Computing gluon TMDs at small-x in the Color Glass Condensate

Cyrille Marquet

Centre de Physique Théorique
Ecole Polytechnique & CNRS

Contents of the talk

- Known issues with TMD factorization

  - process dependence, loss of universality, factorization breaking

- Case study for gluon TMDs at small-x: forward di-jets

  - TMD factorization is contained in the CGC framework, as the leading power in the hard scale

- Small-x evolution of the gluon TMDs

  - obtained from the JIMWLK equation
  - numerical results

Factorization breaking

consider a hadronic collision $A + B \to C + D + \cdots + X$

- ”hard” factorization breaking: Collins and Qiu (2007)

a maximum of two transverse-momentum-dependent “hadrons” (parton distributions or fragmentation functions) may be considered

for 3 or more TMD hadrons, factorization cannot be established

- ”soft” factorization breaking: Boer and Mulders (2000), Belitsky, Ji and Yuan (2003)

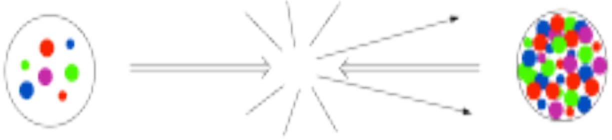
for one or two transverse-momentum-dependent hadrons, TMD factorization can be obtained, but different processes involve different TMDs

universality is lost

note: even then TMD factorization is broken at some order in perturbation theory, here I am only discussing the validity at leading-order

- in our forward di-jet study, due to the asymmetry, only the target nucleus will be described with TMDs

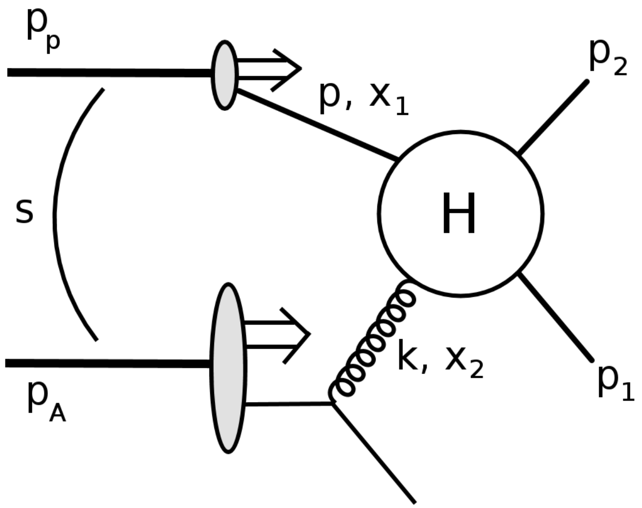
$\langle k_T \rangle \sim \Lambda_{QCD}$



$\langle k_T \rangle \sim Q_s$

Dilute-dense kinematics

- large-x projectile (proton) on small-x target (proton or nucleus)



$$\hat{s} = (p+k)^2$$
$$\hat{t} = (p_2 - p)^2$$
$$\hat{u} = (p_1 - p)^2$$

Incoming partons' energy fractions:

$$\begin{aligned} x_1 &= \tfrac{1}{\sqrt{s}}\left(|p_{1t}|e^{y_1} + |p_{2t}|e^{y_2}\right) \\ x_2 &= \tfrac{1}{\sqrt{s}}\left(|p_{1t}|e^{-y_1} + |p_{2t}|e^{-y_2}\right) \end{aligned} \quad \xrightarrow{y_1, y_2 \gg 0} \quad \begin{aligned} x_1 &\sim 1 \\ x_2 &\ll 1 \end{aligned}$$

Gluon's transverse momentum (p_{1t}, p_{2t} imbalance):

$$|k_t|^2 = |p_{1t} + p_{2t}|^2 = |p_{1t}|^2 + |p_{2t}|^2 + 2|p_{1t}||p_{2t}| \cos \Delta\phi$$

The back-to-back regime

$$|p_{1t}|, |p_{2t}| \gg |k_t|, Q_s$$

this is the regime of validity of TMD factorization:

$$\frac{d\sigma^{pA\to \mathrm{dijets}+X}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1x_2s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2) \sum_{i=1}^{2} K^{(i)}_{ag\to cd} \Phi^{(i)}_{ag\to cd} \frac{1}{1+\delta_{cd}}$$

it involves six unpolarized gluon TMDs $\Phi^{(i)}_{ag\to cd}(x_2,k_t^2)$ (2 per channel)

their associated hard matrix elements $K^{(i)}_{ag\to cd}$ are on-shell (i.e. $k_t = 0$)

it can be derived in two ways:

from the generic TMD factorization framework (valid up to power corrections): by taking the small-x limit

