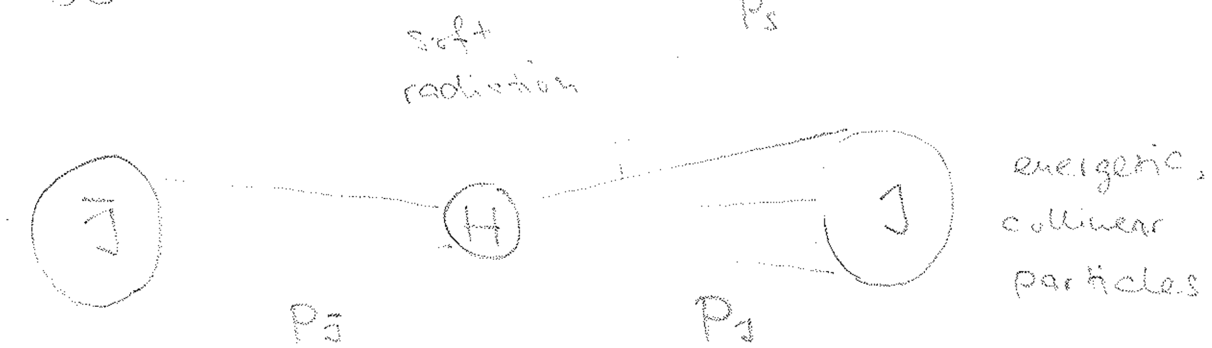
Soft - Collinear Effective Theory

SCET is the EFT for processes with energetic particles, e.g. jet production at high-E colliders



Typical scale hierarchy:

$$\underbrace{Q^2 = (p_J + p_{\bar{J}})^2}_{\substack{\text{hard-scattering part} \\ H,\ \text{integrate out}}} \gg \underbrace{p_J^2 \sim p_{\bar{J}}^2 \gg p_s^2}_{\substack{\text{Soft and collinear particles} \\ \text{degrees of freedom of EFT.}}}$$

Result of EFT analysis is typically factorization theorem, schematically:

$$\sigma = H \cdot J \cdot \bar{J} \cdot S$$

Benefits:

* Ingredients fulfill RG equations: Resummation of large logs using RG-improved PT, see Mathias Neubert's lecture. LL: $\alpha_s^n \ln^{2n}\left(\frac{\mu^2}{\mu_s^2} \cdot\right)$
* Sometimes soft and collinear scales are so low that these contributions are nonperturbative (NP). Theorem then separates perturbative from NP physics.

Traditionally, the analysis of factorization in high-E processes was done purely diagrammatically. SCET provides efficient EFT framework for factorization and resummation.

Compared to traditional EFTs, such as Fermi Theory, there are several complications:

* Cannot simply integrate out particles: quarks & gluons are still present at low E!

  Instead, one splits field into modes

$$\underset{q,g}{\phi} = \underset{\text{off-shell, integrate out}}{\overset{H}{\phi_h}} + \underbrace{\overset{J}{\phi_c} + \overset{\bar{J}}{\phi_{\bar{c}}} + \overset{S}{\phi_s}}_{\text{low-E modes, SCET fields}}$$

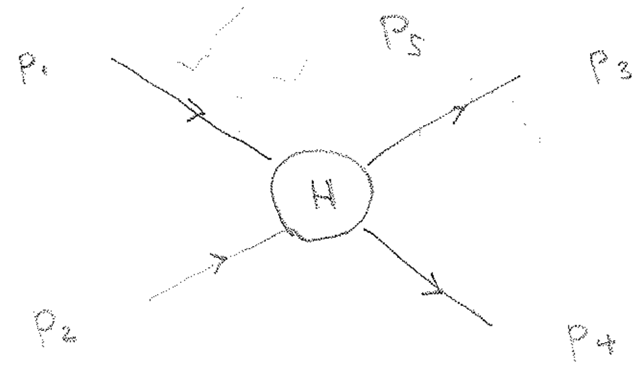
* Different momentum components scale differently. Momentum components along jet direction are large, transverse momenta small.

  Need reference vectors $n^\mu \propto p_J^\mu$; $\bar{n}^\mu \propto p_{\bar{J}}^\mu$ to construct SCET.

* Non-localities associated with directions of large momentum flow. Wilson lines, ...

I. Soft Effective Theory

As a warm up, consider e^-e^- scattering



with $E_1, E_2 \sim m_e$, $E_5 \ll m_e$.

