (f)^2 |F_{SI}(E_R)|^2$$

$$f = Z (f_P^{(S)} + f_P^{(V)} - e b_X - e \frac{M_X}{2M_P})$$

$$+ (A-Z) (f_N^{(S)} + f_N^{(V)})$$

$$E_{th} < E \leq E_{max} \quad (E_{max} = 2 \frac{M_P^2}{M_X} v^2)$$

When Dirac DM has weak charge, it is severely constrained.
($Q = T_{3X} + Y_X = 0$)

$$\mathcal{L} = + \frac{g^2}{m_X^2} Y_X \bar{\chi}_R \chi \left[\underbrace{\left(\frac{1}{4} - g_W^2 \right)}_{\approx 0} \bar{\chi}_R \chi + \left(-\frac{1}{4} \right) \bar{\chi}_R \chi \right]$$

$$\rightarrow \sigma_n^{SI} \approx \frac{1}{4\pi} m_n^2 \frac{g^2}{m_X^2} Y_X^2 \approx 7 \times 10^{-40} \text{ cm}^2$$

XENON IT $\sigma_n \lesssim 10^{-45} \text{ cm}^2$ ($M_X \approx 1 \text{ TeV}$)

$$\rightarrow M_X \gtrsim 10^5 \text{ TeV}$$

Pseudo Dirac DM

$$\mathcal{L} = -M \bar{X} X - \frac{1}{2} \mu (\bar{X}^c X + \text{h.c.})$$

$$= -\frac{1}{2} \left[(\bar{X}_L)^c, \bar{X}_R \right] \begin{bmatrix} M & \mu \\ \mu & M \end{bmatrix} \begin{bmatrix} X_L \\ (X_R)^c \end{bmatrix} + \text{h.c.}$$

\rightarrow

$$\begin{aligned} M_1 &= M + \mu \\ M_2 &= M - \mu \end{aligned} \quad \Delta M = 2\mu$$

When $\Delta M \gtrsim m_{\nu}^2 \approx 10^2 \text{ KeV}$, elastic scattering ($X, N \rightarrow X, N$) is suppressed.

III, Direct detection experiments.

Dark matter around us.

Density: $\rho_x = (0.3 \sim 0.7) \text{ GeV/cm}^3$

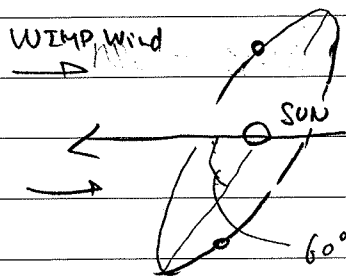
Velocity distribution.

(assume Maxwellian.)

$$f(\vec{v}) = \frac{1}{(2\pi v_0^2)^{3/2}} e^{-\frac{(\vec{v} + \vec{v}_\oplus)^2}{v_0^2}}$$

$$v_0 \approx 230 \text{ km/s}$$

Velocity of earth.



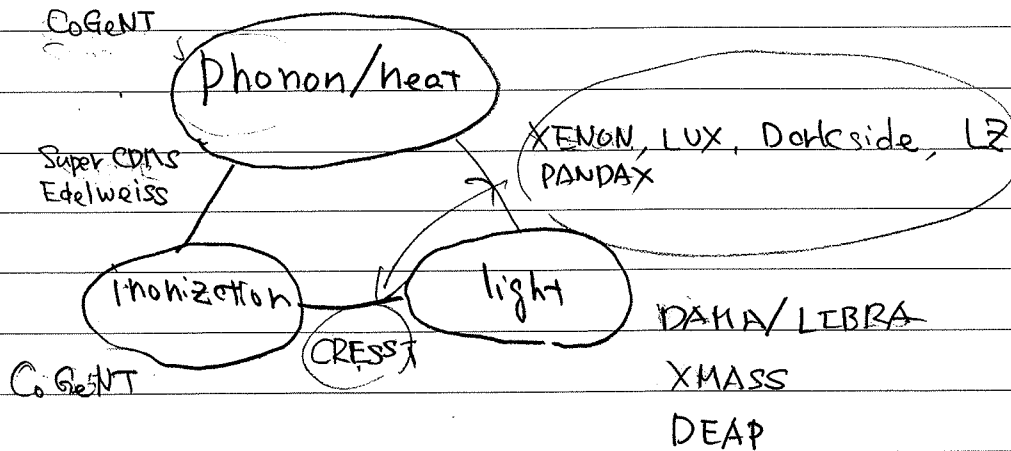
$$|\vec{v}_{\oplus}| \approx 244 + 15 \sin(2\pi y) \text{ (km/s)}$$

y : elapsed time from 2nd, March.