Bomhof, Mulders and Pijlman (2006)

Kotko, Kutak, CM, Petreska, Sapeta and van Hameren (2015)

from the CGC framework (valid at small-x): by extracting the leading power

Dominguez, CM, Xiao and Yuan (2011)

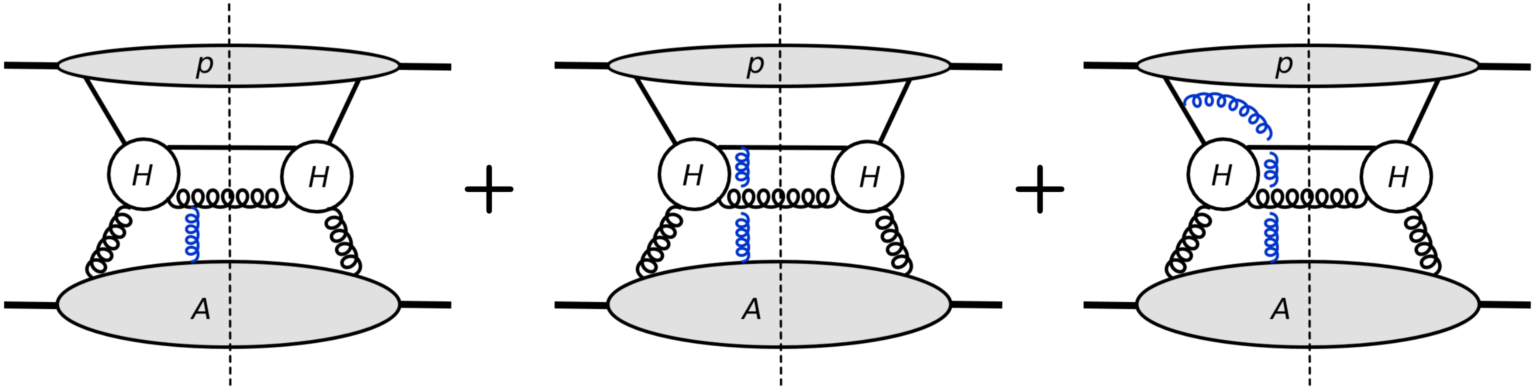
CM, Petreska, Roiesnel (2016)

TMD gluon distributions

- the naive operator definition is not gauge-invariant

$$\mathcal{F}_{g/A}(x_2,k_t) \overset{\text{naive}}{=} 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}_t}{(2\pi)^3 p_A^-}\, e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}_t} \left\langle A\right|\mathrm{Tr}\left[F^{i-}\left(\xi^+,\boldsymbol{\xi}_t\right)F^{i-}(0)\right]\left|A\right\rangle$$

- a theoretically consistent definition requires to include more diagrams



$+$ similar diagrams with $2,3,\ldots$ gluon exchanges

They all contribute at leading power and need to be resummed.

this is done by including gauge links in the operator definition

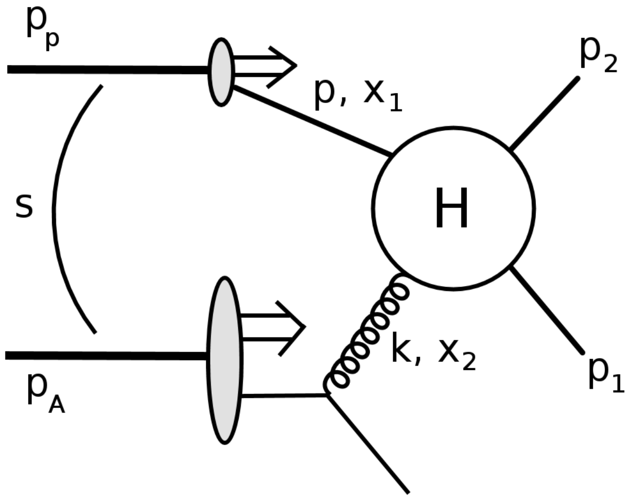
Process-dependent TMDs

- the proper operator definition(s)

some gauge link $\mathcal{P}\exp\left[-ig\int_{\alpha}^{\beta}d\eta^{\mu}A^{a}(\eta)T^{a}\right]$

$$\mathcal{F}_{g/A}(x_2,k_t)=2\int\frac{d\xi^{+}d^{2}\boldsymbol{\xi}_{t}}{(2\pi)^{3}p_A^{-}}e^{ix_2p_A^{-}\xi^{+}-ik_t\cdot\boldsymbol{\xi}_t}\left\langle A\right|\mathrm{Tr}\left[F^{i-}\left(\xi^{+},\boldsymbol{\xi}_t\right)U_{[\xi,0]}F^{i-}(0)\right]\left|A\right\rangle$$

