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Introduction to EFT: Problems

1. Show that for a *connected* graph, $V - I + L = 1$. What is the formula if the graph has n connected components?

2. Work out the properties of fermion bilinears $\overline{\psi}(\mathbf{x},t)\Gamma P_L \chi(\mathbf{x},t)$ under C, P, T, where $\Gamma = 1, \gamma^\mu, \sigma^{\mu\nu}$. The results for $P_L \to P_R$ can be obtained by using $L \leftrightarrow R$.

3. Fierz identities are relations of the form

$$(\overline{A}\Gamma B)(\overline{C}\Gamma D) = \sum_i (\overline{C}\Gamma_i B)(\overline{A}\Gamma_i D)$$

   where A, B, C, D are fermion fields. They are much simpler if written in terms of chiral fields, where $\Gamma_i = 1, \gamma^\mu, \sigma^{\mu\nu}$. Work out the Fierz relations for

$$(\overline{A}P_L B)(\overline{C}P_L D),\ (\overline{A}P_L B)(\overline{C}P_R D),\ (\overline{A}\gamma^\mu P_L B)(\overline{C}\gamma_\mu P_L D), (\overline{A}\gamma^\mu P_L B)(\overline{C}\gamma_\mu P_R D),$$
$$(\overline{A}\sigma^{\mu\nu} P_L B)(\overline{C}\sigma_{\mu\nu} P_L D),\ (\overline{A}\sigma^{\mu\nu} P_L B)(\overline{C}\sigma_{\mu\nu} P_R D)$$

   The $P_R\ P_R$ identities are given by using $L \leftrightarrow R$ on the $L\ L$ identities. Do not forget the Fermi minus sign.

4. In $d = 4$ spacetime dimensions, work out the field content of Lorentz-invariant operators with dimension n for $n = 1, \ldots, 6$. At this point, do not try and work out which operators are independent, just the possible structure of allowed operators. Use the notation ϕ for a scalar, ψ for a fermion, $X_{\mu\nu}$ for a field strength, and D for a derivative. For example, an operator of type $\phi^2 D$ such as $\phi D_\mu \phi$ is not allowed because it is not Lorentz-invariant. An operator of type $\phi^2 D^2$ could be either $D_\mu \phi D^\mu \phi$ or $\phi D^2 \phi$, so a $\phi^2 D^2$ operator is allowed, and we will worry later about how many independent operators $\phi^2 D^2$ we can construct.

5. For $d = 2, 3, 4, 5, 6$ dimensions, work out the field content of operators with dimension $n \leq d$, i.e. the renormalizable operators.