Instead of SCET, we deal with "SET" (spoiler alert: SET $\equiv$ HQET, see T. Mannel's lecture).

This allows us to

1.) Derive a classical QED factorization theorem by Yennie, Frautschi & Suura '61.

2.) Deal with some of the aforementioned complications in a simple setting.

Note: One cannot avoid the presence of soft photons in QED processes. No matter how sensitive our detectors are, γ's below some threshold will always go unnoticed. It was realized very early (Bloch and Nordsieck '37) that one needs to include them in computations to get sensible results.

EFT for soft photons (by themselves)

$$\mathcal{L}^{\gamma}_{eff} = \mathcal{L}^{(4)}[A_\mu] + \frac{1}{m_e^2}\mathcal{L}^{(6)} + \frac{1}{m_e^4}\mathcal{L}^{(8)} + \dots$$

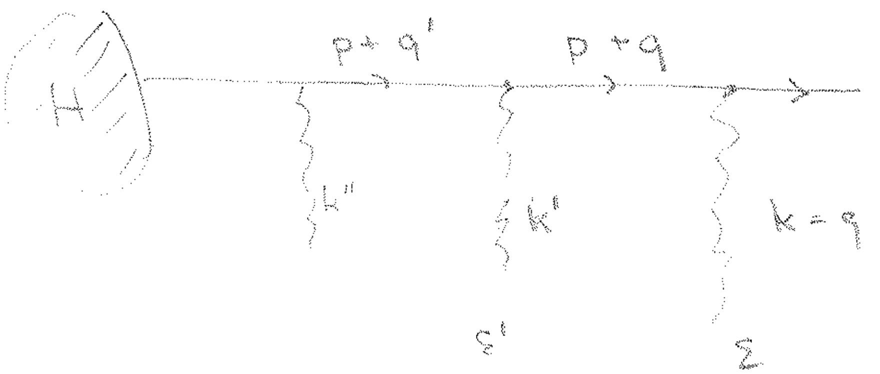
$$= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + 0 + \frac{c_1}{m_e^4}(F_{\mu\nu}F^{\mu\nu})^2 + \dots$$

(Euler-Heisenberg theory)

The leading power $\mathcal{L}^{(4)}$ will be good enough for us: we will drop all E_γ/m_e corrections.

However, we also need to include the e^-'s which radiate the photons. (Due to fermion number conservation, they remain even as $E_\gamma \to 0$.)

Consider outgoing e^-, together with soft photons



$p = m_e v \, ; \; v^2 = 1$

Propagator:

$$\Delta_F(p+q) = i \frac{(\not{p}+\not{q}) + m_e}{(p+q)^2 - m_e^2 + i\varepsilon} = i \frac{\not{p} + m_e}{2p\cdot q + i\varepsilon} + \ldots$$

$$= \frac{1+\not{v}}{2} \frac{i}{v\cdot q + i\varepsilon} = P_v \frac{i}{v\cdot q + i\varepsilon}$$

Note: $P_v^2 = P_v \; ; \; \not{v} P_v = P_v$

$P_v \not{\varepsilon} P_+ = P_v \, \varepsilon \cdot v$ (exercise)

The diagram becomes

$$\bar{u}(p)\,(-ie\,\varepsilon\cdot v)\,P_v\,\frac{i}{v\cdot q}\,(-ie\,\varepsilon'\cdot v)\,P_v\,\frac{i}{v\cdot q'}\cdots$$

Can we get this from an effective Lagrangian?

yes:
$$\mathcal{L}^{v}_{eff} = \bar{h}_v\, iv\cdot D\, h_v$$

where h_v is Dirac field, subject to

$$P_v h_v = h_v \;;\; \slashed{v} h_v = h_v$$

* $\mathcal{L}^{v}_{eff}$ depends on reference vector v^μ!

* Due to $\slashed{v} h_v = h_v$, field has two indep. components, propagator $\dfrac{i}{v\cdot q + i\varepsilon}$ has <u>single</u> pole. Field describes only electron close to mass-shell.