  - $U_{[\alpha,\beta]}$ renders gluon distribution gauge invariant



however, the precise structure of the gauge link is process-dependent:

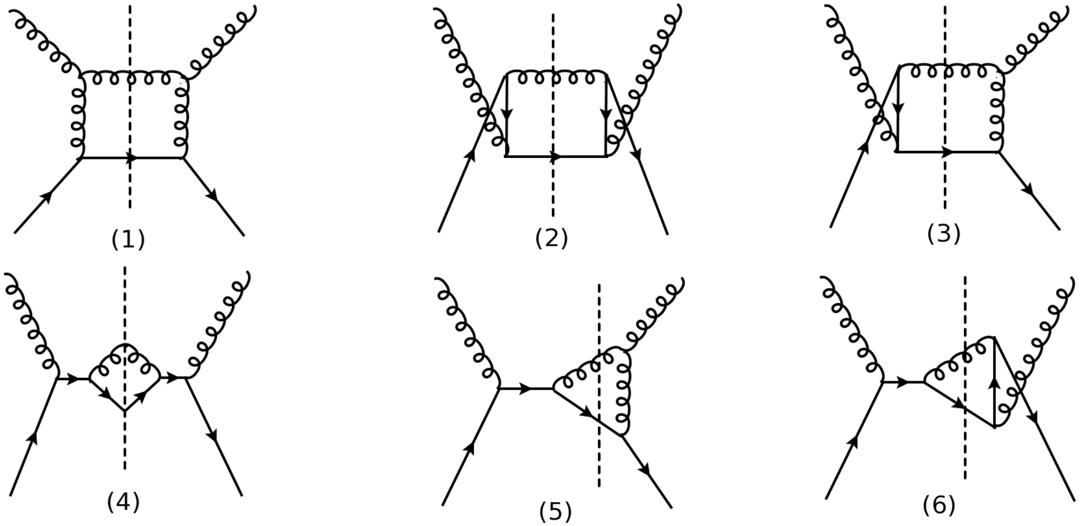
it is determined by the color structure of the hard process H

- in the large k_t limit, the process dependence of the gauge links disappears (like for the integrated gluon distribution), and a single gluon distribution is sufficient

TMDs for forward di-jets

- several gluon distributions are needed already for a single partonic sub-process

example for the $qg^* \to qg$ channel



each diagram generates a different gluon distribution

2 unintegrated gluon distributions per channel (i=1,2): $\Phi^{(i)}_{ag\to cd}(x_2,k_t^2)$

$$qg^* \to qg \qquad gg^* \to q\bar{q} \qquad gg^* \to gg$$

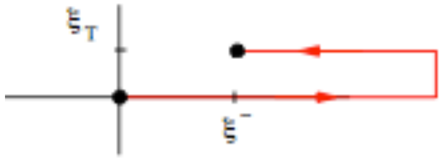
The six TMD gluon distributions

- correspond to a different gauge-link structure

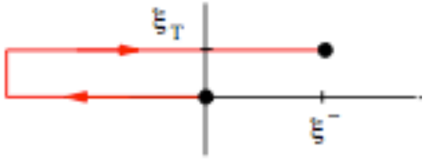
$$\mathcal{F}_{g/A}(x_2, k_t) = 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}_t}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}_t} \left\langle A\right|\mathrm{Tr}\left[F^{i-}\left(\xi^+, \boldsymbol{\xi}_t\right) U_{[\xi,0]} F^{i-}(0)\right]\left|A\right\rangle$$

several paths are possible for the gauge links

examples :



$\mathcal{U}^{[+]}$



$\mathcal{U}^{[-]}$

- when integrated, they all coincide

$$\int^{\mu^2} d^2k_t\ \Phi^{(i)}_{ag\to cd}(x_2, k_t^2) = x_2 f(x_2, \mu^2)$$

- they are independent and in general they all should be extracted from data

only one of them has the probabilistic interpretation of the number density of gluons at small x_2

TMDs from the CGC

- the gluon TMDs involved in the di-jet process are:

