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Introduction to EFT: Problems

1. Show that for a *connected* graph, $V - I + L = 1$. What is the formula if the graph has n connected components?

2. Work out the properties of fermion bilinears $\overline{\psi}(\mathbf{x},t)\Gamma P_L \chi(\mathbf{x},t)$ under C, P, T, where $\Gamma = 1, \gamma^\mu, \sigma^{\mu\nu}$. The results for $P_L \to P_R$ can be obtained by using $L \leftrightarrow R$.

3. Fierz identities are relations of the form

$$(\overline{A}\Gamma B)(\overline{C}\Gamma D) = \sum_i (\overline{C}\Gamma_i B)(\overline{A}\Gamma_i D)$$

where A, B, C, D are fermion fields. They are much simpler if written in terms of chiral fields, where $\Gamma_i = 1, \gamma^\mu, \sigma^{\mu\nu}$. Work out the Fierz relations for

$$(\overline{A}P_L B)(\overline{C}P_L D),\ (\overline{A}P_L B)(\overline{C}P_R D),\ (\overline{A}\gamma^\mu P_L B)(\overline{C}\gamma_\mu P_L D), (\overline{A}\gamma^\mu P_L B)(\overline{C}\gamma_\mu P_R D),$$
$$(\overline{A}\sigma^{\mu\nu} P_L B)(\overline{C}\sigma_{\mu\nu} P_L D),\ (\overline{A}\sigma^{\mu\nu} P_L B)(\overline{C}\sigma_{\mu\nu} P_R D)$$

The $P_R\ P_R$ identities are given by using $L \leftrightarrow R$ on the $L\ L$ identities. Do not forget the Fermi minus sign.

4. In $d = 4$ spacetime dimensions, work out the field content of Lorentz-invariant operators with dimension n for $n = 1, \ldots, 6$. At this point, do not try and work out which operators are independent, just the possible structure of allowed operators. Use the notation ϕ for a scalar, ψ for a fermion, $X_{\mu\nu}$ for a field strength, and D for a derivative. For example, an operator of type $\phi^2 D$ such as $\phi D_\mu \phi$ is not allowed because it is not Lorentz-invariant. An operator of type $\phi^2 D^2$ could be either $D_\mu \phi D^\mu \phi$ or $\phi D^2 \phi$, so a $\phi^2 D^2$ operator is allowed, and we will worry later about how many independent operators $\phi^2 D^2$ we can construct.

5. For $d = 2, 3, 4, 5, 6$ dimensions, work out the field content of operators with dimension $n \le d$, i.e. the "renormalizable" operators.

6. Show that if $\alpha_s(\mu)$ is fixed at some high scale, say $\mu = 1\,\text{TeV}$, then $m_p \propto m_t^{2/27}$, where m_p is the proton mass and m_t is the top quark mass.

7. Compute in dimensional regularization in $d = 4 - 2\epsilon$ dimensions

$$I_F = -i\mu^{2\epsilon} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{1}{(k^2 - m^2)(k^2 - M^2)}$$
$$I_M = -i\mu^{2\epsilon} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{1}{(k^2 - M^2)} \left[\frac{1}{k^2} + \frac{m^2}{k^4} + \ldots\right]$$
$$I_{\text{EFT}} = -i\mu^{2\epsilon} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{1}{(k^2 - m^2)} \left[-\frac{1}{M^2} - \frac{k^2}{M^4} + \ldots\right]$$

and determine $I_F - I_M - I_{\text{EFT}}$.

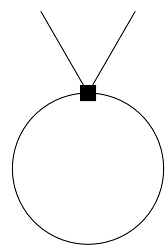
8. Show that for $SU(N)$,

$$[T^A]^\alpha{}_\beta [T^A]^\lambda{}_\sigma = \frac{1}{2}\delta^\alpha_\sigma \delta^\lambda_\beta - \frac{1}{2N}\delta^\alpha_\beta \delta^\lambda_\sigma$$

and the color Fierz identities

$$\delta^\alpha{}_\beta \delta^\lambda{}_\sigma = \frac{1}{N}\delta^\alpha_\sigma \delta^\lambda_\beta + 2[T^A]^\alpha{}_\sigma [T^A]^\lambda{}_\beta$$
$$[T^A]^\alpha{}_\beta [T^A]^\lambda{}_\sigma = \frac{N^2-1}{2N^2}\delta^\alpha_\sigma \delta^\lambda_\beta - \frac{1}{N}[T^A]^\alpha{}_\sigma [T^A]^\lambda{}_\beta$$

9. Compute the one-loop scalar graph with a scalar of mass m and interaction $-\lambda\phi^4/4!$.



10. Compute the anomalous dimension mixing matrix of

$$O_1 = (\overline{b}^\alpha \gamma^\mu P_L c_\alpha)(\overline{u}^\alpha \gamma^\mu P_L d_\alpha)$$
$$O_2 = (\overline{b}^\alpha \gamma^\mu P_L c_\beta)(\overline{u}^\beta \gamma^\mu P_L d_\alpha)$$

Another basis often used is

$$Q_1 = (\overline{b}\gamma^\mu P_L c)(\overline{u}\gamma^\mu P_L d)$$
$$Q_2 = (\overline{b}\gamma^\mu P_L T^A c)(\overline{u}\gamma^\mu P_L T^A d)$$

So let

$$\mathcal{L} = c_1 O_1 + c_2 O_2 = d_1 Q_1 + d_2 Q_2$$

and work out the transformation between c_i and d_i, and the anomalous dimension for d_i using the anomalous dimensions for c_i.