Full EFT (with $h_i \equiv h_{v_i}$)

$$\mathcal{L} = \sum_{i=1}^{4} \bar{h}_i \, i v_i \cdot D \, h_i - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \Delta\mathcal{L}$$

interactions

Leading power interaction for $e^- e^- \to e^- e^-$

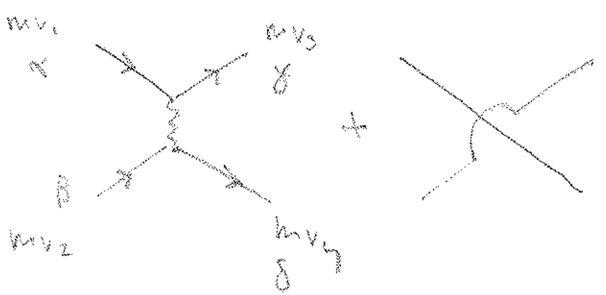
$$\Delta\mathcal{L} = C_{\alpha\beta\gamma\delta}(v_1, v_2, v_3, v_4) \, h_1^\alpha \, h_2^\beta \, \bar{h}_3^\gamma \, \bar{h}_4^\delta$$

or, more elegantly

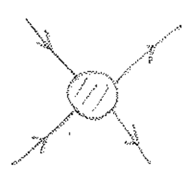
$$= \sum_i C_i(v_1, v_2, v_3, v_4) \, \bar{h}_3 \Gamma_i h_1 \, \bar{h}_4 \Gamma_i h_2$$

To obtain the Wilson coefficient C, we perform a matching computation.

QED



EFT



$= C_{\alpha\beta\gamma\delta}(v_1, v_2, v_3, v_4)$

Wilson coeff. is on-shell scattering amplitude!

Now consider the "Wilson line"

$$S_i(x) = \exp\left[-ie\int_0^\infty ds\, v_i\cdot A(x+sv_i)\right]$$

This fulfills (exercise)

$$v_i\cdot D\, S_i(x) = 0$$

This is useful, because we can perform a field redefinition ("decoupling transformation")

$$h_i(x) = S_i(x)\, h_i^{(0)}(x)$$

and

$$\bar h_i\, iv\cdot D\, h_i = \bar h_i^{(0)} S_i^\dagger\, iv\cdot D\, S_i\, h_i^{(0)}$$

$$= \bar h_i^{(0)} S_i^\dagger S_i\, iv\cdot\partial h_i^{(0)} = \bar h_i^{(0)}\, iv\cdot\partial h_i^{(0)}$$

↑ free field!

Interaction:

$$\Delta\mathcal{L} = \sum_i C_i\, \bar h_3^{(0)} S_3^\dagger\, \Gamma_i\, S_1 h_1^{(0)}\; \bar h_4^{(0)} S_4^\dagger \Gamma_i S_1 h_1^{(0)}$$

After all this preparation, we can now compute

$$e^-(p_1)+e^-(p_2) \to e^-(p_3)+e^-(p_4)+\underbrace{\gamma(k_1)+\ldots+\gamma(k_n)}_{X_s\,,\;P_{X_s}}$$

In the EFT, we get

$$\mathcal{A} = \sum_i C_i\;\bar{u}(p_3)\,\Gamma_i\,u(p_1)\;\bar{u}(p_4)\,\Gamma_i\,u(p_2)$$

$$\cdot\,\langle X_s|\,S_3^\dagger S_1\, S_4^\dagger\, S_2\,|0\rangle$$

$$= \mathcal{A}(e^-e^-\to e^-e^-)\cdot\langle X_s|\ldots|0\rangle \quad \text{factorization!}$$

The cross section, including arbitrary many photons, with energy below E_0, is

$$d\sigma = \frac{1}{2E_1\,2E_2\,|\vec{v}_1-\vec{v}_2|}\;\frac{d^3p_3}{2E_3(2\pi)^3}\;\frac{d^3p_4}{2E_4(2\pi)^3}\;|\mathcal{A}(e^-e^-\to e^-e^-)|^2$$

$$*\;\sum\!\!\!\!\!\!\int_{X_s} |\langle X_s|S_3^\dagger S_1\,S_4^\dagger\,S_2|0\rangle|^2\;\theta(E_0-E_{X_s})$$

$$*\;(2\pi)^4\,\delta^{(4)}(p_1+p_2-p_3-p_4-p_{X_s})$$

power correction

$p_{X_s}^\mu \ll$ the

Define $\mathcal{S}(\{\underline{v}\}, E_0) = \mathcal{S}(\{v_1, v_2, v_3, v_4\}, E_0)$

$$= \sum\!\!\!\!\!\!\int_{X_S} |\langle X_S | S_3^+ S_1 S_4^+ S_2 | 0 \rangle|^2 \, \theta(E_0 - E_{X_S})$$

$$d\sigma = d\hat{\sigma} \cdot \mathcal{S}(\{\underline{v}\}, E_0)$$

Factorization theorem for σ.