(showing here the $qg^* \to qg$ channel TMDs only)

$$\mathcal{F}_{qg}^{(1)} = 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}} \left\langle \mathrm{Tr}\left[F^{i-}(\xi)\,\mathcal{U}^{[-]\dagger}F^{i-}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{qg}^{(2)} = 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}} \left\langle \mathrm{Tr}\left[F^{i-}(\xi)\,\frac{\mathrm{Tr}\left[\mathcal{U}^{[\Box]}\right]}{N_c}\mathcal{U}^{[+]\dagger}F^{i-}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

- at small x they can be written as:

$$U_{\mathbf{x}} = \mathcal{P}\exp\left[ig\int_{-\infty}^{\infty} dx^+ A_a^-(x^+,\mathbf{x})t^a\right]$$

$$\mathcal{F}_{qg}^{(1)}(x_2,|k_t|) = \frac{4}{g^2}\int \frac{d^2x d^2y}{(2\pi)^3}\; e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})} \left\langle \mathrm{Tr}\left[(\partial_i U_{\mathbf{y}})(\partial_i U_{\mathbf{x}}^\dagger)\right]\right\rangle_{x_2}$$

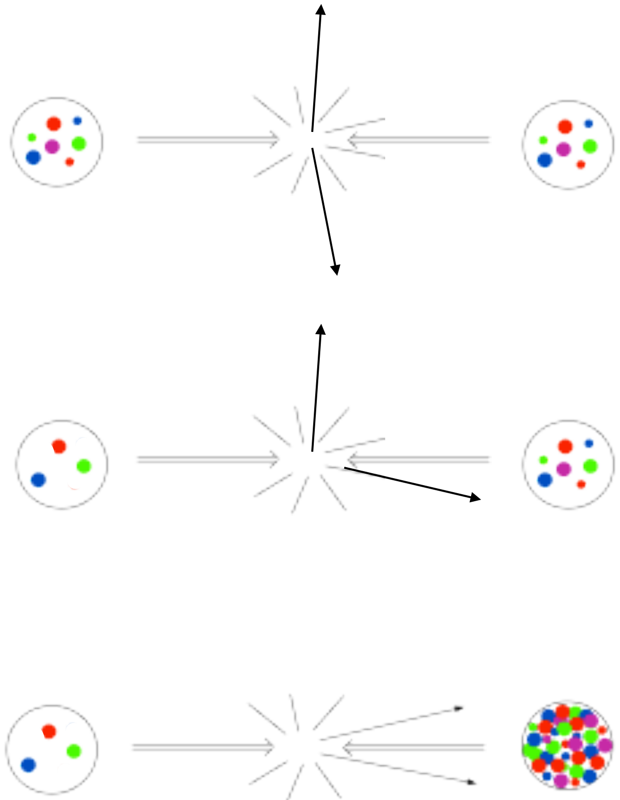
$$\mathcal{F}_{qg}^{(2)}(x_2,|k_t|) = -\frac{4}{g^2}\int \frac{d^2x d^2y}{(2\pi)^3}\; e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})} \frac{1}{N_c} \left\langle \mathrm{Tr}\left[(\partial_i U_{\mathbf{x}})U_{\mathbf{y}}^\dagger(\partial_i U_{\mathbf{y}})U_{\mathbf{x}}^\dagger\right]\mathrm{Tr}\left[U_{\mathbf{y}}U_{\mathbf{x}}^\dagger\right]\right\rangle_{x_2}$$

these Wilson line correlators also emerge directly of the CGC formulae

Di-jet final-state kinematics

final state : $k_1, y_1 \qquad k_2, y_2$ $\qquad x_p = \dfrac{k_1\, e^{y_1} + k_2\, e^{y_2}}{\sqrt{s}} \qquad x_A = \dfrac{k_1\, e^{-y_1} + k_2\, e^{-y_2}}{\sqrt{s}}$

scanning the wave functions:



$$x_p \sim x_A < 1$$

central rapidities probe moderate x

x_p increases | x_A ~ unchanged

$$x_p \sim 1,\, x_A < 1$$

forward/central doesn’t probe much smaller x

x_p ~ unchanged | x_A decreases

$$x_p \sim 1,\, x_A << 1$$

forward rapidities probe small x

Outline of the derivation

- using $\langle p|p'\rangle = (2\pi)^3\ 2p^-\delta(p^- - p'^-)\delta^{(2)}(p_t - p'_t)$ and translational invariance

$$\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}}\ \langle A|O(0,\xi)|A\rangle = \frac{2}{\langle A|A\rangle}\int \frac{d^3\xi d^3\xi'}{(2\pi)^3} e^{ix_2 p_A^-(\xi^+ - {\xi'}^+) - ik_t\cdot(\boldsymbol{\xi}-\boldsymbol{\xi}')}\ \langle A|O(\xi',\xi)|A\rangle\ \ .$$