- Annual modulation of event rate $\sim 3\%$
- Directional detection is still challenging

Detectional technique



Low BG = underground
coincidence

Event rate

$$dR = \frac{N_0}{A} \sigma v dx$$

\swarrow Avogadro #
 \nwarrow atomic #

: event rate/unit target mass

Assuming zero-momentum transfer cross section $\sigma \approx \text{const} = \sigma_0$

$$R_0 = \frac{2}{\pi} \frac{N_0}{A} \frac{\rho_x}{M_x} \sigma_0 v_0 \quad (\text{escape velocity } v_E \rightarrow 0)$$

$$\approx \frac{540}{A M_x} \left(\frac{\sigma_0}{10^{-36} \text{cm}^2} \right) \left(\frac{\rho_0}{0.4 \text{GeV/cm}^3} \right) \left(\frac{v_0}{230 \text{km/s}} \right) \text{event/kg/day}$$

\uparrow GeV unit.

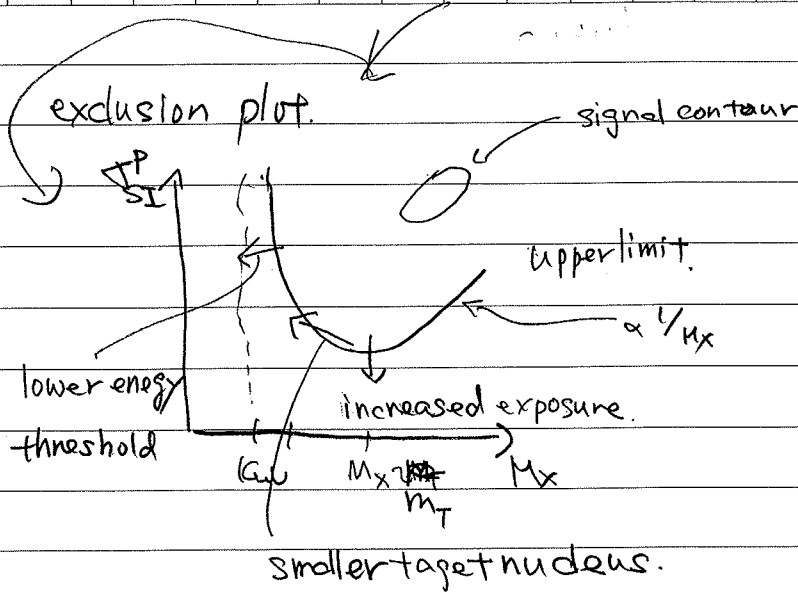
Event spectrum. (E_R : recoil energy)

$$\frac{dR}{dE_R} \approx \frac{R_0}{E_0} e^{-E_R/E_0}$$

where

$$E_0 = \frac{1}{2} M_x v^2 \ll \frac{M_x M_T}{(M_x + M_T)^2}$$

~ assuming isoscalar SI interaction



Background from ν coherent scattering.

• Solar ν .

$$\sigma_{SI}^P \leq 10^{-44} \text{ cm}^2 \quad (M_x \leq \text{several GeV})$$

• Atmospheric ν .

$$\sigma_{SI}^P \leq 10^{-43} \text{ cm}^2 \quad (M_x \leq 100 \text{ GeV})$$

$$10^{-48} \text{ cm}^2 \quad (M_x \geq 1 \text{ TeV})$$

If directional detection is available, we may detect WIMP with smaller cross section than BG limits.

Current limit.

PandaX, LUX, XENON-1T

$$\sigma_{SI}^p \sim 10^{-46} \text{ cm}^2 \quad (M_x \approx 50 \text{ GeV})$$

Future experiments.