Wilson line matrix elements have some surprising properties. They are especially simple in QED. One finds exponentiation

$$\mathcal{S}(\{\underline{v}\}, E_0) = \exp\left[\frac{\alpha}{4\pi}\mathcal{S}^{(1)}\right]$$

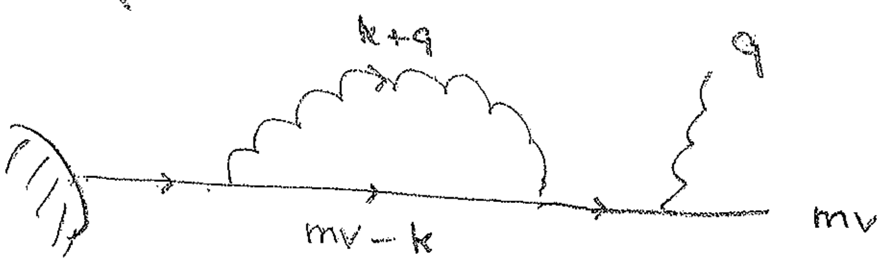
No higher order corrections to exponent!

II. Method of regions

When constructing $\mathcal{L}_{eff}$, we have expanded in the small photon momenta. OK at tree level, but how about loops?

Of course Taylor expansion does not commute with loop integration. To correct for this one performs matching computations.

It is instructive to consider scalar example integral



$$F = \int d^dk \, \frac{1}{(mv-k)^2\,(k+q)^2}$$

In the low-E theory, we assumed that $k \sim q \ll m_e$, which gives the integral

$$F_{low} = \int d^d k \, \frac{1}{(k+q)^2} \, \frac{1}{-2mv\cdot k} \left\{ 1 + \frac{k^2}{2mv\cdot k} + \dots \right\}$$

In a Wilsonian framework, we would have put a cutoff $E_\gamma \ll \Lambda \ll m_e$ on the loop integrals, but we use dim. reg and integrate $\int_{-\infty}^{\infty} dk$. Of course, for $k_\mu \sim m_e$, the expansion of the propagator was not justified!

To correct for this, consider

$F_{high} = F - F_{low} \;\hat{=}\;$ "matching correction"

$$= \int d^d k \, \frac{1}{(k+q)^2} \left[\frac{1}{(mv-k)^2} - \frac{1}{-2mv\cdot k} \left\{ 1 + \frac{k^2}{2mv\cdot k} + \dots \right\} \right]$$

By construction, the integrand has only support for $k^\mu \gg q^\mu$ since the $[\;] \to 0$ for $k \sim q$.

We can therefore expand the integrand around $q^\mu = 0$:

$$F_{high} = \int d^dk \, \frac{1}{k^2} \left\{ 1 - \frac{q^2}{2q\cdot k} + \ldots \right\}$$
$$\left[\frac{1}{(mv-k)^2} - \frac{1}{-2mvk} \left\{ 1 + \ldots \right\} \right]$$

Now for the crucial observation: Integrals of the form

$$I[\alpha,\beta,\gamma] = \int d^dk \, (k^2)^\alpha (v\cdot k)^\beta (q\cdot k)^\gamma = 0$$

because they are scaleless!

Proof: $k \to \lambda k$

$$\to I(\alpha,\beta,\gamma) = \lambda^{d+2\alpha+\beta+\gamma} \, I(\alpha,\beta,\gamma)$$

for any $\lambda > 0$

$$\to I(\alpha,\beta,\gamma) = 0.$$

$$F_{high} = \int d^dk \, \frac{1}{(mv-k)^2 \, k^2} \left\{ 1 - \frac{q^2}{2q\cdot k} + \dots \right\}$$

This is the expansion of the integrand for $k^\mu \sim m_e \gg q^\mu$

→ Method of regions: Expansion of full integral is obtained

a.) Consider all relevant scalings ("regions") of loop momenta.

For us $k_\mu \sim m_e$, $k_\mu \sim q_\mu$

b.) Expand in each region.

c.) Integrate over full phase space $\int d^dk$.

d.) Add up contributions

If one expands in each region, the overlaps correspond to scaleless integrals.

This method of region technique is a very general method to expand loop integrals around different limits and has been used in many different kinematical situations (see Smirnov, Springer Tracts. Mod. Phys. 177, 2002.)

In the following we will apply it to the Sudakov form factor.