- setting $\exp[ix_2 p_A^-(\xi^+ - {\xi'}^+)] = 1$ and denoting $\dfrac{\langle A|O(\xi',\xi)|A\rangle}{\langle A|A\rangle} = \langle O(\xi',\xi)\rangle_{x_2}$

we obtain e.g.

$$\mathcal{F}_{qg}^{(1)}(x_2,k_t) = 4\int \frac{d^3x d^3y}{(2\pi)^3}\ e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})}\left\langle \mathrm{Tr}\left[F^{i-}(x)\ \mathcal{U}^{[-]\dagger}F^{i-}(y)\ \mathcal{U}^{[+]}\right]\right\rangle_{x_2}$$

- then performing the x^- and y^- integrations using

$$\partial_i U_{\mathbf{y}} = ig\int_{-\infty}^{\infty} dy^+ U[-\infty,y^+;\mathbf{y}]F^{i-}(y)U[y^+,+\infty;\mathbf{y}]$$

we finally get $\mathcal{F}_{qg}^{(1)}(x_2,|k_t|) = \dfrac{4}{g^2}\displaystyle\int \frac{d^2x d^2y}{(2\pi)^3}\ e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})}\left\langle \mathrm{Tr}\left[(\partial_i U_{\mathbf{y}})(\partial_i U_{\mathbf{x}}^\dagger)\right]\right\rangle_{x_2}$

The other (unpolarized) TMDs

- involved in the $gg^* \to q\bar{q}$ and $gg^* \to gg$ channels

$$\mathcal{F}_{gg}^{(1)}(x_2,k_t) = 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}}\ \frac{1}{N_c}\left\langle A\right|\mathrm{Tr}\left[F^{i-}(\xi)\,\mathcal{U}^{[-]\dagger}F^{i-}(0)\,\mathcal{U}^{[+]}\right]\mathrm{Tr}\left[\mathcal{U}^{[\square]\dagger}\right]\left|A\right\rangle\ ,$$

$$\mathcal{F}_{gg}^{(2)}(x_2,k_t) = 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}}\ \frac{1}{N_c}\left\langle A\right|\mathrm{Tr}\left[F^{i-}(\xi)\,\mathcal{U}_{\xi}^{[\square]\dagger}\right]\mathrm{Tr}\left[F^{i-}(0)\,\mathcal{U}_{0}^{[\square]}\right]\left|A\right\rangle\ ,$$

$$\mathcal{F}_{gg}^{(3)}(x_2,k_t) = 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}}\left\langle A\right|\mathrm{Tr}\left[F^{i-}(\xi)\,\mathcal{U}^{[+]\dagger}F^{i-}(0)\,\mathcal{U}^{[+]}\right]\left|A\right\rangle\ ,$$

$$\mathcal{F}_{gg}^{(4)}(x_2,k_t) = 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}}\left\langle A\right|\mathrm{Tr}\left[F^{i-}(\xi)\,\mathcal{U}^{[-]\dagger}F^{i-}(0)\,\mathcal{U}^{[-]}\right]\left|A\right\rangle\ ,$$

$$\mathcal{F}_{gg}^{(5)}(x_2,k_t) = 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}}\left\langle A\right|\mathrm{Tr}\left[F^{i-}(\xi)\,\mathcal{U}_{\xi}^{[\square]\dagger}\mathcal{U}^{[+]\dagger}F^{i-}(0)\,\mathcal{U}_{0}^{[\square]}\mathcal{U}^{[+]}\right]\left|A\right\rangle\ ,$$

$$\mathcal{F}_{gg}^{(6)}(x_2,k_t) = 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}}\ \frac{1}{N_c^2}\left\langle A\right|\mathrm{Tr}\left[F^{i-}(\xi)\,\mathcal{U}^{[+]\dagger}F^{i-}(0)\,\mathcal{U}^{[+]}\right]\mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]\mathrm{Tr}\left[\mathcal{U}^{[\square]\dagger}\right]\left|A\right\rangle\ .$$