~~XENON1T (16-18)~~ \rightarrow XENONnT (19-)

$$\sigma_{SI}^p \sim 10^{-47} \text{ cm}^2$$

$$\sigma_{SI}^p \sim 10^{-48} \text{ cm}^2$$

~~LUX (~16)~~ \rightarrow LZ (19-)

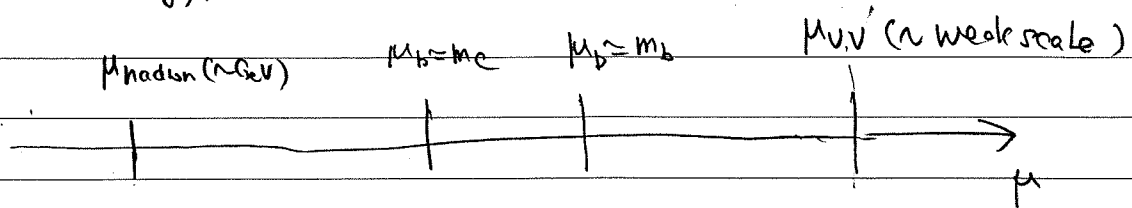
~~PandaX~~ \rightarrow PandaX-xT

\downarrow
DAWN (24?)

DarksidedM (?)

IV. Evaluation of effective interaction of WIMP with nucleons.

IV-I Strategy



Standard procedure

(i) Construct effective theory of WIMP and quark/gluon.
(relevant to the direct detection)

$$\mathcal{L} = \bar{Q}_i \not{\partial} P_i + \bar{\psi} P_i \psi$$

(ii) Evaluate Wilson coefficients for the effective operator

$C_i^{UV}(\mu_{UV})$ by integrating out massive particles in UV theory.

(iii) Evaluate $C_i^{hadron}(\mu_{hadron})$ by RG equations.

Include threshold correction @ μ_b or m_c if necessary
from viewpoint of accuracy.

(iv) Evaluate effective coupling in nucleon level using
matrix elements.

$$\langle P | O_i(\mu_{hadron}) | P \rangle$$

If we get to know matrix element @ μ_{UV} , we can skip step (iii).

$$\langle P | \not{\partial} C_i(\mu_{UV}) O_i(\mu_{UV}) | P \rangle = \langle P | \not{\partial} C_i(\mu_{had}) O_i(\mu_{had}) | P \rangle$$

IV-II effective interaction @ parton level

X: Majorana fermion

- up to eq of motion
- up to D=4 for quark/gluon operators
- Neglect terms suppressed DM velocity or momentum transfer

$$\mathcal{L}_{eff} = \sum_{P=q\bar{q}} C_S^P O_S^P + \sum_{i=1,2} \sum_{P=q\bar{q}} C_{T_i}^P O_{T_i}^P + \sum_f C_{AV}^f O_{AV}^f$$

$$O_S^f = \bar{X} X \frac{m_f}{\Lambda^2} \bar{f} f \quad (D=7)$$

$$O_S^g = \frac{ds}{\Lambda^2} \bar{X} X G_{\mu\nu}^A G_{\mu\nu}^A \quad (D=7)$$

$$O_{T_1}^P = \frac{1}{\Lambda^2} \bar{X} i \partial_\mu \delta_\nu X O_{\mu\nu}^P \quad (D=8)$$

$$O_{T_2}^P = \frac{1}{\Lambda^2} \bar{X} i \partial_\mu i \partial_\nu X O_{\mu\nu}^P \quad (D=9)$$

$$O_{AV}^f = \bar{X} \delta_\nu \delta_5 X \bar{f} \delta_\nu \delta_5 f \quad (D=6)$$

SI

SD

where Spin-2 twist-2 operator

$$O_{\mu\nu}^f = \frac{1}{2} \bar{f} i (D_\mu \delta_\nu + D_\nu \delta_\mu - \frac{1}{2} g_{\mu\nu} \not{D}) f$$

$$O_{\mu\nu}^g = G_{\mu\rho}^A G_{\nu\rho}^A - \frac{1}{4} g_{\mu\nu} G_{\mu\sigma}^A G^{\sigma\nu A}$$

twist = dim - spin

① In NR limit of X

$$O_{T1}^P \approx X^\dagger X O_{00}^P \quad O_{T2}^P = \bar{X} X O_{00}^P$$

② Trace anomaly in QCD

$$\Theta_\mu^\mu = \frac{\beta(g_s)}{4g_s} G_{\mu\nu}^A G^{\mu\nu A} + (1 - \gamma_m) \sum_f m_f \bar{f} f$$