- Note: for the $gg^* \to q\bar{q}$ channel, we have assumed massless quarks

however, when the quark mass is non-negligible, polarized gluon TMDs appear, even in un-polarized collisions

see talk by Pieter Taels later

The other TMDs at small-x

- involved in the $gg^* \to q\bar{q}$ and $gg^* \to gg$ channels

$$\mathcal{F}_{gg}^{(1)}(x_2,k_t) = \frac{4}{g^2}\int\frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3}\ e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})}\frac{1}{N_c}\left\langle \mathrm{Tr}\left[(\partial_i U_\mathbf{y})(\partial_i U_\mathbf{x}^\dagger)\right]\mathrm{Tr}\left[U_\mathbf{x}U_\mathbf{y}^\dagger\right]\right\rangle_{x_2}\ ,$$

$$\mathcal{F}_{gg}^{(2)}(x_2,k_t) = -\frac{4}{g^2}\int\frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3}\ e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})}\frac{1}{N_c}\left\langle \mathrm{Tr}\left[(\partial_i U_\mathbf{x})U_\mathbf{y}^\dagger\right]\mathrm{Tr}\left[(\partial_i U_\mathbf{y})U_\mathbf{x}^\dagger\right]\right\rangle_{x_2}\ ,$$

$$\mathcal{F}_{gg}^{(4)}(x_2,k_t) = -\frac{4}{g^2}\int\frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3}\ e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})}\left\langle \mathrm{Tr}\left[(\partial_i U_\mathbf{x})U_\mathbf{x}^\dagger(\partial_i U_\mathbf{y})U_\mathbf{y}^\dagger\right]\right\rangle_{x_2}\ ,$$

$$\mathcal{F}_{gg}^{(5)}(x_2,k_t) = -\frac{4}{g^2}\int\frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3}\ e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})}\left\langle \mathrm{Tr}\left[(\partial_i U_\mathbf{x})U_\mathbf{y}^\dagger U_\mathbf{x}U_\mathbf{y}^\dagger(\partial_i U_\mathbf{y})U_\mathbf{x}^\dagger U_\mathbf{y}U_\mathbf{x}^\dagger\right]\right\rangle_{x_2}\ ,$$

$$\mathcal{F}_{gg}^{(6)}(x_2,k_t) = -\frac{4}{g^2}\int\frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3}\ e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})}\frac{1}{N_c^2}\left\langle \mathrm{Tr}\left[(\partial_i U_\mathbf{x})U_\mathbf{y}^\dagger(\partial_i U_\mathbf{y})U_\mathbf{x}^\dagger\right]\mathrm{Tr}\left[U_\mathbf{x}U_\mathbf{y}^\dagger\right]\mathrm{Tr}\left[U_\mathbf{y}U_\mathbf{x}^\dagger\right]\right\rangle_{x_2}\ .$$

with a special one singled out: the Weizsäcker-Williams TMD

$$\mathcal{F}_{gg}^{(3)}(x_2,k_t) = -\frac{4}{g^2}\int\frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3}\ e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})}\left\langle \mathrm{Tr}\left[(\partial_i U_\mathbf{x})U_\mathbf{y}^\dagger(\partial_i U_\mathbf{y})U_\mathbf{x}^\dagger\right]\right\rangle_{x_2}$$

x evolution of CGC correlators

the evolution of the gluon TMDs with decreasing x can be computed from the so-called JIMWLK equation

$$\frac{d}{d\ln(1/x_2)}\langle O\rangle_{x_2}=\langle H_{JIMWLK}\ O\rangle_{x_2}$$

Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner

a functional RG equation that resums the leading logarithms in

$$y=\ln(1/x_2)$$

- the JIMWLK “Hamiltonian” reads:

$$H_{JIMWLK}=\frac{d}{d\log(1/x_2)}=\int\frac{d^2\mathbf{x}}{2\pi}\frac{d^2\mathbf{y}}{2\pi}\frac{d^2\mathbf{z}}{2\pi}\frac{(\mathbf{x}-\mathbf{z})\cdot(\mathbf{y}-\mathbf{z})}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2}\frac{\delta}{\delta A_c^-(\mathbf{x})}\left[1+V_{\mathbf{x}}^\dagger V_{\mathbf{y}}-V_{\mathbf{x}}^\dagger V_{\mathbf{z}}-V_{\mathbf{z}}^\dagger V_{\mathbf{y}}\right]^{cd}\frac{\delta}{\delta A_d^-(\mathbf{y})}$$

with the adjoint Wilson line $V_{\mathbf{x}}=\mathcal{P}\exp\left[ig\int_{-\infty}^{\infty}dx^+A_a^-(x^+,\mathbf{x})T^a\right]$

Evolution of the "dipole" TMD

(in a mean-field type approximation)

- the Balitsky-Kovchegov (BK) evolution

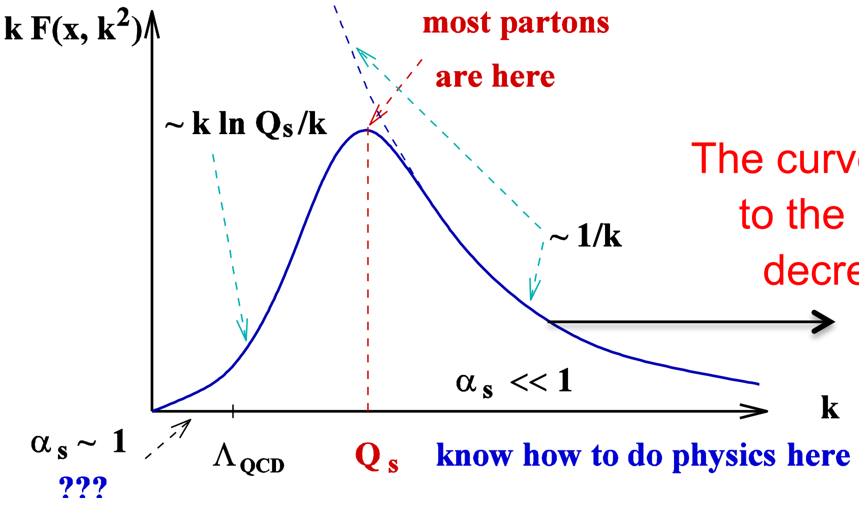
Balitsky (1996), Kovchegov (1998)

$$\frac{d}{dY} f_Y(k) = \bar{\alpha} \underbrace{\int \frac{dk'^2}{k'^2}\left[\frac{k'^2 f_Y(k') - k^2 f_Y(k)}{|k^2 - k'^2|} + \frac{k^2 f_Y(k)}{\sqrt{4k'^4 + k^4}}\right]}_{\text{BFKL}} - \bar{\alpha} f_Y^2(k)$$

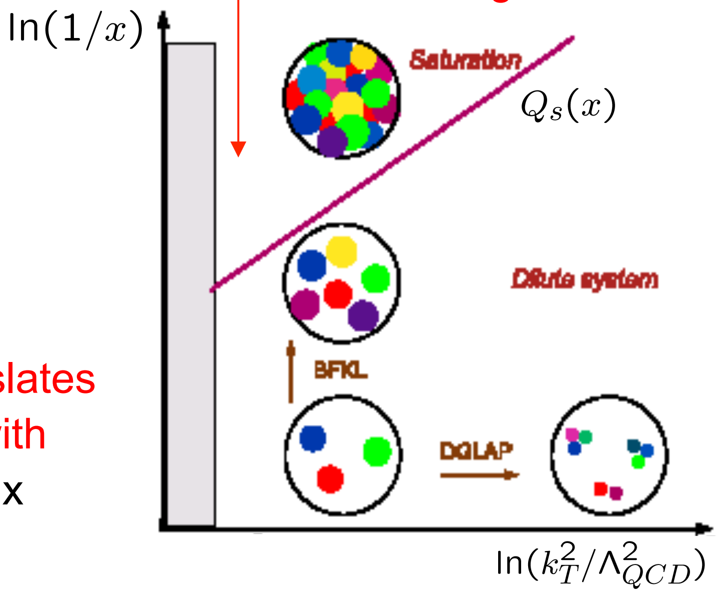
$$Y = \ln\left(\frac{1}{x}\right) \qquad \bar{\alpha} = \frac{\alpha_s N_c}{\pi}$$