$$\text{where } \beta(g_s) \equiv \mu \frac{\partial}{\partial \mu} g_s = 2b_1 \frac{g_s^2}{4\pi} + 2b_2 \frac{g_s^3}{(4\pi)^2}$$

$$b_1 = -\frac{11}{3} N_c + \frac{2}{3} N_f$$

$$b_2 = -\frac{34}{3} N_c^2 + \frac{10}{3} N_c N_f + 2 N_f$$

$$\gamma_m m_f \equiv \mu \frac{\partial}{\partial \mu} m_f = -6 G \frac{g_s}{4\pi}$$

$$\begin{cases} O_S^f = m_f \bar{f} f & \text{RG inv (MS)} \\ O_S^g = \frac{g_s}{\pi} G^2 & \text{" @ } O(d_s) \end{cases}$$

$$\mu \frac{\partial}{\partial \mu} \Theta_\mu^\mu = 0$$

$$\mu \frac{\partial}{\partial \mu} \begin{pmatrix} O_S^f \\ O_S^g \end{pmatrix} = \begin{bmatrix} 0 & 0 \\ -\frac{4g_s^2}{\pi} \frac{\partial \gamma_m}{\partial g_s} & \frac{\partial}{\partial g_s} \frac{\partial}{\partial \mu} \left(\frac{\beta(g_s)}{g_s^2} \right) \end{bmatrix} \begin{pmatrix} O_S^f \\ O_S^g \end{pmatrix}$$

$\sim O(d_s^1) \qquad \qquad \qquad \sim O(d_s^2)$

IV-III Matrix elements.

① $\langle N | m_q \bar{q} q | N \rangle \equiv f_{Tq}^{(N)} m_N$ ← *meas factor*

QCD lattice (N_f=2+1), ETM collab. 1601.01624)

$f_u^{(CP)} = 0.0149(17) \binom{21}{16}$

$f_u^{(N)} = 0.0117(15) \binom{18}{12}$

$f_d^{(CP)} = 0.0234(23) \binom{27}{16}$

$f_d^{(N)} = 0.0298(23) \binom{20}{16}$

$f_s^{(CP)} = 0.0440(88) \binom{92}{31}$

$f_c^{(CP)} = 0.085(22) \binom{11}{7}$

↑ statistical ↑ systematic

$(r = m_u/m_d = 0.50(9))$

② $\langle N | \frac{d_s}{\pi} G^2 | N \rangle$

trace anomaly $m_N = \langle N | \Theta_F^T | N \rangle$

$\langle N | \frac{d_s}{\pi} G^2 | N \rangle = m_N \frac{4d_s^2}{2\beta(d_s)} [1 - (1-\delta_m) \sum_f f_{Tf}^{(N)}]$

$(= \frac{8}{b_1} m_N (1 - \sum_f f_{Tf}^{(N)})) \quad O(d_s^0)$

$= - \frac{8}{9} m_N (1 - \underbrace{\sum_f f_{Tf}^{(N)}}_{\lesssim 0.1})$

$\therefore \langle N | \frac{d_s}{\pi} G^2 | N \rangle$ is $O(1)$

$$\langle N | m_0 \bar{Q} Q | N \rangle \quad Q = \text{heavy quark}$$

$$\langle N | \Theta_F^P | N \rangle |_{N_H} = \langle N | \Theta_F^P | N \rangle |_{N_H-1}$$

∴

$$\langle N | m_0 \bar{Q} Q | N \rangle \simeq -\frac{1}{12} \left(1 + 11 \frac{d_S}{d_T} \right) \langle N | \frac{d_S}{\Lambda^2} G^2 | N \rangle$$

$$\textcircled{Q} \quad \mu \simeq m_0$$

$$\simeq \frac{2}{27} m_N$$

~ 0.074 (consistent with lattice)

Integrating out Q is ok?

$$-\frac{d_S}{12\Lambda^2} G^2 + \frac{d_S}{64\pi m_0^2} (D^\nu G_{\nu\mu}^A)(D^\mu G_{\rho\mu}^A) - \frac{g_S d_S (m_0)}{1720 m_0^2} f_{ABC} G_{\mu\nu}^A G^{\mu\rho B} G^{\rho\sigma C}$$

$$\frac{\Lambda_{QCD}^2}{m_c^2} \simeq 0(10\%) \quad \text{parametrically} \quad \Delta \text{ atew\%}$$