non-linearity important when the gluon density becomes large

- solutions: qualitative behavior



The curve translates to the right with decreasing x



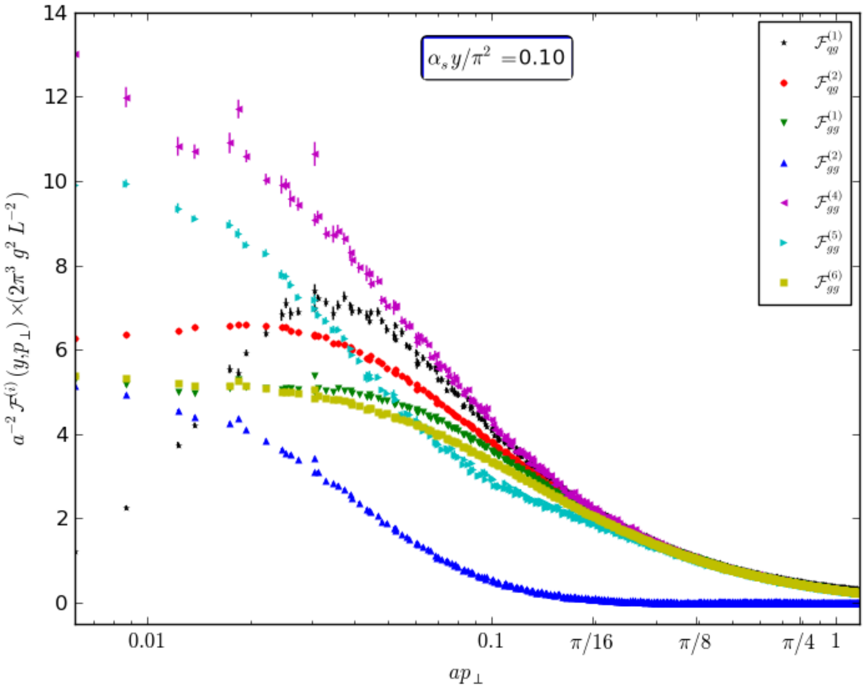
the distribution of partons as a function of x and k_T

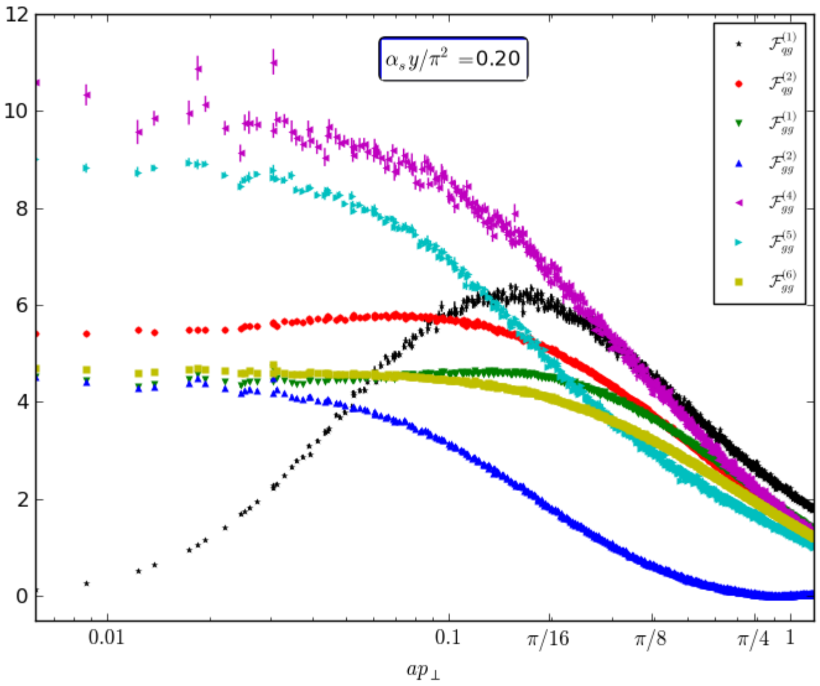
JIMWLK numerical results

using a code written by Claude Roiesnel

initial condition at y=0 : MV model
evolution: JIMWLK at leading log

CM, Petreska, Roiesnel (2016)





saturation effects impact the various gluon TMDs in very different ways

Conclusions

- for forward di-jet production, TMD factorization and CGC calculations are consistent with each other in the overlapping domain of validity

  small *x* and leading power of the hard scale $|p_{1t}|, |p_{2t}| \gg |k_t|, Q_s$

- saturation physics is relevant if the di-jet transverse momentum imbalance $|k_t|$ is of the order of the saturation scale Qs

- at small-x, the ”soft” factorization breaking is expected, understood, and is not a issue in saturation calculations:

  the more appropriate description of the parton content in terms of classical fields allows to use information extracted from a process to predict another

- given an initial condition, all the gluon TMDs can be obtained at smaller values of x, from the JIMWLK equation

  the scale dependence of the TMDs, which at small x boils down to Sudakov logarithms, can also be implemented (future work)