③ Spin-2 twist-2 operator

$$\langle N\Phi | O_{\mu\nu}^{(M)} | N\Phi \rangle = m_N \left(\frac{P_\mu P_\nu}{m_N^2} - \frac{1}{4} g_{\mu\nu} \right) (f^{(N)}(2, M) + \bar{f}^{(N)}(2, M))$$

$$\langle N\Phi | O_{\mu\nu}^{(M)} | N\Phi \rangle = -m_N (\dots) f^{(N)}(2, M)$$

where

$$f^{(N)}(n, M) = \int_0^1 dx x^{n-1} f^{(N)}(x, M)$$

$$\bar{f}^{(N)}(n, M) = \int_0^1 dx x^{n-1} \bar{f}^{(N)}(x, M) \quad (n\text{-th moment of PDF})$$

$$g^{(N)}(n, M) = \int_0^1 dx x^{n-1} g^{(N)}(x, M)$$

$f^{(N)}(x, M)$, $\bar{f}^{(N)}(x, M)$, $g(x, M)$: PDF of M .

Proof)

($\delta^+ N \rightarrow X$)

• OPE for DIS, optical theorem ($\delta^+ N \rightarrow \delta^+ N$) and, contour integral

• See Reskin & Schweder Ch 18

• Assume integral for $|w| = |\frac{1}{z}| \rightarrow \infty$ is zero.

• DGE of twist-2 ops are consistent with Altarelli-Parisi evolution of PDF

$$\{P^{h_1} P^{h_2} \dots P^{h_J}\}_{TS} \langle N | O_J^{(M)} | N \rangle = \frac{1}{2} \langle N | \bar{f}_J (\delta^{h_1} D^{h_2} \dots D^{h_J})_{TS} f_J | N \rangle$$

↑ traceless sym.

$$\langle N | O_J^{(M)} | N \rangle = \int_0^1 dx x^{J-1} (f_J^{(M)}(x, M) + (-)^J \bar{f}_J^{(M)}(x, M)) \frac{1}{2m_N}$$

total term for DIS ($\delta^+ N \rightarrow X$)

2nd moments of PDF of Proton ($\mu = M_Z$; $N_f = 5$)

CTEQ-Jefferson Lab. collaboration

$$g(z) = 0.464(z)$$

$$u(z) = 0.223(z)$$

$$\bar{u}(z) = 0.036(z)$$

$$d(z) = 0.118(z)$$

$$\bar{d}(z) = 0.037(z)$$

$$s(z) = 0.0258(z)$$

$$\bar{s}(z) = s(z)$$

$$c(z) = 0.0187(z)$$

$$\bar{c}(z) = c(z)$$

$$b(z) = 0.0117(z)$$

$$\bar{b}(z) = b(z)$$

valence quarks and gluon line sizable

RGEs of twist op

$$\mu \frac{d}{d\mu} \begin{pmatrix} O_{\mu}^q \\ O_{\mu}^g \end{pmatrix} = - \begin{bmatrix} \gamma_{qq} & \gamma_{qg} \\ \gamma_{gq} & \gamma_{gg} \end{bmatrix} \begin{bmatrix} O_{\mu}^q \\ O_{\mu}^g \end{bmatrix}$$

$$\gamma_{qq} = \frac{16}{3} C_F \frac{d_s}{4Z} + \left(-\frac{208}{27} C_F N_f - \frac{224}{27} C_F^2 + \frac{752}{27} C_F N_c \right) \left(\frac{d_s}{4Z} \right)^2$$

$$\gamma_{qg} = \frac{4}{3} \frac{d_s}{4Z} + \left(\frac{168}{27} C_F + \frac{70}{27} N_c \right) \left(\frac{d_s}{4Z} \right)^2$$

$$\gamma_{gq} = \frac{16}{3} C_F \frac{d_s}{4Z} + \left(-\frac{208}{27} C_F N_f - \frac{224}{27} C_F^2 + \frac{752}{27} C_F N_c \right) \left(\frac{d_s}{4Z} \right)^2$$

$$\gamma_{gg} = \frac{4}{3} N_f \frac{d_s}{4Z} + \left(\frac{168}{27} C_F N_f + \frac{70}{27} N_c N_f \right) \left(\frac{d_s}{4Z} \right)^2$$

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